Eq. (A4). We take

$$\psi(u,s) = \sum_{m=0}^{n} s^{m} M_{m}(u) + \phi(u,s),$$

where

$$\phi(u,s) \xrightarrow[s \to \infty]{} 0$$

Then  $\phi$  can be treated separately and from Eq. (A8) gives a vanishing contribution in the limit of large  $\Lambda$ . We further require that  $\psi(u,s)$  be symmetric in u so that the term in I which goes as  $\Lambda^{2n}$  vanishes as in

Eq. (A9). The term which goes as  $\Lambda^{2n-2}$  receives contributions from both  $M_n$  and  $M_{n-1}$ . Now the point which Schroer and Stichel make is that there exist a class of  $\psi$ 's for which these two contributions cancel one another so that there is no  $\Lambda^{2n-2}$  term. In fact, this process is continued until all divergent terms are canceled. This then leaves a finite ETC which in general is a polynomial in  $|\mathbf{q}|$  of order *n*. In the Appendix we considered  $\psi$ 's which behaved only as  $s^n M(u)$ and thus did not consider the possibility of this cancellation.

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# Possible $|\Delta T| = \frac{3}{2}$ Amplitudes in $\Lambda^0$ Decay\*

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Using  $K_2^0 \to \pi\pi$  and  $K^+ \to \pi\pi$  as a guide, estimates are made on the magnitudes of  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitudes and possible CP violation in the decay  $\Lambda^0 \rightarrow n\pi^0$ . It is found that, consistent with current experimental data on  $\Lambda^0$  decay, a small  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitude could be reflected in an observable departure of  $\alpha_0/\alpha_-$  from unity, and that large CP-violating phases could be present.

 $\mathbf{R}^{\mathrm{ECENT}}$  measurements<sup>1</sup> of the rate  $K_{2^{0}} \rightarrow \pi^{0}\pi^{0}$  indicate that  $|\Delta \mathbf{T}| \geq \frac{3}{2}$  amplitudes are responsible for the CP violation exhibited by the decays  $K_{2^{0}} \rightarrow \pi \pi$ .<sup>2,3</sup> The only other known  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitude is found in  $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}$ . From this rate a ratio of S-wave  $|\Delta \mathbf{T}| = \frac{3}{2}$ and  $|\Delta \mathbf{T}| = \frac{1}{2}$  amplitudes can be calculated:  $|A_2/A_0|$  $\approx 10\%$ .<sup>4</sup> The *CP*-violating phase angle  $\varphi$  can be estimated by assuming that only the  $|A_2|/|A_0|$  term contributes to the ratio  $\eta_{+-} = \operatorname{amp.}(K_2^0 \to \pi^+ \pi^-)/$ amp. $(K_1^0 \rightarrow \pi^+\pi^-)$ . The resulting equation,  $|\eta_{+-}=1/$  $(\sqrt{10})(|A_2|/|A_0|) \times \sin\varphi, 5$  gives  $\varphi = 57$  mrad for  $\eta_{+-}$ =  $(1.83 \pm .12 \times 10^{-3})$ .<sup>6</sup> A possible SU<sub>3</sub> suppression of the  $K_1^0 \rightarrow \pi\pi$  rate of unknown magnitude would decrease  $|A_2|/|A_0|$  and increase  $\varphi$  correspondingly.<sup>3</sup> An additional uncertainty in the amplitude and phase estimates comes from the possible presence of both  $|\Delta \mathbf{T}| = \frac{3}{2}$  and  $|\Delta \mathbf{T}| = \frac{5}{2}$  terms. Thus it is possible that both the  $|\Delta \mathbf{T}| = \frac{3}{2}$  and  $|\Delta \mathbf{T}| = \frac{5}{2}$  amplitudes are large and have

large phase angles relative to  $|\Delta \mathbf{T}| = \frac{1}{2}$ , but the  $K_{2^{0}} \rightarrow \pi\pi$  rate is suppressed through a cancellation.<sup>7</sup> With these reservations in mind, it is the purpose of this paper to point out that  $|A_2|/|A_0| \approx 10\%$  and  $\varphi \approx 0$ could be reflected in the nonleptonic decay of the  $\Lambda^0$ and could be easily detected. In addition, if large  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitude and phase were allowed, a large *CP* violation in  $\Lambda^0 \rightarrow n\pi^0$  could occur.

The data on nonleptonic decay of the  $\Lambda^0$  are summarized in Table I. Since the  $|\Delta \mathbf{T}| = \frac{1}{2}$  rule requires the same amplitudes for  $\Lambda^0 \rightarrow p\pi^-$  and  $\Lambda^0 \rightarrow n\pi^0$ , this decay is particularly suited to a search for the presence of  $|\Delta T| = \frac{3}{2}$ . The ratio of asymmetry parameters  $\alpha_0/\alpha_-$  is the most sensitive available test. Possible values for  $\alpha_0$  and  $\beta_0$  have been studied using the constraints in Table I except for the ratio  $|P_0|/|S_0|$  and

TABLE I. Experimental data on  $\Lambda^0$  decay.

Mode	Rate	$lpha^{ m a}$	β	P / S
$\pi^- p \pi^0 n$	1 0.48±0.019⁰	$+0.65\pm0.05^{b}$ $+0.70\pm0.20^{d}$	0.10±0.10 <sup>b</sup>	$\begin{array}{c} 0.36{\pm}0.06^{\rm b} \\ 0.39_{-0.12}{}^{+0.26} \end{array}$

<sup>a</sup> The spin parameters are defined in the same way as Samios, Ref. c.
<sup>b</sup> O. Overseth and R. Roth (to be published).
<sup>e</sup> N. P. Samios, in Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory Report ANL No. 7130, 1965, p. 206 (unpublished).
<sup>d</sup> B. Cork, L. Kerth, W. Wenzel, J. Cronin, and R. Cool. Phys, Rev. 120, 1000 (1960).
<sup>e</sup> R. H. Dalitz, in International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory Report No. BNL 837 (C-39), 1963, p. 385 (unpublished).

<sup>7</sup> D. Cline, Nuovo Cimento 48, 566 (1967).

<sup>\*</sup> Work supported in part by the U. S. Atomic Energy Com-mission under Contract No. AT (11-1)-881, COO-881-102. <sup>1</sup> J. M. Gaillard, F. Krienen, W. Galbraith, A. Hussri, M. Jane, N. Lipman, G. Manning, T. Ratcliffe, P. Day, A. Parham, B. Payne, A. Sherwood, H. Faissner, and H. Reither, Phys. Rev. Letters 18, 20 (1967); J. W. Cronin, P. Kunz, W. Risk, and P. Wheeler, *ibid.* 18, 25 (1967). <sup>2</sup> T. Truong, Phys. Rev. Letters 13, 358 (1964). <sup>8</sup> S. Barshay, Phys. Rev. 149, 1229 (1966). <sup>4</sup> G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts), p. 446. <sup>5</sup> See, for example, L. Wolfenstein, Nuovo Cimento 42A, 17 (1966). Our definition of  $A_2$  differs by  $1/\sqrt{5}$ . <sup>6</sup> V. Fitch, in *Proceedings of the XIIIth International Conference* on High Energy Physics, Berkeley, 1966 (University of California

on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967),

the value of  $\alpha_0$ , which were allowed to vary within the errors. The results of this study are summarized in Table II. For fixed  $|P_0|/|S_0|$  ratio,  $\alpha_0$  and  $\beta_0$  satisfy the relation  $\alpha_0^2 + \beta_0^2 = 4(|P_0|/|S_0|)^2/[1+(|P_0|/|S_0|)^2]^2$ , ignoring final-state interactions. If no restrictions are placed on the magnitudes and phases of the  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitudes, then for  $|P_0|/|S_0| = 0.36$ ,  $+0.5 \le \alpha_0 + 0.65$ ; for  $|P_0|/|S_0| = 0.6$ ,  $+0.5 \le \alpha_0 \le +0.85$ ; and for  $|P_0|/|S_0| = 0.25$ ,  $\alpha_0 \approx 0.5$ . Note that  $\alpha_0 > \alpha_-$  is characterized by very large  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitudes. Also large values of  $\beta_0$  are possible, but are accompanied by phases at least an order of magnitude larger than 57 mrad. If the ratio  $|P_0|/|S_0|$  is small (~0.25), then  $\alpha_0/\alpha_-=0.8$ , and the  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitudes are  $\sim 15\%$  or less.

We conclude that in spite of the apparent agreement with the  $|\Delta \mathbf{T}| = \frac{1}{2}$  rule,  $|\Delta \mathbf{T}| = \frac{3}{2}$  components could be present in  $\Lambda^0$  decay within current experimental errors. A measurement of  $\alpha_0/\alpha_-$  to 5% would be sensitive to amplitude admixtures of the same order of magnitude as those observed in  $K^+$  decay. The possibility of a

TABLE II. Limits on  $|\Delta \mathbf{T}| = \frac{3}{2}$  amplitudes and phases for  $\Lambda^{\circ}$  decay. The  $|\Delta \mathbf{T}| = \frac{3}{2}$  and  $|\Delta \mathbf{T}| = \frac{1}{2}$  amplitudes are related to the  $n\pi^{\circ}$  and  $p\pi^{-}$  amplitudes by  $S_0 = \sqrt{2S_3 - S_1}$ ,  $S_- = \sqrt{2S_1 + S_3}$ .  $S_1$  is chosen real >0.  $\theta_3$  is the phase angle (in radians) of  $S_3$  relative to  $S_1$ .  $\varphi_1$  and  $\varphi_3$  are the phase angles of  $P_1$  and  $P_3$  relative to  $S_1$ .  $\beta_-$  is zero,  $\alpha_-$  is 0.65, and  $(|S_0|^2 + |P_0|^2)/(|S_-|^2 + |P_-|^2) = 0.5$  for each case.

$P_0 / S_0 $	$\alpha_0   \alpha_0$	$\beta_0$	$ S_3  /  S_1 $	$ P_{3} / P_{1} $	$\theta_3$	$arphi_1$	$\varphi_3$
0.36	0.65	0	0	0	0	0	0
0.36	0.50	0.4	2.0	1.0	0.8	0	1.2
0.6	0.85	0	2.4	13.7	0	π	0
0.6	0.50	0.74	1.3	0.6	1.0	0	1.9
0.25	0.50	0	0.01	0.15	$\pi$	0	0

large  $\beta_0$  is not ruled out at the present time either by the experimental data or by any reliable connection between *CP*-violating phases in *K* and *Y* decay.<sup>8</sup>

<sup>8</sup> See, however, B. Kenny [Enrico Fermi Institute Report No. EFINS 66-69, 1966 (unpublished)] for a discussion of a model which makes such a connection.

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## Isospin Crossing Matrices\*†

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The rules for expressing the isospin crossing matrix for a two-body scattering process as a 6-J symbol are summarized and derived. It is shown that rules for doing this previously derived in the literature are incomplete, besides being too complicated. The appendices gather together from the widely scattered literature a number of results useful for calculating and checking crossing matrices, not only for isospin but for the higher groups as well.

### I. INTRODUCTION

THE first to point out that the isospin crossing matrix for two-body scattering is proportional to a 6-J symbol was Dyson<sup>1</sup>:

$$\mathbb{C}(I_d, I_c) = (-1)^{I_d + I_c + I_\beta - I_\gamma} N_c \begin{cases} I_\alpha & I_\beta & I_d \\ I_\delta & I_\gamma & I_c \end{cases}.$$
(1.1)

 $N_j \equiv 2I_j + 1$ . The direct and crossed reactions are

$$I_{\alpha}\bar{I}_{\beta} \to I_{d} \to \bar{I}_{\gamma}I_{\delta}, \qquad (1.2a)$$

$$I_{\alpha}I_{\gamma} \to I_{c} \to I_{\beta}I_{\delta}. \tag{1.2b}$$

The  $\mathcal{C}(I_d, I_c)$  are the coefficients which appear on crossing  $A(I_c)$  to  $A(I_d)$ , A(I) being the amplitude for scat-

tering of states having definite total isospin I

$$A(I_d) = \sum_{I_c} \mathfrak{C}(I_d, I_c) A(I_c).$$
(1.3)

Ordinarily we would use the symbol C for a crossing matrix; but here we shall use a script letter because there exists an extensive literature in which the letter C is assigned to a matrix to be introduced at Eqs. (1.5)-(1.6) below.

The phase  $(-1)^{I_d+\cdots}$  given in Eq. (1.1) is not the appropriate one to use under every circumstance. In this introduction the rules governing choice of phase are listed, with a minimum of discussion, for the convenience of persons who wish to use Eq. (1.1) in conjunction with a table of 6-J brackets to compute a given  $\mathcal{C}(I_d, I_c)$ .<sup>2,3</sup> Persons interested in a detailed discussion

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<sup>&</sup>lt;sup>†</sup> Part of this work was done while the author was an Air Force Office of Scientific Research Fellow at CERN, Geneva, Switzerland.

Switzerland. <sup>1</sup> F. J. Dyson, Phys. Rev. 100, 344 (1955). Dyson used Racah coefficients, which are 6-J symbols apart from a phase.

<sup>&</sup>lt;sup>2</sup> M. Rotenberg, R. Bivens, N. Metropolis, and J. K. Wooten, Jr., 3-J and 6-J Symbols (Technology Press, Cambridge, Massachusetts, 1959).

<sup>&</sup>lt;sup>3</sup> Tables of Racah Coefficients, edited by Ishidzu Takehiko (Pan-Pacific Press, Tokyo, 1960).