verse of $(D^{\dagger}D)$ is now simple and, using (3.15), we obtain the equation for the form factors

$$F_i(s)=[F_i(0)+\sum_k d_{ik}F_k(0)-i\sum_k \rho_i N_{ik}F_k(0)]/(1+\bar{\sigma}_i+\sigma_i). \quad (3.28)$$

For the single-channel or the finite-channel case, N, d, and σ all go to zero as $s\to\infty$, so that $F_i(s)\to F_i(0)\neq 0$ $s\to\infty$. Now σ_i and $\bar{\sigma}_i$ are positive-definite and can be made to develop any power behavior in energy, s^p $(p>0)$, by letting the number of channels increase appropriately fast. Then, because the numerator in the right-hand side of Eq. (3.28) is a sum over random quantities, $F_i(s)\to 0$ as $s\to\infty$. The scattering amplitude can also be computed easily, and we obtain for λ and μ,

$$\lambda=(N_{11}+\sum_k N_{1k}d_{k1})/(1+\sigma_1+\bar{\sigma}_1), \quad (3.29)$$

$$\mu=\frac{\sigma_1}{1+\sigma_1+\bar{\sigma}_1}. \quad (3.30)$$

Now clearly $\lambda\to 0$ as $s\to\infty$. Also, from (3.22), (3.26), and (3.27) we find that $\sigma_i/\bar{\sigma}_i\to 0$ in this limit, since $d(s)$ is a principal-value integral over $\rho N(s)$.[10] Clearly then, $\mu\to 0$ and if we assume that $\sigma_i\to\infty$, then $\xi\to\infty$ (diffraction scattering) and $R\to\infty$. However, in each case $\eta(\infty)=1$.

[10] We have assumed the same asymptotic bound for $N(s)$ as in footnote 9.

IV. CONCLUSIONS AND DISCUSSION

It is well known that with elastic unitarity the form factors may go to constants asymptotically. We have examined the asymptotic behavior of the form factors in the framework of coupled two-particle channels under the assumption that an infinite number of channels open up as we go into the asymptotic region. We have shown, in the context of three specific models, that if the density of inelastic channels increases sufficiently rapidly with energy, the form factors can be made to vanish asymptotically. We observe that this behavior of the form factors is consistent with $\eta(\infty)=1$. We do not find a purely imaginary asymptotic amplitude $(\lambda/\mu=0)$ to be a necessary condition for the vanishing behavior for the form factors as $s\to\infty$, though a purely imaginary amplitude, which need not necessarily vanish asymptotically, would certainly force the form factors to vanish as $s\to\infty$. We conclude by remarking that the dynamics of opening channels seems to provide a possible mechanism within the framework of dispersion theory for the asymptotic vanishing of form factors; the conditions for diffraction scattering seem to be sufficient (Sec. III C) but not necessary (Sec. III A and III B).

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One-Pion Exchange and Single-Isobar Production in pp Collisions between 3 and 30 GeV

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The one-pion exchange (OPE) amplitude with sharp cutoff at $b=1$ F explains well the reaction $pp\to pN^*(1238)$ between 3 and 15 GeV/c. Similar good results are obtained with the absorption model. The unmodified OPE amplitudes violate unitarity in the entire 3–15-GeV/c momentum range, mainly for impact parameters below $b=0.5$ F. The sharp-cutoff and absorption amplitudes satisfy the unitarity condition in the same momentum range. The one-pion exchange with absorption corrections cannot explain the reactions $pp\to pN^*(1410)$, $pp\to pN^*(1518)$, and $pp\to pN^*(1688)$ between 4 and 30 GeV/c.

1. INTRODUCTION

THE one-pion exchange (OPE) amplitudes, with absorption modifications,[1] were used successfully by Alexander *et al.*[2] to explain both the differential cross section and the decay parameters of the $N^*(1238)$ isobar produced in the reaction $pp\to nN^{*++}(1238)$ at 5.5 GeV/c. The purpose of this paper is to find out whether the results at 5.5 GeV/c on the OPE mechanism were only accidental, or are of a more general nature.

In this paper the differential cross sections of the reaction

$$pp\to pN^{*+}(1238) \quad (1)$$

are calculated according to Alexander *et al.*,[2] and are compared with the available experimental results between 3 and 15 GeV/c.[3]

[1] See for instance N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962); A. Dar, M. Kugler, Y. Dothan, and S. Nussinov, Phys. Rev. Letters **12**, 82 (1964); K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964); L. Durand and Y. T. Chiu, Phys. Rev. **137**, B1530 (1965) and references given therein.

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[3] See also B. Margolis and A. Rotstein, Nuovo Cimento **45**, 1010 (1966); and P. C. M. Yock, *ibid.* **44**, 777 (1966).

The OPE calculation and the comparison with experimental observations are extended also to the reactions

$$pp \to pN^{*+}(1410)\,, \qquad (2)$$

$$pp \to pN^{*+}(1518)\,, \qquad (3)$$

and

$$pp \to pN^{*+}(1688) \qquad (4)$$

in the momentum range 4–30 GeV/c. The data on the reactions (1)–(4) quoted here are mostly from counter experiments on inelastic pp scattering by Chadwick *et al.*,[4] Cocconi *et al.*,[5] Ankenbrandt *et al.*,[6] Anderson *et al.*,[7] and Blair *et al.*[8]

2. OPE AND ABSORPTION CORRECTIONS

The unmodified OPE amplitude of reaction (1) violates unitarity; it also fails to describe the differential cross section for the reaction, and the decay parameters of the isobar $N^{*++}(1238) \to p\pi^+$. Guided by the peripheral nature of reaction (1) and the observation that only the low-J partial waves of the OPE amplitude violate unitarity, an attempt was made to describe reaction (1) by the sharp-cutoff model.[2] In this model it is assumed that absorption processes completely suppress the OPE amplitude at low angular momentum ($J \leq J_c$), and do not affect the partial waves at higher values of J.

The differential cross section of reaction (1), and also those of reactions (2), (3), and (4),[9] are given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s q^2} \times \tfrac{1}{4} \sum_{\lambda} |M_{\lambda N^*,\lambda_{p''},\lambda_p,\lambda_{p'}}|^2. \qquad (5)$$

$M_{\lambda N^*,\lambda_{p''},\lambda_p,\lambda_{p'}}$ is the normalized amplitude of reaction (1) and the λ's are the helicities of the four baryons in (1); s and q are the total energy squared and the incoming momentum in the c.m. system of reaction (1) and t is the momentum transfer squared.

The OPE Born amplitude of (1) can be written as[10]

$$B_{\lambda\mu}(s,x) = \left(\frac{1+x}{2}\right)^{(\lambda+\mu)/2} \left(\frac{1-x}{2}\right)^{(\lambda-\mu)/2} \times \left[\frac{A_{\lambda\mu}(s,z)}{z-x} + p_{\lambda\mu}{}^n(s,x)\right], \qquad (6)$$

where $\lambda = \lambda p' - \lambda p$, $\mu = \lambda N^* - \lambda p''$, and $x = \cos\theta$; $z = 1 + (m_\pi{}^2 - t_1)/2qq'$, and t_1 is the momentum transfer squared for $x=1$. The function $p_{\lambda\mu}(s,x)$ is a polynomial of the nth degree in x. For reaction (1) where a $J=\frac{3}{2}^+$ isobar is formed, it can be demonstrated that $n<2$.

The partial-wave expansion of (6) will be

$$B_{\lambda\mu}(s,x) = A_{\lambda\mu}(s,z) \sum_J (2J+1) C_{\lambda\mu}{}^J(z) d_{\lambda\mu}{}^J(x) + \sum_J^N (2J+1) R_{\lambda\mu}{}^J(z) d_{\lambda\mu}{}^J(x). \qquad (7)$$

With the help of (7) and according to the sharp-cutoff (SCO) model the reaction amplitude of (1) is

$$M_{\lambda\mu}{}^{\mathrm{SCO}} = A_{\lambda\mu} \sum_{J>J_c} (2J+1) C_{\lambda\mu}{}^J(z) d_{\lambda\mu}{}^J(x). \qquad (8)$$

The cutoff value, J_c in (8), is a free parameter of the model, and it is found by best fit between the theoretical and experimental differential cross sections.

In the following sections the predictions of the sharp-cutoff model for the reactions (1)–(4) are compared with the experimental data.

The results of the sharp-cutoff model can be obtained also by the absorption model.[1] In this model the OPE amplitude is modified by absorption effects, in the incoming and outgoing channels of the reactions (1)–(4). The absorption effects are expressed by the elastic scattering (absorptive) amplitudes of those channels.

The modified OPE amplitude for the reactions (1) and also for (2)–(4) is[10,11]

$$M_{\lambda p'',\lambda N^*,\lambda p,\lambda p'}{}^{\mathrm{ABS}} = \sum (2J+1) (S_{p''N^*}{}^J)^{1/2} B_{\lambda\mu}{}^J (S_{pp'}{}^J)^{1/2} d_{\lambda\mu}{}^J(x). \qquad (9)$$

The $S_{pp'}$ and $S_{p''N^*}$ are the elastic scattering amplitudes of the pp' and $p''N^*$ pairs; $B_{\lambda\mu}{}^J$ is the J component of the Born term of the one-pion-exchange diagram.

The pp elastic scattering differential cross section in the momentum range 2.8–15.0 GeV/c can be described by

$$\frac{d\sigma}{dt} = A e^{-t/2\nu^2}.$$

The values of ν deduced from the experimental results[12,13] decrease slightly with increasing incident momentum and are summarized in Table I. This form of the differential elastic scattering is that of diffractive

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TABLE I. Elastic scattering parameters ν, and ν'; cutoff J_c, and unitarity J_u, angular momenta, as functions of the incident momentum.

P_{inc} (GeV/c)	ν (GeV) $pp \to pp$	ν' (GeV) $pN^*(1238) \to pN^*(1238)$	Cutoff J_c	Unitarity J_u
2.85	0.27	0.14	5	3
3.70	0.27	0.10	7	3
4.55	0.27	0.14	6	4
5.52	0.24	0.12	8	4
6.06	0.26	0.10	9	5
7.10	0.26	0.10	9	5
7.88	0.25	0.10	10	5
10.00	0.25	0.09	13	6
15.00	0.24	0.11	15	8

scattering (strong absorption) with the following partial J amplitudes:

$$S_{pp}{}^J = 1 - \frac{\nu^2 \sigma_{\text{tot}}}{2\pi} \exp[-(J\nu/q)^2] \tag{10}$$

and

$$\frac{d\sigma}{dt} = \frac{\sigma_{\text{tot}}{}^2}{16\pi} \exp(-t/2\nu^2).$$

There is not any direct observation on the pN^* elastic scattering. We shall tentatively assume that the partial waves of pN^* are described also by

$$S_{pN^*}{}^J = 1 - \frac{\nu'^2 \sigma_{\text{tot}}}{2\pi} \exp[-(J\nu'/q)^2],$$

with an unknown parameter ν'.

With the help of (7), (9), and (10) the reaction amplitude according to the absorption model will be

$$M_{\lambda\mu}{}^{\text{ABS}} = A_{\lambda\mu}(s,z) \sum (2J+1)(1-e^{-(J\nu/q)^2})^{1/2} \times C_{\lambda\mu}{}^J(z)(1-e^{-(J\nu'/q)^2})^{1/2} d_{\lambda\mu}{}^J(x), \tag{11}$$

where the contributions of the polynomial $R_{\lambda\mu}(z)$ were neglected, and $\sigma_{\text{tot}} \simeq 2\pi/\nu^2$.

As in the study of the reaction $pp \to nN^{*++}(1238)$, the differential cross sections of reaction (1), in the momentum range 2.85–15.0 GeV/c, will be evaluated for two versions of the absorption model, namely:

(a) Ordinary absorption model (ABS), assuming the same scattering amplitude for the incoming (pp') and outgoing ($p''N^*$) pairs of particles, i.e., $\nu = \nu'$, as given in Table I.

(b) Absorption, with a different scattering amplitude (ABD) for the outgoing pair of particles: $S_{p''N^*}{}^J \neq S_{pp'}{}^J$. The scattering parameter of pN^*, ν', will be found by best fit between theory and experiment.

The differential cross sections of the reactions (2), (3), and (4) were calculated only according to the ordinary absorption model with $\nu = \nu'$.

3. THE REACTION $pp \to pN^*(1238)$

The production of the $N^*(1238)$ isobar in pp collision was studied in various counter and bubble chamber experiments. The observed differential cross section value $d\sigma/dt$ of reaction (1) at nine different incident momenta, in the range 2.85–15.0 GeV/c, are compared on Fig. 1 with the predictions of the sharp-cutoff and the absorption models. The OPE Born term $B_{\lambda N^*, \lambda p'', \lambda p, \lambda p'}$ for reaction (1) is

$$B_{\lambda N^*, \lambda p'', \lambda p', \lambda p} = \frac{iG_{pp\pi^0} G_{pN^*\pi^0}}{m_\pi(m_\pi{}^2 - t)} \bar{U}^{\lambda p''} \gamma_5 U^{\lambda p'} \psi_\mu{}^{\lambda N} q_\mu U^{\lambda p}, \tag{12}$$

where the U's are the nucleon spinors and ψ is the wave function of a spin-$\frac{3}{2}^+$ $N^*(1238)$ isobar; m_π and q_μ are the mass and the four momentum of the exchanged pion, and $t = q_\mu \cdot q_\mu$. The OPE amplitude (12) was evaluated using the explicit wave functions for the spin-$\frac{3}{2}$ particle and with the coupling-constant values[2] $G_{pp\pi^0}{}^2 = 14.5$ and $G_{N^*(1238)\, p\pi^0}{}^2 = 3.2$.

With the help of (12), (5), (6), and (8), the results of the sharp-cutoff model (SCO) were calculated with J_c as a free parameter, and they are given on Fig. 1 (solid lines). The best values of J_c are given in Table I. In this simple model the cutoff impact parameter $b_c = J_c/q$ changes little with energy, and practically a single cutoff impact parameter at $b = 1$ F in (6) accounts for the entire 2.85–15.0 GeV/c momentum range. The differential cross section of (1) was calculated also according to the absorption model with the help of (1) and Table I for (a) $\nu = \nu'$ and (b) $\nu' \neq \nu$, in the momentum range 2.85–15.0 GeV/c. The results are shown in Fig. 1. The predictions of the ordinary absorption model $\nu' = \nu$ (ABS) are higher than the observed differential cross section in this range of momentum. Better agreement between theory and experiment is obtained by treating ν' ($\neq \nu$) as a free parameter. The values of ν' as function of the incident momentum, in (11), that best fit the experimental results are given in Table I, and their corresponding (dashed) curves are plotted on Fig. 1. The ν' values obtained in this way change a little with energy and it is remarkable that with almost one free parameter ν', the absorption model predicts the differential cross section of reaction (1) in the 2.85–15.0 GeV/c momentum range.

The amplitudes of reaction (1), according to the sharp-cutoff model (8) and the absorption model (11), do not violate the unitarity bound. Integratihg the partial-wave expansion of $M_{\lambda\mu}$ in (5), the total reaction cross section of the Jth wave is $[q^2/(64\pi sq)](2J+1) \times |\sum M_{\lambda\mu}{}^J|^2$. This cross section cannot exceed the upper limit $(\pi/q^2)(2J+1)$. Therefore

$$[qq'/(64\pi^2 s)] \sum |M_{\lambda\mu}{}^J|^2 \leq 1 \tag{13}$$

is the bound set by unitarity on the reaction amplitude.

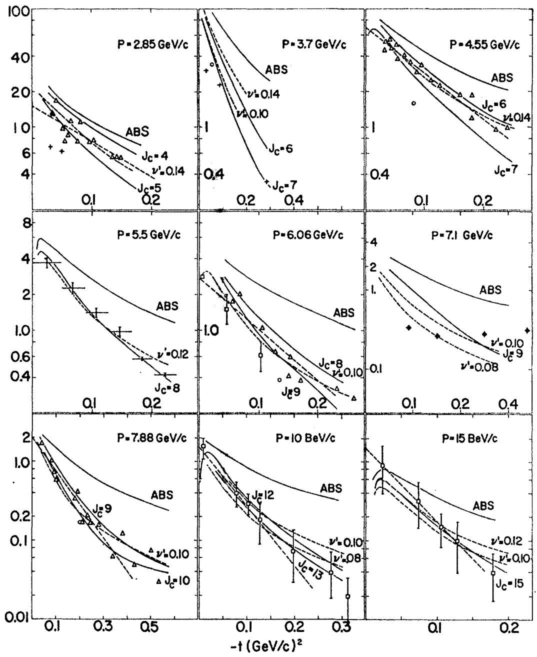

FIG. 1. Differential cross sections of the reaction $pp \to pN^* \times(1238)$ between 2.85 and 15.0 GeV/c, according to (a) sharp-cutoff model, with best values of J_c and R (F), full lines; (b) absorption model (ABS), full lines, and (c) absorption model with ν' fitted by experiment (dashed lines). The experimental points are by: (△) Blair *et al.* (Ref. 8); (+) Chadwick *et al.* (Ref. 4); (○) Cocconi *et al.* (Ref. 5); (---) Alexander *et al.* (Ref. 2) (5.5 GeV/c); and (□) Anderson *et al.* (Ref. 7). The dashed straight lines describe the results of Anderson *et al.* (Ref. 7) and Blair *et al.* (Ref. 8) represented by the expression $d\sigma/dt = Ae^{-bt}$.

With the help of (7), (11), and (13), the OPE amplitude of reaction (1) was tested in the momentum range 2.85–15.0 GeV/c.

The lowest angular momentum values J_u, which satisfy (13) found in this way, are given in Table I.

It appears from Table I that in the momentum range 3–15 GeV/c, the OPE amplitude of (1) violates unitarity. The violation is due to the partial waves with $J \leq J_u$. With the help of the relation $J = qb$, one finds that the J_u values in Table I in the range 3–15 GeV/c correspond to a single impact parameter $b = 0.5$ F. In the sharp-cutoff model the OPE amplitude is cut below $b = 1$ F, and $J_u < J_c$, therefore the SCO amplitude satisfies the unitarity bound (13).

With the help of (11) and (13) it was found that the reaction amplitude of (1) evaluated according to the absorption model for (a) $\nu = \nu'$ and (b) $\nu \neq \nu'$ also satisfies the unitarity condition (13) in the momentum range 2.85–15.0 GeV/c.

4. THE REACTIONS $pp \to pN^*(1410)$, $pp \to pN^*(1518)$, AND $pp \to pN^*(1688)$

Several isobars in the mass range 1400–1700 MeV were found in the phase-shift analysis of $\pi^+ p$ and $\pi^- p$ scattering experiments.[14,15] Not all these isobars were found in pp reactions, or in other production experiments. The missing-mass spectrum of the reaction $pp \to pX$ between 4.0 and 30.0 GeV/c shows clear evidence for the production of three $I = \frac{1}{2}$ isobars in the 1400–1700 mass range. They are identified with the following isobars found by phase-shift analysis: (a) $N^*_{1/2^+}(1410)$, (b) $N^*_{3/2^-}(1518)$, and the two isobars of the same mass, (c) $N^*_{5/2^+}(1688)$, and (d) $N^*_{5/2^-}(1688)$.

Anderson *et al.*[7] and Blair *et al.*[8] measured the differential cross section of the reactions (2), (3), and (4) between 4 and 30 GeV/c. It was shown that these cross sections can be well approximated by the expression $d\sigma/dt = ae^{-bt}$. The observed differential cross section of reactions (2), (3), and (4) are compared with the prediction of the absorption and sharp-cutoff models on Figs. 2 and 3.

The one-pion-exchange Born terms for the reactions (2), (3), and (4) are

$$B_{\lambda N^*, \lambda p'', \lambda p, \lambda p'} = \frac{iG_{pp\pi^0} G_{pN^*\pi^0}}{m_\pi^2 - t} \bar{U}^{\lambda N^*} Y_5 U^{\lambda p} \bar{U}^{\lambda p''} Y_5 U^{\lambda p'} \quad (14a)$$

for $N^*_{1/2^+}(1410)$ production in reaction (2), where the U's are $J = \frac{1}{2}$ spinors.

$$B_{\lambda N^*, \lambda p'', \lambda p', \lambda p} = \frac{iG_{pp\pi^0} G_{pN^*\pi^0}}{m_\pi (m_\pi^2 - t)} \bar{U}^{\lambda p} \gamma_5 U^{\lambda p'} \bar{\psi}_\mu^{\lambda N^*} q_\mu \gamma_5 U^{\lambda p} \quad (14b)$$

for $N^*_{3/2^-}(1518)$ production in reaction (3),[16] where ψ_μ is the $J = \frac{3}{2}$ wave function.

$$B_{\lambda N^*, \lambda p'', \lambda p', \lambda p} = \frac{iG_{p\pi\pi^0}{}^2 G_{N^* p\pi^0}{}^2}{m_\pi^2 (m_\pi^2 - t)} \bar{U}^{\lambda p''} \gamma_5 \bar{\psi}_{\mu\nu}^{\lambda N^*} q_\mu q_\nu U^{\lambda p} \quad (14c)$$

for $N^*_{5/2^+}(1688)$ production in reaction (4),[16] where $\psi_{\mu\nu}$ is the $J = \frac{5}{2}$ wave function and

$$B_{\lambda N^*, \lambda p'', \lambda p', \lambda p} = \frac{iG_{pp\pi^0}{}^2 G_{N^* p\pi^0}{}^2}{m_\pi^2 (m_\pi^2 - t)} \bar{U}^{\lambda p''} U^{\lambda p'} \bar{\psi}_{\mu\nu}^{\lambda N^*} q_\mu q_\nu U^{\lambda p} \quad (14d)$$

for the production of the $N^*_{5/2}(1688)$ isobar in reaction (4).

The four different coupling constants $Gp\pi^0 N^*$, in (14) were estimated from the decay widths $\Gamma(N^* \to p\pi^0)$ of the four respective isobars summarized in Table II.

With the help of (14), (6), and (7) the differential cross sections (5) were valuated for the four isobars produced in reactions (2), (3),[16] and (4),[16] according to

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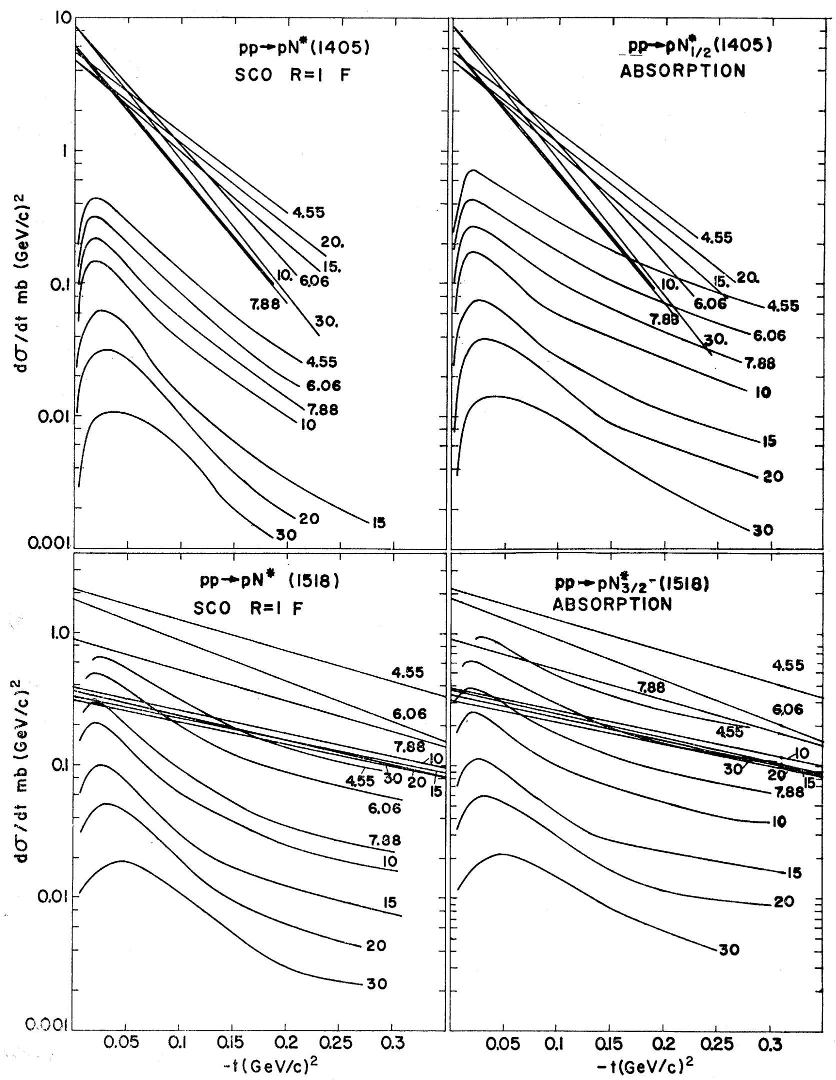

FIG. 2. Differential cross sections for the reactions $pp \to pN^*(1410)$ and $pp \to pN^*(1518)$ between 4.55 and 30.0 GeV/c according to the (a) sharp-cutoff model (SCO) at $R=1$ F and (b) absorption model. The straight lines describe the experimental results of Anderson *et al.* (Ref. 7) and Blair *et al.* (Ref. 8) represented by the expression $d\sigma/dt = ae^{-bt}$.

(a) the sharp-cutoff model (8) with a cutoff at $b=1$ F and (b) the absorption model (11) with $\nu=\nu'$. The calculated differential cross sections are shown in Figs. 2 and 3.

It appears from Fig. 2 that both the sharp-cutoff model and the absorption models predict cross sections for the two reactions (2) and (3) which are too low and with the wrong energy dependence.

The differential cross sections for the production of

TABLE II. Coupling constants squared, $G_{p\pi^0N^*}{}^2$, for the reaction $pp \to pN^*$, estimated from the decay widths $\Gamma(N^* \to p\pi^0)$ of four isobars: $N^*_{1/2^+}(1410)$, $N^*_{3/2^-}(1518)$, $N^*_{5/2^+}(1688)$, and $N^*_{5/2^-}(1688)$.

Mass (MeV)	J^P	(MeV)	Elasticity	$G_{p\pi^0N^*}{}^2$
1410	$\frac{1}{2}^+$	120	0.70	19.50
1518	$\frac{3}{2}^-$	120	0.50	2.70
1688	$\frac{5}{2}^+$	120	0.50	0.110
1688	$\frac{5}{2}^-$	120	0.35	0.006

the even and odd $N^*_{5/2}(1688)$ isobars in reaction (4) are given on Fig. 3. The cross-section values for each isobar alone are too low, and with the wrong energy dependence, to explain the observed differential cross sections of reaction (4). However, when the cross sections of the two $N^*(1688)$ isobars are added together and compared with the experimental results, better agreement is obtained.

The absorption model predicts cross-section values for reaction (4) at $P=4.55$ GeV/c for the two isobars which are slightly higher than observed by Blair *et al.*,[8] while at higher incident momentum, the absorption values are lower than the observed ones, and the discrepancy increases with increasing incident momentum.

5. CONCLUSIONS

Of the four reactions (1)–(4) studied, only the reaction $pp \to pN^*_{3/2^+}(1238)$ can be explained in the range 3–15.0 GeV/c by the one-pion-exchange (OPE) mechanism with absorption corrections. In the other

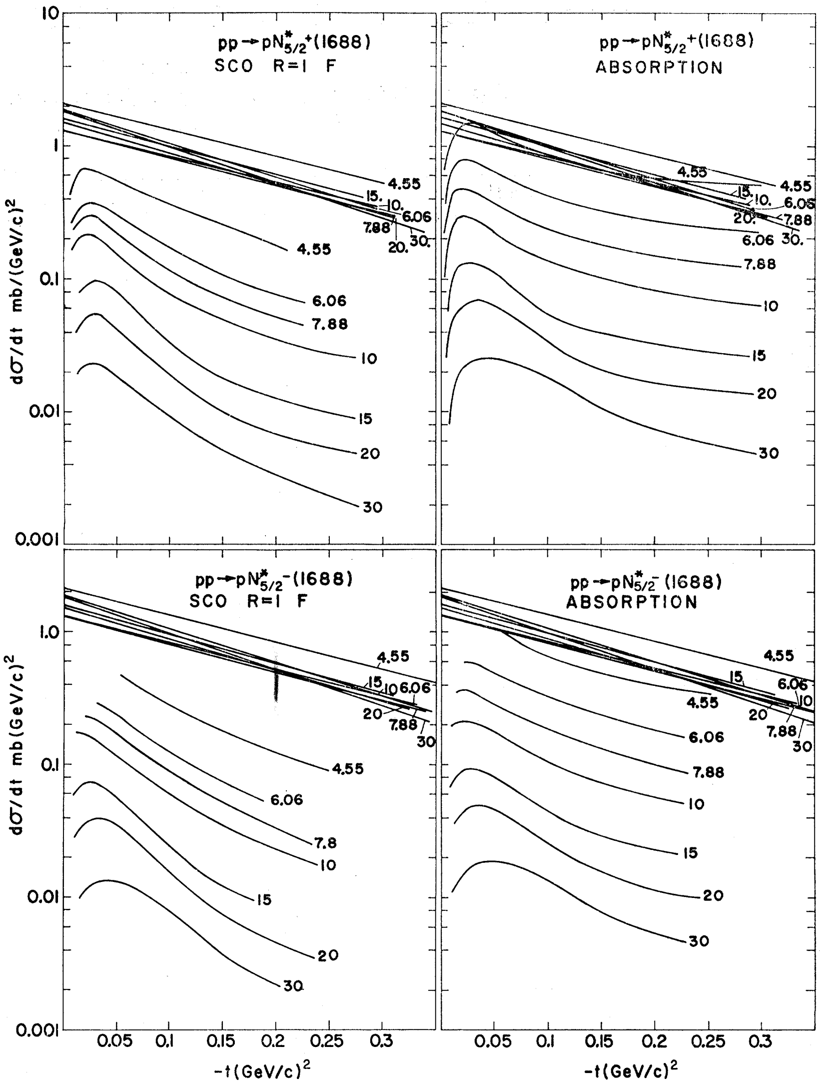

FIG. 3. Differential cross sections for the reaction $pp \to pN^*(1688)$ between 4.55 and 30 GeV/c for the odd and even $J=\frac{5}{2}$ isobars, according to the (a) sharp-cutoff model (SCO) at $R=1$ F and (b) absorption model. The straight lines are the experimental results of Anderson *et al.* (Ref. 7) and Blair *et al.* (Ref. 8) represented by the expression $d\sigma/dt = ae^{-bt}$.

cases, when $I=\frac{1}{2}$ isobars are produced, the failure of the OPE model, even with absorption corrections, is evident. The OPE model predicts cross sections that decrease strongly with incident momentum, while the observed cross sections of the reactions (3) and (4) have a completely different incident momentum dependence. The cross sections of reactions (2) and (4) change very little with incident momentum and are practically constant between 4 and 30 BeV/c, while those of reaction (3) decrease between 3 and 10 GeV/c and level off above 10 GeV/c. The only exception, which could be accidental, is that the OPE model explains the cross section of reaction (4) near 5 GeV, as the result of two $J=\frac{5}{2}$ isobars, one even and one odd, with $M=1688$ MeV. The failure of the OPE calculations for reactions (2)–(4), and the energy dependence of their cross sections, suggest for them quite a different reaction mechanism. The similarity with elastic scattering may serve as a guide to attempt a Regge-pole formalism with Pomeranchuk exchange.

The agreement between the OPE model, and the observed cross sections for reaction (1) over the entire 3–15 GeV/c range is significant. This good agreement was obtained for the simple sharp-cutoff model with practically a single cutoff impact parameter $b \sim 1$ F. Similar good agreement is obtained by the absorption model, when an absorption parameter of $\nu' \sim 0.1$ GeV/c is used for the outgoing $pN^*(1238)$ pair.

The unmodified OPE amplitude of reaction (1) violates unitarity in the entire 2.85–15.0 GeV/c range. Partial-wave expansion of the OPE amplitude shows that unitarity is violated only for small impact parameters $b \leq 0.5$ F, and that the sharp-cutoff model and the absorption model satisfy the unitarity condition (13).

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