Equal-Time Commutation Relations of the Isovector Current Densities*

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The one-particle matrix elements of the local equal-time commutators of the isovector currents are derived by applying the Dyson representation to the causal parts of the invariant absorptive parts. Assuming the equal-time commutation relation between the total charge and charge density and also assuming certain asymptotic limits for the Dyson spectral function, we can generate the local equal-time commutation relations between various components of the currents. It is shown that, for a reasonable asymptotic behavior of the spectral function, the charge-space component has no antisymmetric (in isospin) Schwinger terms, but involves two possible q-numer symmetric Schwinger terms, and that the space-space components can have no antisymmetric Schwinger terms. It is pointed out that as the asymptotic behavior becomes worse, we can no longer define the equal-time commutation relations uniquely.

1. INTRODUCTION AND SUMMARY

RECENTLY, much interest has been shown in the so-called Schwinger terms[1] in connection with the validity of certain sum rules derived from local current-current commutation rules. One of the main questions has been whether or not there exist q-number Schwinger terms other than the original Schwinger term which is the vacuum expectation value of the current commutators, and which, therefore, appears in any diagonal matrix element of current commutators as the contribution from the disconnected vacuum-vacuum transitions. (In this paper, we call any derivative of a three-dimensional δ function in the equal-time commutators a Schwinger term.) There have been a few general formulations[2,3] of Schwinger terms without using any specific models, but no concrete answer to the above question has been given from these formulations. Okubo and others[4] used the Jacobi identity to show that the spatial current-current equal-time commutator must have antisymmetric q-number Schwinger terms. However, the validity of the use of the Jacobi identity is doubtful as pointed out by Johnson and Low.[5] The latter authors undertook the investigation of the Schwinger terms using specific field-theoretic models, and their conclusion is that there exist all sorts of q-number Schwinger terms.

The simplest way to obtain the vacuum Schwinger term is to use the spectral representation for the vacuum expectation value of the current commutators, which was the original method Imamura and Goto[6] used to show the discrepancy of the blind application of the canonical commutation relations to the current commutators.[7] The method we use in this paper to analyze the diagonal one-particle matrix element of the isovector current commutators (to be specific, one-fermion matrix element averaged over spin) is a generalization of this procedure, and is based on the application of the Dyson representation[8] of local commutators to certain amplitudes.[9,10] In the case of the conserved isovector currents, their commutator, which is a second rank tensor, is determined in terms of two invariant amplitudes. We first separate out from these invariant amplitudes the noncausal part which does not vanish outside the light cone, so that the remainder is now causal. (Of course the components of the tensor amplitudes themselves are causal.) The separation is not unique, and we do it in such a way that the separated term will give the quark-model equal-time commutators (ETC) obtained using canonical commutation relations. Then, the causal part to which we apply the Dyson representation gives whatever terms are present in the ETC in addition to the canonical terms in the quark model, including possible q-number Schwinger terms.

In deriving the equal-time commutation relations of the current densities, we have used, besides the relativistic invariance, the causality and the conservation of current, two important inputs. One is the ordinary ETC between the *total* isovector charge and the charge density. This means that we have not made any specific assumptions about the Schwinger terms. The other input is the behavior of the spectral function of the Dyson representation at the infinite end of the spectrum, which

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[9] The first use of the Dyson representation to charge algebras was made by B. Schroer and P. Stichel, Comm. Math. Phys. 3, 258 (1966).
[10] Since submitting this paper we have become aware of another paper on this subject by Jean-Loup Gervais and M. Le Bellac [Nuovo Cimento (to be published)]. They do not consider a conserved current and assume that the case of a conserved current can be obtained by a suitable limit at the end of any calculation. They obtain the incorrect conclusion that the quark model ETC cannot be fitted into the framework of invariant amplitudes represented by a Dyson representation. The reason they reach this conclusion is that their assumption about the behavior of the Dyson spectral functions are much too strong.

in turn is closely related to the asymptotic behavior of the invariant absorptive parts. An especially simple case is obtained if we assume that the spectral functions vanish at infinity, which is a plausible assumption for any reasonable field theoretic model. In this case a formal manipulation using the Dyson representation is permissible and we obtain the following results:

(i) Charge-density—current ETC has no Schwinger term antisymmetric with respect to isospin. Thus, the sum rule of the usual type is possible. (ii) The same ETC has two possible q-number Schwinger terms, symmetric in isospin. (iii) The spatial current-spatial current ETC has one symmetric Schwinger term, which is related to one of the two terms discussed above in (ii). (iv) The same ETC has no antisymmetric Schwinger term, but there exist, other than the quark-model canonical terms, two possible terms directly proportional to $\delta^3(\mathbf{x})$. These terms should be there as the space-space part of the ETC is model-dependent.

As is pointed out in the Appendix, the ETC, precisely speaking, should be defined as the limit of the average of the commutator over a short time interval. Knowing the asymptotic behavior of the spectral function, we can calculate this average, as long as the spectral function is bounded by a power of the spectral variable. The higher the power, the more Schwinger terms are generated besides those discussed above. These Schwinger terms are polynomial in the cutoff momentum used for the averaging, and are not unique, depending on the cutoff function used. Thus, if the spectral functions are ill behaved at infinity, as would certainly be the case for a very singular interaction, the equal-time commutation relations can no longer be uniquely defined.

2. SEPARATION OF THE NONCAUSAL PART

We consider the one-particle matrix element of the commutator of two isovector currents

$$A_{\mu\nu}{}^{ab}(x)=\langle p|[j_\mu{}^a(x),j_\nu{}^b(0)]|p\rangle, \qquad (1)$$

where the state $|p\rangle$ represents either a scalar particle of momentum p or a fermion. In the latter case Eq. (1) means the average over two spin states, so that $A_{\mu\nu}{}^{ab}$ is a tensor in both cases. For simplicity we call the state $|p\rangle$ the proton. The Fourier transform of $A_{\mu\nu}{}^{ab}(x)$,

$$\tilde{A}_{\mu\nu}{}^{ab}(q)=\int d^4x\, e^{iq\cdot x}\langle p|[j_\mu{}^a(x),j_\nu{}^b(0)]|p\rangle, \qquad (2)$$

is the absorptive part for the Compton scattering of a vector meson of mass $\lambda=q^2$ by the proton.

Because of the conservation law $\partial^\mu j_\mu{}^a=0$, we have

$$q^\mu\tilde{A}_{\mu\nu}{}^{ab}=0 \qquad (3)$$

and $\tilde{A}_{\mu\nu}{}^{ab}$ can be expressed by two independent covarient tensors, which we choose as

$$\tilde{A}_{\mu\nu}{}^{ab}(q)=\tilde{L}_{\mu\nu}{}^{(1)}(q)\tilde{A}_1{}^{ab}(q)+\tilde{L}_{\mu\nu}{}^{(2)}(q)\tilde{A}_2{}^{ab}(q) \qquad (4)$$

with

$$\tilde{L}_{\mu\nu}{}^{(1)}(q)=(1/m^2)[p_\mu p_\nu q^2-(p_\mu q_\nu+q_\mu p_\nu)p\cdot q+g_{\mu\nu}(p\cdot q)^2]$$
$$\tilde{L}_{\mu\nu}{}^{(2)}(q)=[q^2g_{\mu\nu}-q_\mu q_\nu],$$

where m represents the proton mass. From the structure of $\tilde{L}_{\mu\nu}{}^{(2)}$, it is evident that $\tilde{A}_2(q)$ can have no kinematical singularities, because otherwise $\tilde{A}_{\mu\nu}$ will have the same singularities. On the other hand, $\tilde{A}_1(q)$ can have a singularity of the form

$$\delta(q^2-\alpha p\cdot q)/q^2 \qquad (5)$$

without inducing any new singularity in $\tilde{A}_{\mu\nu}$ itself. Such a term will give a noncausal function, as we see easily by considering the region near $q^2=0$ and taking the system $\mathbf{p}=0$. Then we have

$$\delta(p\cdot q)/q^2=-\delta(q_0)/m\mathbf{q}^2 \qquad (6)$$

the Fourier transform of which is $1/|\mathbf{x}|$. Hence we have to separate such noncausal part of A_i in order to have a Dyson representation for A_i's.

We choose the separation specifically as follows.

$$\tilde{A}_1{}^{ab}(q)=\tilde{A}_{1c}{}^{ab}(q)+\tilde{A}_{1n}{}^{ab}(q) \qquad (7)$$

$$\tilde{A}_{1n}{}^{ab}(q)=(2\pi m/q^2)[\epsilon(q_0+p_0)\delta(q^2+2p\cdot q)-\epsilon(q_0-p_0)\delta(q^2-2p\cdot q)]i\epsilon_{abc}\langle T_c\rangle, \qquad (8)$$

where T_c is the isospin operator for the proton. $\tilde{A}_{1n}{}^{ab}(q)$ is exactly what we obtain for the antisymmetric (in isospin) part of $A_1{}^{ab}$ from the quark-model current $j_\mu{}^a=\bar{\psi}\gamma_\mu(\tau_a/2)\psi$ using the commutation relations for the free field ψ.[11]

Conversely, if we insert (8) into (4) and take the Fourier transform for $x_0=0$, we obtain

$$A_{n,0\mu}{}^{ab}(\mathbf{x},0)\equiv L_{0\mu}{}^{(1)}A_{1n}{}^{ab}(x)|_{x_0=0}=i\epsilon_{abc}\langle T_c\rangle\delta^3(\mathbf{x})p_\mu/m \qquad (9)$$

and

$$A_{n,ij}{}^{ab}(\mathbf{x},0)\equiv L_{ij}{}^{(1)}A_{1n}{}^{ab}(x)|_{x_0=0}=i\epsilon_{abc}\langle T_c\rangle\delta_{ij}\delta^3(\mathbf{x})p_0/m. \qquad (10)$$

Thus, $A_{1n}{}^{ab}$ generates the quark-model ETC.

With this choice of $\tilde{A}_{1n}{}^{ab}$, we can prove that $A_{1c}{}^{ab}(x)$ is a causal function and also that its Fourier transform $\tilde{A}_{1c}{}^{ab}(q)$ has the right support required for the Dyson representation. These two requirements do not uniquely determine $\tilde{A}_{1n}{}^{ab}$ because we can add to it any causal function with the right support. Therefore the specific form (8) is to a certain extent a matter of convenience. From (8) we can calculate $A_{1n}{}^{ab}(x)$, a convenient repre-

[11] This is of course equivalent to taking one-particle and disconnected three-particle intermediate states in (2) and setting the charge form factor equal to 1 and the moment form factor 0.

sentation of which is given by

$$A_{1n}{}^{ab}(x)=-i\epsilon_{abc}\langle T_c\rangle\left\{\frac{m}{4\pi[(p\cdot x)^2-m^2x^2]^{1/2}}+\frac{p\cdot x}{m}\int_0^1 d\alpha\,\Delta(X(\alpha))\cos(\alpha p\cdot x)\right\},\quad(11)$$

with $X_\mu(\alpha)=x_\mu-p_\mu(1-\alpha)p\cdot x/m^2$. Since $X^2=x^2-(p\cdot x)^2\times(1-\alpha^2)/m^2<0$ for $x^2<0$ and $\alpha\leqslant 1$, the second term of Eq. (11) vanishes for $x^2<0$, so that

$$A_{1n}{}^{ab}(x)=\frac{-i\epsilon_{abc}\langle T_c\rangle m}{4\pi[(p\cdot x)^2-m^2x^2]^{1/2}}\quad\text{for}\quad x^2<0.\quad(12)$$

From (8), we have $\tilde{A}_{1n}{}^{ab}(q)=0$ for

$$m-\mathbf{q}^2+m^2<q_0<-m+\mathbf{q}^2+m^2,\quad \mathbf{p}=0.\quad(13)$$

Since m is the minimum mass of our system, $\tilde{A}_{\mu\nu}{}^{ab}(q)$ and hence $\tilde{A}_1{}^{ab}(q)$ and $\tilde{A}_2{}^{ab}(q)$ vanish in the same region (13). Then $\tilde{A}_{1c}{}^{ab}$ vanishes in the same region, which is the required property for the Dyson representation.[12]

We will prove that

$$A_{1c}{}^{ab}(x)=0\quad\text{and}\quad A_2{}^{ab}(x)=0,\quad\text{for}\quad x^2<0,\quad(14)$$

in the rest frame $\mathbf{p}=0$, as Eq. (14) is an invariant statement. In this frame, Eq. (4) gives

$$A_{00}{}^{ab}(x)=\nabla^2[A_1{}^{ab}(x)+A_2{}^{ab}(x)].\quad(15)$$

The solution which vanishes as $|\mathbf{x}|\to\infty$ is

$$A_1{}^{ab}(x)+A_2{}^{ab}(x)=-\frac{1}{4\pi}\int d^3x'\frac{A_{00}{}^{ab}(\mathbf{x}',x_0)}{|\mathbf{x}-\mathbf{x}'|}.\quad(16)$$

For $x_0{}^2<\mathbf{x}^2$, the integrand is nonvanishing only for $\mathbf{x}'^2\leqslant x_0{}^2<\mathbf{x}^2$, since $A_{00}(\mathbf{x},x_0)$ vanishes for spacelike x. Since $A_i(x)$ can depend only upon $p\cdot x$ and x^2, we note from Eq. (4) that in the frame $\mathbf{p}=0$, $A_{00}{}^{ab}(\mathbf{x}',x_0)$ is spherically symmetric in $\mathbf{x}'$. Hence we can safely put $1/|\mathbf{x}-\mathbf{x}'|\to 1/|\mathbf{x}|$ in Eq. (16). Then, from the conservation law $\partial^\mu j_\mu{}^a=0$,

$$\begin{aligned}\int d^3x' A_{00}{}^{ab}(\mathbf{x}',x_0)&=\langle p|\left[\int d^3x' j_0{}^a(x'),j_0{}^b(0)\right]|p\rangle\\&=\langle p|[Q^a,j_0{}^b(0)]|p\rangle\\&=i\epsilon_{abc}\langle T_c\rangle,\quad(\mathbf{p}=0),\end{aligned}\quad(17)$$

where we have assumed that $[Q^a,j_0{}^b]=i\epsilon_{abc}j_0{}^c$. Thus

$$A_1{}^{ab}(x)+A_2{}^{ab}(x)=-i\epsilon_{abc}\langle T_c\rangle/4\pi|\mathbf{x}|,\quad x^2<0.\quad(18)$$

The right-hand-side is exactly equal to $A_{1n}{}^{ab}(x)$ as seen from Eq. (12) with $\mathbf{p}=0$. Thus

$$A_{1c}{}^{ab}(x)+A_2{}^{ab}(x)=0,\quad x^2<0.\quad(19)$$

Next we show that $A_2{}^{ab}(x)$ is causal. Equation (4) gives in the rest system

$$\nabla^2 A_2{}^{ab}(x)=(1/2)\textstyle\sum_i A_{ii}{}^{ab}(x)-(3/2)\partial_0{}^2(A_1{}^{ab}+A_2{}^{ab}),$$

which yields

$$A_2{}^{ab}(x)=-\frac{1}{8\pi}\int d^3x'\frac{1}{|\mathbf{x}-\mathbf{x}'|}\{\textstyle\sum_i A_{ii}{}^{ab}(\mathbf{x}',x_0)-3\partial_0{}^2[A_1{}^{ab}(\mathbf{x}',x_0)+A_2{}^{ab}(\mathbf{x}',x_0)]\}.\quad(20)$$

Since the noncausal part of $A_1{}^{ab}+A_2{}^{ab}$ as given by Eq. (18) is independent of time in the rest system $\mathbf{p}=0$, $\partial_0{}^2(A_1{}^{ab}+A_2{}^{ab})$ no longer contains the noncausal part, and vanishes outside the light cone. Again the integrand is nonvanishing only for $\mathbf{x}'^2<\mathbf{x}^2$, if $x^2<0$, and

$$A_2{}^{ab}(x)=-\frac{1}{8\pi|\mathbf{x}|}\int d^3x'\times\{\textstyle\sum_i A_{ii}{}^{ab}-3\partial_0{}^2(A_1{}^{ab}+A_2{}^{ab})\},\quad x^2<0.\quad(21)$$

Equations (15) and (3) give the relation

$$\begin{aligned}\nabla^2\partial_0{}^2(A_1{}^{ab}+A_2{}^{ab})&=\partial_0{}^2A_{00}{}^{ab}(x)\\&=\textstyle\sum_{i,j}\nabla_i\nabla_jA_{ij}{}^{ab}(x)\end{aligned}$$

from which

$$\begin{aligned}\int d^3x'\partial_0{}^2(A_1+A_2)&=-\frac{1}{4\pi}\int d^3x'\int d^3x''\frac{1}{|\mathbf{x}'-\mathbf{x}''|}\times\textstyle\sum_{i,j}\nabla_i''\nabla_j''A_{ij}{}^{ab}(\mathbf{x}'',x_0)\\&=\int d^3x'\int d^3x''\times\textstyle\sum_{i,j}A_{ij}{}^{ab}(\mathbf{x}'',x_0)\nabla_i'\nabla_j'\left(\frac{1}{-4\pi|\mathbf{x}'-\mathbf{x}''|}\right)\\&=(\tfrac{1}{3})\int d^3x''\textstyle\sum_i A_{ii}{}^{ab}(\mathbf{x}'',x_0),\end{aligned}$$

where we have used

$$-\frac{1}{4\pi}\int d^3x'\nabla_i'\nabla_j'\left(\frac{1}{|\mathbf{x}'-\mathbf{x}''|}\right)=(\tfrac{1}{3})\delta_{ij}.$$

From (21),

$$A_2{}^{ab}(x)=0,\quad x^2<0\quad(22)$$

which, combined with (19), gives (14).

3. DERIVATION OF EQUAL-TIME COMMUTATION RELATION

We have seen in the last section that the noncausal function $A_{1n}{}^{ab}$ generates the quark-model ETC (9) and (10). We now calculate the contribution of the causal part $A_{1c}{}^{ab}$ and $A_{2c}{}^{ab}=A_2{}^{ab}$ to the ETC. Since $A_{ic}{}^{ab}(x)$

[12] If we had taken only the first term of (11) as $A_{in}{}^{ab}(x)$, then $\tilde{A}_{in}{}^{ab}$ would be $i\epsilon_{abc}\langle T_c\rangle 2\pi\delta(p\cdot q)/(mq^2)$, which does not vanish in the domain (13). Then $\tilde{A}_{1c}{}^{ab}$ would not satisfy the Dyson representation.

$(i=1,2)$ vanishes for $x^2<0$, and the Fourier transform $\tilde{A}_{ic}{}^{ab}(q)$ vanishes in the domain (13), it satisfies the Dyson representation

$$\tilde{A}_{ic}{}^{ab}(q)=(2\pi)^{-3}\int_0^\infty ds\int d^4u\,\epsilon(q-u)\times\delta((q-u)^2-s)\psi_i{}^{ab}(u,s)\,, \quad (23)$$

where $\psi_i{}^{ab}(u,s)$ is nonvanishing in a finite domain in u space. The Fourier transform of (23) is given by

$$A_{ic}{}^{ab}(x)=i\int_0^\infty ds\,\Delta(x,s)\Phi_i{}^{ab}(x,s)\,, \quad (24)$$

where

$$\Phi_i{}^{ab}(x,s)=(2\pi)^{-4}\int d^4u e^{-iu\cdot x}\psi_i{}^{ab}(u,s)\,. \quad (25)$$

From Eq. (1) we have the crossing symmetry

$$A_{\mu\nu}{}^{ab}(x)=-A_{\nu\mu}{}^{ba}(-x)\,, \quad (26)$$

which requires from (4) that

$$A_i{}^{ab}(x)=-A_i{}^{ba}(-x)\,. \quad (27)$$

If we decompose $A_i{}^{ab}$ into symmetric and antisymmetric parts defined by

$$A_i{}^{ab}(x)=\delta_{ab}A_i{}^{(s)}+i\epsilon_{abc}\langle T_c\rangle A_i{}^{(a)} \quad (28)$$

we have from (27)

$$A_i{}^{(s)}(-x)=-A_i{}^{(s)}(x) \quad (29)$$
$$A_i{}^{(a)}(-x)=A_i{}^{(a)}(x)\,.$$

$A_{1n}{}^{ab}$ is antisymmetric, so that $A_{ic}{}^{(s),(a)}$ must also satisfy (29). In terms of the Dyson spectral function we have

$$\Phi_i{}^{(s)}(x,s)=\Phi_i{}^{(s)}(-x,s)$$

and

$$\Phi_i{}^{(a)}(x,s)=-\Phi_i{}^{(a)}(-x,s)\,. \quad (30)$$

Introducing (24) into

$$A_{c,\mu\nu}{}^{ab}(x)\equiv L_{\mu\nu}{}^{(1)}(\partial)A_{1c}{}^{ab}+L_{\mu\nu}{}^{(2)}(\partial)A_2{}^{ab}\,, \quad (31)$$

we obtain derivatives of up to second order with respect to time of $A_{ic}{}^{ab}(x)$. Using (24) and the crossing relation (30), we have, for the symmetric and antisymmetric parts,

$$A_{ic}{}^{(a,s)}(\mathbf{x},0)=0 \quad (32)$$

$$\dot{A}_{ic}{}^{(a)}(\mathbf{x},0)=0 \quad (33)$$

$$\dot{A}_i{}^{(s)}(\mathbf{x},0)=-ia_i\delta^3(\mathbf{x}), \quad (34)$$

where

$$a_i=\int\Phi_i(0,s)ds=(2\pi)^{-4}\int ds\int d^4u\,\psi_i{}^{(s)}(u,s) \quad (35)$$

$$\frac{d^2A_i{}^{(s)}}{dt^2}(\mathbf{x},0)=0 \quad (36)$$

$$\frac{d^2A_{ic}{}^{(a)}}{dt^2}(\mathbf{x},0)=-2b_i\delta^3(\mathbf{x})\,, \quad (37)$$

where

$$b_i=i\int ds\,\dot{\Phi}_i{}^{(a)}(0,s)=(2\pi)^{-4}\int ds\int d^4u\,u_0\psi_i{}^{(a)}(u,s) \quad (38)$$

In the above we denoted $A_{ic}{}^{(s)}$ by $A_i{}^{(s)}$ because the latter has no noncausal part. The above relations are equivalent in momentum space to

$$\int_{-\infty}^{\infty}dq_0\tilde{A}_{ic}{}^{(a,s)}(q)=0 \quad (32')$$

$$\int_{-\infty}^{\infty}dq_0q_0\tilde{A}_{ic}{}^{(a)}=0 \quad (33')$$

$$\int_{-\infty}^{\infty}dq_0q_0\tilde{A}_i{}^{(s)}(q)=2\pi a_i \quad (34')$$

$$\int_{-\infty}^{\infty}dq_0q_0{}^2\tilde{A}_i{}^{(s)}=0 \quad (36')$$

and

$$\int_{-\infty}^{\infty}dq_0q_0{}^2\tilde{A}_{ic}{}^{(a)}=4\pi b_i\,. \quad (37')$$

These relations do not follow unconditionally from (23), since the interchange of the order of q_0 and s integrations is necessary to derive them. (This point was first considered by Schroer and Stichel.[9]) [The domain of u where $\psi(u,s)$ is nonvanishing is finite; hence u integration gives no difficulty.] In the Appendix we will discuss the precise definitions of Eqs. (32′)–(37′) and the conditions for their validity. There we find that the necessary and sufficient condition for all the above relations (32)–(36) to hold is

$$\lim_{s\to\infty}\psi_i{}^{(s)(a)}(u,s)=0\,. \quad (39)$$

If (39) holds, we have from (23)

$$\lim_{q_0\to\infty}\tilde{A}_i{}^{(s,a)}(q)=0\,, \quad (40)$$

which is a necessary condition for (32)–(36) to hold. [If we consider Eq. (37′) as the definition of b_i, (38) is valid only when $s\psi_i{}^{(a)}\to 0$, instead of (39). However, the structure of ETC does not change if we take (39).]

From (4) we have

$$L_{00}^{(1)} = m^{-2}[p_0^2\nabla^2 - (\mathbf{p}\cdot\nabla)^2]$$
$$L_{00}^{(2)} = \nabla^2$$
$$L_{0i}^{(1)} = m^{-2}[(p_0^2\nabla_i - p_i\mathbf{p}\cdot\nabla)\partial_0 + (p_0p_i\nabla^2 - p_0\nabla_i\mathbf{p}\cdot\nabla)]$$
$$L_{0i}^{(2)} = \nabla_i\partial_0$$
$$L_{ij}^{(1)} = m^{-2}\{(p_0^2\delta_{ij} - p_ip_j)\partial_0^2 + [p_0(p_i\nabla_j + p_j\nabla_i) - 2p_0\delta_{ij}\mathbf{p}\cdot\nabla]\partial_0 + p_ip_j\nabla^2 - \mathbf{p}\cdot\nabla(p_i\nabla_j + p_j\nabla_i) + \delta_{ij}(\mathbf{p}\cdot\nabla)^2]\}$$
$$L_{ij}^{(2)} = \delta_{ij}\partial_0^2 + (\nabla_i\nabla_j - \delta_{ij}\nabla^2). \quad (41)$$

From (32), (33) and (41) we immediately obtain

$$A_{c,0\mu}^{(a)}(\mathbf{x},0) = 0,$$

that is, there is no antisymmetric Schwinger term if (39) holds. Combining this result with (9), we have

$$A_{0\mu}^{(a)}(\mathbf{x},0) = \delta^3(x)p_\mu/m, \quad (42)$$

or in momentum space,

$$\int_{-\infty}^{\infty} dq_0 A_{0\mu}^{(a)}(q) = 2\pi p_\mu/m. \quad (42')$$

For the symmetric part, (32), (34) and (41) give

$$A_{0i}^{(s)}(\mathbf{x},0) = -im^{-2}(p_0^2\nabla_i - p_i\mathbf{p}\cdot\nabla)\times\delta^3(\mathbf{x})a_1 - i\nabla_i\delta^3(\mathbf{x})a_2. \quad (43)$$

We will discuss the magnitudes of a_1 and a_2 later.

For the space-space antisymmetric parts we use (32), (33) and (37), and add the noncausal part contribution (10) to obtain

$$A_{ij}^{(a)}(\mathbf{x},0) = [(p_0/m)\delta_{ij} + (2/m^2)\times(p_ip_j - p_0^2\delta_{ij})b_1 - 2\delta_{ij}b_2]\delta^3(\mathbf{x}). \quad (44)$$

Here we have no derivatives of δ functions, but we do get extra δ-function terms, which we expect to be there as $A_{ij}^{(a)}(\mathbf{x},0)$ should be model-dependent. (32), (34), (36), and (41) give

$$A_{ij}^{(s)}(\mathbf{x},0) = im^{-2}p_0[2\delta_{ij}\mathbf{p}\cdot\nabla - p_i\nabla_j - p_j\nabla_i]\delta^3(\mathbf{x})a_i. \quad (45)$$

If the condition (39) is not satisfied, as we could expect for some field-theoretic model with highly-singular interaction, then we can have more Schwinger terms. However, most of these additional Schwinger terms are dependent on the choice of the averaging function necessary to define the q_0 integral. Then the ETC involving such terms is not uniquely defined. (See Appendix.) For instance, $A_{00}^{ab}(x)$ has the antisymmetric Schwinger terms

$$A_{00}^{(a)}(\mathbf{x},0) = p_0m^{-1}\delta^3(\mathbf{x}) + m^{-2}(p_0^2\nabla^2 - (\mathbf{p}\cdot\nabla)^2)\times\delta^3(\mathbf{x})c_1 + \nabla^2\delta^3(\mathbf{x})c_2, \quad (46)$$

where $c_i = 0(\Lambda^{2\alpha})$, $(\Lambda \to \infty)$. (46) is correct for $0 \leqslant \alpha < 1$. For $\alpha \geqslant 1$, other Schwinger terms appear on the right-hand side of (46), which are of order $\Lambda^{2\alpha-2}$, $\Lambda^{2\alpha-4}$, $\cdots$, but dependent on the choice of averaging function. Hence, for $\alpha \geqslant 1$, $A_{00}^{(a)}(\mathbf{x},0)$ is not uniquely definable.

We will now discuss what sum rules we could obtain from the ETC's (42), (43), (44) and (45). The conventional sum rule (Dashen–Gell-Mann[13] and Fubini[14]) is obtained from (42) by taking the limit $|\mathbf{p}| \to \infty$. In terms of the external boson mass $\lambda = q^2$ and the energy variable $\nu = p\cdot q$, we write

$$\tilde{A}_i^{(a)}(q) = \tilde{A}_i^{(a)}(\lambda,\nu)$$

and change the integration variable from q_0 to ν. Then in the limit $|\mathbf{p}| \to \infty$, $\lambda = (\nu + \mathbf{p}\cdot\mathbf{q})^2/p_0^2 - \mathbf{q}^2 \to -\mathbf{q}^2 \sin^2\theta$, where θ is the angle between $\mathbf{p}$ and $\mathbf{q}$.

Using the crossing symmetry $\tilde{A}_i^{(a)}(\lambda, -\nu) = \tilde{A}_1^{(a)}(\lambda,\nu)$, we obtain

$$\int_0^\infty d\nu\, \lambda\tilde{A}_1^{(a)}(\lambda,\nu) = \pi m, \quad (47)$$

where $\lambda = -\mathbf{q}^2\sin^2\theta$ is a fixed mass independent of ν. This is the usual sum rule since λA_1 is the coefficient of $p_\mu p_\nu$ in $A_{\mu\nu}$.

We cannot take the limit $|\mathbf{p}| \to \infty$ in the Fourier transform of Eq .(43), as Eq. (43) is equivalent to the set of equations (32′) and (34′). But the same limit applied to (34′) gives

$$a_i = (1/\pi p_0^2)\int_0^\infty d\nu\, \nu\tilde{A}_i^{(s)}(\lambda,\nu),$$

which would indicate $a_i = 0$, contrary to the conclusion $a_2 \neq 0$ shown later. Thus the above integral must diverge. If we arbitrarily assume $a_1 = 0$ and let the components of $\mathbf{p}$ perpendicular to $\mathbf{q}$ go to infinity in the Fourier transform of (43), we would obtain, as do Amati *et al.*,[15]

$$\int d\nu\, \nu A_1^{(s)}(-\mathbf{q}^2, \nu) = -a_2.$$

However, this equation does not seem justified.

Similarly, Eq. (44) is equivalent, as we see by decomposing (44) into independent tensorial parts, to the set of Eqs. (32′), (33′) and (42′) with $\mu = 0$. Equation (45) is equal to (32′), (34′), and (36′).

Thus, the sum rules we can derive from the equal-time commutation relation under the assumption (39) are limited to (47) and a set of relations (32′)–(37′). The latter, especially (32′), (33′) and (36′) can give physically meaningful sum rules, which we will discuss elsewhere.[16]

Finally we will derive some useful representation for a_1 and a_2 as defined in (35) and (34′). If we take $\mathbf{p}$ per-

[13] R. F. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1966).

[14] S. Fubini, Nuovo Cimento **43A**, 475 (1966).

[15] D. Amati, R. Jengo, and E. Remiddi, CERN Report TH 759, 1967 (unpublished).

[16] J. W. Meyer and H. Suura, Phys. Rev. Letters **18**, 479 (1967).

pendicular to **q**, then from (29), $\tilde{A}_i^{(s)}(q)$ is antisymmetric with respect to q_0 and we have from (34′)

$$a_i=(1/\pi)\int_\beta^\infty dq_0 q_0 \tilde{A}_i^{(s)}(q^2,\nu=p_0q_0)\,,$$

where $\beta=-p_0+(\mathbf{p}^2+\mathbf{q}^2+m^2)^{1/2}$. As we will show later, the integrand can be continued analytically to $\nu=0$ by letting $|\mathbf{p}|^2\to -m^2$ and $p_0\to 0$. Then changing the integration variable to $\lambda=q^2$, we obtain

$$a_i=(1/2\pi)\int_0^\infty d\lambda\,\tilde{A}_i^{(s)}(\lambda,0)\,. \tag{48}$$

We now express $\tilde{A}_i^{(s)}$ in terms of positive definite quantities, which are essentially cross sections for hypothetical charged photons of mass λ. If we denote the amplitudes for these hypothetical charged photons by $\tilde{A}_i^{(\pm)}$, $\tilde{A}_{\mu\nu}^{(\pm)}$, etc., we obviously have

$$\tilde{A}_i^{(s)}=\tfrac{1}{2}(A_i^{(+)}+A_i^{(-)})\,. \tag{49}$$

The positive definite quantities are

$$\begin{aligned}a^{(\pm)}&=\epsilon^\mu\epsilon^\nu\tilde{A}_{\mu\nu}^{(\pm)}\\&=(1/m^2)[(\epsilon\cdot p)^2\lambda-\nu^2]\tilde{A}_1^{(\pm)}-\lambda\tilde{A}_2^{(\pm)}\,,\end{aligned} \tag{50}$$

where we have used $\epsilon^2=-1$ and $\epsilon\cdot q=0$. In the rest system, $p\cdot\epsilon=0$ for the transverse polarization, so that

$$a_T^{(\pm)}=-[(\nu^2/m^2)\tilde{A}_1^{(\pm)}+\lambda\tilde{A}_2^{(\pm)}]\,. \tag{51}$$

For the longitudinal polarization we have

$$p\cdot\epsilon=[(\nu^2-\lambda m^2)/\lambda]^{1/2}\,,$$

and

$$a_L^{(\pm)}=-\lambda(\tilde{A}_1^{(\pm)}+\tilde{A}_2^{(\pm)})\,. \tag{52}$$

For $q_0>0$,

$$a_{L,T}^{(\pm)}=(2\pi)^4\sum_n |\langle p|\epsilon\cdot j^{(\pm)}(0)|n\rangle|^2 \times\delta^{(4)}(p+q-p_n)\,. \tag{53}$$

Although $p_0=0$ is unphysical with respect to the initial state, it is still in the physical region of the intermediate states, as long as $q_0\geqslant -p_0+[(\mathbf{p}+\mathbf{q})^2+m^2]^{1/2}$ as in Eq. (48), since this is equivalent to $s=(p+q)^2\geqslant m^2$. In other words, $\sum_n\delta(p+q-p_n)$ in Eq. (53) is still well defined in the limit $\nu=0$. Since the matrix element $\langle p|\epsilon\cdot j^{(+)}|n\rangle$ would in general be an analytic function of ν, we can take the limit $\nu\to 0$ in (53) and conclude

$$a_{L,T}^{(\pm)}(\lambda,0)>0\,. \tag{54}$$

Thus, Eqs. (51) and (52) give

$$a_2=-\frac{1}{4\pi}\int_0^\infty\frac{d\lambda}{\lambda}[a_T^{(+)}(\lambda,0)+a_T^{(-)}(\lambda,0)]$$

and

$$a_1+a_2=-\frac{1}{4\pi}\int_0^\infty\frac{d\lambda}{\lambda}[a_L^{(+)}(\lambda,0)+a_L^{(-)}(\lambda,0)]\,. \tag{55}$$

We conclude that $a_2\neq 0$. a_1 involves the difference a_L-a_T and no conclusion can be drawn about it. It should be stressed, however, that the disconnected vacuum contribution is still included in Eq. (55) for a_2. This is because in Eq. (53) for $a_{L,T}$, $\langle p|\epsilon\cdot j(0)|n\rangle$ can be a disconnected matrix element in which the state $|n\rangle$ contains the initial particle p, plus the rest of the particles which are produced by the momentum q independently of p. Such a disconnected matrix element would vanish for $\lambda=q^2$ smaller than a certain threshold because of the energy-momentum conservation factor

$$\delta(q-\sum_{i=1}^{n-1}p_i)\,,$$

but in Eq. (55) the whole range of λ is involved. To prove that the q-number Schwinger term is nonvanishing, we have to subtract from a_2 the vacuum Schwinger term, which is contained in the product of the disconnected matrix elements, and show that the rest is nonvanishing. If we do this subtraction, we obtain, besides the positive definite terms involving only connected matrix elements, the interference terms between connected and disconnected matrix elements. So far we have not been able to prove the non-vanishing of the q-number Schwinger term, although the exact cancellation of direct and interference terms is very unlikely.

APPENDIX

Consider an invariant $\tilde{A}(q)$ which satisfies the Dyson representation

$$\tilde{A}(q)=(2\pi)^{-3}\int_{s_0}^\infty ds\int d^4u\,\epsilon(q-u) \times\delta((q-u)^2-s)\psi(u,s)\,. \tag{A1}$$

The equation

$$\int_{-\infty}^\infty\tilde{A}(q)dq_0=0 \tag{A2}$$

should be interpreted as[17]

$$\lim_{\Lambda\to\infty}\int_{-\infty}^\infty dq_0\exp(-q_0^2/\Lambda^2)\tilde{A}(q)=0\,. \tag{A3}$$

This equation does not necessarily follow from (A1), but depends on the behavior of $\psi(u,s)$ as $s\to\infty$. (The domain of u where ψ is nonvanishing is finite, so that there is no problem in the u integration.) The factor $\exp(-q_0^2/\Lambda^2)$ and the limit $\Lambda\to\infty$ arise from defining the ETC as

$$A_{\mu\nu}(\mathbf{x},0)\equiv\lim_{\tau\to 0}(1/\tau\pi^{1/2})\int_{-\infty}^\infty dx_0\exp(-x_0^2/\tau^2)A_{\mu\nu}(\mathbf{x},x_0)\,,$$

where $\tau=\Lambda^{-1}$ is the half-width of the time measurement.

[17] For a discussion of the meaning of the formal expression of the Dyson representation Eq. (A1) see: A. S. Wightman, in *Dispersion Relations and Elementary Particles* (Hermann, Paris, 1960), p. 304.

We have chosen a Gaussian to define the ETC, but the general qualitative features of the ETC do not depend on the shape of the cutoff function, provided it is symmetric in q_0, although some of the Schwinger terms are dependent on the cutoff function as will be indicated later.

We introduce the notation

$$I[q_0^n\tilde{A};\Lambda]\equiv\int_{-\infty}^{\infty}dq_0\exp(-q_0^2/\Lambda^2)q_0^n\tilde{A}(q). \quad (A4)$$

Introducing (A1) into (A4),

$$I[\tilde{A};\Lambda]=(2\pi)^{-3}\int_{s_0}^{\infty}ds\int d^4u(1/2\beta)\{\exp[-(u_0+\beta)^2/\Lambda^2]-\exp[-(u_0-\beta)^2/\Lambda^2]\}\psi(u,s)$$

$$=-(2\pi)^{-3}\int d^4u\exp[-(u_0^2+b)/\Lambda^2]\sum_{n=0}^{\infty}\frac{(\Lambda^2)^{-(2n+1)}}{(2n+1)!}(2u_0)^{2n-1}\int_{s_0}^{\infty}ds(s+b)^n\exp(-s/\Lambda^2)\psi(u,s), \quad (A5)$$

where we have used the abbreviations

$$\beta=(s+b)^{1/2}\quad\text{and}\quad b=(\mathbf{q}-\mathbf{u})^2. \quad (A6)$$

(A5) involves the integral

$$\int_{s_0}^{\infty}ds\,s^m\exp(-s/\Lambda^2)\psi(u,s)=\Lambda^{2(m+1)}\int_{s_0/\Lambda^2}^{\infty}dx\,x^me^{-x}\psi(u,x\Lambda^2)\begin{array}{ll}\to o(\Lambda^{2(m+\alpha+1)}) & \text{if}\ \psi(u,s)/s^\alpha\to 0\\ \to O(\Lambda^{2(m+\alpha+1)}) & \text{if}\ \psi(u,s)/s^\alpha\to\text{constant}.\end{array} \quad (A7)$$

We can easily see the series in (A5) is absolutely convergent, and we can take the limit $\Lambda\to\infty$ in each term. If $\psi(u,s)$ is bounded by a power of s as $s\to\infty$, then obviously only the first few terms of the expansion in Λ^{-2} in (A5) will survive in the limit $\Lambda\to\infty$. From (A7), the leading term in Λ^2 is given by the $n=0$ term, and we obtain

$$\begin{aligned}\lim_{\Lambda\to\infty}I[\tilde{A};\Lambda]&=0 &&\text{if}\ \psi(u,s)\to 0\\ &=O(\Lambda^{2\alpha}) &&\text{if}\ \psi(u,s)\to s^\alpha.\end{aligned} \quad (A8)$$

For the symmetric part $\tilde{A}^{(s)}$, $\psi(u,s)$ is symmetric in u [see Eqs. (29) and (30)]. Hence, the leading term in (A5) vanishes from $\int u_0\psi^{(s)}(u,s)d^4u=0$. The next leading term is obtained by again taking $n=0$, but taking the first term of the expansion of $\exp[-(u_0^2+b)/\Lambda^2]$ in $1/\Lambda^2$. Thus

$$\lim_{\Lambda\to\infty}I[\tilde{A}^{(s)},\Lambda]\sim-(2\pi)^{-3}\Lambda^{-4}\int d^4u\int_{s_0}^{\infty}ds(-2\mathbf{q}\cdot\mathbf{u})(2u_0)\psi^{(s)}(u,s)\exp(-s/\Lambda^2).$$

Therefore, for $\tilde{A}^{(s)}$, we have besides (A8),

$$\begin{aligned}\lim_{\Lambda\to}I[A^{(s)},\Lambda]&=0 &&\text{if}\ \psi^{(s)}(u,s)/s\to 0\\ &=O(\Lambda^{2\alpha-2}) &&\text{if}\ \psi^{(s)}\to O(s^\alpha),\ \alpha\geqslant 1.\end{aligned} \quad (A9)$$

Similarly, we can analyze the limit of $I[q_0A^{(a,s)};\Lambda]$ and $I[q_0^2A^{(a,s)};\Lambda]$. The results are as follows:

$$\begin{aligned}\lim_{\Lambda\to\infty}I[q_0\tilde{A}^{(a)};\Lambda]&=0, &&\psi^{(a)}(u,s)\to 0\\ &=O(\Lambda^{2\alpha}), &&\psi^{(a)}(u,s)\to s^\alpha\end{aligned} \quad (A10)$$

$$\begin{aligned}\lim_{\Lambda\to\infty}I[q_0\tilde{A}^{(s)};\Lambda]&=\text{constant}, &&\psi^{(s)}(u,s)s\to 0\\ &=O(\Lambda^{2\alpha+2}), &&\psi^{(s)}(u,s)\to s^\alpha\\ & &&(\alpha\geqslant -1)\end{aligned} \quad (A11)$$

$$\begin{aligned}\lim_{\Lambda\to\infty}I[q_0^2\tilde{A}^{(a)};\Lambda]&=\text{constant}, &&\psi^{(a)}(u,s)\ s\to 0\\ &=O(\Lambda^{2\alpha+2}), &&\psi^{(a)}(u,s)\to s^\alpha\\ & &&(\alpha\geqslant -1)\end{aligned} \quad (A12)$$

$$\begin{aligned}\lim_{\Lambda\to\infty}I[q_0^2\tilde{A}^{(s)};\Lambda]&=0, &&\psi^{(s)}(u,s)\to 0\\ &=O(\Lambda^{2\alpha}), &&\psi^{(s)}(u,s)\to s^\alpha\\ & &&(\alpha\geqslant 0).\end{aligned} \quad (A13)$$

The ETC, when the condition (39) does not hold, can be easily read from Eqs. (A10)–(A13). We do not bother to write them down, but the following remark may be important. $I[q_0{}^m\tilde{A};\Lambda]$, as given by (A5), is a power series in Λ^{-2}, starting from some highest-power $\Lambda^{2\alpha+m}$ if $\psi\to s^\alpha$. This leading term $O(\Lambda^{2\alpha+m})$ is independent of our choice of averaging or cutoff function, which we took as $\exp(-q_0{}^2/\Lambda^2)$ for convenience. The next leading term, $O(\Lambda^{2\alpha+m-2})$ will be dependent on the choice of the averaging function, and generally linear in **q**. From the symmetry property of $\psi(u,s)$, the leading term can vanish, as is the case for $I[A^{(s)};\Lambda]$, $I[q_0A^{(a)};\Lambda]$, and $I[q_0{}^2A^{(s)};\Lambda]$. Then, these quantities are entirely model-dependent, and hence the corresponding ETC has no definition without specifying the method of averaging over time. We find the following when $\psi^{(a,s)}(u,s)\to s^\alpha$:

$$A_{00}{}^{(a)}(\mathbf{x},0)=p_0m^{-1}\delta^3(\mathbf{x})+m^{-2}[p_0{}^2\nabla^2-(\mathbf{p}\cdot\nabla)^2]\times\delta^3(\mathbf{x})C_1+\nabla^2\delta^3(\mathbf{x})C_2, \quad \text{(A14)}$$

where

$$C_i=(2\pi)^{-1}\int_{-\infty}^{\infty}A_i{}^{(a)}(q)dq =O(\Lambda^{2\alpha}).$$

For $0\leqslant\alpha<1$, (A14) is exact. For $\alpha\geqslant1$, there will be additional terms of order $\Lambda^{2(\alpha-1)}$, $\Lambda^{2(\alpha-2)}$, $\cdots$, on the right-hand side of (A14), making the ETC undefined. $A_{0i}{}^{(a)}(\mathbf{x},0)$ is undefined for $\alpha\geqslant0$. $A_{00}{}^{(s)}(\mathbf{x}\rho)$ is undefined for $\alpha\geqslant1$. $A_{0i}{}^{(s)}(\mathbf{x},0)$ is given by (43) as long as $\alpha<1$. For $\alpha\geqslant1$, $A_{0i}{}^{(s)}(\mathbf{x},0)$ is undefined. $A_{ij}{}^{(a)}(\mathbf{x},0)$ is given by (44) for $\alpha<0$, and undefined for $\alpha\geqslant0$. $A_{ij}{}^{(s)}(\mathbf{x},0)$ is given by (45) for $\alpha<0$, and is undefined for $\alpha\geqslant0$.

In terms of the transversal and longitudinal amplitudes $a_{T,L}$ [Eqs. (44) and (45)], we have

$$\tilde{A}_1=-\frac{m^2}{\nu^2-m^2\lambda}(a_T-a_L)$$
$$\tilde{A}_2=-\frac{1}{\lambda(\nu^2-m^2\lambda)}(\nu^2a_L-m^2\lambda a_T), \quad \text{(A15)}$$

where $a_{T,L}$ are related to the total cross section of the transverse or longitudinal vector mesons of mass $\lambda=q^2$, by

$$\sigma_{L,T}(\mathrm{W},\lambda)=a_{L,T}/(\nu^2-m^2\lambda)^{1/2}, \quad \text{(A16)}$$

with the total energy in the center-of-mass system $\mathrm{W}^2=m^2+2\nu+\lambda$. Combining (A15) and (A16) we see that

$$\lim_{q_0\to\infty}\tilde{A}_1(q)=-m^2p^{-1}\lim_{q_0\to\infty}[\sigma_T(\mathrm{W},\lambda)-\sigma_L(\mathrm{W},\lambda)]/q_0$$

$$\lim_{q_0\to\infty}\tilde{A}_2(q)=-p^{-1}\lim_{q_0\to\infty}[p_0{}^2\sigma_L(\mathrm{W},\lambda)-m^2\sigma_T(\mathrm{W},\lambda)]/q_0.$$

Note that for the symmetric parts $\tilde{A}_i{}^{(s)}$, $\sigma_{T,L}$ should mean the sum of $\sigma_{T,L}{}^+$ and $\sigma_{T,L}{}^-$, cross sections for $\pm$ charged mesons, while for the antisymmetric parts $\sigma_{T,L}$ represents the difference of $\sigma_{T,L}{}^+$ and $\sigma_{T,L}{}^-$. Since $q_0\to\infty$ means that the incident vector-meson mass $\lambda\to\infty$, we may not assert that $\sigma_{T,L}{}^\pm(\mathrm{W}=\infty,\lambda=\infty)\to$ constant, even if we assume that

$$\sigma_{T,L}{}^\pm(\mathrm{W}=\infty,\lambda=\text{finite})\to\text{constant}.$$

However, $\sigma_{T,L}(\infty,\infty)\to$ constant does not seem contradictory to field-theoretic models with not too singular interactions. Assuming this, we have $\lim_{q_0\to\infty}q_0\tilde{A}_i{}^{(s)}(q)\to$ constant, and $\lim_{q_0\to\infty}q_0\tilde{A}_i{}^{(a)}(q)\to0$, the latter following from $\sigma^+/\sigma^-\to1$. These asymptotic behaviors are obtained if we assume that $\lim_{s\to\infty}s^{1/2}\psi_i{}^{(a),(s)}(u,s)=M(u)$, i.e., $\alpha=-\frac{1}{2}$ in Eq. (A10′). The better convergence for $\tilde{A}_i{}^{(a)}$ is due to

$$\int d^4u\,M^{(a)}(u)=0.$$

Note added in proof. It was shown in the Appendix that if the spectral function goes like s^α as $s\to\infty$, then the ETC is in general a power series in Λ^{-2} (Λ being the cutoff momentum used to define the time averaging) starting from some highest power $\Lambda^{2(m+\alpha)}$ ($m\geqq0$). Based on this analysis a statement was made in the text to the effect that the terms in ETC, except for the most divergent term, would no longer be unique depending on the cutoff function if the spectral function behaves too singularly as $s\to\infty$, and therefore that the ETC can no longer be defined uniquely in such cases. The first part of this statement must be corrected and the second part needs qualification.

On the uniqueness of the terms in the ETC, a detailed analysis shows that the terms of order $\Lambda^\epsilon(\epsilon>0)$ are obviously dependent on the cutoff function including the highest-power term and that the terms of order $O(1)$ are always unique. This brings about for example the following situation. If $\psi\to s$ ($s\to\infty$), the ETC like (A4) would be of order Λ^2. However, this term can vanish if $\psi(u,s)$ is even in u as shown in (A9). Then we are left with terms of order Λ^0, which are independent of the choice of the averaging function. Thus the ETC is well defined in this case contrary to the previous statement. Similarly, in the statements following (A14) the critical case when the leading nonvanishing terms is of order Λ^0, must be included in the case of the well-defined ETC.

On the general statement that the ETC can not be defined for singular ψ, there are exceptions for this statement. It has been brought to our attention by B. Schroer and P. Stichel (private communication, see also Ref. 9) that a well-defined ETC can be obtained for a certain class of Dyson spectral functions even though the spectral function goes as s^n for large s. The situation can be explained in the language of our paper in the following way. Consider $I[A,\Lambda]$ as defined in

Eq. (A4). We take

$$\psi(u,s)=\sum_{m=0}^{n} s^m M_m(u)+\phi(u,s)\,,$$

where

$$\phi(u,s)\underset{s\to\infty}{\longrightarrow} 0\,.$$

Then ϕ can be treated separately and from Eq. (A8) gives a vanishing contribution in the limit of large Λ. We further require that $\psi(u,s)$ be symmetric in u so that the term in I which goes as Λ^{2n} vanishes as in Eq. (A9). The term which goes as Λ^{2n-2} receives contributions from both M_n and M_{n-1}. Now the point which Schroer and Stichel make is that there exist a class of ψ's for which these two contributions cancel one another so that there is no Λ^{2n-2} term. In fact, this process is continued until all divergent terms are canceled. This then leaves a finite ETC which in general is a polynomial in $|\mathbf{q}|$ of order n. In the Appendix we considered ψ's which behaved only as $s^n M(u)$ and thus did not consider the possibility of this cancellation.

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Possible $|\Delta\mathbf{T}|=\frac{3}{2}$ Amplitudes in Λ^0 Decay*

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Using $K_2^0\to\pi\pi$ and $K^+\to\pi\pi$ as a guide, estimates are made on the magnitudes of $|\Delta\mathbf{T}|=\frac{3}{2}$ amplitudes and possible CP violation in the decay $\Lambda^0\to n\pi^0$. It is found that, consistent with current experimental data on Λ^0 decay, a small $|\Delta\mathbf{T}|=\frac{3}{2}$ amplitude could be reflected in an observable departure of α_0/α_- from unity, and that large CP-violating phases could be present.

RECENT measurements[1] of the rate $K_2^0\to\pi^0\pi^0$ indicate that $|\Delta\mathbf{T}|\geq\frac{3}{2}$ amplitudes are responsible for the CP violation exhibited by the decays $K_2^0\to\pi\pi$.[2,3] The only other known $|\Delta\mathbf{T}|=\frac{3}{2}$ amplitude is found in $K^\pm\to\pi^\pm\pi^0$. From this rate a ratio of S-wave $|\Delta\mathbf{T}|=\frac{3}{2}$ and $|\Delta\mathbf{T}|=\frac{1}{2}$ amplitudes can be calculated: $|A_2/A_0|\approx 10\%$.[4] The CP-violating phase angle φ can be estimated by assuming that only the $|A_2|/|A_0|$ term contributes to the ratio $\eta_{+-}=\text{amp.}(K_2^0\to\pi^+\pi^-)/\text{amp.}(K_1^0\to\pi^+\pi^-)$. The resulting equation, $|\eta_{+-}=1/(\sqrt{10})(|A_2|/|A_0|)\times\sin\varphi$,[5] gives $\varphi=57$ mrad for $\eta_{+-}=(1.83\pm.12\times10^{-3})$.[6] A possible SU_3 suppression of the $K_1^0\to\pi\pi$ rate of unknown magnitude would decrease $|A_2|/|A_0|$ and increase φ correspondingly.[3] An additional uncertainty in the amplitude and phase estimates comes from the possible presence of both $|\Delta\mathbf{T}|=\frac{3}{2}$ and $|\Delta\mathbf{T}|=\frac{5}{2}$ terms. Thus it is possible that both the $|\Delta\mathbf{T}|=\frac{3}{2}$ and $|\Delta\mathbf{T}|=\frac{5}{2}$ amplitudes are large and have large phase angles relative to $|\Delta\mathbf{T}|=\frac{1}{2}$, but the $K_2^0\to\pi\pi$ rate is suppressed through a cancellation.[7] With these reservations in mind, it is the purpose of this paper to point out that $|A_2|/|A_0|\approx 10\%$ and $\varphi\approx 0$ could be reflected in the nonleptonic decay of the Λ^0 and could be easily detected. In addition, if large $|\Delta\mathbf{T}|=\frac{3}{2}$ amplitude and phase were allowed, a large CP violation in $\Lambda^0\to n\pi^0$ could occur.

The data on nonleptonic decay of the Λ^0 are summarized in Table I. Since the $|\Delta\mathbf{T}|=\frac{1}{2}$ rule requires the same amplitudes for $\Lambda^0\to p\pi^-$ and $\Lambda^0\to n\pi^0$, this decay is particularly suited to a search for the presence of $|\Delta\mathbf{T}|=\frac{3}{2}$. The ratio of asymmetry parameters α_0/α_- is the most sensitive available test. Possible values for α_0 and β_0 have been studied using the constraints in Table I except for the ratio $|P_0|/|S_0|$ and

Table I. Experimental data on Λ^0 decay.

Mode	Rate	α[a]	β	$\lvert P\rvert/\lvert S\rvert$
π^-p	1	$+0.65\pm0.05$[b]	0.10 ± 0.10[b]	0.36 ± 0.06[b]
π^0n	0.48 ± 0.019[c]	$+0.70\pm0.20$[d]		$0.39_{-0.12}^{+0.2}$[e]

[a] The spin parameters are defined in the same way as Samios, Ref. c.
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