Reggeization of Elementary Particles

T. PRADHANN AND J. N. PASSI
Saha Institute of Nuclear Physics, Calcutta, India
(Received 9 March 1967)

Except in the scattering of vector bosons by spin-$\frac{1}{2}$ nucleons, elementary particles of conventional field theory correspond, in general, to Kronecker-delta singularities in the complex angular-momentum plane of the scattering amplitude, and if there is a Regge trajectory, it does not pass through the elementary-particle pole. However, this Regge trajectory induces a pole in the vertex function and a zero in the propagator of the elementary particle. These induced effects are such that under certain conditions on the renormalization constants, all the Kronecker-delta singularities cancel each other and the Regge trajectory moves to the elementary-particle pole. We demonstrate this Reggeization of elementary particles in a soluble-model field theory.

1. INTRODUCTION

It has been suggested by several authors[1] that the elementary particle of conventional field theory may be regarded as a composite particle when its wave-function renormalization constant Z_3 is set equal to zero. This can be demonstrated in field-theory models such as the Lee model and the Zachariasen model where the vertex function is simple. Although the elementary particles in these cases become composite, it is trivially seen that they do not lie on Regge trajectories but correspond to Kronecker-delta singularities in the complex angular-momentum plane.

In theories with nontrivial vertex functions, the condition $Z_3=0$ is not sufficient to make the elementary particle composite; Gerstein and Despande[2] have shown that additional restrictions must be imposed on the vertex renormalization constant. On the basis of the work of Jin and MacDowell,[3] they conjecture that the composite particle, to which the elementary particle passes, corresponds to a Regge pole. The mechanism by which the elementary-particle pole becomes a Regge pole is as follows: The Regge pole arising from the irreducible part of the scattering amplitude induces Kronecker-delta poles at the position where the Regge trajectory passes through $l=0$, in the vertex function and the inverse propagation function of the elementary particle. As Z_3 is made to approach zero keeping the self-mass of the particle $\delta\mu^2$ finite, the vertex pole moves towards the elementary-particle pole and in the limit coincides with it and cancels it; at the same time the Regge pole moves to the position of the elementary-particle pole. An almost similar type of Reggeization of the elementary particles has been shown in a field theory with two-particle unitarity[4] by Kaus and Zachariasen.[5] In the present paper we shall demonstrate this type of Reggeization in a soluble-model field theory not restricted by the two-particle unitarity. We consider a Lee model with U, V, N, and θ particles,[6] where the heavy particles U, V, and N are not fixed in space, but can have nonrelativistic motion. We show that the sum of the irreducible V-θ scattering diagrams has a Regge asymptotic behavior and that the Regge pole at $l=0$ induces poles in the $UV\theta$ vertex function and the inverse U-particle propagator. The Jin-MacDowell cancellation is exhibited and conditions under which the Kronecker-delta singularities cancel are examined. It is found that under these conditions, the Regge pole at $l=0$ moves to the position of the particle pole, thus Reggeizing this elementary particle.

2. THE SOLUBLE-MODEL FIELD THEORY

Our soluble-model field theory is described by the Hamiltonian

$$
\begin{aligned}
H &= H_0+H_I\,,\\
H_0 &= \int d^3p\left(\frac{p^2}{2m_V{}^{(0)}}+m_V{}^{(0)}\right)\psi_V{}^\dagger(\mathbf{p})\psi_V(\mathbf{p})+\int d^3p\left(\frac{p^2}{2M_0}+M_0\right)\psi_U{}^\dagger(\mathbf{p})\psi_U(\mathbf{p})\\
&\quad+\int d^3q\left(\frac{q^2}{2m_N}+m_N\right)\psi_N{}^\dagger(\mathbf{q})\psi_N(\mathbf{q})+\int d^3k\left(\frac{k^2}{2\mu}+\mu\right)\phi^\dagger(\mathbf{k})\phi(\mathbf{k})\,,\\
H_I &= \frac{g_0}{(2\pi)^{3/2}}\int d^3p\int d^3k\,\psi_V{}^\dagger(\mathbf{p})\psi_N(\mathbf{p}-\mathbf{k})F(p)f(k)\phi(\mathbf{k})+\text{H.c.}\\
&\quad+\frac{\lambda_0}{(2\pi)^{3/2}}\int d^3p\int d^3k\,\psi_U{}^\dagger(\mathbf{p}+\mathbf{k})\psi_V(\mathbf{p})F(p)f(k)\phi(\mathbf{k})+\text{H.c.}
\end{aligned}
\tag{1}
$$

[1] A. Salam, Nuovo Cimento **25**, 224 (1960); M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961); S. Weinberg, *ibid.* **130**, 776 (1963); B. W. Lee, K. T. Mahanthappa, I. S. Gerstein, and M. L. Whippman, Ann. Phys. (N. Y.) **28**, 466 (1964).
[2] I. S. Gerstein and N. G. Despande, Phys. Rev. **140**, B1643 (1965).
[3] Y. S. Jin and S. W. MacDowell, Phys. Rev. **137**, B688 (1965).
[4] It appears to be true quite generally in any local-field theory. See T. Saito, Phys. Rev. **152**, 1339 (1966).
[5] P. E. Kaus and F. Zachariasen, Phys. Rev. **138**, B1304 (1965).
[6] A Lee model of this type with recoilless U, V, and N has been solved by Bronzan; see J. Bronzan, Phys. Rev. **139**, B751 (1965).

The V-θ scattering can be represented by Feynman diagrams (Fig. 1), where the irreducible part, Fig. 1(b), consists of the ladder diagrams (Fig. 2). In these diagrams the double lines represent complete V-particle propagator whose integral equation is diagrammatically represented in Fig. 3. The complete V-particle propagator, therefore, is given by

$$\Delta_{FV}'(p_0,p)=Z_V[h(p_0,p)]^{-1}, \tag{2}$$

with

$$Z_V=1-\frac{g^2}{(2\pi)^3}\int\frac{d^3k\, f^2(k)F^2(p)}{([p^2/2m_V]+m_V-\omega_k-[(\mathbf{p}-\mathbf{k})^2/2m_N]-m_N)^2}$$

and

$$h(p_0,p)=\left(p_0-\frac{p^2}{2m_V}-m_V\right)\times\left[1+\frac{g^2}{(2\pi)^3}\int\frac{d^3k\, f^2(k)F^2(p)([p^2/2m_V]+m_V-p_0)}{([p^2/2m_V]+m_V-\omega_k-[(\mathbf{p}-\mathbf{k})^2/2m_N]-m_N)^2(p_0-\omega_k-[(\mathbf{p}-\mathbf{k})^2/2m_N]-m_N)}\right],$$

where

$$\omega_k=(k^2/2\mu)+\mu.$$

Here m_V is the renormalized V-particle mass and g is the renormalized $VN\theta$ coupling constant. The form factor $F(p)$ is chosen in such a fashion so as to make Z_V, the V-particle wave-function renormalization constant, independent of p.[7]

The integral equation for the U-particle propagator, appearing in the reducible part of the scattering matrix, is represented by Fig. 4. Therefore the complete propagator of the U particle, in its rest frame, is given by

$$\Delta_{FU}'(E)=[E-M_0-\Sigma(E)]^{-1}, \tag{3a}$$

where

$$\Sigma(E)=\frac{\lambda_0^2Z_V}{(2\pi)^3}\left[\int\frac{d^3k\, f^2(k)F^2(k)}{h(E-\omega_k,\, k)}+\int\int\frac{d^3kd^3k'T_1(E,\mathbf{k}\cdot\mathbf{k}')f(k)f(k')F(k)F(k')}{h(E-\omega_k,\, k)h(E-\omega_{k'},\, k')}\right]. \tag{3b}$$

$T_1(E,\mathbf{k}\cdot\mathbf{k}')$ is the irreducible part of the V-θ scattering amplitude. The U-particle wave-function renormalization constant and the renormalized mass M are given by

$$M=M_0+\Sigma(M), \tag{4a}$$

$$Z_U^{-1}=1-\left.\frac{\partial\Sigma(E)}{\partial E}\right|_{E=M}. \tag{4b}$$

This is equivalent to saying that the complete propagator has a pole at $E=M$ with residue Z_U. The $UV\theta$ vertex function appearing in the reducible part of the scattering amplitude, of Fig. 1(a), can be expressed in terms of T_1, as shown in Fig. 5. Therefore for the U particle at rest

$$\Gamma\left(E,\frac{p^2}{2m_V}+m_V\right)=\frac{\lambda_0f(p)F(p)}{(2\pi)^{3/2}}\left[1+\int\frac{d^3k\, T_1(E,\mathbf{p}\cdot\mathbf{k})f^{-1}(p)f(k)F^{-1}(p)F(k)}{h(E-\omega_k,\, k)}\right]. \tag{5}$$

The vertex-function renormalization constant, by definition, is

$$Z_1^{-1}=\frac{(2\pi)^{3/2}\Gamma(E=M,\,[p^2/2m_V]+m_V)}{\lambda_0f(p)F(p)}=1+\int\frac{d^3k\, T_1(M,\mathbf{p}\cdot\mathbf{k})f^{-1}(p)f(k)F^{-1}(p)F(k)}{h(M-\omega_k,\, k)}. \tag{6}$$

The renormalized $UV\theta$ coupling constant is given by

$$\lambda=Z_V^{1/2}Z_U^{1/2}Z_1^{-1}\lambda_0. \tag{7}$$

[7] This type of introduction of a form factor has been suggested by Fried and Sartori; see D. L. Fried and L. Sartori, Phys. Rev. **128**, 2879 (1962).

3. ASYMPTOTIC BEHAVIOR OF THE IRREDUCIBLE V-θ SCATTERING DIAGRAMS

In this section we shall consider the behavior of the irreducible part of V-θ scattering amplitude in the limit of large momentum transfer in each order of perturbation expansion and show that it exhibits Regge asymptotic behavior. It is easy to see that in this limit the lowest-order graph, Fig. 2(a), tends to

$$T_1^{(2)}(E,z) \to -\frac{g^2 f^2(p) F^2(p) m_N}{(2\pi)^3 p^2}\frac{1}{z}, \tag{8}$$

where

$$z = \mathbf{p}_i \cdot \mathbf{p}_f / p^2 .$$

For the next higher order given in Fig. 2(b), the Feynman matrix element is

$$T_1^{(4)} = \frac{g^4}{(-i)(2\pi)^7}\int \frac{d^4k\, f^2(p) f^2(k) F^2(p) F^2(k)}{(k_0-\omega_k)\left(\dfrac{p^2}{2m_V}+m_V-k_0-\dfrac{(\mathbf{p}_i-\mathbf{k})^2}{2m_N}-m_N\right)\left(\dfrac{p^2}{2m_V}+m_V-k_0-\dfrac{(\mathbf{p}_f-\mathbf{k})^2}{2m_N}-m_N\right)h(E-k_0,k)} .$$

The integration can be performed with the aid of Feynman parameters and the final result in the limit of large z is

$$T_1^{(4)} \to \frac{g^2 f^2(p) F^2(p) m_N}{(2\pi)^3 p^2}\frac{1}{z} \times\left[\frac{g^2 m_N}{2\pi^2}\int \frac{dk\, f^2(k) F^2(k)}{h(E-\omega_k,k)} \ln z\right]. \tag{9}$$

For the sixth-order graph given in Fig. 2(c), the integrals occuring in the Feynman matrix elements cannot be carried out analytically. However, the large-z limit can be obtained by the standard methods discussed, for example, by Federbush and Grisaru,[8] Polkinghorne,[9] Tiktopoulos,[10] and many others. The details of the procedure for our sixth-order matrix element are worked out in Appendix A. The result of this calculation is

$$T_1^{(6)} \to -\frac{g^2 f^2(p) F^2(p) m_N}{(2\pi)^3 p^2}\frac{1}{z}\frac{1}{2!} \times\left[\frac{g^2 m_N}{2\pi^2}\int \frac{dk\, f^2(k) F^2(k)}{h(E-\omega_k,k)} \ln z\right]^2 . \tag{10}$$

Examination of the asymptotic limits of $T_1^{(2)}$, $T_1^{(4)}$, and $T_1^{(6)}$ reveals that they are the first three terms of the series expansion of the function

$$T_1 = -\frac{g^2 f^2(p) F^2(p) m_N}{(2\pi)^3 p^2}(z)^{\alpha(E)}, \tag{11}$$

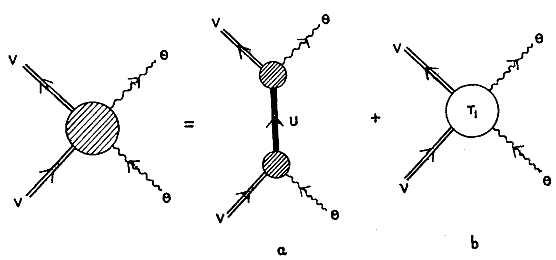

FIG. 1. Complete V-θ scattering amplitude.

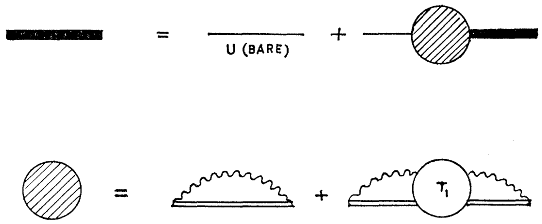

FIG. 2. Integral equation of U-particle propagator.

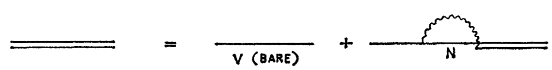

FIG. 3. Integral equation of V-particle propagator.

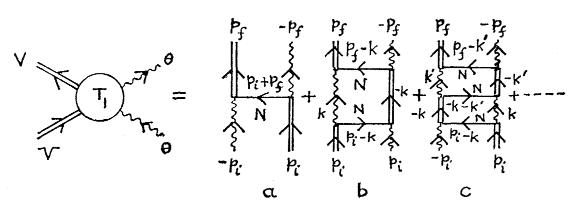

FIG. 4. Perturbation expansion of the irreducible part of V-θ scattering amplitude.

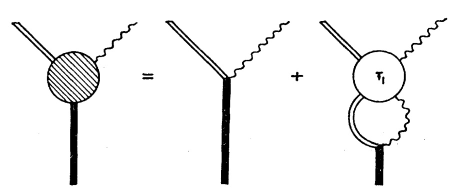

FIG. 5. Vertex function expressed in terms of irreducible part of V-θ scattering amplitude.

[8] P. Federbush and M. Grisaru, Ann. Phys. (N. Y.) **22**, 263 (1963); **22**, 299 (1963).
[9] J. Polkinghorne, J. Math. Phys. **4**, 503 (1963).
[10] G. Tiktopoulos, Phys. Rev. **131**, 2373 (1963).

where

$$\alpha(E)=-1-\frac{m_N g^2}{2\pi^2}\int\frac{dk\, f^2(k)F^2(k)}{h(E-\omega_k,\,k)}. \qquad (12)$$

We shall presume that the asymptotic limits of the higher-ordered diagrams will form the higher members of this series expansion. In that case it is quite clear that the irreducible part of the V-θ scattering amplitude exhibits Regge asymptotic behavior. This result can also be obtained by an independent method which consists of taking partial-wave projection of the integral equation for T_1 represented diagrammatically in Fig. 6, and then solving it. This is done in the Appendix B. This method gives us the residues of the Regge poles. It is easy to check that $\alpha(E)$, the Regge-pole parameter, is real below threshold and complex above threshold. The zero of $\alpha(E)$ occurs below threshold showing the existence of an S-wave bound-state pole in T_1.

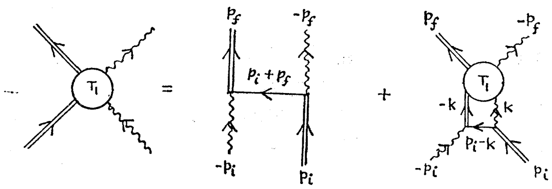

FIG. 6. Integral equation for the irreducible part of the V-θ scattering amplitude.

4. INDUCED POLES OF VERTEX AND INVERSE PROPAGATION FUNCTIONS

We shall show in this section that the Regge pole in T_1 gives rise to poles in the exact vertex and inverse propagation functions in such a way that the net contribution of these three poles to the scattering matrix is identically zero. Let the value of E for $\alpha(E)=0$ be m, i.e.,

$$\frac{m_N g^2}{2\pi^2}\int\frac{dk\, f^2(k)F^2(k)}{h(m-\omega_k,\,k)}=-1. \qquad (13)$$

Corresponding to this pole at $\alpha=0$ in the angular-momentum plane, there occurs a pole of T_1 in the energy plane at $E=m$, the residue of which has been calculated in the Appendix B. We therefore have

$$T_1(E\rightarrow m)=\left[\frac{f^2(p)F^2(p)}{(2\pi)^3p^2}\right]\frac{R_1}{E-m}, \qquad (14a)$$

where

$$R_1=g^2m_N/\mathcal{G}(m). \qquad (14b)$$

Our vertex function Γ, given in Sec. 2, has been expressed in terms of T_1. From that expression it is clear that Γ will have a pole at $E=m$ corresponding to the pole of T_1 discussed above. In the neighborhood of this pole, the vertex function is given by

$$\Gamma(E\rightarrow m)=\left[\frac{f(p)F(p)}{(2\pi)^{3/2}p}\right]\frac{R_2}{E-m}, \qquad (15a)$$

where

$$R_2=\frac{\lambda g^2 m_N Z_1}{\mathcal{G}(m)(Z_UZ_V)^{1/2}}\int\frac{d^3k\, f^2(k)F^2(k)}{(2\pi)^3kh(m-\omega_k,\,k)}. \qquad (15b)$$

The inverse U-particle propagation function will also have a pole at $E=m$. From our expression for this function given in Sec. 2, Eq. (3a), we have

$$[\Delta_{FU}'(E\rightarrow m)]^{-1}=-\frac{Z_VR_3}{E-m}, \qquad (16a)$$

where

$$R_3=\left[\frac{\lambda^2g^2m_NZ_1^2}{\mathcal{G}(m)Z_UZ_V}\int\int\frac{d^3kd^3k'\,f^2(k)f^2(k')F^2(k)F^2(k')}{(2\pi)^6kk'h(m-\omega_k,\,k)h(m-\omega_{k'},\,k')}\right]. \qquad (16b)$$

The complete V-θ scattering amplitude can be written as

$$T_l(E)\equiv\frac{f^2(p)F^2(p)}{(2\pi)^3p^2}t_l(E)=\delta_{l0}Z_V\Gamma^2\left(E,\,\frac{p^2}{2m_V}+m_V\right)\Delta_{FU}'(E)+T_{1l}(E). \qquad (17)$$

Using Eqs. (14b), (15b), (16b), and (17) we see that $t_l(E)$ has the following pole structure:

$$t_{l=0}(E)=\frac{Z_VZ_U\Gamma^2(E=M,[p^2/2m_V]+m_V)}{E-M}\left[\frac{(2\pi)^3p^2}{f^2(p)F^2(p)}\right]-\frac{R_2^2/R_3}{E-m}+\frac{R_1}{E-m}+\cdots$$

$$=\frac{Z_VZ_U\Gamma^2(E=M,[p^2/2m_V]+m_V)}{E-M}\left[\frac{(2\pi)^3p^2}{f^2(p)F^2(p)}\right]+\frac{(R_1R_3-R_2^2)/R_3}{E-M}+\cdots.$$

It is easy to check that $R_2^2=R_1R_3$. We thus have the well-known Jin-MacDowell cancellation[4] between the vertex pole and the Regge pole at $\alpha=0$.

5. REGGEIZATION OF THE U PARTICLE

In the energy plane, the S-wave vertex pole and the Regge pole at $\alpha=0$ cancel each other, and $t_{l=0}(E)$ is left with only the elementary-particle pole at $E=m$. In the complex angular-momentum plane, however, the situation is different. The elementary-particle- and the vertex-pole terms of the scattering amplitude are Kronecker-delta singularities, whereas the irreducible part has a Regge pole, i.e.,

$$t_l(E)=\delta_{l0}\left[\frac{R_E}{E-M}-\frac{R_2^2/R_3}{E-m}\right]+\frac{\beta(E)}{l-\alpha(E)}+\cdots, \tag{18}$$

where

$$R_E=Z_VZ_U\Gamma^2\left(E=M,\frac{p^2}{2m_V}+m_V\right)\left[\frac{(2\pi)^3p^2}{f^2(p)F^2(p)}\right]. \tag{19}$$

$T_{1l}(E)$ has a pole at energy where the Regge trajectory passes $l=0$. Inspection of Eq. (6) reveals that the position of this pole of $T_{1l}(E)$ depends on the renormalization constant Z_1. Therefore we can make both the Regge pole and the vertex pole move by changing Z_1. From Eq. (6) we get

$$Z_1\sim(M-m)\mathcal{G}(M)\Bigg/\left[(M-m)\mathcal{G}(M)+\frac{m_Ng^2}{p}\int\frac{d^3kf^2(k)F^2(k)}{(2\pi)^3kh(M-\omega_k,k)}\right]\quad\text{for } m\sim M. \tag{20}$$

It is thus clear that $m\to M$ as $Z_1\to 0$ and in the limit the vertex pole, the Regge pole with $\alpha=0$ and the elementary-particle pole coincide. If in addition, the residues of the elementary-particle pole and the vertex pole could be made equal, i.e.,

$$R_E=R_2^2/R_3,$$

then the Kronecker-delta singularities would completely disappear, leaving a Regge pole at the position of the elementary-particle pole. Since the residue of the pole of $t_{l=0}(E)$ in the energy plane corresponding to the Regge pole also equals R_2^2/R_3 (note that this leads to the Jin-MacDowell cancellation) as discussed in the previous section, it can be said that the Regge pole has completely replaced the elementary-particle pole which gets cancelled by the vertex pole. In other words, the elementary particle gets Reggeized under these conditions: $Z_1=0$ and $R_E=R_2^2/R_3$. Using Eqs. (15b), (16b), and (19) we see that the latter condition is equivalent to

$$1=\lambda^2p^2\mathcal{G}(M)/m_Ng^2. \tag{21}$$

By further making use of Eq. (20) we see that for $Z_1\to 0$ Eq. (21) becomes

$$1=\frac{\lambda^2m_NZ_1^2g^2}{\mathcal{G}(M)(M-m)^2}\int\int\frac{d^3kd^3k'f^2(k)f^2(k')F^2(k)F^2(k')}{(2\pi)^6kk'h(M-\omega_k,k)h(M-\omega_{k'},k')}. \tag{22}$$

From Eqs. (4b) and (14a) we see that when $Z_1\to 0$, Eq. (22) is equivalent to $Z_U=0$. Hence the latter condition is fulfilled if $Z_U=0$ in addition to $Z_1=0$. Although the elementary-particle pole gets replaced by the Regge pole under these conditions, the theory is not yet completely dynamical. This is because the contribution of the reducible part of $t_l(E)$ at points other than $E=M$ still remains. For the theory to be completely dynamical these contributions must also disappear, leaving only the Regge-pole contributions. Since the reducible part in these regions can be shown to be proportional to Z_1^2/Z_U, we must demand that $Z_1^2/Z_U=0$ in addition to $Z_1=0$ and $Z_U=0$ for complete Reggeization of the theory.

ACKNOWLEDGMENT

One of us (JNP) is grateful to the Department of Atomic Energy, Government of India, for financial support.

APPENDIX A

In order to show the required Regge asymptotic behavior of $T_1^{(6)}(E,z)$, we shall pick up only the asymptotic leading term of the form $(\ln z)^2/z$ from the sixth-order graph, Fig. 2(c). The sixth-order matrix element is given by

$$T_1^{(6)}=\frac{-g^6}{(2\pi)^{11}}\int\int\left[\frac{d^4kd^4k'f^2(p)f^2(k)f^2(k')F^2(p)F^2(k)F^2(k')h^{-1}(E-k_0,k)h^{-1}(E-k_0',k')}{(k_0-\omega_k)\left(\frac{p^2}{2m_V}+m_V-k_0-\frac{(\mathbf{p}_i-\mathbf{k})^2}{2m_N}-m_N\right)(k_0'-\omega_{k'})\left(\frac{p^2}{2m_V}+m_V-k_0'-\frac{(\mathbf{p}_f-\mathbf{k}')^2}{2m_N}-m_N\right)}\right.$$
$$\left.\times\left(E-k_0-k_0'-\frac{(-\mathbf{k}-\mathbf{k}')^2}{2m_N}-m_N\right)^{-1}\right]. \quad \text{(A1)}$$

Carrying out k_0 and k_0' integrations and separating out the angular integrations, we get

$$T_1^{(6)}=\frac{g^6}{(2\pi)^9}\int\int\frac{k^2dkk'^2dk'f^2(p)f^2(k)f^2(k')F^2(p)F^2(k)F^2(k')}{h(E-\omega_k,k)h(E-\omega_{k'},k')}$$
$$\times\int\int\frac{d\Omega_kd\Omega_{k'}}{\left(\frac{p^2}{2m_V}+m_V-\omega_k-\frac{(\mathbf{p}_i-\mathbf{k})^2}{2m_N}-m_N\right)\left(E-\omega_k-\omega_{k'}-\frac{(-\mathbf{k}-\mathbf{k}')^2}{2m_N}-m_N\right)\left(\frac{p^2}{2m_V}+m_V-\omega_{k'}-\frac{(\mathbf{p}_f-\mathbf{k}')^2}{2m_N}-m_N\right)}. \quad \text{(A2)}$$

The $d\Omega_k$ integration can be performed by introducing Feynman parameters and similarly the $d\Omega_{k'}$ integration can be done. The result for the angular integration part of $T_1^{(6)}$ is

$$=-16\pi^2\int_0^1\int_0^1\frac{dx_1dx_2}{4x_1(1-x_1)x_2(1-x_2)m_N^{-3}p^2k^2k'^2z-D(x_1,x_2)}, \quad \text{(A3)}$$

where

$$D(x_1,x_2)=a_2^2-\{4(1-x_1)^2x_1^2(1-x_2)^2k^4p^2m_N^{-4}+p^2x_2^2m_N^{-2}\}k'^2,$$
$$a_2^2=\left\{a_1^2-\frac{k^2}{m_N^2}[p^2x_1^2+k'^2(1-x_1)^2]\right\}(1-x_2)+\left\{\frac{p^2}{2m_V}+m_V-\frac{p^2+k'^2}{2m_N}-m_N-\omega_{k'}\right\}x_2,$$
$$a_1^2=\left\{\frac{p^2}{2m_V}+m_V-\omega_k-\frac{p^2+k^2}{2m_N}-m_N\right\}x_1+\left\{E-\omega_k-\omega_{k'}-\frac{k^2+k'^2}{2m_N}-m_N\right\}(1-x_1).$$

We shall confine our interest only in the dominant terms of the expression (A3) for large z. The only range of parameters which will extract the desired behavior are from 0 to ϵ_1 and from $1-\epsilon_1$ to 1 for x_1 and from 0 to ϵ_2 and from $1-\epsilon_2$ to 1 for x_2, where $\epsilon_1\ll1$ and $\epsilon_2\ll1$. Writing the expression (A3) for the desired range of parameters, we get

$$=-16\pi^2\left[\int_0^{\epsilon_1}\int_0^{\epsilon_2}\frac{dx_1dx_2}{4x_1x_2(1-x_1)(1-x_2)m_N^{-3}p^2k^2k'^2z-D(x_1,x_2)}\right.$$
$$+\int_0^{\epsilon_1}\int_{1-\epsilon_2}^1\frac{dx_1dx_2}{4x_1x_2(1-x_1)(1-x_2)m_N^{-3}p^2k^2k'^2z-D(x_1,x_2)}+\int_{1-\epsilon_1}^1\int_0^{\epsilon_2}\frac{dx_1dx_2}{4x_1x_2(1-x_1)(1-x_2)m_N^{-3}p^2k^2k'^2z-D(x_1,x_2)}$$
$$\left.+\int_{1-\epsilon_1}^1\int_{1-\epsilon_2}^1\frac{dx_1dx_2}{4x_1x_2(1-x_1)(1-x_2)m_N^{-3}p^2k^2k'^2z-D(x_1,x_2)}\right]. \quad \text{(A4)}$$

In the first term of the expression (A4) we put $x_1=0$ and $x_2=0$ in $D(x_1,x_2)$. This term reduces to

$$-16\pi^2\int_0^{\epsilon_1}\int_0^{\epsilon_2}\frac{dx_1dx_2}{4x_1x_2m_N^{-3}p^2k^2k'^2z-D(x_1=0,\,x_2=0)}.$$

After performing x_1 and x_2 integrations in the limit of large z we get

$$=-\frac{2\pi^2m_N}{p^2k^2k'^2}\frac{(\ln z)^2}{z}.$$

The other terms of (A4) can be evaluated similarly. Substituting these in expression (A2), we get

$$T_1^{(6)}=-\frac{g^2f^2(p)F^2(p)m_N}{(2\pi)^3p^2}\frac{1}{z}\frac{1}{2!}\left[\frac{g^2m_N}{2\pi^2}\int\frac{dkf^2(k)F^2(k)}{h(E-\omega_k,k)}\ln z\right]^2. \tag{A5}$$

APPENDIX B

The integral equation for $T_1(E,z)$ represented by Fig. 6 is

$$T_1(E,z)=\frac{g^2f^2(p)F^2(p)}{(2\pi)^3\left[\frac{p^2}{2m_V}+m_V-\omega_p-\frac{(\mathbf{p}_i+\mathbf{p}_f)^2}{2m_V}-m_N\right]}+\frac{g^2}{(2\pi)^4(-i)}\int\frac{d^4kf(p)f(k)F(p)F(k)T_1(E,\mathbf{p}_f\cdot\mathbf{k})h^{-1}(E-k_0,k)}{(k_0-\omega_k)\left(\frac{p^2}{2m_V}+m_V-k_0-\frac{(\mathbf{p}_i-\mathbf{k})^2}{2m_N}-m_N\right)}. \tag{B1}$$

The partial-wave amplitude can be obtained by using

$$\left[\frac{p^2}{2m_V}+m_V-\omega_p-\frac{(\mathbf{p}_i+\mathbf{p}_f)^2}{2m_N}-m_N\right]^{-1}=\frac{m_N}{p^2}\sum_l(2l+1)Q_l\left(\frac{[p^2/2m_V]+m_V-\omega_p-[p^2/m_N]-m_N}{2p^2}\right)P_l(z) \tag{B2}$$

and

$$T_1(E,\mathbf{p}_i\cdot\mathbf{p}_f)=\sum_l(2l+1)T_{1l}(E)P_l(z), \tag{B3}$$

and carrying out angular integrations. This integral equation for the partial-wave amplitude is

$$T_{1l}(E)=\frac{g^2f^2(p)F^2(p)m_N}{(2\pi)^3p^2}Q_l\left(\frac{[p^2/2m_V]+m_V-\omega_p-[p^2/m_N]-m_N}{2p^2}\right)+\frac{2m_Ng^2}{i(2\pi)^3p}\int\int\frac{dkdk_0kf(p)f(k)F(p)F(k)T_{1l}(E,k)}{(k_0-\omega_k)h(E-k_0,k)}Q_l\left(\frac{[p^2/2m_V]+m_V-k_0-[(p^2+k^2)/2m_N]-m_N}{2kp}\right). \tag{B4}$$

The analytic continuation of $T_{1l}(E)$ in complex angular-momentum plane can be done, since the analyticity of Q_l functions in the l plane is well known. We are interested in the solution of this equation in the neighborhood of $l=-1$. Since for $l\sim-1$,

$$Q_l\sim1/(l+1),$$

the kernel of Eq. (B4) becomes the separable type, the solution can be done by the method of iteration. The desired solution is

$$T_{1\alpha}(E)=\frac{g^2f^2(p)F^2(p)m_N}{(2\pi)^3p^2D(\alpha,E)}, \tag{B5}$$

where

$$D(\alpha,E)=\alpha(E)+1+\frac{m_Ng^2}{2\pi^2}\int\frac{dk\,f^2(k)F^2(k)}{h(E-\omega_k,k)}. \tag{B6}$$

Hence the Regge trajectory is given by

$$\alpha(E)=-1-\frac{m_Ng^2}{2\pi^2}\int\frac{dk\,f^2(k)F^2(k)}{h(E-\omega_k,k)}.$$

The existence of the S-wave V-θ bound state at $E=m$ implies that the Regge trajectory passes through the point

$l=0$ at $E=m$,

$$\alpha(E=m)=0=-1-\frac{m_N g^2}{2\pi^2}\int\frac{dk\, f^2(k)F^2(k)}{h(m-\omega_k,\, k)}.$$

Corresponding to this Regge pole of T_1 there is a pole in the energy plane at $E=m$. The residue at the pole at $E=m$ of the irreducible part of the V-θ scattering amplitude, as seen from Eq. (B5), is

$$\frac{g^2 f^2(p)F^2(p)m_N}{(2\pi)^3 p^2 \mathcal{G}(m)}, \tag{B7}$$

where

$$\mathcal{G}(E)=\frac{D(E)}{E-m}.$$

PHYSICAL REVIEW VOLUME 160, NUMBER 5 25 AUGUST 1967

Meson-Baryon Scattering in Broken $SU(6)_W$

SACHCHIDA NANDA GUPTA*
Department of Physics, University of Delhi, Delhi, India
and
Theoretical Physics Division, National Physical Laboratory, New Delhi, India

(Received 12 January 1967)

We have studied the scattering processes like $P+B\to P+B$ and $P+B\to P+D$ within the framework of broken $SU(6)_W$, the breaking being provided by a W-spin scalar spurion having $I=0=Y$ and belonging to the 35-dimensional adjoint representation of $SU(6)_W$. It is found that many of the bad results of the exact $SU(6)_S$ and exact $SU(6)_W$ schemes are absent. At the same time, some of the results which agree with experiment are retained. The Johnson-Treiman relations, however, are no longer valid.

I. INTRODUCTION

In a recent paper[1] we studied the meson-baryon couplings within the framework of exact and broken $SU(6)_W$ symmetry. It was found that whereas the predictions of exact symmetry are not very good, those of broken symmetry are in very good agreement with experiment in the case of decuplet decays; in the case of meson-baryon couplings, the predictions seem to be consistent with present experimental knowledge about them.

It was shown by Jackson[2] that the $SU(6)_W$ predictions for meson-baryon scattering in the exact symmetry[3] are in very poor agreement with experiment except for the Johnson-Treiman[4] relations. In view of the better agreement of the couplings with experiment as predicted by the broken $SU(6)_W$, we are encouraged to investigate the scattering process within the framework of broken $SU(6)_W$ and examine whether there is any improvement for the scattering predictions. We shall confine our attention to the processes of the type

$$\text{(A)}\qquad P+B\to P+B,$$

and

$$\text{(B)}\qquad P+B\to P+D,$$

where P, B, and D are taken to mean pseudoscalar meson, baryon, and baryon resonance, respectively.

We break the symmetry by a W-spin scalar spurion having $I=0=Y$ and belonging to the 35-dimensional adjoint representation of the group $SU(6)_W$. Such a spurion is given by

$$S_\alpha{}^{\alpha'}=\delta_i{}^{i'}(\lambda_8)_A{}^{A'}, \tag{1.1}$$

where

$$\lambda_8\sim\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}. \tag{1.2}$$

II. MESON-BARYON SCATTERING

In exact symmetry the scattering is described by the following 4 invariant amplitudes:

$$f_1\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\gamma}\phi_\rho{}^\delta\phi_\delta{}^{\dagger\rho}, \tag{2.1}$$

$$f_2\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\delta}\phi_\delta{}^\rho\phi_\rho{}^{\dagger\gamma}, \tag{2.2}$$

$$f_3\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\beta\delta}\phi_\rho{}^\gamma\phi_\delta{}^{\dagger\rho}, \tag{2.3}$$

$$f_4\psi_{\alpha\beta\gamma}{}^\dagger\psi^{\alpha\delta\rho}\phi_\delta{}^\beta\phi_\rho{}^{\dagger\gamma}. \tag{2.4}$$

---

* Present address: Roorkee University, Roorkee, India.

[1] Sachchida Nanda Gupta, Phys. Rev. **151**, 1235 (1966). We have followed the notation of this paper. Other references on the application of the group $SU(6)_W$ are listed in this paper.

[2] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[3] J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters **15**, 373 (1965). These authors have studied the various scattering processes in the exact $SU(6)_W$ symmetry using the Clebsch-Gordan coefficients. Our results, obtained here by using the tensor techniques are in agreement with theirs for the exact symmetry if we put all g's$=0$.

[4] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965).