Current Commutator and CPT

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It is shown that Gupta's commutator theory of CP violation violates the CPT theorem and that the anticommutator form for the current–current interaction is necessitated by CPT invariance alone, irrespective of whether CP is conserved or not.

IN order to take into account the observed[1] CP violation in the decay $K_L^0 \to 2\pi$, Gupta[2] has constructed the following weak-interaction Lagrangian:

$$\mathfrak{L}=\tfrac{1}{2}G\{J_\alpha,J_\alpha\dagger\}+\tfrac{1}{2}G[J_\alpha,J_\alpha\dagger]=GJ_\alpha J_\alpha\dagger\,, \tag{1}$$

where J_α is the weak interaction current. The anticommutator $\{J_\alpha,J_\alpha\dagger\}$ and the commutator $[J_\alpha,J_\alpha\dagger]$ are supposed to give rise to the CP-conserving and CP-violating processes, respectively. Pestieau and Weyers[3] have applied partially conserved axial-vector current (PCAC) and current commutation relations to the $\mathfrak{L}$ in Eq. (1) and obtained some results on the neutral K decays.

The purpose of this paper is to point out (a) that the $\mathfrak{L}$ given by Eq. (1) is *not* CPT-invariant and (b) that the necessity of the anticommutator form $\{J_\alpha,J_\alpha\dagger\}$ to describe weak interactions is dictated by CPT invariance rather than CP invariance, contrary to what may appear to be the case from the statements found in recent literature.[4]

For convenience, let us choose the quark model and write[5] the current J_α in terms of the quark fields (p,n,λ) and the lepton fields:

$$J_\alpha=i\bar{p}\gamma_\alpha(1+\gamma_5)n\cos\theta+i\bar{p}\gamma_\alpha(1+\gamma_5)\lambda\sin\theta +i\bar{\nu}_e\gamma_\alpha(1+\gamma_5)e+i\bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\mu\,, \tag{2}$$

where θ is the Cabibbo angle. Further, let us ignore all the phase factors arising in the operations C, P, or T. It is then quite straightforward to show[6] that the J_α given by Eq. (2) satisfies the following transformation properties:

$$CPJ_\alpha(CP)^{-1}=+J_\alpha\dagger\,,\quad TJ_\alpha T^{-1}=-J_\alpha\,, \tag{3}$$

so that

$$CPTJ_\alpha(CPT)^{-1}=-J_\alpha\dagger$$

and[7]

$$CPTJ_\alpha J_\alpha\dagger(CPT)^{-1}=+J_\alpha\dagger J_\alpha\,. \tag{4}$$

Thus, the anticommutator and commutator satisfy:

$$CPT\{J_\alpha,J_\alpha\dagger\}(CPT)^{-1}=+\{J_\alpha,J_\alpha\dagger\}\,, \tag{5a}$$

$$CPT[J_\alpha,J_\alpha\dagger](CPT)^{-1}=-[J_\alpha,J_\alpha\dagger] \tag{5b}$$

and hence $\mathfrak{L}$ given by Eq. (1) is *not* CPT-invariant. These equations clearly show that CPT invariance requires $\mathfrak{L}$ to be of the form $\{J_\alpha,J_\alpha\dagger\}$. Note that one does not have to invoke CP invariance as seems to be implied by the statements in the literature.[4] This conclusion is actually independent of the choice of model as well as the choice of phase factors.[8] For, clearly the essential point is just that under CPT operation any Dirac bilinear goes into its Hermitian conjugate (apart from phase factors) and hence commutators formed out of a Dirac bilinear and its Hermitian conjugate can *never* be invariant under CPT.

Why does the apparently innocuous $\mathfrak{L}$ in Eq. (1) violate the CPT theorem? The reason is that this $\mathfrak{L}$ does not satisfy one of the assumptions[9] used in proving the CPT theorem in conventional local-field theory, namely, that in the Lagrangian all products are antisymmetrized with respect to Fermi fields and symmetrized with respect to Bose fields. For instance, $\mathfrak{L}$ in Eq. (1) contains the term $\bar{p}_i n_j \bar{\lambda}_k p_l$ (where i, j, k, l are the spinor indices) but does not contain the term with $\bar{p}_i p_l$ in reverse order, namely, $-p_l\bar{p}_i n_j\bar{\lambda}_k$. As noted by Lüders,[9] an alternative assumption which again leads to the CPT theorem is that the Lagrangian contains only Wick's normal products. This assumption will at once rule out the commutator $[J_\alpha,J_\alpha\dagger]$, since $N(J_\alpha J_\alpha\dagger)=N(J_\alpha\dagger J_\alpha)$ where N denotes the normal product. So,

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[2] V. Gupta, Phys. Rev. Letters **14**, 838 (1965).

[3] J. Pestieau and J. Weyers, Nuovo Cimento **43A**, 759 (1966).

[4] For instance, J. S. Bell, in Proceedings of the 1966 CERN School of Physics (unpublished); and S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. **146**, 1118 (1966).

[5] Our γ matrices, C, P, T operators etc. are the same as in J. J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1964).

[6] See, for instance, Ref. 5.

[7] We have used the Wigner time-reversal operator. If we use the Schwinger time-reversal operators T_s (see Sakurai, Ref. 5, p. 86), then, $T_sJ_\alpha T_s^{-1}=-J_\alpha\dagger$ and so $CPT_sJ_\alpha(CPT_s)^{-1}=-J_\alpha$. But this operator T_s changes the order of the operators $J_\alpha J_\alpha\dagger$ in $\mathfrak{L}$ also so that $CPT_sJ_\alpha J_\alpha\dagger(CPT_s)^{-1}=J_\alpha\dagger J_\alpha$ which is the same result as before.

[8] The possibility that the weak current is a linear combination of two currents J_α and K_α such that

$$CPTJ_\alpha(CPT)^{-1}=e^{i\theta}J_\alpha\dagger$$

and

$$CPTK_\alpha(CPT)^{-1}=e^{i\varphi}K_\alpha\dagger$$

with $\theta\neq\varphi$ can be readily ruled out. For this purpose, one writes out the individual terms of the commutator and the anticommutator of the weak current $(J_\alpha+K_\alpha)$ explicitly and verifies that the commutator can never be CPT invariant and that the CPT invariance of the anticommutator requires that $\theta=\varphi$. The crucial point in this argument is that J and K should have different structures in terms of some basic fields; otherwise we cannot allow the different phases in the first place.

[9] G. Lüders, Ann. Phys. (N.Y.) **2**, 1 (1957).

in order to satisfy the CPT theorem, $\mathcal{L}$ should have one of the following forms: (a) $\mathcal{L}=GA(J_\alpha J_\alpha\dagger)$ where A denotes antisymmetrization with respect to Fermi fields, (b) $\mathcal{L}=GN(J_\alpha J_\alpha\dagger)$, or (c) $\mathcal{L}=\frac{1}{2}G\{J_\alpha,J_\alpha\dagger\}$. Needless to say, if one regards the weak currents themselves as the fundamental entities, the alternative (c) seems to be the most natural choice; the currents being bosonlike, in the Lagrangian the product should be symmetrized with respect to the currents. On the other hand, the operations A and N in the first two alternatives have to be defined in terms of the fields rather than the currents themselves.

Note added in proof. If a nonlocal generalization of the commutator is used to describe CP violation, the conclusions in Ref. 2 that depend only on the symmetry properties of the commutator will still be valid for such a theory.

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Castillejo-Dalitz-Dyson Poles and Asymptotic Fields*†

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The meaning of Castillejo-Dalitz-Dyson poles (CDD poles) in the field-theoretic context of a boson separable-potential model is studied. Because the asymptotic-field operators for this model depend upon the D function, it is possible to insert CDD poles into these fields and to gauge their effect. It is found that CDD poles have a profound influence: Their presence prevents the in-fields from satisfying canonical commutation relations. A return to the free-particle algebra is possible, however, if new particles are inserted into the theory, one for every pole. These new particles are unstable, thus confirming the link between CDD poles and instability. A further consequence of this work is the development of a method for constructing Hamiltonians which, *a priori*, yield scattering amplitudes containing CDD poles.

I. INTRODUCTION

TRADITIONALLY, Castillejo-Dalitz-Dyson poles[1] (CDD poles) have been associated with unstable particles.[2] The reason is that unstable particles may be identified with the poles of the scattering amplitude in the lower half plane of the second Riemann sheet[3]; CDD poles can also cause these to appear. Of course not all such unstable particle poles can be attributed to CDD poles.[4] What is wanted is a precise connection between instability and the presence of a CDD pole.

The purpose of the present work is to examine this connection in the context of field theory. This is most conveniently done by a study of the infields of a field theoretic model. The program is to explore the effects that CDD poles have on the algebra of asymptotic fields.

In a previous paper[5] one of us investigated the asymptotic fields of a boson separable-potential model.[6,7] In Sec. II CDD poles are inserted into the $\theta_{\rm in}{}^\dagger$ fields of this model. It is found that the operator algebra is radically changed with CDD poles present; no longer do the in-fields obey canonical commutation relations. A return to free-boson algebra is possible, if additional particles are added to the theory. These new particles are shown to be unstable regardless of the existence of a pole in the scattering amplitude on the second Riemann sheet. Furthermore, a method of constructing Hamiltonians containing any number of CDD poles is found.

II. ASYMPTOTIC FIELDS OF THE SEPARABLE POTENTIAL MODEL AND CDD POLES

This model describes a system with two types of particles in it, a static heavy boson φ of mass M, and a boson θ which may move with three momentum $\mathbf{p}$; energy ω_p. The operators which create these particles are labeled $\varphi^\dagger$ and $\theta^\dagger(\mathbf{p})$; vector symbols for momentum indices on boson operators will hereafter be suppressed.

The Hamiltonian for this model in momentum space

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[1] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956).
[2] See, for instance, S. Mandelstam, *Weak Interactions and Topics in Dispersion Physics* (W. A. Benjamin, Inc., New York, 1963).
[3] R. E. Peierls, in *Proceedings of the Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, Ltd., London, 1955), p. 296.
[4] G. F. Chew, Lawrence Radiation Laboratory Report No. UCRL-9289, 1960 (unpublished).
[5] E. Kazes, Phys. Rev. **135**, B477 (1964).
[6] M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961).
[7] J. D. Childress and J. Urrechaga-Altuna, Phys. Rev. **148**, 1359 (1966).