Superconvergent Relations for the Process $\pi N \rightarrow \rho N$†

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We consider the process $\pi N \rightarrow \rho N$. The high-energy behavior of the invariant amplitudes following from the exchange of Regge trajectories allows us to write down three consistent superconvergent sum rules. Five other sum rules, obtained by equating to zero the coefficients of higher powers of t, lead to inconsistencies. In the degenerate-mass limit, partial consistency with the $U(6,6)$ theory is obtained.

I. INTRODUCTION

RECENTLY de Alfaro, Fubini, Rossetti and Furlan[1] have obtained strong interaction sum rules at fixed t from the consideration of the high-energy behavior of scattering amplitudes. If a particular amplitude falls off fast enough with increasing energy at fixed momentum transfer, then the integral over energy of the imaginary part of the amplitude vanishes. Such expressions have come to be known as superconvergent relations. The saturation of the superconvergent relations with one-particle intermediate states gives rise to sum rules relating the coupling constants and masses of the particles involved. By considering $\pi\rho$ scattering, de Alfaro *et al.*,[1] obtained in this way sum rules connecting the coupling constants $g_{\rho\pi\pi}$, $g_{\omega\rho\pi}$, $g_{\phi\rho\pi}$ and the masses of the ϕ, ρ, ω and, π mesons. Subsequently, this technique has been applied to meson-baryon scattering,[2,3] Compton scattering,[4] and photoproduction processes.[5]

Here we consider the process $\pi N \rightarrow \rho N$ and obtain three nontrivial superconvergent relations. By saturating these with nucleon and N^* resonance, we obtain a set of consistent sum rules. In the degenerate-mass case ($m_\rho = m_\pi$, $m_N = m_{N^*}$) we find partial consistency with the $U(6,6)$ theory.[6,7] Additional sum rules may be obtained by equating to zero the coefficients of t and t^2 (t is the momentum transfer squared). When these are combined with the original sum rules, we find inconsistent results.

In Sec. II we determine the asymptotic behavior of the amplitudes describing $\pi N \rightarrow \rho N$ by considering exchange of Regge trajectories in the t channel. The sum rules are obtained in Sec. III.

II. ASYMPTOTIC BEHAVIOR OF THE INVARIANT AMPLITUDES

The process $\pi N \rightarrow \rho N$ can be described in terms of six invariant amplitudes. These are taken to be the following:

$$T = i\epsilon^\mu \bar{U}(p')[P_\mu(A_1{}^{(\pm)} + \gamma\cdot Q B_1{}^{(\pm)})\gamma_5 + Q_\mu(A_2{}^{(\pm)} + \gamma\cdot Q B_2{}^{(\pm)})\gamma_5 + \gamma_\mu\gamma_5 A_3{}^{(\pm)} + i\sigma_{\mu\nu}Q^\nu\gamma_5 B_3{}^{(\pm)}]\ U(p) \quad (2.1)$$

where,

$$P = \tfrac{1}{2}(p'+p), \quad Q = \tfrac{1}{2}(q'+q)$$

with $p(p')$ as the incoming (outgoing) nucleon momentum and q (q') as the pion (ρ meson) momentum. The polarization vector for the ρ meson is denoted by ϵ. The superscript ($\pm$) labels the isotopic spin channels defined by

$$A_i{}^{\alpha\beta} = \delta_{\alpha\beta} A_i{}^{(+)} + \tfrac{1}{2}[\tau_\alpha, \tau_\beta] A_i{}^{(-)}, \qquad (2.2)$$

with a similar expression for B_i. Here $\alpha(\beta)$ is the isotopic spin index of the pion (ρ meson).

We can define the scalar invariants, $s = (p+q)^2$, $u = (p-q')^2$, $t = (q-q')^2$ and

$$\nu = P\cdot Q = \tfrac{1}{4}(s-u).$$

The invariant amplitudes are written as $A_i(\nu,t)$, $B_i(\nu,t)$. The transformation $\nu \rightarrow -\nu$ can be achieved by interchanging the two nucleons. Then making use of the invariance property of the scattering amplitude under G conjugation we obtain the following crossing-

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[5] R. Musto and F. Nicodemi, Nuovo Cimento **49A**, 333 (1967); L. K. Pande, Nuovo Cimento **48A**, 839 (1967); M. B. Halpern, Phys. Rev. **160**, 1441 (1967).

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[7] Jones and Scadron (Ref. 3) considering the process $\pi N \rightarrow \pi N^*$ find sum rules which are in complete agreement with the $U(6,6)$ theory.

symmetry relations:

$$\begin{aligned} A_1^{(\pm)}(\nu,t)&=\pm A_1^{(\pm)}(-\nu,t)\,, & A_2^{(\pm)}(\nu,t)&=\mp A_2^{(\pm)}(-\nu,t)\,, & A_3^{(\pm)}(\nu,t)&=\mp A_3^{(\pm)}(-\nu,t)\,,\\ B_1^{(\pm)}(\nu,t)&=\pm B^{(\pm)}(-\nu,t) & B_2^{(\pm)}(\nu,t)&=\mp B_2^{(\pm)}(-\nu,t) & B_3^{(\pm)}(\nu,t)&=\pm B_3^{(\pm)}(-\nu,t)\,. \end{aligned} \tag{2.3}$$

The high-energy behavior of the invariant amplitudes in the s channel are determined by the exchange of the leading Regge trajectories in the t channel. To find out the contributions of various Regge trajectories to the six invariant amplitudes we follow the helicity formalism of Jacob and Wick.[8] We find

$$\begin{aligned}
A&\equiv tA_1-(\rho^2-\mu^2)\{mB_1-B_3\}=\sum_J(2J+1)P_J'(\cos\theta)\beta^J\,,\\
B&\equiv\nu B_1+A_3=\frac{m}{p}\sum_J(2J+1)\{[P_J'(\cos\theta)+\cos\theta P_J''(\cos\theta)]\gamma^J-P_J''(\cos\theta)\alpha^J\}\,,\\
C&\equiv p^2B_1+mB_3=\frac{m}{q}\sum_J(2J+1)\{-[P_J'(\cos\theta)+\cos\theta P_J''(\cos\theta)]\alpha^J+P_J''(\cos\theta)\gamma^J\}\,,\\
D&\equiv B_3=\sum_J(2J+1)P_J'(\cos\theta)\alpha'^J\,,\\
E&\equiv\nu\left\{-\frac{t+\rho^2-\mu^2}{4m}A_1+\frac{(\rho^2-\mu^2)(t+\rho^2-\mu^2)}{4t}B_1+\frac{t-3\rho^2-\mu^2}{4m}B_3\right\}\\
&\qquad+q^2\left\{\frac{t}{4m}A_2-\frac{\rho^2-\mu^2}{4}B_2+A_3\right\}=\sum_J(2J+1)P_J(\cos\theta)\beta'^J\,,\\
F&\equiv\frac{-p\omega_2}{m}\{\nu B_1+A_3\}+\frac{t^{1/2}pq^2}{2m}B_2=\sum_J(2J+1)P_J'(\cos\theta)\gamma'^J\,,
\end{aligned}\tag{2.4}$$

where m, ρ and, μ are the nucleon, ρ meson and pion masses, respectively. And

$$p^2=\tfrac{1}{4}t-m^2\,,\qquad q^2=\frac{[t-(\rho+\mu)^2][t-(\rho-\mu)^2]}{4t}$$

$$\omega_1=(q^2+\mu^2)^{1/2}\,,\quad \omega_2=(q^2+\rho^2)^{1/2}\,.$$

The amplitudes with definite total angular momentum and definite parity are defined as follows:

$$\begin{aligned}
\alpha^J&=-\frac{1}{\sqrt{2}}\frac{\langle\frac12,-\frac12|1\rangle^J-\langle\frac12,-\frac12|-1\rangle^J}{J(J+1)}\,,\\
\alpha'^J&=\frac{1}{\sqrt{2}}\frac{m}{qE}\frac{\langle\frac12,\frac12|1\rangle^J-\langle\frac12,\frac12|-1\rangle^J}{[J(J+1)]^{1/2}}\,,\\
\beta^J&=-\sqrt{2}\frac{m\sqrt{t}}{p}\frac{\langle\frac12,\frac12|1\rangle^J+\langle\frac12,\frac12|-1\rangle^J}{[J(J+1)]^{1/2}}\,,\\
\beta'^J&=q\rho\langle\tfrac12,\tfrac12|0\rangle^J\,,\\
\gamma^J&=-\frac{1}{\sqrt{2}}\frac{\langle\frac12,-\frac12|1\rangle+\langle\frac12,-\frac12|-1\rangle}{J(J+1)}\,,\\
\gamma'^J&=\rho\frac{\langle\frac12,-\frac12|0\rangle^J}{[J(J+1)]^{1/2}}\,.
\end{aligned}\tag{2.5}$$

[8] M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) **7**, 404 (1959).

We have used the shorthand notation $\langle\lambda_1,\lambda_2|\lambda_3\rangle^J$ to denote the scattering amplitude T^J with definite total angular momentum J, and we have evaluated this amplitude between the helicity states λ_1, λ_2, and λ_3 for the nucleon, antinucleon, and ρ meson, respectively.

The right-hand sides in Eq. (2.4) are in suitable forms for Reggeization.[9] The G parity of the initial state is -1. The fact that the $N\bar{N}$ system must also have $G=-1$ leads to selection rules. We display the allowed channels in Table I. The last column of the table shows the experimentally known Regge trajectories with $G=-1$ which can be exchanged.

The intercepts at $t=0$ for the trajectories ω, ϕ, A_1, and A_2 are taken to be positive while for the pion trajectory and for channels with no trajectories we take the intercepts to be negative.

TABLE I. Allowed channels and Regge trajectories.

Amplitudes	Parity	J	Allowed I (isotopic spin)	Known Regge trajectories
α^J, α'^J	$(-1)^J$	even	$I=1$	A_2
		odd	$I=0$	ω, ϕ
β^J, β'^J	$(-1)^{J-1}$	even	$I=1$	π
		odd	$I=0$	None
γ^J, γ'^J	$(-1)^{J-1}$	even	$I=0$	None
		odd	$I=1$	A_1

[9] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

If we go through the usual Reggeization procedure for the right-hand sides of Eq. (2.4), we find the following asymptotic behavior at large s for the amplitudes on the left-hand sides:

$$\begin{aligned}
&A\approx s^{\beta(t)-1},\\
&B\approx as^{\gamma(t)-1}+bs^{\alpha(t)-2},\\
&C\approx a's^{\alpha(t)-1}+b's^{\gamma(t)-2},\\
&D\approx s^{\alpha(t)-1},\\
&E\approx s^{\beta(t)},\\
&F\approx s^{\gamma(t)-1},
\end{aligned}\tag{2.6}$$

where a, b, a', and b' are arbitrary functions of t. The trajectory parameters α, β, and γ refer to the amplitudes (α^J,α'^J), (β^J,β'^J), and (γ^J,γ'^J), respectively.

In the next section we shall show that some of the above amplitudes satisfy superconvergent relations. These relations are assumed to be saturated by N and N^* states and coupling-constant sum rules result.

III. SUPERCONVERGENT RELATIONS AND THEIR SATURATIONS

If a particular invariant amplitude $A(\nu,t)$ falls off faster than ν^{-1} for $\nu\to\infty$ at fixed t, it follows that

$$\int_{-\infty}^{\infty} d\nu\,\mathrm{Im}A(\nu,t)=0.\tag{3.1}$$

In the following, we will assume that t is small. From Eqs. (2.4), (2.6) and our assumptions about the intercepts of the exchanged Regge trajectories, we see that the amplitudes $A^{(0,1)}(\nu,t)$, $B^{(0)}(\nu,t)$, and $B_2^{(0)}(\nu,t)$ are superconvergent;[10]

$$\int_{-\infty}^{\infty} d\nu\{t\,\mathrm{Im}A_1^{(1)}(\nu,t)-(\rho^2-\mu^2)[m\,\mathrm{Im}B_1^{(1)}(\nu,t)-\mathrm{Im}B_3^{(1)}(\nu,t)]\}=0\tag{3.2}$$

$$\int_{-\infty}^{\infty} d\nu\{\nu\,\mathrm{Im}B_1^{(0)}(\nu,t)+\mathrm{Im}A_3^{(0)}(\nu,t)\}=0\tag{3.3}$$

$$\int_{-\infty}^{\infty} d\nu\,\mathrm{Im}B_2^{(0)}(\nu,t)=0.\tag{3.4}$$

We will attempt to saturate our superconvergent relations Eqs. (3.2)–(3.4) with N and N^* intermediate states. Notice that the validity of Eq. (3.2) depends on the pion trajectory having a negative intercept at $t=0$. Experimentally it seems that this intercept is negative, but quite close to the origin. Therefore the sum rules resulting from Eq. (3.2) by saturation with only a small number of intermediate states could be badly violated.

The relevant couplings in momentum space are taken as

$$\pi NN:\quad g_{\pi NN}\bar{U}\gamma_5\tau^i U\pi^i,\tag{3.5}$$

$$\pi NN^*:\quad \frac{1}{m^*}g_{\pi NN^*}q^\mu\epsilon_{ab}\bar{\psi}_\mu{}^{bcd}(\tau^i)_d{}^aU_c\pi^i,\tag{3.6}$$

$$\rho NN:\quad g^c{}_{\rho NN}\bar{U}\gamma^\mu\tau^iU\rho_\mu{}^i+\frac{i}{m}g^m{}_{\rho NN}\bar{U}\sigma^{\mu\nu}\tau^iU\rho_\mu{}^iq_\nu,\tag{3.7}$$

$$\rho NN^*:\quad \epsilon^{ab}\bar{U}^c\left\{g_{\rho NN^*}g_{\mu\lambda}+\frac{1}{m^*}g'_{\rho NN^*}\gamma_\mu q_\lambda+\frac{1}{2m^{*2}}g''_{\rho NN^*}P_\mu q_\lambda\right\}\gamma_5(\tau^i)_a{}^d\psi_{bcd}{}^\lambda\rho^{i,\mu},\tag{3.8}$$

where $P=p'+p$ and $q=p'-p$, with p (p') being the momentum of the incoming (outgoing) baryon. The masses of N and N^* are denoted by m and m^*, respectively. The resulting sum rules are polynomials in t. We set the coefficients of the various powers of t equal to zero and from Eq. (3.2) we obtain

$$t^0:\quad 3g^c{}_{\rho NN}g_{\pi NN}-\frac{2}{3m^*}g_{\pi NN^*}\left\{\left[\frac{m}{m^{*2}}(-2m^{*2}+m^2-mm^*-\mu^2)+\frac{b}{2m^*}\right]g_{\rho NN^*}+\left[\frac{a}{m^*}-\frac{3m^2}{m^*}+\frac{m}{2m^{*2}}b\right]g'_{\rho NN^*}+\left[a-3m^2+\frac{m}{2m^*}b\right]\frac{mg''_{\rho NN^*}}{m^{*2}}\right\}=0,\tag{3.9}$$

$$t^1:\quad 3g^c{}_{\rho NN}g_{\pi NN}-\frac{2}{3m^*}g_{\pi NN^*}\left\{\frac{3}{2m^*}(m^{*2}-m^2+\mu^2)g_{\rho NN^*}+\left[\frac{a}{m^*}-\frac{3m^2}{m^*}+\frac{2m^*-m}{2m^{*2}}b\right]g'_{\rho NN^*}+\frac{1}{m^{*2}}\left[3m^*m^2-c+\frac{m}{2}b\right]g''_{\rho NN^*}-(\rho^2-\mu^2)\frac{3}{2m^*}\left[g'_{\rho NN^*}+\frac{m}{m^*}g''_{\rho NN^*}\right]\right\}=0,\tag{3.10}$$

$$t^2:\quad g_{\pi NN^*}[g'_{\rho NN^*}-g''_{\rho NN^*}]=0.\tag{3.11}$$

[10] Note that the eigenamplitudes for total isotopic spin are $A_i=(\sqrt{6})A_i^{(+)}$, $A_i^{(1)}=2A_i^{(-)}$ and similarly for the B_i. The superconvergent relation for $\{tA_1-(\rho^2-\mu^2)[mB_1-B_3]\}$ with $I=0$ is trivially satisfied by crossing symmetry [Eq. (2.3)].

From Eq. (3.3), we obtain

$$t^0:\quad \frac{3(\rho^2+\mu^2)}{2m}g^m{}_{\rho NN}g_{\pi NN}+\frac{4}{3m^*}g_{\pi NN^*}\Bigg\{\left[\frac{(2m^{*2}-2m^2-\rho^2-\mu^2)(-2m^{*2}+m^2-mm^*-\mu^2)}{4m^{*2}}+\frac{m^*-m}{2m^*}b\right]g_{\rho NN^*}$$
$$+\left[\frac{3m^2(m+m^*)}{m^*}-\frac{(m+m^*)}{m^*}a-\frac{2m^2+2mm^*-\rho^2+\mu^2}{4m^{*2}}b\right]g'{}_{\rho NN^*}$$
$$+\frac{2m^{*2}-2m^2-\rho^2-\mu^2}{4m^{*2}}\left[-3m^2+a+\frac{m}{2m^*}b\right]g''{}_{\rho NN^*}\Bigg\}=0, \quad (3.12)$$

$$t^1:\quad -\frac{3}{2m}g^m{}_{\rho NN}g_{\pi NN}+\frac{4}{3m^*}g_{\pi NN^*}\Bigg\{\frac{-2m^{*2}+m^2-mm^*-\mu^2}{4m^{*2}}g_{\rho NN^*}+\left[\frac{b}{4m^{*2}}-\frac{3(m^*+m)}{2m^*}\right]g'{}_{\rho NN^*}$$
$$+\frac{1}{4m^{*2}}\left[-3m^2+a+\frac{m}{2m^*}b+\tfrac{3}{2}(2m^{*2}-2m^2-\rho^2-\mu^2)\right]g''{}_{\rho NN^*}\Bigg\}=0, \quad (3.13)$$

$$t^2:\quad g_{\pi NN^*}g''{}_{\rho NN^*}=0. \quad (3.14)$$

From Eq. (3.4)

$$t^0:\quad -\frac{3}{m}g^m{}_{\rho NN}g_{\pi NN}+\frac{2}{3m^*}g_{\pi NN^*}\Bigg\{\frac{4m^{*2}-mm^*+m^2-\mu^2}{m^{*2}}g_{\rho NN^*}+\frac{b}{m^{*2}}g'{}_{\rho NN^*}$$
$$+\frac{1}{m^{*2}}\left[-3m^2+a+\frac{m}{2m^*}b\right]g''{}_{\rho NN^*}\Bigg\}=0, \quad (3.15)$$

$$t^1:\quad g_{\pi NN^*}g''{}_{\rho NN^*}=0, \quad (3.16)$$

where

$$a=\frac{(m^{*2}+m^2-\rho^2)(m^{*2}+m^2-mm^*-\mu^2)}{2m^{*2}},\quad b=m^{*2}+2mm^*+m^2-\mu^2,\quad c=\frac{(m^{*2}+m^2-\rho^2)(2m^{*2}+2m^2+mm^*-2\mu^2)}{2m^*}.$$

We have also made use of the following results. In the s channel

$$A_i^{(+)}=\tfrac{1}{3}[A_i^{(1/2)}+2A_i^{(3/2)}],\quad A_i^{(-)}=\tfrac{1}{3}[A_i^{(1/2)}-A_i^{(3/2)}].$$

Similar results hold for B_i. The N and N^* pole terms have isotopic spin projections $\binom{3}{0}$ and $\binom{0}{2}$ respectively for the $\binom{\frac{1}{2}}{\frac{3}{2}}$ states. The sum rules obtained by equating the coefficients of t^n, to zero corresponds to the derivative sum rule at $t=0$. We notice that only N^* contributes to the sum rules (3.11), (3.14), and (3.16). We, therefore, expect these sum rules are not as well saturated as others.

From Eqs. (3.11), (3.14), and (3.16), we find that either $g_{\pi NN^*}=0$ or $g'{}_{\rho NN^*}=g''{}_{\rho NN^*}=0$. The first alternative leads to $g^m{}_{\rho NN}g_{\pi NN}=g^c{}_{\rho NN}g_{\pi NN}=0$. The second alternative, when used in Eqs. (3.9), (3.10), (3.12), (3.13), and (3.15), leads to inconsistent results. So, we disregard these sum rules.

We now consider the case where $m^*=m$, and $\rho=\mu$. Then our sum rules become

$$t^0:\quad 9g^c{}_{\rho NN}g_{\pi NN}=-\frac{\mu^2}{m^2}g_{\pi NN^*}\left\{3g_{\rho NN^*}+\frac{4m^2-\mu^2}{m^2}[g'{}_{\rho NN^*}+g''{}_{\rho NN^*}]\right\}, \quad (3.17)$$

$$9g^m{}_{\rho NN}g_{\pi NN}=-\frac{2}{m^2}g_{\pi NN^*}\left\{(2m^2+\mu^2)g_{\rho NN^*}+(4m^2-\mu^2)\left[2g'{}_{\rho NN^*}+\frac{\mu^2}{2m^2}g''{}_{\rho NN^*}\right]\right\}, \quad (3.18)$$

$$9g^m{}_{\rho NN}g_{\pi NN}=\frac{2}{m^2}g_{\pi NN^*}\left\{(4m^2-\mu^2)g_{\rho NN^*}+(4m^2-\mu^2)\left[g'{}_{\rho NN^*}-\frac{\mu^2}{2m^2}g''{}_{\rho NN^*}\right]\right\}. \quad (3.19)$$

$$t^1:\quad 9g^c{}_{\rho NN}g_{\pi NN}=\frac{\mu^2}{m^2}g_{\pi NN^*}\left\{3g_{\rho NN^*}+\frac{4m^2-\mu^2}{m^2}[-g'{}_{\rho NN^*}+2g''{}_{\rho NN^*}]\right\}, \tag{3.20}$$

$$9g^m{}_{\rho NN}g_{\pi NN}=-\frac{2}{m^2}g_{\pi NN^*}\left\{(2m^2+\mu^2)g_{\rho NN^*}+(8m^2+\mu^2)g'{}_{\rho NN^*}+\frac{1}{2m^2}[10m^2\mu^2-\mu^4]g''{}_{\rho NN^*}\right\}. \tag{3.21}$$

Comparing Eqs. (3.18) and (3.19), we obtain

$$g_{\rho NN^*}=-\frac{4m^2-\mu^2}{2m^2}g'{}_{\rho NN^*}. \tag{3.22}$$

In the Appendix we write down the relevant vector currents of the $U(6,6)$ theory and express them in suitable forms. From Eq. (A6) we note that Eq. (3.22) is a result of the $U(6,6)$ theory. To determine whether our sum rules are completely consistent with the $U(6,6)$ theory, we will assume the additional $U(6,6)$ relation $g'{}_{\rho NN^*}=g''{}_{\rho NN^*}$ [see Eq. (A6)]. Then Eqs. (3.17)–(3.19) become

$$9g^c{}_{\rho NN}g_{\pi NN}=\frac{\mu^2}{m^2}g_{\pi NN^*}g_{\rho NN^*}, \tag{3.23}$$

$$9g^m{}_{\rho NN}g_{\pi NN}=4g_{\pi NN^*}g_{\rho NN^*}. \tag{3.24}$$

These equations lead to the ratio

$$g^m{}_{\rho NN}/g^c{}_{\rho NN}=4m^2/\mu^2. \tag{3.25}$$

In the $U(6,6)$ theory the charge and the magnetic coupling of ρNN vertex is written as

$$\rho NN:\quad g'^c{}_{\rho NN}\bar{U}\frac{P^\mu}{2m}\tau^iU\rho_\mu{}^i+g'^m{}_{\rho NN}\bar{U}\frac{r^\mu}{4m^2}\tau^iU\rho_\mu{}^i. \tag{3.7$'$}$$

Then Eqs. (3.23) and (3.24) become

$$9\left(g'^c{}_{\rho NN}+\frac{\mu^2}{4m^2}g'^m{}_{\rho NN}\right)g_{\pi NN}=\frac{\mu^2}{m^2}g_{\pi NN^*}g_{\rho NN^*} \tag{3.26}$$

$$9(g'^c{}_{\rho NN}+g'^m{}_{\rho NN})g_{\pi NN}=-8g_{\pi NN^*}g_{\rho NN^*}. \tag{3.27}$$

These equations yield the ratio

$$g'^m{}_{\rho NN}/g'^c{}_{\rho NN}=-\frac{8m^2+\mu^2}{3\mu^2}. \tag{3.28}$$

From Eq. (A1) we see that the $U(6,6)$ result is

$$g'^m{}_{\rho NN}/g'^c{}_{\rho NN}=-\frac{5}{3}\frac{2m}{\mu}. \tag{3.29}$$

So our ratio of the magnetic to charge coupling for the ρ_{NN} vertex differs from that predicted by the $U(6,6)$ theory. However, it is amusing to note that if we assume an average mass of $2.4\,\mu$ for the N and N^* (μ is the average mass of ρ meson and pion), we find that the right-hand sides of Eqs. (3.28) and (3.29) become ≈ -16 and ≈ -8, respectively. This is not a very large discrepancy considering the crudeness of our approximation.

It is important to notice that the above disagreement with $U(6,6)$ depends ultimately on our use of Eq. (3.2). As was remarked earlier Eq. (3.2) may not lead to reliable sum rules. However, we will now show that Eq. (3.27), which does not depend on Eq. (3.2), by itself contradicts $U(6,6)$. Using the $U(6,6)$ relations

$$g_{\pi NN}=-\frac{5}{3}\left(1-\frac{\mu^2}{4m^2}\right)g_{\pi NN^*}, \tag{3.30}$$

$$g'^c{}_{\rho NN}=-\frac{3\mu}{10m}g'^m{}_{\rho NN}, \tag{3.31}$$

we find

$$g_{\rho NN^*}/g'^c{}_{\rho NN}=\frac{15}{8}\left(1-\frac{10m}{3\mu}\right)\left(1-\frac{\mu^2}{4m^2}\right), \tag{3.32}$$

while the $U(6,6)$ result (See Appendix)

$$g_{\rho NN^*}/g'^c{}_{\rho NN}=\frac{4m}{\mu}\left(1-\frac{\mu^2}{4m^2}\right) \tag{3.33}$$

is considerably different.

We now examine the "derivative" sum rules (3.20) and (3.21). We find that Eq. (3.20) when combined with Eqs. (3.17)–(3.19) forms a consistent set leading to the full $U(6,6)$ result for the ρNN^* vertex:

$$-\frac{2m^2}{4m^2-\mu^2}g_{\rho NN^*}=g'{}_{\rho NN^*}=g''{}_{\rho NN^*}. \tag{3.34}$$

However, the inclusion of Eq. (3.21) leads to inconsistent results. For example, comparing Eqs. (3.21) and (3.18), we find $g'{}_{\rho NN^*}=-g''{}_{\rho NN^*}$, which contradicts Eq. (3.30). Finally, we note that one of our rejected sum rules, Eq. (3.11) agrees with (3.30).

IV. CONCLUSIONS

We have seen that certain combinations of the invariant amplitudes describing $\pi N\to\rho N$ scattering satisfy superconvergent relations. By assuming that these relations are saturated by N and N^* intermediate states, we obtain coupling-constant sum rules of both the derivative and nonderivative types. Some of the derivative-type sum rules receive contributions only from the N^* state and yield inconsistent results. We have rejected these sum rules on the ground that they are not as well saturated as the sum rules which receive contributions from both N and N^* states.

In the degenerate mass case ($m^*=m$, $\rho=\mu$) the three nonderivative-type sum rules are consistent with each other and are in partial agreement with the $U(6,6)$ theory. There are two derivative sum rules which receive contributions from both N and N^* states. One of these [Eq. (3.20)], when combined with the non-derivative sum rules improves the agreement with $U(6,6)$. The other [Eq. (3.21)] leads to a result which disagrees with $U(6,6)$. Hence these two derivative sum rules contradict each other. Since there is no *a priori* way of choosing one derivative sum rule over the other and since derivative sum rules are expected to require saturation by higher-spin intermediate states, we should not take Eq. (3.34) [obtained by using Eq. (3.20) too seriously].

In contrast to Jones and Scadron,[3] who find superconvergent sum rules for the process $\pi N \to \pi N^*$ which are in full agreement with $U(6,6)$, our results show partial agreement. It should be noticed that Eq. (3.22) which is an $U(6,6)$ result depends only on the superconvergent relations Eqs. (3.3) and (3.4). The disagreement with $U(6,6)$ expressed in Eq. (3.28) depends ultimately on Eq. (3.2), which is a consequence of our assumption that the intercept of the pion trajectory is negative. Since the intercept is very close to the origin, the resulting sum rule may be badly violated. We have shown however that disagreement with $U(6,6)$ may be obtained without making use of Eq. (3.2). We are therefore led to believe that superconvergent sum rules of this type should not lead in general to the results of a higher symmetry.

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APPENDIX

In the $U(6,6)$ theory the $N\bar{N}$ part of the vector current is

$$J^\mu(N\bar{N})=g\left\{\left(1+\frac{\mu}{2m}\right)\bar{U}\frac{P^\mu}{2m}U-\frac{5}{3}\left(1+\frac{2m}{\mu}\right)\bar{U}\frac{r^\mu}{4m^2}U\right\}, \quad \text{(A1)}$$

where

$$r^\mu=\epsilon^{\mu\nu\kappa\lambda}P_\nu q_\kappa\gamma_\lambda\gamma_5, \quad \text{with} \quad \epsilon^{0123}=1.$$

Note that our definition of q differs from that of Ref. 6 by a sign. We have the identity

$$\bar{U}\frac{P^\mu}{2m}U=\bar{U}\gamma^\mu U-\frac{i}{2m}\bar{U}\sigma^{\mu\nu}Uq_\nu. \quad \text{(A2)}$$

Next using

$$\epsilon^{\mu\nu\kappa\lambda}\gamma_\lambda\gamma_5=g^{\nu\kappa}\gamma^\mu-g^{\mu\kappa}\gamma^\nu+i\sigma^{\mu\nu}\gamma^\kappa,$$

we find

$$\bar{U}\frac{r^\mu}{4m^2}U=\bar{U}\left\{\frac{\mu^2}{4m^2}\gamma^\mu-\frac{i}{2m}\sigma^{\mu\nu}q_\nu\right\}U. \quad \text{(A3)}$$

Equation (A2) and (A3) allow us to go back and forth between the forms (A1) and (3.7).

The $N^*\bar{N}$ part of the vector current is

$$J^\mu(N^*\bar{N})=g\frac{1}{2m^2}\left(1+\frac{2m}{\mu}\right)\epsilon^{\mu\nu\kappa\lambda}P_\nu q_\kappa\bar{U}\psi_\lambda. \quad \text{(A4)}$$

Using the identity

$$\epsilon^{\mu\nu\kappa\lambda}=\{i[g^{\mu\kappa}\sigma^{\nu\lambda}+g^{\nu\lambda}\sigma^{\mu\kappa}-g^{\mu\lambda}\sigma^{\nu\kappa}-g^{\nu\kappa}\sigma^{\mu\lambda}]+g^{\nu\kappa}g^{\mu\lambda}-g^{\mu\kappa}g^{\nu\lambda}+\sigma^{\mu\nu}\sigma^{\kappa\lambda}\}\gamma_5, \quad \text{(A5)}$$

we obtain

$$J^\mu(N^*\bar{N})=g\left(1+\frac{2m}{\mu}\right)\bar{U}\left\{\frac{4m^2-\mu^2}{2m^2}g^{\mu\lambda}-\frac{q^\lambda}{m}\gamma^\mu-\frac{q^\lambda P^\mu}{2m^2}\right\}\gamma_5\psi_\lambda. \quad \text{(A6)}$$

In all of the above equations we have omitted the isotopic spin part of the couplings. They are the same as chosen in Eqs. (3.5)–(3.8).