$l=0$ at $E=m$,

$$\alpha(E=m)=0=-1-\frac{m_N g^2}{2\pi^2}\int\frac{dk\, f^2(k)F^2(k)}{h(m-\omega_k,\, k)}.$$

Corresponding to this Regge pole of T_1 there is a pole in the energy plane at $E=m$. The residue at the pole at $E=m$ of the irreducible part of the V-θ scattering amplitude, as seen from Eq. (B5), is

$$\frac{g^2 f^2(p)F^2(p)m_N}{(2\pi)^3 p^2 \mathcal{G}(m)}, \tag{B7}$$

where

$$\mathcal{G}(E)=\frac{D(E)}{E-m}.$$

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Meson-Baryon Scattering in Broken $SU(6)_W$

SACHCHIDA NANDA GUPTA*
Department of Physics, University of Delhi, Delhi, India
and
Theoretical Physics Division, National Physical Laboratory, New Delhi, India

(Received 12 January 1967)

We have studied the scattering processes like $P+B\rightarrow P+B$ and $P+B\rightarrow P+D$ within the framework of broken $SU(6)_W$, the breaking being provided by a W-spin scalar spurion having $I=0=Y$ and belonging to the 35-dimensional adjoint representation of $SU(6)_W$. It is found that many of the bad results of the exact $SU(6)_S$ and exact $SU(6)_W$ schemes are absent. At the same time, some of the results which agree with experiment are retained. The Johnson-Treiman relations, however, are no longer valid.

I. INTRODUCTION

In a recent paper[1] we studied the meson-baryon couplings within the framework of exact and broken $SU(6)_W$ symmetry. It was found that whereas the predictions of exact symmetry are not very good, those of broken symmetry are in very good agreement with experiment in the case of decuplet decays; in the case of meson-baryon couplings, the predictions seem to be consistent with present experimental knowledge about them.

It was shown by Jackson[2] that the $SU(6)_W$ predictions for meson-baryon scattering in the exact symmetry[3] are in very poor agreement with experiment except for the Johnson-Treiman[4] relations. In view of the better agreement of the couplings with experiment as predicted by the broken $SU(6)_W$, we are encouraged to investigate the scattering process within the framework of broken $SU(6)_W$ and examine whether there is any improvement for the scattering predictions. We shall confine our attention to the processes of the type

$$\text{(A)} \qquad P+B\rightarrow P+B,$$

and

$$\text{(B)} \qquad P+B\rightarrow P+D,$$

where P, B, and D are taken to mean pseudoscalar meson, baryon, and baryon resonance, respectively.

We break the symmetry by a W-spin scalar spurion having $I=0=Y$ and belonging to the 35-dimensional adjoint representation of the group $SU(6)_W$. Such a spurion is given by

$$S_\alpha^{\alpha'}=\delta_i^{i'}(\lambda_8)_A^{A'}, \tag{1.1}$$

where

$$\lambda_8\sim\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}. \tag{1.2}$$

II. MESON-BARYON SCATTERING

In exact symmetry the scattering is described by the following 4 invariant amplitudes:

$$f_1\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\gamma}\phi_\rho{}^\delta\phi_\delta{}^{\dagger\rho}, \tag{2.1}$$

$$f_2\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\delta}\phi_\delta{}^\rho\phi_\rho{}^{\dagger\gamma}, \tag{2.2}$$

$$f_3\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\delta}\phi_\rho{}^\gamma\phi_\delta{}^{\dagger\rho}, \tag{2.3}$$

$$f_4\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\delta\rho}\phi_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}. \tag{2.4}$$

* Present address: Roorkee University, Roorkee, India.

[1] Sachchida Nanda Gupta, Phys. Rev. **151**, 1235 (1966). We have followed the notation of this paper. Other references on the application of the group $SU(6)_W$ are listed in this paper.

[2] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[3] J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters **15**, 373 (1965). These authors have studied the various scattering processes in the exact $SU(6)_W$ symmetry using the Clebsch-Gordan coefficients. Our results, obtained here by using the tensor techniques are in agreement with theirs for the exact symmetry if we put all g's$=0$.

[4] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965).

The breaking gives rise to the following (eighteen) additional invariant amplitudes

$$g_1'\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\delta}{}^{\gamma}\phi_{\lambda}{}^{\dagger\rho}S_{\rho}{}^{\lambda}, \tag{2.5}$$

$$g_2'\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\lambda}{}^{\rho}\phi_{\delta}{}^{\dagger\gamma}S_{\rho}{}^{\lambda}, \tag{2.6}$$

$$g_1\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\gamma}\phi_{\rho}{}^{\delta}(Y\phi^{\dagger})_{\delta}{}^{\rho}, \tag{2.7}$$

$$g_2\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\gamma}\phi_{\rho}{}^{\delta}(Z\phi^{\dagger})_{\delta}{}^{\rho}, \tag{2.8}$$

$$g_3(Y\psi^{\dagger})_{\alpha\beta\gamma}\psi^{\alpha\beta\gamma}\phi_{\rho}{}^{\delta}\phi_{\delta}{}^{\dagger\rho}, \tag{2.9}$$

$$g_4\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\delta}{}^{\rho}(Y\phi^{\dagger})_{\rho}{}^{\gamma}, \tag{2.10}$$

$$g_5\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\delta}{}^{\rho}(Z\phi^{\dagger})_{\rho}{}^{\gamma}, \tag{2.11}$$

$$g_6\psi_{\alpha\beta\gamma}{}^{\dagger}(Y\psi)^{\alpha\beta\delta}\phi_{\delta}{}^{\rho}\phi_{\rho}{}^{\dagger\gamma}, \tag{2.12}$$

$$g_7(Y\psi^{\dagger})_{\alpha\beta\gamma}\psi^{\alpha\beta\delta}\phi_{\delta}{}^{\rho}\phi_{\rho}{}^{\dagger\gamma}, \tag{2.13}$$

$$g_8\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\rho}{}^{\gamma}(Y\phi^{\dagger})_{\delta}{}^{\rho}, \tag{2.14}$$

$$g_9\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\beta\delta}\phi_{\rho}{}^{\gamma}(Z\phi^{\dagger})_{\delta}{}^{\rho}, \tag{2.15}$$

$$g_{10}\psi_{\alpha\beta\gamma}{}^{\dagger}(Y\psi)^{\alpha\beta\delta}\phi_{\rho}{}^{\gamma}\phi_{\delta}{}^{\dagger\rho}, \tag{2.16}$$

$$g_{11}(Y\psi^{\dagger})_{\alpha\beta\gamma}\psi^{\alpha\beta\delta}\phi_{\rho}{}^{\gamma}\phi_{\delta}{}^{\dagger\rho}, \tag{2.17}$$

$$g_{12}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}\phi_{\delta}{}^{\beta}(Y\phi^{\dagger})_{\rho}{}^{\gamma}, \tag{2.18}$$

$$g_{13}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}\phi_{\delta}{}^{\beta}(Z\phi^{\dagger})_{\rho}{}^{\gamma}, \tag{2.19}$$

$$g_{14}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}(Y\phi)_{\delta}{}^{\beta}\phi_{\rho}{}^{\dagger\gamma}, \tag{2.20}$$

$$g_{15}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}(Z\phi)_{\delta}{}^{\beta}\phi_{\rho}{}^{\dagger\gamma}, \tag{2.21}$$

$$g_{16}(Y\psi^{\dagger})_{\alpha\beta\gamma}\psi^{\alpha\delta\rho}\phi_{\delta}{}^{\beta}\phi_{\rho}{}^{\dagger\gamma}. \tag{2.22}$$

Note that the terms with g_1' and g_2' do not contribute to any of the processes of interest to us. It is also to be noted that the terms with f_1 and with g_1, g_2, and g_3 contribute to elastic scattering only.

In writing these amplitudes we have adopted the convention[5]

$$\begin{aligned}
(Y\psi^{\dagger})_{\alpha\beta\gamma}&=S_{\alpha}{}^{\alpha'}\psi_{\alpha'\beta\gamma}{}^{\dagger}+S_{\beta}{}^{\alpha'}\psi_{\alpha\alpha'\gamma}{}^{\dagger}+S_{\gamma}{}^{\alpha'}\psi_{\alpha\beta\alpha'}{}^{\dagger},\\
(Y\psi)^{\alpha\beta\gamma}&=S_{\alpha'}{}^{\alpha}\psi^{\alpha'\beta\gamma}+S_{\alpha'}{}^{\beta}\psi^{\alpha\alpha'\gamma}+S_{\alpha'}{}^{\gamma}\psi^{\alpha\beta\alpha'},\\
(Y\phi)_{\alpha}{}^{\beta}&=S_{\alpha}{}^{\alpha'}\phi_{\alpha'}{}^{\beta}-S_{\alpha'}{}^{\beta}\phi_{\alpha}{}^{\alpha'},\\
(Z\phi)_{\alpha}{}^{\beta}&=S_{\alpha}{}^{\alpha'}\phi_{\alpha'}{}^{\beta}+S_{\alpha'}{}^{\beta}\phi_{\alpha}{}^{\alpha'}.
\end{aligned} \tag{2.23}$$

Thus after contracting with the spurion we have projected out the same representation and one can see that with a spurion like the one given by Eq. (1.1), the tensors $(Y\psi^{\dagger})_{\alpha\beta\gamma}$, $(Y\psi)^{\alpha\beta\gamma}$, and $(Y\phi)_{\alpha}{}^{\beta}$ are obtainable by multiplying every component of the original tensor by the associated hypercharge. As an illustration we have, for example,

$$(Y\phi)_{\alpha}{}^{\alpha'}\sim(\sigma_3 p_3)_i{}^{i'}W_A{}^{A'}, \tag{2.24}$$

where

$$W=\begin{bmatrix} 0 & 0 & K^+ \\ 0 & 0 & K^0 \\ -K^- & -\bar{K}^0 & 0 \end{bmatrix}. \tag{2.25}$$

[5] P. Babu, Nuovo Cimento **40**, 261 (1965).

In a similar manner the tensor $(Z\phi)_{\alpha}{}^{\alpha'}$ is given by

$$(Z\phi)_{\alpha}{}^{\alpha'}\sim(\sigma_3 p_3)_i{}^{i'}X_A{}^{A'}, \tag{2.26}$$

where

$$X=\begin{bmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & \pi^+ & -\frac{1}{2}K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & -\frac{1}{2}K^0 \\ -\frac{1}{2}K^- & -\frac{1}{2}\bar{K}^0 & \dfrac{4\eta}{\sqrt{6}} \end{bmatrix}. \tag{2.27}$$

III. RESULTS AND COMPARISON WITH EXPERIMENTS

With these remarks we now give below the important results of our computation.

(i) The Johnson-Treiman relations are no longer satisfied in the broken scheme. Nor is there any relation among the pion- and kaon-initiated reactions. In other words, the Barger-Rubin[8] version of Johnson-Treiman relations does not hold in the broken symmetry.

(ii) There is a simple relation for the (Ξ) production; we write it in terms of the total cross sections:

$$\frac{\sigma(K^-p\to K^+\Xi^-)}{\sigma(K^-p\to K^0\Xi^0)}=4. \tag{3.1}$$

Experimentally this ratio is known[6] to be 2.3 ± 0.5. In view of the large experimental error, we may conclude that the agreement is fairly satisfactory. Equation (3.1) is also valid both in exact[6] $SU(6)_S$ and $SU(6)_W$ schemes.

(iii) One of the bad predictions of exact $SU(6)_S$ and $SU(6)_W$, viz.,

$$\frac{\sigma(K^-p\to K^+\Xi^-)}{\sigma(K^-n\to K^0\Xi^-)}=4, \tag{3.2}$$

s however, also retained in our broken scheme. This relation is in very poor agreement with experiment.[6]

(iv) In static[6] $SU(6)$ and in exact $SU(6)_W$ the following relation holds:

$$\frac{\sigma(K^-p\to K^+\Xi^-)}{\sigma(\pi^-p\to K^+\Sigma^-)}=4, \tag{3.3}$$

which is also known to be in very poor agreement with experiment.[7] It is, however, interesting to note that (3.3) is absent in our scheme of broken $SU(6)_W$.

(v) Two other relations which follow in exact[6] $SU(6)$

[6] V. Barger and M. H. Rubin, Phys. Rev. Letters **14**, 713 (1965); J. C. Carter, J. J. Coyne, and S. Meshkov, *ibid.* **14**, 523 (1965).

[7] T. Binford, D. Cline, and M. Olsson, *ibid.* **14**, 715 (1965).

and also in exact $SU(6)_W$, are the following:

$$\sigma(K^-p\to K^+\Xi^-)=4\sigma(K^-p\to\pi^+\Sigma^-)\,, \tag{3.4}$$

and

$$\sigma(K^-p\to K^0\Xi^0)=\sigma(K^-p\to\pi^+\Sigma^-)\,. \tag{3.5}$$

[Note that Eq. (3.5) is a pure $SU(3)$ relation.] Both these relations are again in very poor agreement with experiment as is shown by Jackson. Significantly, none of these bad predictions exists in broken $SU(6)_W$. A comparison of our results for the processes of the type considered in this section with those obtained by breaking the $\tilde{U}(12)$ symmetry in various ways and as also of the exact $U(3)\otimes U(3)$, collinear symmetry[8] is instructive. By breaking the $SU(3)$ part alone, Babu[5] finds, for example, that the Johnson-Treiman relations do not hold any more. Oehme,[9] on the other hand, by breaking only the space-time part of $\tilde{U}(12)$ finds that the Johnson-Treiman relations retain their validity.

Recently Ruegg and Volkov[8] have applied the $U(3)\otimes U(3)$ symmetry for collinear processes to the study of baryon-meson scattering. These authors find that a linear combination of the Johnson-Treiman relations still holds. Further, the relations (3.1) through (3.5) are also valid.

In contrast to these situations we summarize our results of this section by noting that the important feature of this broken symmetry is that many of the bad predictions of the exact $SU(6)_S$ and of exact $SU(6)_W$ schemes are now absent, while some of the good ones are retained. Thus the broken $SU(6)_W$ symmetry has provided considerable improvement over the previous results although the Johnson-Treiman relations are no longer valid.

IV. SCATTERING PROCESSES LIKE $P+B\to P+D$

In this section we study reaction processes in which the final particles are a pseudoscalar meson and a baryon resonance. We have already listed in Sec. II all the invariant amplitudes which can contribute to these processes. It is, however, to be noted that there is only one invariant amplitude, viz. (2.4), in the exact symmetry which contributes to the processes $P+B\to P+D$ and out of the 18 invariant amplitudes which arise on breaking, only five, viz. (2.18) through (2.22), can give any contribution. Thus the scattering operator is

$$\begin{aligned}S=f_4\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}\phi_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}&+g_{12}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}\phi_\delta{}^\beta(Y\phi^\dagger)_\rho{}^\gamma\\&+g_{13}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}\phi_\delta{}^\beta(Z\phi^\dagger)_\rho{}^\gamma\\&+g_{14}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}(Y\phi)_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}\\&+g_{15}\psi_{\alpha\beta\gamma}{}^{\dagger}\psi^{\alpha\delta\rho}(Z\phi)_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}\\&+g_{16}(Y\psi^\dagger)_{\alpha\beta\gamma}\psi^{\alpha\delta\rho}\phi_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}.\end{aligned} \tag{4.1}$$

Thus we have 6 parameters connecting the various reactions. It is interesting to note that these same amplitudes also contribute to processes like $K^-p\to K^0\Xi^0$, etc. We find the following important results:

(i) Firstly we point out that in the exact $SU(6)_W$ symmetry the reactions $\langle K^-p|\pi^+\Sigma^-\rangle$, $\langle K^-p|K^0\Xi^0\rangle$, $\langle K^-p|K^+\Xi^-\rangle$, $\langle K^-p|\pi^-Y_1{}^{*+}\rangle$, $\langle K^-p|\pi^+Y_1{}^{*-}\rangle$, and $\langle\pi^+p|\pi^0N^{*++}\rangle$ are all expressible in terms of one parameter only. Jackson[2] has compiled data for these reactions and concludes that the disagreement with experiment is very profound. We have already mentioned in Sec. III(v) that $(K^-p\to\pi^+\Sigma^-)$ is no longer related to these reactions in broken $SU(6)_W$. However, we do find that

$$\sigma(K^-p\to Y_1{}^{*-}\pi^+)=4\sigma(K^-p\to\pi^-Y_1{}^{*+}) \tag{4.2}$$

still holds, and this is in disagreement with experiment.[2] However, the relation

$$\sigma(\pi^+p\to\pi^0N^{*++})=\tfrac{3}{2}\sigma(K^-p\to Y_1{}^{*+}\pi^-)\,, \tag{4.3}$$

which holds in exact $SU(6)_W$ and is in violent disagreement with experiment, no longer holds in broken symmetry.

(ii) The reaction amplitude for a few processes vanishes identically; e.g.,

$$\langle K^+p|N^{*++}K^0\rangle=0\,. \tag{4.4}$$

The above relation seems to be satisfied[3,10] at 1.14 BeV/c.

(iii) The following equality holds:

$$\langle\pi^+p|\pi^0N^{*++}\rangle=\sqrt{3}\langle\pi^+p|\eta N^{*++}\rangle\,. \tag{4.5}$$

(iv) Meshkov *et al.*[11] derived the following sum rule in $SU(3)$:

$$\begin{aligned}|\langle K^+p|K^0N^{*++}\rangle|^2=|\langle\pi^+p|\pi^0N^{*++}\rangle|^2\\+3|\langle\pi^+p|N^{*++}\eta\rangle|^2-3|\langle\pi^+p|K^+Y_1{}^{*+}\rangle|^2.\end{aligned} \tag{4.6}$$

From Eq. (4.4), the left side of (4.6) is zero. Now in broken $SU(6)_W$ we have only a relation of the type (4.5), i.e. (4.6) is no longer satisfied in broken $SU(6)_W$.

(v) By using the U-spin technique, the following simple $SU(3)$ equalities were derived[12]:

$$\begin{aligned}\tfrac{1}{3}|\langle\pi^-p|\pi^+N^{*-}\rangle|^2&=|\langle K^-p|K^+\Xi^{*-}\rangle|^2\\&=|\langle\pi^-p|K^+Y_1{}^{*-}|^2=|\langle K^-p|\pi^+Y_1{}^{*-}\rangle|^2.\end{aligned} \tag{4.7}$$

These also hold in exact $SU(6)_W$, but no longer hold in broken $SU(6)_W$. The only equality in broken $SU(6)_W$ connecting these four amplitudes is

$$\begin{aligned}\langle\pi^-p|N^{*-}\pi^+\rangle-\sqrt{3}\langle\pi^-p|K^+Y_1{}^{*-}\rangle\\=\sqrt{3}[\langle K^-p|Y_1{}^{*-}\pi^+\rangle-\langle K^-p|K^+\Xi^{*-}\rangle].\end{aligned} \tag{4.8}$$

[8] H. Ruegg and D. V. Volkov, Nuovo Cimento 43, 84 (1966). See also V. Barger and M. H. Rubin, Phys. Rev. 140, B1365 (1965).

[9] R. Oehme, Phys. Rev. Letters 14, 664 (1965).

[10] E. Boldt, J. Duboe, N. H. Duong, P. Eberhard, R. George, V. P. Henri, F. Levy, J. Poyen, M. Pripstein, J. Crussard, and A. Tran, Phys. Rev. 133, B220 (1964).

[11] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters 12, 87 (1964).

[12] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters 10, 361 (1963).

Equation (4.8) reduces to an identity both in exact $SU(3)$ and in exact $SU(6)_W$; this relation has also been derived in broken $SU(3)$.[13] It is known that Eq. (4.8) is very well satisfied experimentally.[13]

(vi) Another relation which follows in the broken and exact $SU(6)_W$ is the following:

$$\langle \pi^- p | \pi^- N^{*+} \rangle = (1/2\sqrt{3}) \langle \pi^- p | \pi^+ N^{*-} \rangle . \qquad (4.9)$$

Such a simple type of relation does not exist in $SU(3)$. It has been shown by Olsson[14] that a relation of the type (4.9) is very well satisfied experimentally.

V. CONCLUSION

In conclusion, we wish to emphasize that the broken $SU(6)_W$ successfully eliminates many of the bad predictions of exact $SU(6)_W$ and of exact $SU(6)_S$ and retains some of the good results as far as baryon-meson scattering is concerned. Similarly when one of the final scattered particles is a baryon resonance, some of the bad features disappear; at the same time some of the good results retain their validity. It is important to notice that the symmetry-breaking interaction which we have used to break the $SU(6)_W$ symmetry is isospin conserving and can not alter the exact $SU(6)_W$ ratio between two isospin amplitudes for processes in which the members of the same isomultiplets are involved. Thus the nonvalidity of the Johnson-Treiman relations involving reactions with different isospin multiplets is easily understood. The validity or nonvalidity of all other relations like the ones discussed in Secs. III and IV can similarly be explained.

[13] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **13**, 212 (1964).

[14] M. G. Olsson, Phys. Rev. Letters **15**, 710 (1965).

ACKNOWLEDGMENTS

The author would like to express his deep gratitude to Dr. Gyan Mohan for his guidance throughout the course of this investigation. He is extremely grateful to Professor R. C. Majumdar for his constant encouragement and for the facilities provided in the Physics Department. He wishes to express his sincere gratitude to Professor S. N. Biswas for several helpful discussions. Finally he would like to thank Dr. L. K. Pande for his generous collaboration in the early stages of this calculation and for pointing out an error in the manuscript.

PHYSICAL REVIEW VOLUME 160, NUMBER 5 25 AUGUST 1967

Observable Effects of the Leading Landau Singularity of the Box Graph*

PETER COLLAS AND RICHARD E. NORTON

Department of Physics, University of California, Los Angeles, California

(Received 19 December 1966; revised manuscript received 20 March 1967)

The effect of the leading Landau singularity of the fourth-order single-loop graph in a two-particle to four-particle amplitude is investigated. The amplitude for the graph is evaluated from a dispersion relation in the mass squared of the two particles interacting in the final state, and its behavior is studied using a simple model. The reactions $\bar{p}p \to \bar{K}K\pi\pi$ and $\bar{p}p \to \bar{\Lambda}\Lambda\pi^+\pi^-$ are examined in some detail, and peaks are predicted for the amplitude squared in a certain range of the external variables.

I. INTRODUCTION

It is of some interest to investigate the possibility that the singularities associated with some graphs may under certain circumstances have *direct* physical manifestations.

The possibility of an experimentally observable effect arising from triangle graphs has been the subject of intense investigation.[1] Unfortunately, the leading Landau singularity of the triangle graph is a logarithmic branch point and is thus rather "weak"; attempts to reinforce the amplitude enhancement by combining this logarithmic singularity with the inverse square root non-Landau branch point of the triangle (see, for example, Landshoff, Treiman, Halpern, and Watson[1]) have met with limited success (see, for example, Month, but also Cason *et al.*, in Ref. 1). This is due to the fact that the non-Landau branch point is always on an unphysical Riemann sheet and only under special circumstances can it approach the normal threshold; one has in fact to impose stringent mass constraints at

* Partially supported by the National Science Foundation. Based, in part, on a thesis submitted by P. Collas to the University of California at Los Angeles in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1] P. V. Landshoff and S. B. Treiman, Phys. Rev. **127**, 649 (1962); R. Aaron, Phys. Rev. Letters **10**, 32 (1963); F. R. Halpern and H. L. Watson, Phys. Rev. **131**, 2674 (1963); I. J. R. Aitchison, *ibid.* **133**, B1257 (1964); V. V. Anisovich and L. G. Dakhno, Phys. Letters **10**, 221 (1964); C. Goebel, *ibid.* **13**, 143 (1964); C. Kascer, *ibid.* **12**, 269 (1964); Y. F. Chang and S. F. Tuan, Phys. Rev. **136**, B741 (1964); M. Month, Phys. Letters **18**, 357 (1965), Phys. Rev. **139**, B1093 (1965); *ibid.* **151**, 1302 (1966); N. M. Cason, S. Mikamo, and A. Subramanian, Phys. Rev. Letters **17**, 838 (1966); C. Schmid, Phys. Rev. **154**, 1363 (1967); J. B. Bronzan, *ibid.* **134**, B687 (1964).