Models of $\eta^0 \to \pi^0 + 2\gamma$ Decay*

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Recent experimental results on the $\eta^0 \to \pi^0 2\gamma$ decay indicate that the branching ratio $B \equiv \Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$ is of the order of (or compatible with) 1. Using the vector-meson-dominant model, we have computed the width and the pion energy spectrum of the $\eta^0 \to \pi^0 2\gamma$ decay, using the relevant $SU(3)$-symmetric coupling constants, including the ω-ϕ mixing hypothesis as given by the static $SU(6)$ theory, which demands $\Gamma_\phi(\pi^0\gamma) \approx 0$. We have then studied the effect of varying the mixing angle and the effective coupling constant $f_{\phi\pi^0\gamma}$ around the static $SU(6)$ value over a wide range compatible with the present experimental information, using as input the experimental $\Gamma_\omega(\pi^0\gamma)$ as before. The model gives $B \approx 0.331 \times 10^{-2}$ at the static $SU(6)$ value, which increases by a factor of 10 at the mixing angle of 20°. It therefore appears that unless there is a drastic decrease in the experimental branching ratio, the vector-meson-dominant model cannot provide a dominant mechanism for the $\eta^0 \to \pi^0 2\gamma$ decay. A remark is also made about the relation between the value of the ω-ϕ mixing angle and the ratios of various observed decay rates. We also study an alternative simple mechanism for the $\eta^0 \to \pi^0 2\gamma$ decay which may be suggested by the tadpole model of the electromagnetic mass difference of hadrons. We assume the existence of the $I=1$, normal 0^+ $(\eta\pi)$ resonance ϵ, and consider the intermediary of the ϵ meson $\eta^0 \to \pi^0\epsilon^0 \to \pi^0 2\gamma$. Some consequences of this model are discussed.

I. INTRODUCTION

THE study of the decays of the pion and of the pionic resonances has attracted considerable attention in the last two decases.[1] The η-meson occupies a special position among the mesonic resonances; it decays by isospin- or G-parity-violating electromagnetic interactions.

Essentially three dynamical models have been used to study the η (and π^0) decays: They are named, according to the dominant intermediate states, the baryon-antibaryon loop model,[2] the intermediate-vector-meson model,[3] and the intermediate-pion model[4] for the $\eta^0 \to 3\pi$ decay.

Recent experiments[5,6] have considerably changed the over-all picture of the η-decay branching ratios. One observes that the $(2\gamma)/(\pi^+\pi^-\gamma)$ ratio has changed slightly and the $(3\pi^0)/(\pi^+\pi^-\pi^0)$ ratio has become smaller than 1. The cause of these changes is the proof of the existence of the $\eta \to \pi^0 2\gamma$ mode and its surprisingly large branching ratio. Okubo and Sakita[7] made an estimate based on an effective Hamiltonian of the simplest form,

$$H = \xi F_{\mu\nu} F_{\mu\nu} \eta^0 \pi^0 . \tag{1}$$

By assuming that the transition mass between η^0 and π^0 can be obtained by contracting photon lines in the above Hamiltonian, they estimated $\Gamma_\eta(\pi^0 2\gamma) \simeq 8$ eV. Since, as they stated, this sort of estimate (by introducing a cutoff) may easily be wrong by a factor $\simeq 10$, the large rate of $\eta^0 \to \pi^0 2\gamma$ decay observed need not be very puzzling. We may also recall that the decays $\eta^0 \to \pi^0 2\gamma$ and 3π are A allowed, whereas $\eta^0 \to 2\gamma$ and $\pi^+\pi^-\gamma$ are A forbidden.[8]

The vector-meson-dominant model seems to have achieved a fair success in explaining the branching

* For a preliminary report of part of this work, see G. Oppo, University of Maryland, Department of Physics and Astronomy, Technical Report No. 627, 1966 (unpublished); also G. Oppo and S. Oneda, Bull. Am. Phys. Soc. **12**, 127 (1967).

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[1] See, for example, for the earlier period, H. A. Bethe and F. de Hoffmann, *Mesons and Fields*, (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. II; R. E. Marshak, *Meson Physics* (Dover Publications, Inc., New York, 1964). For a review article covering the work up to 1962, see G. Puppi, Ann. Rev. Nucl. Sci. **13**, 287 (1963).

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[5] C. Di Giugno, R. Querzoli, G. Troise, F. Vanoli, M. Giorgi, P. Schiavon, and V. Silvestrini [Phys. Rev. Letters **16**, 767 (1966)] give $R = (\eta^0 \to \pi^0 2\gamma)/(\eta \to 2\gamma) = 0.9 \pm 0.1$. M. A. Whalig, E. Shibata, and I. Mannelli [Phys. Rev. Letters **17**, 221 (1966)] give an upper limit 0.50 for Z. S. Strugalski *et al.* give $R = 0.86 \pm 0.40$ [*Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967), p. 315]. J. Grunhaus gives $R = 0.61 \pm 0.24$ [Columbia University Report No. Nevis 156, 1966 (unpublished)].

[6] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[7] S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963).

[8] J. B. Bronzan and F. Low, Phys. Rev. Letters **12**, 522 (1964).

ratios $\Gamma_\omega(\pi\gamma)/\Gamma_\omega(3\pi)$ and $\Gamma_\eta(\pi^+\pi^-\gamma)/\Gamma_\eta(2\gamma)$. Therefore, it seems worth while to study whether or not the same model also works for $\Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$. In this model, it turns out that the matrix elements obtained differ from those given by the above simplest one, Eq. (1), in two respects: First, ξ is, of course, not constant; second, there is an additional term which cannot be reduced to the form $F_{\mu\nu}F_{\mu\nu}$.[9] As regards the rate $\Gamma_\eta(\pi^0 2\gamma)$ we have, from the outset, some pessimistic feeling about this model. Namely, if, for instance, we choose the ρ meson as an intermediate state, like $\eta\to\rho+\gamma\to(\pi+\gamma)+\gamma$ and $\eta\to\rho+\gamma\to(\pi^++\pi^-)+\gamma$, we expect, crudely speaking, that

$$\Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)\simeq|G_{\rho\pi\gamma}/G_{\rho\pi\pi}|2\lesssim 0.01\,.$$

However, these arguments could easily be wrong by an order of magnitude. Furthermore, we are not justified in neglecting the diagrams involving the ω and ϕ mesons. Therefore, a detailed calculation may be useful. In order to test only the vector-meson-dominant model, one has to divorce it as much as possible from additional assumptions. For this purpose, although we use the $SU(3)$-symmetric couplings with the ω-ϕ mixing hypothesis, we vary the input parameters, the ω-ϕ mixing angle, and $\Gamma_\phi(\pi^0\gamma)$ over as wide a range as is compatible with the present experimental information.

We treat the intermediate unstable-particle graphs in the first Born approximation with renormalized coupling constants, corresponding to the pole approximation of a dispersion-theoretic calculation. For definiteness we first use $SU(3)$-symmetric coupling constants with ω-ϕ mixing parameter θ and f/g ratio (ratio of singlet to octet coupling) as obtained from the assumed vanishing of $\Gamma_\phi(\pi^0\gamma)$ [which are[10] the same as in static $SU(6)$], together with the known width $\Gamma_\omega(\pi^0\gamma)$. Then we discuss the effect of possible departures from the static-$SU(6)$ values and of the experimental uncertainty in $\Gamma_\omega(\pi^0\gamma)$ and $\Gamma_\phi(\pi^0\gamma)$. The branching ratio $B\equiv\Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$ turns out to be $\simeq 0.33\times 10^{-2}$ for the static-$SU(6)$ values, but increases by a factor of 10 if the mixing angle is changed to 20°. Therefore, if the present experimental indication $B\gtrsim 1$ is correct, this model does not seem to provide a dominant mechanism of $\eta\to\pi 2\gamma$ decay.[9,11] We also note a feature of the correlation between the ω-ϕ mixing angle and the ratios of various decay rates. As a result, we are led to consider in Sec. III an alternative approach which seems quite natural if the tadpole model[12] of the electromagnetic mass difference of hadrons is correct. Namely, we assume the existence of an $I=1$, normal 0^+ $\eta\pi$ resonance ϵ $(\epsilon^+,\epsilon^0,\epsilon^-)$, and consider the mechanism

$$\eta^0\to\pi^0+\epsilon^0\to\pi^0+2\gamma\,.$$

The plausibility and the consequences of this model will be discussed. We emphasize that the pion spectrum of $\eta\to\pi^0 2\gamma$ decay is very different in the vector-meson intermediate model and in the ϵ-meson model.

II. THE VECTOR-MESON-DOMINANT MODEL

A. The Matrix Element

The graphs which contribute to the decay are shown in Fig. 1. We consider the intermediate vector mesons as stable particles.

We explain the notation in detail for the first diagram in Fig. 1, as there are only obvious changes for the others. We take the following as invariant couplings for the $\eta\rho\gamma$ and $\rho\pi^0\gamma$ vertices, respectively:

$$\frac{f_{\eta\rho\gamma}}{m_\pi}\epsilon_{\alpha\beta\mu\nu}q_\alpha k_\beta e_\mu \rho_\nu \quad\text{and}\quad \frac{f_{\rho\pi^0\gamma}}{m_\pi}\epsilon_{\gamma\rho\sigma\tau}\rho_\gamma k_\rho' q_\sigma' e_\tau'\,.$$

The $f_{\eta\rho\gamma}$ and $f_{\rho\pi^0\gamma}$ are dimensionless effective coupling constants, m_π is the pion mass, and $\epsilon_{\alpha\beta\gamma\rho}$ is the totally antisymmetric unit tensor density of rank 4. Finally, q_α, k_β, k_ρ', and q_σ' are 4-momentum vector components of the η, γ, γ', and π^0, respectively; and e_μ, ρ_ν, and e_τ' are covariant polarization vector componensts of the γ, ρ, and γ', respectively.

Thus the invariant matrix element for the ρ-intermediate diagram is, up to kinematical factors,

$$(f_\rho/m_\pi^2)\epsilon_{\gamma\delta\sigma\tau}\epsilon_{\alpha\beta\mu\gamma}q_\alpha k_\beta e_\mu k_\delta' q_\sigma' e_\tau'(p_\rho^2+m_\rho^2)^{-1}.\quad(2)$$

Here $f_\rho\equiv f_{\eta\rho\gamma}f_{\rho\pi\gamma}$. (In the following, we shall denote by f_ω and f_ϕ the similar coupling constants for the intermediate ω and ϕ diagrams.) The last factor in (2) comes from the ρ-meson propagator. (We use the convention $ab=\mathbf{a}\cdot\mathbf{b}-a_0b_0$.)

Using 4-momentum conservation, we can eliminate q' by rewriting Eq. (2) in a form containing only the three

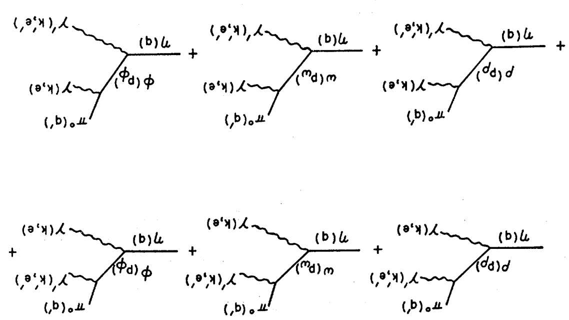

FIG. 1. Feynman diagrams for the decay $\eta\to\pi^0 2\gamma$ in the vector-meson-dominant model. The letters inside the parentheses indicate 4-momenta and polarizations.

[9] G. Oppo, University of Maryland, Department of Physics and Astronomy, Technical Report No. 627, 1966 (unpublished).

[10] C. D. Soloviev, Phys. Letters **16**, 345 (1965); V. V. Anisovich *et al.*, *ibid.* **16**, 194 (1965).

[11] See also H. Pietchmann and W. Thirring, Phys. Letters **21**, 713 (1966); Möbius and H. Pietchmann, *ibid.* **22**, 684 (1966); W. Alles, A. Baracca, and A. T. Romos, Nuovo Cimento **45**, 272 (1966).

[12] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B681 (1964).

independent 4-momenta q, k', and k:

$$-(f_\rho/m_\pi^2)\epsilon_{\gamma\alpha\beta\mu}\epsilon_{\gamma\delta\sigma\tau}(q_\alpha k_\beta e_\mu k_\delta' k_\sigma e_\tau' - q_\alpha k_\beta e_\mu k_\delta' q_\sigma e_\tau') \times (p_\rho^2+m_\rho^2)^{-1}. \quad (3)$$

By the use of the relation $\epsilon_{\gamma\alpha\sigma\tau}\epsilon_{\gamma\delta\sigma\tau}=\delta_{\alpha\delta}\delta_{\beta\sigma}\delta_{\mu\tau}+\delta_{\alpha\tau}\delta_{\beta\delta}\delta_{\mu\sigma}+\delta_{\alpha\sigma}\delta_{\beta\tau}\delta_{\mu\delta}-\delta_{\alpha\delta}\delta_{\beta\tau}\delta_{\mu\sigma}-\delta_{\alpha\tau}\delta_{\beta\sigma}\delta_{\mu\delta}-\delta_{\alpha\sigma}\delta_{\beta\delta}\delta_{\mu\tau}$ one obtains from (3) the gauge- and Lorentz-invariant expression:

$$\begin{aligned}-(f_\rho/m_\rho^2)(p_\rho^2+m_\rho^2)^{-1}\{&[(k\cdot e')(e\cdot k')-(k\cdot k')(e\cdot e')]\\ &\times(q^2-q\cdot k)+(q\cdot k')(k\cdot q)(e\cdot e')+(q\cdot e')(k\cdot k')(e\cdot q)\\ &-(q\cdot k')(k\cdot e')(e\cdot q)-(q\cdot e')(k\cdot q)(e\cdot k')\}, \quad (4)\end{aligned}$$

while the other terms vanish by virtue of the relations $k^2=k\cdot e=k'\cdot e'=0$. The term inside the square bracket in (4) is equal to $\frac{1}{2}F_{\mu\nu}(k)F_{\mu\nu}(k')$. Therefore, this term corresponds to the simplest effective Hamiltonian given by (1). However, as already mentioned,[9] we get additional contributions in this model; this can be seen explicitly from Eq. (4).

The presence of these additional terms leads to an energy spectrum of the pion which is very different from the one predicted by the simplest Hamiltonian (1) or by a model introduced in Sec. III. We now take a gauge, $e_4=e_4'=0$. Then the expression (4) reduces (in the rest frame of the η meson) to

$$\begin{aligned}-(f_\rho/m_\pi^2)(p_\rho^2+m_\rho^2)^{-1}\{&(-m_\eta^2+m_\eta k)[(\mathbf{k}\cdot\mathbf{e}')(\mathbf{k}'\cdot\mathbf{e})\\ &-(k\cdot k')(\mathbf{e}\cdot\mathbf{e}')]+m_\eta^2kk'(\mathbf{e}\cdot\mathbf{e}')\}\\ &=-(f_\rho/m_\pi^2)D_\rho(k)(ST+NU). \quad (5)\end{aligned}$$

Above, expressions like $k\cdot k'$ indicate 4-products, while kk' is the product of the magnitudes of 3-momenta. $\mathbf{k}\cdot\mathbf{k}'$ is the scalar product of 3-vectors. $D_\rho(k)=(p_\rho^2+m_\rho^2)^{-1}=(m_\rho^2-m_\eta^2+2m_\eta k)^{-1}$, and D_ω, D_ϕ below are the corresponding expressions for the ω, ϕ graphs. Also we have used the abbreviations

$$S\equiv -m_\eta^2+m_\eta k, \quad T\equiv(\mathbf{k}\cdot\mathbf{e}')(\mathbf{k}'\cdot\mathbf{e})-(k\cdot k')(\mathbf{e}\cdot\mathbf{e}'),$$
$$N\equiv m_\eta^2kk', \qquad U\equiv\mathbf{e}\cdot\mathbf{e}'.$$

The complete matrix element μ from the six diagrams of Fig. 1 will be given (up to kinematical factors) by

$$\mu=-(g_iD_iS+g_iD_i'S')T-(g_iD_i+g_iD_i')NU \quad (i=\rho,\omega,\phi), \quad (6)$$

where the primed terms come from the second line of Fig. 1, and $S'\equiv -m_\eta^2+m_\eta k'$; $D_i'=(m_i^2-m_\eta^2+2m_\eta k')^{-1}$. For simplicity we also put $g_i\equiv f_i/m_\pi^2$; an expression like g_iD_i means $g_\rho D_\rho+g_\omega D_\omega+g_\phi D_\phi$.

B. Polarization Sums, Phase-Space Integral, and Pion Energy Spectrum

We need for later use the expression for $\sum_{\text{pol}}|\mu|^2$. We have

$$\begin{aligned}\sum_{\text{pol}}|\mu|^2=&(g_iD_iS+g_iD_i'S')^2\sum_{\text{pol}}T^2\\ &+(g_iD_i+g_iD_i')^2N^2\sum_{\text{pol}}U^2+2(g_iD_iS+S_iD_i'S_i')\\ &\times(g_iD_i+g_iD_i')N\sum_{\text{pol}}TU. \quad (7)\end{aligned}$$

The three photon polarization sums on the right-hand side of Eq. (7) can be evaluated in a standard way. After some algebra,[9] one obtains the expression (symmetric in k and k' as expected)

$$\begin{aligned}\sum_{\text{pol}}|\mu|^2=m_\eta^2k^2k'^2\{&[2(g_iD_i)^2(k-m_\eta)k\\ &+2(g_iD')^2(k'-m_\eta)k']+2(g_iD_i)(g_jD_j')\\ &\times[2(k-m_\eta)(k'-m_\eta)+m_\eta(k-m_\eta)\\ &+m_\eta(k'-m_\eta)](1-\cos\beta)^2\\ &+m_\eta^2(g_iD_i+g_iD_i')^2(1+\cos^2\beta)\}, \quad (8)\end{aligned}$$

where β is the angle between the 3-momentum vectors $\mathbf{k}$ and $\mathbf{k}'$.

We get the total decay rate by performing the nine-dimensional integration over the final 3-momenta. We get

$$\Gamma_\eta(\pi^0 2\gamma)=\frac{m_\eta}{(64)(2\pi)^3}\left(\frac{m_\eta}{m_\pi}\right)^4F, \quad (9)$$

where F is a dimensionless integral given by

$$F\equiv\sum_{i,j}f_if_jI_{ij} \quad (i,j=\rho,\omega,\phi). \quad (10)$$

The f_i are the dimensionless coupling constants introduced earlier. I_{ij} is defined as follows:

$$I_{ij}=-\int_{m_\pi}^{(1+m_\pi^2)/2}dE_\pi\int_{\frac{1}{2}[1-E_\pi-(E_\pi^2-m_\pi^2)^{\frac{1}{2}}]}^{\frac{1}{2}[1-E_\pi+(E_\pi^2-m_\pi^2)^{\frac{1}{2}}]}dk\,M_{ij}, \quad (11)$$

with

$$M_{ij}\equiv(\alpha\beta^2-\gamma)D_iD_j+(\delta\beta^2-\gamma)D_i'D_j'+(\epsilon\beta^2-\gamma)D_iD_j', \quad (12)$$

where

$$\begin{aligned}&\alpha\equiv 2k(1-k), \quad \beta\equiv(1+m_\pi^2-2E_\pi),\\ &\gamma\equiv 4k^2(1-E_\pi-k)^2+[2k^2-2k(1-E_\pi)+\beta]^2,\\ &\delta\equiv 2(1-E_\pi-k)(E_\pi+k),\\ &\epsilon\equiv -2(1-k)(E_\pi+k)+1+E_\pi,\\ &D_i=(m_i^2-1+2k)^{-1}, \quad D_i'=(m_i^2+1-2E_\pi-2k)^{-1}.\end{aligned} \quad (13)$$

All the physical quantities in (10), (11), (12), and (13) are dimensionless ($m_\eta=1$). The indices i, j stand for any of the intermediate vector mesons ρ, ω, and ϕ. The dk integral can be evaluated in a closed form. However, it has been found more convenient to evaluate the double

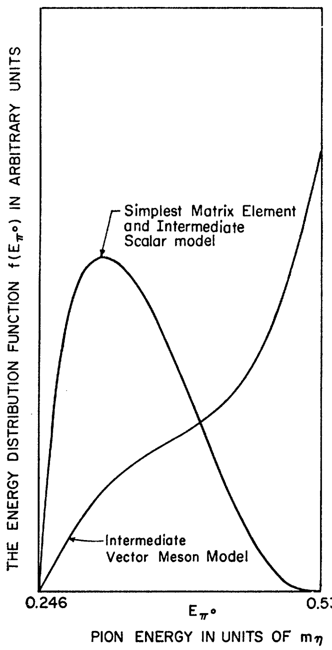

FIG. 2. The pion energy spectrum for the $\eta\rightarrow\pi^0 2\gamma$ decay. The spectrum for the vector-dominant models is characteristic of the model and does not depend on additional assumptions. (See also footnote 32.)

integral numerically, with the following results:

$$\begin{aligned} &I_{\rho\rho}=1.83\times10^{-3},\ I_{\omega\omega}=1.52\times10^{-3},\ I_{\phi\phi}=0.414\times10^{-3},\\ &I_{\rho\omega}=I_{\omega\rho}=1.67\times10^{-3},\ I_{\phi\rho}=I_{\rho\phi}=0.871\times10^{-3}, \qquad (14)\\ &I_{\omega\phi}=I_{\phi\omega}=0.793\times10^{-3}. \end{aligned}$$

The k integral $f(E_\pi)$ gives the energy spectrum of the pion in the vector-meson-dominant model. Its shape does not depend appreciably on the particular value of the input parameters (see next section) or the graph considered. Figure 2 gives a plot of this spectrum as obtained for the $\rho\rho$ case. $f(E_\pi)$ increases monotonically from 0 at the lowest energy to a maximum at the highest pion energy. This behavior is to be compared with the cases of the simplest matrix element given by (1) and of the intermediate ϵ model (see the discussion in Sec. III) which are also plotted in Fig. 2. It is seen that the pion energy spectrum will be useful to distinguish the vector-meson-dominant model from other simple models.

C. Coupling Constants and the Decay Rate

In order to get a final numerical value for $\Gamma_\eta(\pi^0 2\gamma)$, we need estimates of the effective coupling constants f_i. We use the experimental $\Gamma_\omega(\pi^0\gamma)$ width to obtain[13] $(f_{\omega\pi^0\gamma}{}^2/4\pi)=0.15\alpha$ [corresponding to $\Gamma_\omega(\pi^0\gamma)=1.08$ MeV], where α is the fine-structure constant. We assume the usual $SU(3)$-symmetric[14] electromagnetic vertices P-V-γ and introduce ω-ϕ mixing for the vector meson.[15] The relations thus obtained turn out to be the same as those obtained[16] by using $SU(3)$-symmetric V-V-P couplings and V-γ couplings with ω-ϕ mixing. We take first a value of mixing angle $\arcsin\theta=\sqrt{\tfrac{1}{3}}$ i.e., $\theta=35°\ 18'$. This is[10] very close to the $SU(6)$ value and is in reasonable agreement with the various estimates[17] of the ω-ϕ mixing angle. We then assume the vanishing[18] of the $\Gamma_\phi(\pi^0\gamma)$ width, to obtain for the f/g ratio [which is the same again as in static $SU(6)$] the value $1/\sqrt{2}$.

We now sketch the procedure used. We write

$$|\phi\rangle=\cos\theta\ |\omega_8\rangle-\sin\theta\ |\omega_1\rangle, \qquad (15)$$

$$|\omega\rangle=\sin\theta\ |\omega_8\rangle+\cos\theta\ |\omega_1\rangle, \qquad (16)$$

where $|\phi\rangle$ and $|\omega\rangle$ are the physical particle states, and $|\omega_1\rangle$ and $|\omega_8\rangle$ are the pure singlet and octet states.

The general V-P-γ interaction may be assumed to be a linear combination of $\mathrm{Tr}(V_8P_8Q)$, $\mathrm{Tr}(V_8QP_8)$, and $V_1\,\mathrm{Tr}(P_8Q)$. Space-time indices are understood, V_1 represents the vector-meson unitary singlet, and the matrices P_8 and V_8 represent the pseudoscalar and the vector octet, respectively. Because the particles of interest are only in the diagonal, the matrices P_8 (and V_8) are effectively diagonal and effectively commute with Q. We obtain, therefore, only two independent interactions with coupling constants f and g, respectively. The following relations result:

$$f_{\rho\pi^0\gamma}=(1/\sqrt{3})f_{\rho\eta\gamma}=(1/\sqrt{3})f_{\omega_8\pi^0\gamma}=-f_{\eta\omega_8\gamma}=f \qquad (17a)$$

and

$$f_{\omega_1\eta\gamma}=(1/\sqrt{3})f_{\omega_1\pi^0\gamma}=g, \qquad (17b)$$

from which, by the use of (15) and (16), one obtains

$$f_{\omega\pi^0\gamma}=\sqrt{3}(f\sin\theta+g\cos\theta), \qquad (18a)$$

$$f_{\phi\pi^0\gamma}=\sqrt{3}(f\cos\theta-g\sin\theta), \qquad (18b)$$

$$f_{\omega\eta\gamma}=-f\sin\theta+g\cos\theta, \qquad (18c)$$

$$f_{\phi\eta\gamma}=-f\cos\theta-g\sin\theta. \qquad (18d)$$

[13] This corresponds to a $\Gamma_\omega(\pi^0\gamma)$ of 1.08 MeV, and is obtained from the expression $\Gamma_\omega(\pi^0\gamma)=(1/96)(f_{\omega\pi^0\gamma}{}^2)m_\omega(m_\omega/m_\pi)^2[1-(m_\pi/m_\omega)^2]^3$. The minimum and maximum values of $\Gamma_\omega(\pi^0\gamma)$, as from A. H. Rosenfeld *et al.*'s latest compilation [Rev. Mod. Phys. **39**, 1 (1967)] are 0.902 and 1.27 MeV, respectively.

[14] M. Gell-Mann, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964).

[15] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); S. L. Glashow, *ibid.* **11**, 48 (1963); R. F. Dashen and D. H. Sharp, Ref. 5; Y. S. Kim, S. Oneda, J. C. Pati, Ref. 3.

[16] S. L. Glashow, Ref. 15.

[17] See Ref. 15. Also S. Okubo, Phys. Letters **5**, 165 (1963); J. J. Sakurai, Phys. Rev. **132**, 434 (1963); T. Massam and A. Zichichi, Nuovo Cimento **41A**, 310 (1966).

[18] This mode has not, to our knowledge, been found. See J. S. Lindsey and G. A. Smith, Phys. Rev. **147**, 913 (1966). We actually assume that $\Gamma_\phi(\pi^0\gamma)\leq0.008$ MeV, corresponding to the values of $\epsilon\leq0.2$ as quoted in this reference. There are several arguments indicating that this width is indeed very small. For instance the $\phi\pi^0\gamma$ phase space is about 2.9 times larger than that of $\omega\pi^0\gamma$; therefore, $(f_{\phi\pi^0\gamma}{}^2/4\pi)\ll(f_{\omega\pi^0\gamma}{}^2/4\pi)(2.9)^{-1}$. Another argument is based on the smallness of $\Gamma_\phi(\rho\pi)$. The latest Rosenfeld tabulation (Ref. 6) gives $\Gamma_\phi(3\pi+\rho\pi)\simeq0.540$ MeV, which leads to an estimate of $\Gamma_\phi(\pi^0\gamma)$ of the same order.

The assumed relation[18] $f_{\phi\pi\gamma} \sim 0$ gives $f/g = 1/\sqrt{2}$. With this ratio and $\arcsin\theta = 1/\sqrt{3}$ we get

$$f_{\pi^0\omega\gamma} = 3f;\quad f_{\phi\pi^0\gamma} = 0;\quad f_{\omega\eta\gamma} = 1/\sqrt{3}f;\quad f_{\phi\eta\gamma} = -(2\sqrt{\tfrac{2}{3}})f;$$
$$f_{\rho\eta\gamma} = \sqrt{3} f_{\rho\pi^0\gamma} = \sqrt{3} f. \tag{19}$$

According to our definition

$$f_\rho = \sqrt{3} f^2;\quad f_\omega = f_\rho;\quad f_\phi = 0. \tag{20}$$

Finally

$$(f_{\omega\pi\gamma}^2/4\pi) = (3f)^2/4\pi = 0.15\alpha \quad (\text{range } 0.121\alpha\text{–}0.171\alpha)$$

(when we take into account the experimental error), which gives

$$f_\rho = f_\omega = 0.363\alpha \quad (0.295\alpha\text{–}0.413\alpha), \tag{21}$$

where the range has the same meaning as above.

We get for the transition width $\Gamma_\eta(\pi^0 2\gamma)$

$$\Gamma_\eta(\pi^0 2\gamma) = aF = 9.446\times 10^5 F \text{ eV} \tag{22}$$

with F given by

$$F \equiv \sum_{i,j} f_i f_j I_{ij} = f_\rho^2 I_{\rho\rho} + f_\omega^2 I_{\omega\omega} + 2 f_\rho f_\omega I_{\rho\omega}$$
$$= (0.363)^2\alpha^2[1.83 + 2(1.67) + 1.52]\times 10^{-3}$$
$$= 4.667\times 10^{-8}. \tag{23}$$

Thus

$$\Gamma_\eta(\pi^0 2\gamma) = 0.447 \text{ eV} \quad (0.346\text{–}0.574 \text{ eV}). \tag{24}$$

The upper and lower value in the range indicate as before the highest and lowest value compatible with experimental data for $\Gamma_\omega(\pi^0\gamma)$. Using the same parameters, the ρ-dominant model gives for the $\Gamma_\eta(\pi^+\pi^-\gamma)$ width[19,20]

$$\Gamma_\eta(\pi^+\pi^-\gamma) = \left(\frac{f_{\eta\rho\gamma}^2}{4\pi}\right)\left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)\left(\frac{\pi}{4}\right)(4.81\times 10^{-5})\frac{m_\eta^3}{m_\pi^2}\left(1 - \frac{4m_\pi^2}{m_\eta^2}\right)^3$$
$$= 137.0 \text{ eV} \quad (108\text{–}156 \text{ eV}), \tag{25}$$

with $(f_{\rho\pi\pi}^2/4\pi) = 2.51$ from the known $\Gamma_\rho(\pi^+\pi^-)$ width.[6] Thus we obtain

$$\Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma) = 0.326\times 10^{-2}$$
$$(0.315\times 10^{-2}\text{–}0.367\times 10^{-2}). \tag{26}$$

Thus the estimate (26) in the vector-meson-dominant model using the values of coupling constants based on the static-$SU(6)$ symmetry is in sharp disagreement with the experimental result by Di Giugno *et al.*[5] or the compilation by Rosenfeld *et al.*,[6] but is compatible with Wahling, Shibata, and Mannelli's experimental result,[5] which only gives an upper limit.

We have also obtained

$$\Gamma_\eta(\text{total}) = 2490 \text{ eV} \quad (1960\text{–}2820 \text{ eV}) \tag{27}$$

by the use of (25) and the known decay fraction for the $\eta \to \pi^+\pi^-\gamma$ mode[6] (5.5%). Equation (27) is not a new determination of the total η-decay width. It is essentially the same as in Ref. 20. The small discrepancy is due to the different formula used to compute the partial $\Gamma_\eta(\pi^+\pi^-\gamma)$ width, and to the newer values of $\Gamma_\rho(\pi^+\pi^-)$ and m_ρ.[6]

We have also made a study of the dependence of $\Gamma_\eta(\pi^0 2\gamma)$ and $\Gamma_\eta(\text{total})$ on the mixing angle and on the uncertainties in the $\Gamma_\phi(\pi^0\gamma)$ and $\Gamma_\omega(\pi^0\gamma)$ widths, to see if we can obtain an enhancement of $\Gamma_\eta(\pi^0 2\gamma)$ and a reduction of $\Gamma_\eta(\text{total})$ by breaking the static-$SU(6)$ relation. This is discussed in the next section.

D. Dependence of Coupling Constants and Decay Widths on θ, $\Gamma_\omega(\pi^0\gamma)$, and $\Gamma_\phi(\pi^0\gamma)$

From (18b) we obtain

$$(f/g) = \tan\theta + (f_{\phi\pi^0\gamma}\tan\theta)/(\sqrt{3} f\cos\theta - f_{\phi\pi^0\gamma}), \tag{28}$$

where the second term vanishes in the $SU(6)$ limit. Then (18a) gives, by use of (28),

$$f = (f_{\omega\pi^0\gamma} + f_{\phi\pi^0\gamma}\cot\theta)/\sqrt{3}(\sin\theta + \cos\theta\cot\theta), \tag{29}$$

which in the static-$SU(6)$ limit becomes

$$f = f_{\omega\pi^0\gamma}/\sqrt{3}(\sin\theta + \cos\theta\cot\theta). \tag{29'}$$

Thus (29′) can be used to study the dependence of the coupling constants and widths on the input parameters.

We again use as input for $f_{\omega\pi^0\gamma}$ the most likely value as well as the maximum and minimum values compatible with the experimental data available on the $\Gamma_\omega(\pi^0\gamma)$ partial width.

One sees from formula (29) that the effect of nonvanishing $\Gamma_\phi(\pi^0\gamma)$ will be small as long as

$$(f_{\phi\pi^0\gamma}^2/4\pi) \ll (\tan^2\theta)(f_{\omega\pi^0\gamma}^2/4\pi). \tag{30}$$

If, for the sake of an estimate, we take $f_{\omega\pi^0\gamma}^2/4\pi = 0.88\times 10^{-3}$, the lowest experimental value for $\Gamma_\omega(\pi^0\gamma)$, and require, for instance,

$$f_{\phi\pi^0\gamma}^2/4\pi \leq 0.1(f_{\omega\pi^0\gamma}^2/4\pi)\tan^2\theta = 10^{-4}\tan^2\theta, \tag{30'}$$

then we can estimate very conservatively the highest $\Gamma_\phi(\pi^0\gamma)$ that will leave our conclusion essentially unchanged.

One finds that the inequality (30) is practically satisfied for all values of the mixing angle, but especially well for large angles in the sense that the $\Gamma_\phi(\pi^0\gamma)_{\max}$ obtained by the use of Eq. (30′) is larger than the maximum experimental $\Gamma_\phi(\pi^0\gamma) = 0.008$ MeV. $\Gamma_\phi(\pi^0\gamma) = 0.008$ MeV corresponds indeed to a very small strength $f_{\phi\pi^0\alpha}^2/4\pi = 4.76\times 10^{-4}\alpha = 3.48\times 10^{-6}$. Estimates of the

[19] This formula is obtained by use of the intermediate ρ vector-meson diagram. It gives a value for the width which is 1.37 times larger than that of L. M. Brown and P. Singer [Phys. Rev. Letters **8**, 460 (1962); **10**, 424 (1966)] or from formula (2) of F. A. Berends and P. Singer, Ref. 20.

[20] F. A. Berends and P. Singer, Phys. Letters **19**, 249 (1965); **19**, 616 (1965).

same order of magnitude are also obtained with a related model. (See for instance J. Yellin, Ref. 3.)

To test further the effect of the assumption $\Gamma_\phi(\pi^0\gamma) \sim 0$ we successively take as input for $f_{\phi\pi^0\gamma}$ the values 0, 8.2×10^{-3}, and 1.57×10^{-2}. The first of these values is the static-$SU(6)$ limit, and corresponds to the most likely experimental situation.[18] The second corresponds to $\Gamma_\phi(\pi^0\gamma)=3\times0.008$ MeV, which is considered to be an upper estimate for this width.[18] The third corresponds to $\Gamma_\phi(\pi^0\gamma)=10\times0.008$ MeV and takes care of a catastrophic situation.

The mixing angle has been varied up to a value of 50°; the criterion is to use data compatible with the different measurements of the $\Gamma_\phi(K\bar{K})$ width.[21]

The results are given in Figs. 3, 4, and 5.

It appears that the assumption $\Gamma_\phi(\pi^0\gamma)=0$ is quite satisfactory, in the sense that the results are practically unaffected by it. We have also obtained the series of results for the minimum and maximum values of $\Gamma_\omega(\pi^0\gamma)$ compatible with experiments; they are not given here, as they do not differ significantly from the rest.

E. Discussion of the Results

It is not possible to explain the $\Gamma_\eta(\pi^02\gamma)$ in the intermediate-vector-meson model with either the static-$SU(6)$ prediction or just the hypothesis of $SU(3)$ plus ω-ϕ mixing. One obtains a $\Gamma_\eta(\pi^02\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$ of the order of 10^{-2}, compared with an experimental result[5,6] of the order of 1. One sees that an enhancement of the $\pi^02\gamma$ width up to a $\Gamma_\eta(\pi^02\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$ of the order 10^{-1} is obtained with smaller mixing angles; this enhancement, however, is still smaller than needed for agreement with Di Giugno *et al.*,[5,6] or even to get the upper limit of Shibata *et al.*[5] If one interprets both experiments at least as a trend toward higher $\Gamma_\eta(\pi^02\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma)$, one would favor a smaller mixing angle. Furthermore, a smaller angle might be favored for the following reasons:

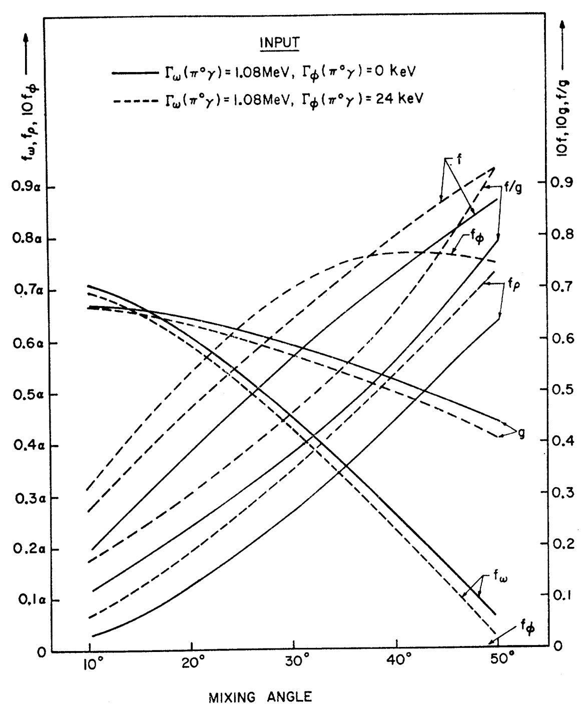

FIG. 3. The figure shows the dependence of some important variables defined in the text on the mixing angle and the $\Gamma_\phi(\pi^0\gamma)$ width [in the vector-meson-dominant model with $SU(3)$]. The static-$SU(6)$ values correspond to $\Gamma_\phi(\pi^0\gamma)=0$ and $\theta=35°\ 18'$. One sees that the effect of the mixing angle is substantial, while the variation of the width $\Gamma_\phi(\pi^0\gamma)$ has negligible effects.

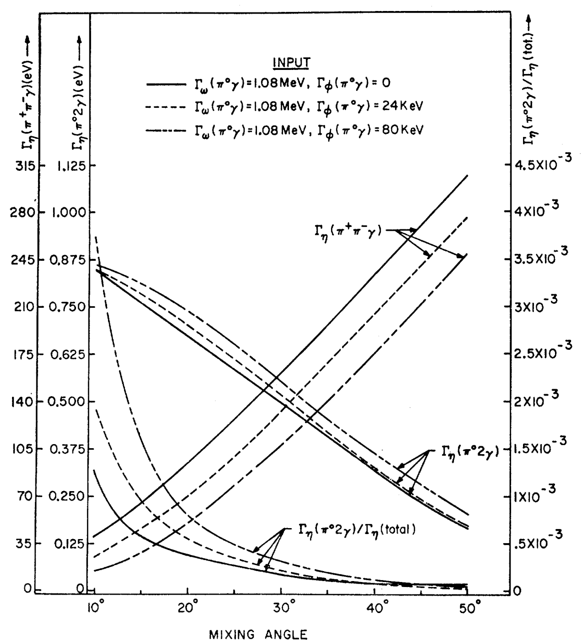

FIG. 4. The figure shows that the combined effect of mixing-angle and $\Gamma_\phi(\pi^0\gamma)$ changes may lead to an order-of-magnitude increase of the $SU(6)$ value of the $\eta\rightarrow\pi^02\gamma$ branching ratio.

(a) As shown in (27), the value of Γ_η(total) based on the values of coupling constants given by the static-$SU(6)$ theory is about an order of magnitude larger[20] than the one which will be obtained if we use the $SU(3)$ relation[22] between $\Gamma_{\pi^0}(2\gamma)$ and $\Gamma_{\eta^0}(2\gamma)$ and experimental η-decay branching ratio. A smaller mixing angle tends to narrow this discrepancy. This is even more so if we take the lower estimates[23] for the $\Gamma_\omega(\pi^0\gamma)$. A discrepancy of the same order of magnitude also appears in the ratio $\Gamma_\omega(\pi^+\pi^-\pi^0)/\Gamma_{\pi^0}(2\gamma)$ in an updated version of the

[21] T. Massam and A. Zichichi, Ref. 18.

[22] N. Cabibbo and R. Gatto, Nuovo Cimento **21**, 872 (1961); S. Okubo, Phys. Letters **4**, 15 (1963).

[23] About 0.9 MeV, see J. S. Lindsey and G. A. Smith, Ref. 22, and Ref. 5 there. There appears to be a trend toward a smaller $\Gamma_\omega(\pi^0\gamma)$, as shown also from a comparison of the latest with the previous Rosenfeld data (Ref. 6).

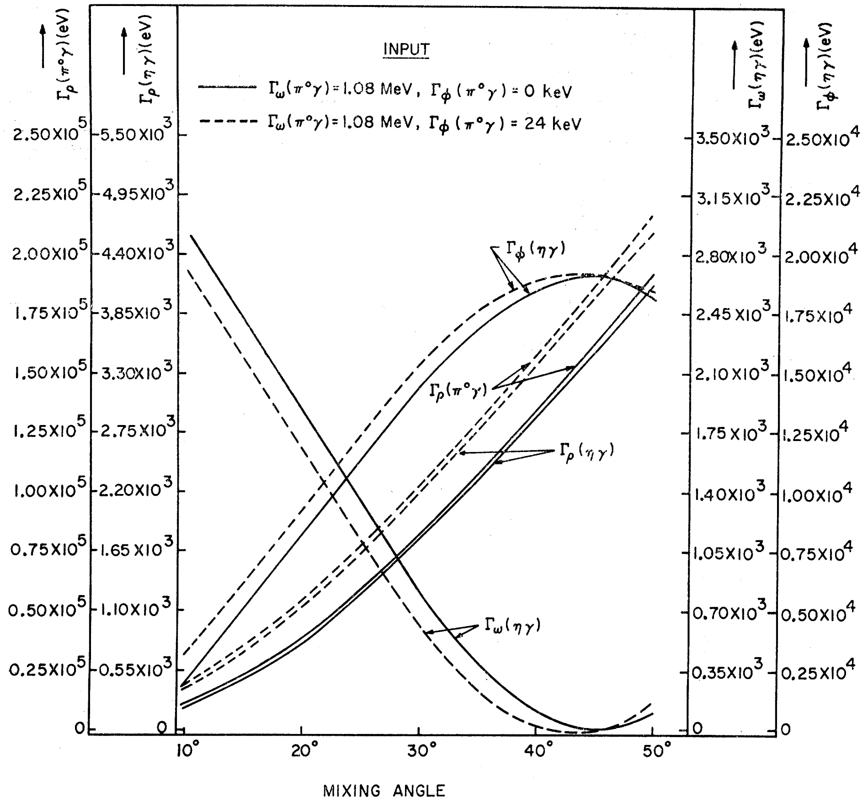

FIG. 5. The figure shows that several widths and their ratios depend strongly on the mixing angle. They could, if measured, be used to determine the mixing angle.

vector-meson-dominant model.[24] Both of them are related to the large experimental $\omega \to \pi + \gamma$ width.

(b) A smaller angle would further reduce (see Fig. 3) the $\Gamma_\rho(\pi^0\gamma)$ width, in qualitative agreement with the recent estimates[25] based on photoproduction with a very small $f_{\rho\pi\gamma}$.

On the other hand, a lower mixing angle, for instance $\theta = 23°$, would give higher values for the $\Gamma_\omega(\eta\gamma)$ width. Larger angles, close to Okubo's value $\theta \simeq 38°$, would require a very small $\Gamma_\omega(\eta\gamma)/\Gamma_\rho(\pi^0\gamma)$, as indicated in Fig. 5.

As soon as new data on $\Gamma_\eta(\pi^0 2\gamma)$ and at least *one* more width [such as $\Gamma_\rho(\pi^0\gamma)$, $\Gamma_\omega(\eta\gamma)$, or $\Gamma_\phi(\bar{K},K)$ for instance] become available, it will be possible to test more definitely the intermediate-vector model and the other assumptions we have used (besides, of course, the already mentioned test based on the pion energy spectrum). We might add the following comments about the $SU(3)$ symmetry breaking. We have assumed the $SU(3)$-symmetric coupling constants and only considered the violation due to ω-ϕ mixing. This may not be justified, especially when photons are involved. It is not inconceivable that the inclusion of this symmetry-breaking effect shifts the observed mixing angle to a smaller value than the static-$SU(6)$ value or the one obtained from the vector-meson mass splitting.

Finally we mention the possibility of an enhancement by η-X^0 mixing.[26] However, if one uses the quadratic mass formula for mesons one gets a small value of $\pm 10°$ for the mixing angle. (A much larger angle is obtained from the linear mass formula.)

It is expected that a detailed calculation including η-X^0 mixing will not change the order of magnitude of the $\Gamma_\eta(\pi^0 2\gamma)$. It might, however, still be of interest because it might shed light on the sign of the η-X^0 mixing angle and on the choice between linear and quadratic mass formulas.[27]

III. INTERMEDIATE-SCALAR-MESON MODEL

A. Introduction

The branching ratio $\Gamma_\eta(\pi^0 2\gamma)/\Gamma_\eta(\pi^+\pi^-\gamma) \simeq 1$ which appears in the table of Rosenfeld *et al.*[6] seems to be clearly out of reach in the vector-meson-dominant

[24] If we update the calculations of Ref. 3 with the parameters given by the static $SU(6)$ [using $\Gamma_{\pi^0}(2\gamma) \simeq 7.5$ eV] we obtain a value of $\Gamma_\omega(\pi^+\pi^-\pi^0)$ which is an order of magnitude smaller than the experimental value.

[25] A. Donnachie and G. Shaw, Ann. Phys. (N.Y.) **37**, 333 (1966). On the other hand, the most probable $\Gamma_\rho(\pi^0\gamma)$ width of 0.7 MeV given by F. R. Hudson *et al.* [Phys. Letters **20**, 91 (1966)] is more than five times larger than even the $SU(6)$ value. See also G. Fidecaro *et al.*, Phys. Letters **23**, 163 (1966).

[26] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); **38**, 1945 (1965).

[27] A. J. Macfarlane and R. M. Socolow, Phys. Rev. **144**, 1194 (1966).

model. However, a branching ratio of the order $\simeq 10^{-1}$ may not be inconsistent with this model, in view of the theoretical uncertainties involved. It is our feeling that this margin is at present too narrow to definitely reject the model, and that more precise experimental determinations of the branching ratio and measurements of the pion energy spectrum are desirable.

Pending a clarification of the experimental situation, it seems worthwhile to look into other possibile mechanisms for the $\eta \to \pi^0 2\gamma$ decay.

We study here the effect of the still hypothetical isovector normal scalar meson ϵ ($\epsilon^+, \epsilon^0, \epsilon^-$). This scalar meson would be assigned to be the $I=1$ member of the scalar $SU(3)$ octet. We propose the following intermediary role of the ϵ meson:

$$\eta^0 \to \pi^0 + \epsilon^0 \to \pi^0 + (\gamma+\gamma).$$

It may be mentioned that this mechanism will be quite natural if the tadpole[12] model of the electromagnetic mass difference of hadrons is correct.

In the following we first study the $\eta \to \pi^0 2\gamma$ decay in this model. Then we study other modes of decay of the ϵ^0 particle and discuss the consistency of the model by applying it to the $\eta \to 3\pi^0$ mode.

B. The $\Gamma_\eta(\pi^0 2\gamma)$ and the Energy Spectrum

We write the strong $\eta^0 \epsilon^0 \pi$ interaction as

$$G\eta^0 \epsilon^0 \pi \tag{31}$$

and the effective $\epsilon^0 \to 2\gamma$ interaction as

$$g\epsilon^0 F_{\mu\nu} F_{\mu\nu}'. \tag{32}$$

The matrix element for the decay $\eta \to \pi^0 2\gamma$ in this model differs from the simplest matrix element given by (1) only by the presence of the propagator of the ϵ meson. We then obtain the pion energy spectrum and the width of the $\eta^0 \to \pi^0 2\gamma$ decay in this model, which are given by

$$\Gamma_\eta(\pi^0 2\gamma) = \frac{1}{16}\left(\frac{1}{\pi}\right)^3 G^2 g^2 \frac{1}{m_\eta} \int_{m_\pi}^{(m_\eta^2+m_\pi^2)/2m_\eta} dE \times \frac{(E^2-m_\pi^2)^{1/2}(m_\eta^2+m_\pi^2-2m_\eta E)^2}{[2m_\eta E+(m_\epsilon^2-m_\eta^2-m_\pi^2)]^2}. \tag{33}$$

The denominator of the integral in Eq. (33) comes from the propagator of the ϵ^0 meson. Thus in order to get a numerical value for the width and the energy spectrum we have to guess the value of m_ϵ.

We first note that $M_\epsilon > m_\eta - m_\pi$. (Otherwise the $\eta \to \epsilon + \pi$ decay will become a strong decay, so that the observed $\eta \to 2\gamma$ decay could hardly compete with it.[28]) Furthermore, if $m_\eta - m_\pi < m_\epsilon < m_K$, it would be hard to explain the presently favored sign of the K_1^0-K_2^0 mass difference ($m_{K_2^0} > m_{K_1^0}$). Moreover, the K_{e4} decay could then take place as $K \to \epsilon + e + \nu \to (\pi+\pi)+e+\nu$ (the last step follows because the ϵ can decay electromagnetically into two pions). The occurrence of this process does not seem to be consistent with present knowledge about the K_{e4} decay. Therefore, it is probably necessary to assume $m_\epsilon \gtrsim m_K$. We will take $500 < m_\epsilon < 750$ MeV as the ϵ-mass range of interest in the following.

In computing the integral in (33), we first approximate the denominator as constant. Then the energy spectrum of the pion has a broad maximum at $E_\pi \simeq m_\eta/3$. (The energy spectrum is given in Fig. 2.) We again stress that, under this approximation, the model gives the same matrix elements as given by (1) and this pion spectrum is very different from that obtained in the vector-meson-dominant model. (See also footnote 32.) We then obtain

$$\Gamma_\eta(\pi^0 2\gamma) = \frac{1}{16}\left(\frac{1}{\pi}\right)^3 G^2 g^2 m_\eta^{-1} [m_\epsilon^2 - (m_\eta^2/3) - m_\pi^2]^{-2} \times I(\eta \to \pi^0 2\gamma), \tag{34}$$

where

$$I(\eta \to \pi^0 2\gamma) = \int_{m_\pi}^{(m_\eta^2+m_\pi^2)/2m_\eta} dE\, [(E^2-m_\pi^2)^{1/2} \times (m_\eta^2+m_\pi^2-2m_\eta E)^2] \simeq 5.3\times 10^{-3} m_\eta^6 \tag{35a}$$

is the phase-space integral based on the matrix element (1). In the same approximation of taking the ϵ-meson propagator as a constant, the spectrum of the directly emitted photon of the $\eta \to \pi^0 2\gamma$ decay is given by

$$\int dk (m_\eta^2 - m_\pi^2 - 2m_\eta k)^3 \frac{k^3}{(m_K - 2k)^3}. \tag{35b}$$

C. Consistency of the Model

In order to get some idea of the branching ratio of the $\eta^0 \to \pi^0 2\gamma$ decay in this model, we next compute the rate of $\eta \to 3\pi^0$ decay by assuming that its amplitude is also dominated by an intermediate ϵ^0 meson: $\eta^0 \to \epsilon^0 + \pi^0 \to (\pi^0+\pi^0)+\pi^0$. We write the effective $\epsilon^0 \to \pi^0 + \pi^0$ interaction (which is, of course, G violating) as

$$g'\epsilon^0 \pi^0 \pi^0. \tag{36}$$

Then the rate of $\eta^0 \to 3\pi^0$ due to this mechanism in the rest frame of the η^0 is given by

$$\Gamma_\eta(3\pi^0) \simeq \left(\frac{1}{2\pi}\right)^5 (\tfrac{3}{8}) G^2 g'^2 m_\eta^{-1} [m_\epsilon^2 - (m_\eta^2/3) - m_\pi^2]^{-2} \times I(\eta \to 3\pi^0), \tag{37}$$

where

$$I(\eta \to 3\pi^0) = \int \frac{d^3p}{E_{\pi^0}(p)} \frac{d^3q}{E_{\pi^0}(q)} \frac{d^3r}{E_{\pi^0}(r)} \delta^4(k-p-q-r) \simeq 8\pi^2 m_\eta^2 (1.5\times 10^{-2}). \tag{38}$$

[28] However, in the literature existence of such a low-mass ϵ meson ($m_\epsilon \simeq 283$ MeV) had been proposed. R. E. Marshak, V. S. Mathur, and L. K. Pandit, Phys. Letters 21, 563 (1966).

In evaluating (37) we have made an expansion of the propagator of ϵ around the symmetry point $[m_\epsilon^2-(m_\eta^2/3)-m_\pi^2]$ and kept only the leading term. (The contribution of the linear term in this expansion vanishes owing to the symmetry properties of the $3\pi^0$ states.)

On the other hand, from (32) and (36) we also obtain

$$\Gamma_{\epsilon^0}(2\gamma)=(g^2/4\pi)m_\epsilon^3 \tag{39}$$

and

$$\Gamma_\epsilon(2\pi^0)=(g'^2/4\pi)(2m_\epsilon)^{-1}[1-(2m_\pi/m_\epsilon)^2]^{1/2}. \tag{40}$$

By using the results (34), (37), (39), and (40), we can obtain a relation between the experimentally known branching ratio $R\equiv\Gamma_\eta(\pi^02\gamma)/\Gamma_\eta(3\pi^0)$ and the unknown ratio $R'\equiv\Gamma_{\epsilon^0}(2\gamma)/\Gamma_{\epsilon^0}(2\pi^0)$, namely,

$$R'\simeq1.25\times10^{-1}(m_\eta/m_\epsilon)^4R. \tag{41}$$

If, for example, we take[6] $R\simeq\frac{2}{3}$, we obtain from (41) for the interesting range of ϵ mass ($500<m_\epsilon<750$ MeV) the relation

$$3.4<R'<15. \tag{42}$$

From the proportionality between R' and R (at constant m_ϵ) it follows that the allowed range for R' would be displaced toward smaller values if R were smaller. If $m_\epsilon>750$ MeV we need $R'\gg1$. Of course, the assumption that the ϵ^0-meson contribution alone will dominate the $\eta^0\to3\pi^0$ amplitude cannot be completely justified. However, the above estimate can at least be taken as an indication that the $\Gamma_\epsilon(2\gamma)$ has to be comparable to or a little larger than the $\Gamma_\epsilon(2\pi^0)$ in order to explain the surprisingly large branching ratio of the $\eta\to\pi^02\gamma$ decay. Therefore, if $m_\epsilon<m_\eta+m_\pi\simeq690$ MeV, we expect[29]

$$\Gamma_{\epsilon^0}(2\gamma)\gtrsim\Gamma_{\epsilon^0}(2\pi^0 \text{ or } \pi^+\pi^-). \tag{43}$$

In the following we give some estimates which indicate that the model is not so unrealistic. Suppose that $\epsilon^+\to\pi^++\pi^0$ decay occurs through the electromagnetic virtual transition $\eta^0\to\pi^0$:

$$\epsilon^+\to\eta^0+\pi^+\to\pi^0+\pi^+.$$

We then obtain

$$\Gamma_\epsilon(\pi^+\pi^0)=(16\pi m_\epsilon)^{-1}G^2[f_{\eta^0\pi^0}/(m_\eta^2-m_\pi^2)]^2\times[1-(2m_\pi/m_\epsilon)^2]^{1/2}, \tag{44}$$

where we have denoted the strength of the η-π^0 vertex by $f_{\eta\pi^0}$. If we use the value of $f_{\eta\pi^0}\simeq-(54\text{ MeV})^2$ obtained from the electromagnetic mass difference of the pseudoscalar mesons,[7] we obtain[30]

$$\Gamma_\epsilon(\pi^+\pi^0)\simeq3.4\text{ keV}. \tag{45}$$

We remark here that if we use this model to evaluate the effective coupling-constant g' of the $\epsilon^0\to2\pi^0$ vertex, we obtain from (37) and (38)

$$\Gamma_\eta(3\pi^0)\simeq163\text{ eV}. \tag{46}$$

If we use the $SU(3)$ symmetry to compute the $\eta\to2\gamma$ rate from the $\pi^0\to2\gamma$ rate,[6] we obtain

$$\Gamma_\eta(2\gamma)\simeq162\text{ eV}. \tag{47}$$

Then using the experimental η^0 branching ratio, we obtain $\Gamma_\eta(3\pi^0)\simeq102$ eV compared with (46). Therefore, the present model does not seem to be so unrealistic. From (45), we expect

$$\Gamma_{\epsilon^0}(2\gamma)\simeq1\text{–}10\text{ keV}. \tag{48}$$

If we compare this with (47), the width of $\epsilon^0\to2\gamma$ decay given by (48) seems reasonable.

We add some remarks on the $\epsilon^0\to\pi^++\pi^-+\gamma$ decay, which might compete with the $\epsilon^0\to2\gamma$ decay. We may use the ρ-dominant model, as in the case $\Gamma_\omega(\pi^+\pi^-\pi^0)/\Gamma_\omega(\pi^0\gamma)$. We write the $\epsilon^0\rho^0\gamma$ and $\rho^0\gamma$ vertex as

$$2f_{\epsilon\rho\gamma}m_\rho^{-1}\rho_{\mu\nu}F_{\mu\nu}\epsilon^0 \tag{49}$$

and

$$f_{\rho\gamma}m_\rho^2\rho_\mu e_\mu, \tag{50}$$

where, for instance, $\rho_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu$, and ρ_μ and e_μ are the polarization vectors of the ρ and photon, respectively. The π^+ and the photon energy spectrum and the rate are given by

$$\Gamma_\epsilon(\pi^+\pi^-\gamma)=m_\epsilon\left(\frac{f_{\epsilon\rho\gamma}^2}{4\pi}\right)\left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)(2\pi)^{-2}F,\quad F=\int_{m_\pi}^{m_\epsilon/2}dE\int_{L+}^{L-}X(E_\pi,k)dk, \tag{51}$$

where

$$L_\mp=(m_\epsilon^2-2m_\epsilon E_\pi)/2[m_\epsilon-E_\pi\mp(E_\pi^2-m_\pi^2)^{1/2}],$$
$$X(E_\pi,k)=k^2(E_\pi^2-m_\pi^2)D^2(\rho)R,$$
$$D(\rho)=(m_\rho^2-m_\epsilon^2+2m_\epsilon k)^{-1},$$

and

$$R=1-\frac{[m_\epsilon^2-2m_\epsilon(E_\pi+k)+2E_\pi k]^2}{4k^2(E^2-m_\pi^2)}\simeq1.$$

The pion energy spectrum behaves in a similar way to that of $\eta\to\pi^02\gamma$ decay in the ϵ^0-meson model, given by Fig. 2. We obtain for the branching ratio

$$R_\epsilon\equiv\Gamma_{\epsilon^0}(\pi^+\pi^-\gamma)/\Gamma_{\epsilon^0}(2\gamma)=\frac{F}{\alpha(2\pi)^2}\left(\frac{m_\rho}{m_\epsilon}\right)^2\left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right), \tag{52}$$

where α is the fine-structure constant. With the relevant mass values m_ϵ, viz. 500, 600, and 700 MeV, the numerical values of F give for R_ϵ the values 1.4×10^{-3}, 7.3×10^{-3}, and 2.8×10^{-2}, respectively. Therefore, if

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[29] The $\epsilon^0\to2\gamma$ decay will involve a higher momentum barrier effect than the $\epsilon^0\to2\pi$ decay, so that we should normally expect that $\Gamma_\epsilon(2\pi)\gg\Gamma_\epsilon(2\gamma)$, contrary to (44). (We note that the $\eta\to3\pi$ decay can complete with the $\eta\to2\gamma$ decay.)

[30] We have used the value of G normalized in such a way that, using the $SU(3)$-symmetric coupling constants, the $I=\frac{1}{2}$ scalar meson κ ($m_\kappa=725$), which belongs to the same octet as the ϵ meson, has the width $\Gamma_\kappa(\text{all})\simeq25$ MeV. See D. Loebbaka, S. Oneda, and J. C. Pati, Phys. Rev. **144**, 1280 (1966).

$m_\epsilon < 700$ MeV, we expect, roughly speaking,[31]

$$\Gamma_{\epsilon^0}(2\gamma) > \Gamma_{\epsilon^0}(\pi^+\pi^-,\pi^0\pi^0) > \Gamma_{\epsilon^0}(\pi^+\pi^-\gamma)$$

and

$$\Gamma_{\epsilon^+}(\pi^+\pi^0) > \Gamma_{\epsilon^+}(\pi^+\pi^0\gamma).$$

If $m_\epsilon > 700$ MeV, the strong decay $\epsilon \to \eta + \pi$ becomes possible.

IV. CONCLUDING REMARKS

In conclusion, if the presently reported large branching ratio of the $\eta^0 \to \pi^0 + 2\gamma$ decay is confirmed, the vector-meson-dominant model will not be able to explain it. An unambiguous check will be provided by the study of the pion energy spectrum.[32] In this case the ϵ model discussed in Sec. III seems promising, although it is not entirely necessary to postulate the existence of the ϵ meson. That is, eliminating the ϵ meson in the present model, one might simply postulate the existence of an effective interaction such as $\eta^0\epsilon^0\rho_{\mu\nu}\omega_{\mu\nu}$ which can lead to the $\eta \to \pi + 2\gamma$ decay. Nevertheless, a search for the ϵ^0 meson seems interesting, particularly from the point of view of the tadpole model of the electromagnetic mass differences of hadrons. If the presently accepted branching ratio of $\eta^0 \to \pi^0 + 2\gamma$ decay is correct, the study of the similar nonleptonic weak decays $K^\pm \to \pi^\pm + 2\gamma$ and $K_2^0 \to \pi^0 + 2\gamma$ will also be very interesting. The branching ratios of these decays may not be as small as we usually expect.

[31] This is in qualitative agreement with another estimate. See D. Loebbaka and J. C. Pati, Phys. Rev. Letters **14**, 929 (1965).

[32] In Fig. 2, the pion spectrum in the ϵ-meson model, Eq. (33), is drawn by approximating the denominator of Eq. (33) as constant. The higher mass of ϵ tends to move the maximum point of the spectrum to the right.

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Asymptotic Behavior of Form Factors*

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The asymptotic behavior of form factors is examined in the presence of inelastic states. It is shown how the form factors can vanish asymptotically when the density of inelastic states assumes a sufficiently rapid power behavior in energy. Three distinct, soluble models for scattering are examined, and the relationship among the asymptotic behavior of form factors, inelasticities, and diffraction scattering amplitudes are discussed. It is pointed out that the scattering amplitudes need not approach their Born amplitudes asymptotically when the density of inelastic states increases sufficiently rapidly with energy.

I. INTRODUCTION

LET us consider, for the sake of definiteness, the $(\pi\pi \to \pi\pi)$ scattering amplitude $(\pi\pi|T|\pi\pi)$, which can be expressed in the N/D form. The pion form factor $F_\pi(s) = \langle\pi\pi|T|\sigma\rangle$, where σ is some one-particle state, e.g., the ρ or the photon, is essentially the inverse of the D function: $F_\pi(s) = C/D(s)$, where C is an arbitrary constant. Because $D(s)$ can be made to go to a constant at infinity, the form factor $F_\pi(s)$ does not vanish asymptotically. However, it appears likely that the nonvanishing behavior of the form factors is connected with the assumption of elastic unitarity on the $\pi\pi$ scattering amplitude, and it is interesting to investigate the behavior of the form factors in the asymptotic region if one allows for the existence of inelastic channels. Recently,[1] the asymptotic behavior of phase shifts (real and imaginary) was investigated under the assumption that an infinite number of two-body channels become available for scattering as we go into the asymptotic region.[2] We shall examine the asymptotic behavior of the form factors under similar assumptions. Ours is a multichannel formalism, and we work with models which we can solve in a closed form so that we can obtain the scattering amplitudes and form factors explicity. These forms are then examined when certain sums over the density of channels increase rapidly with energy. In this limit, the form factors are then shown to go to zero, and the asymptotic limit of the various measures of inelasticities is obtained. It is found that the various measures of inelasticities do not necessarily reach the limiting values dictated by one's naive conception about maximum inelasticity or absorption.[3] For example, in all the models examined we find that $\eta(\infty) = 1$, as previously noted,[1] and the scattering amplitudes need not become purely imaginary. Recently (see Ref. 4), the asymptotic behavior of the form factors was examined according to the same general philosophy. In this work,

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[1] R. E. Kreps and Pran Nath, Phys. Rev. **148**, 1436 (1966).

[2] The number $N(s)$ of inelastic channels increases very rapidly with the number of allowed fundamental thresholds; e.g., if we assume L fundamental thresholds like $\pi\pi$, $K\bar{K}$, $N\bar{N}$, etc., then $N(s) \sim s^{L/2}$ as the energy becomes large.

[3] One might assume that the limits $|1+2iT_{11}| = 0$, $\mathrm{Re}T_{11}/\mathrm{Im}T_{11} = 0$, and $\sum_{n\neq1}|T_{1n}|^2/|T_{11}|^2 \to \infty$ are necessarily reached simultaneously as $s \to \infty$, where T_{ij} is the ij scattering amplitude.

[4] D. Beder, Phys. Rev. **149**, 1208 (1966).