In the same way, the average number of particles is

$$\langle n\rangle=\sum_{n=2}^{\sim\ln s} n\sigma^{(n)}(s)/\sigma_T(s)\sim\ln s. \tag{24}$$

We also deduce that the ratio $\sigma^{(n)}(s)/\sigma^{(n')}(s)$, that is, the ratio of the probability to produce n particles to that to produce n' particles, is independent of s.

The basic ingredient in both (23) and (24) is the assumption that we can neglect all kinematic regions in which any t_i is large. Without this restriction, the maximum number of particles allowed at a given s might be expected to be as much as

$$n=(\sqrt{s})/\mu\,,$$

where μ is the average rest mass of the particle in the final state.

The principal conclusions to be drawn about the multi-Regge-pole hypothesis then are (1) It is internally consistent. (2) It is best tested by looking for diffraction-peak shrinkage and dips in the differential cross section. (3) If the Pomeranchuk trajectory is a Regge pole, then at ultra-high energies, the particle multiplicity grows like $\ln s$, and the cross section to produce n particles falls like $1/\ln a$.

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High-Energy Limit of Photon Scattering on Hadrons*†

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We show that the presence of a fixed pole at $J=1$ in the process $\gamma+\gamma\rightarrow h+\bar{h}$ reinstates the coupling of the γ-γ state to the vacuum trajectory, and hence permits a finite total photon cross section as $s\rightarrow\infty$.

I. INTRODUCTION

It has been observed by Abarbanel and Nussinov[1] and Mur[2] that in a naive Regge-pole model, the Pomeranchuk Regge trajectory with $\alpha(0)=1$ does not contribute to the forward nonhelicity-flip Compton amplitude. Hence, by the optical theorem the total photoabsorption cross section will go to zero as $E\rightarrow\infty$. We will show that as a consequence of the linear unitarity (more precisely, the absence of bilinear unitarity) for processes to low order in the weak and electromagnetic interactions there exist fixed poles at nonsense values of the angular momentum. The fixed pole in the angular-momentum variable J will exist together with the Regge pole (if it exists) in a multiplicative fashion. In particular, there is a fixed pole in the Compton amplitude at the point $J=1$, the first nonsense wrong signature point for the relevant partial-wave amplitude for the crossed channel. The fixed pole at $J=1$ will not contribute to the physical scattering amplitude, but will contribute in such a manner as to restore the contribution of the Pomeranchuk trajectory to the forward nonhelicity-flip Compton amplitude.

Section II is devoted to some kinematic preliminaries for the Compton amplitude. In Sec. III the fixed pole in the Compton case will be derived for the same model which produces a fixed pole in the current commutator case.[3] In Sec. IV we will show the existence of fixed poles in general as a consequence of the unitarity of lowest-order weak and electromagnetic processes. Finally, Sec. V will be devoted to conclusions and speculations.

All of our remarks on Compton scattering will be

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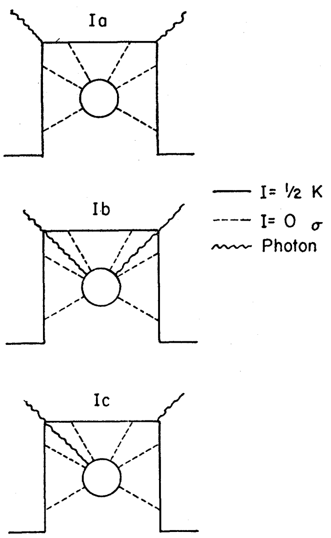

FIG. 1. (a) The diagrams included in our approximation. (b) The diagrams not included in our approximation. (c) The diagrams that do not contribute because of isotopic spin.

demonstrated on scalar hadron targets and to order e^2. The extension to non-spinless targets will be straightforward.

II. KINEMATIC PRELIMINARIES

The Compton amplitude $T_{\mu\nu}$ can be expressed in terms of two invariant amplitudes: $\gamma(q_1)+h(p_1)\to\gamma(q_2)+h(p_2)$.

$$T_{\mu\nu}=T_1(P_\mu P_\nu q_1\cdot q_2-\nu(q_{1\mu}P_\nu+q_{2\nu}P_\mu)+\nu^2\delta_{\mu\nu})+T_2(\delta_{\mu\nu}q_2\cdot q_1-q_{1\mu}q_{2\nu}), \quad (2.1)$$

where

$$P_\mu=\tfrac{1}{2}(p_1+p_2) \quad\text{and}\quad \nu=(s-u)/4.$$

The forward Compton amplitude is obviously

$$H_{\lambda'\lambda}(q_2,p_2;q_1,p_1)=\epsilon_{\lambda'}{}^{\mu *}\epsilon_\lambda{}^{\mu}\nu T_1 \quad (2.2)$$

and its imaginary part is related to the total photoabsorption cross section by

$$\sigma_T=\tfrac{1}{4}s\,\mathrm{Im}T_1. \quad (2.3)$$

The authors of Refs. 1 and 2 have shown that normal Regge behavior of H implies that $\mathrm{Im}T_1(\nu)\to\nu^{\alpha-2}(\alpha-1)b(t)$ in the neighborhood of $\alpha=1$, where α is the leading Regge trajectory in the t channel. Thus, a nonsingular reduced residue b would imply a vanishing contribution of the trajectory near $\alpha=1$ and hence a vanishing cross section as $\nu\to\infty$. On the other hand, if the coefficient $b(t)$ has a pole at $\alpha(t)=1$, then the total cross section would be finite as $\nu\to\infty$. This is precisely the behavior which is predicted by our models.

III. DEGENERATE MODEL

The model we consider consists of $I=\frac{1}{2}$, spin-zero particles (K's) interacting with scalar isoscalars (σ's?) through a $\bar{\mathbf{K}}\cdot\mathbf{K}\sigma$ coupling. We ignore, as usual, the well-known difficulties associated with super-renormalizable theories, since they are irrelevant to the question under consideration. With this model, we consider a further gauge-invariant approximation which excludes both initial and final photons coupling to closed loops of the $I=\frac{1}{2}$ particles. Thus, we would keep the Feynman diagram Fig. 1(a) and discard the diagram Fig. 1(b). Diagrams like Fig. 1(c) we may keep, but they are in any case zero by isotopic-spin conservation.

If we permit the photons to be charged, then the isotopic-spin dependence of all the diagrams of Fig. 1(a) is given by

$$T_{\mu\nu}{}^{\beta\alpha(a)}=\tau_\beta\tau_\alpha T_{\mu\nu}{}^{(a)}(s,t)+\tau_\alpha\tau_\beta T_{\nu\mu}{}^{(a)}(u,t) \quad (3.1)$$

(where we have included the crossed diagrams), whereas that of Fig. 1(b) will be

$$T_{\mu\nu}{}^{\beta\alpha(b)}=\delta_{\beta\alpha}T_{\mu\nu}{}^{(b)}(s,t). \quad (3.2)$$

Thus, the diagrams of Fig. 1(b) do not contribute to the $I=1$ amplitude of the t channel, whereas those of Fig. 1(a) contribute to both $I=1$ and $I=0$ in the t channel.

We now recall the Dashen–Gell-Mann–Fubini sum rule[4]:

$$q_1\cdot q_2\int \mathrm{Im}T_1{}^a(s,t)ds=g(t), \quad (3.3)$$

where $g(t)$ is the form factor of the $I=\frac{1}{2}$ particle. Some caution should be exercised in interpreting Eq. (3.3) in the neighborhood of $t=2q_1\cdot q_2=0$. By current conservation, $g(t)=1$. On the left-hand side, $\mathrm{Im}T_1{}^a(s,t)$ has a kinematic pole at $t=0$ due to the one-particle state:

$$\mathrm{Im}T_1{}^{(a)}(s,t)=4e^2\delta(s-m_K{}^2)/q_1\cdot q_2+\text{nonsingular continuum contributions}, \quad (3.4)$$

where

$$\alpha=e^2/4\pi.$$

Thus, if we let

$$g(t)=1+\tfrac{1}{2}tf(t), \quad (3.5)$$

where $f(t)$ is in general nonsingular at $t=0$, we have

$$\int_{(m_K+m_\sigma)^2}^{\infty}\mathrm{Im}T_1{}^{(a)}(s',t)ds'=f(t). \quad (3.6)$$

Since $f(t)$ is a vector form factor, it will have a pole at the value of t corresponding to $\alpha_-(t)=1$, where α_- is a negative signature trajectory.

We now notice two related and compensating degeneracies of our model.

(a) The $I=0$ part of $\mathrm{Im}T_{\mu\nu}{}^{(a)}$ is equal to the $I=1$ part. This is evident from the factorization of Eq. (3.1).

[4] R. F. Dashen and M. Gell-Mann, in *Proceedings of the Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1966), and S. Fubini, Nuovo Cimento 43, 1 (1966).

Therefore, the same sum rule Eq. (3.6) holds for the $I=0$ part of the absorptive amplitude.

(b) The theory is so degenerate that the $I=0$ trajectories concide with the $I=1$ trajectories. Note that the $I=0$ trajectories have even signature and the $I=1$ trajectories have odd signature. This is because G is $+$ for the two $I=1$ photons, so that $C=(-1)^I=(-1)^J$ for the neutral $K\bar{K}$ state.

Putting (a) and (b) together, we find that the asymptotic form of the $I=0$ absorptive part, in order to give the $\alpha=1$ pole of the form factor, must be

$$\operatorname{Im}T_1^{(a)}(s,t)\to\beta(t)s^{\alpha(t)-2}, \tag{3.7}$$

with a $\beta(t)$ which is not zero as $\alpha(t)\to 1$ (where α is the leading *even* signature trajectory).

Thus, the total photon cross section will be finite as $s\to\infty$, in the absence of accidental vanishing of a coupling constant.

IV. FIXED POLES IN WEAK PROCESSES

General Considerations

The model we have just considered is so degenerate that it is probably not totally convincing. We have presented it here only because it is transparent. Next we will consider a model that satisfies unitarity in some limited region and permits Regge behavior for the hadronic processes. We will then show that when the corresponding approximation is made in the weak and electromagnetic processes and unitarized with respect to the hadrons that fixed poles and Regge behavior will both emerge.[5]

The approximation will consist of keeping a finite number of two-particle channels in the unitarity condition for the hadrons. We will label the various two-hadron states $A\bar{A}$ by i, and the particular weak channel we will study is $\gamma\gamma$. We will use the unitarity in the t channel ($\gamma\gamma\to A\bar{A}$), ($A\bar{A}\to B\bar{B}$), etc., where $A\bar{A}$ is one of the hadronic states with the same quantum numbers as two photons or any other weak channel under consideration. Consequently, we will discuss the Regge poles of the crossed channel ($\gamma\gamma\to A\bar{A}$) and their corresponding contribution to the high-energy behavior of the Compton amplitude ($\gamma A\to\gamma A$). The hadron A is spinless in all that follows.

Since unitarity is expressed most simply in terms of the partial-wave amplitude, we define the partial-wave amplitude in the t channel, a_i, for the process $\gamma\gamma\to A\bar{A}$ by

$$T_{i\,1\,-1}(s,t)=\sum_{J=2}^{\infty} d_{20}{}^J(\theta_t)(2J+1)a_{i\,1\,-1}(J,t), \tag{4.1}$$

where the helicity-two amplitude $T_{1\,-1}$ for $2\gamma\to A\bar{A}$ is related to the coefficient of the gauge-invariant tensor T_1 Eq. (2.3) by

$$T_{i\,1\,-1}(s,t)=q_1\cdot q_2\,p_i{}^2\sin^2\theta_t\,T_{1i}, \tag{4.2}$$

p_i and θ_t being the c.m. final momenta and scattering angle in the t channel. Since we will be working only with the helicity amplitude $a_{i\,1\,-1}(J,t)$, we will drop the photon indices 1, -1 in all of the following. The even-signature amplitude $T_{1i}{}^{(+)}$ is free of kinematic singularities in s, and has the partial-wave expansion

$$T_{1i}{}^{(+)}(s,t)=\sum_{J=2}^{\infty}(2J+1)P_J{}''(z_t)b^{(+)}(J,t)(2p_iq)^{J-2}, \tag{4.3}$$

where

$$b^{(+)}(J,t)=\frac{a^{(+)}(J,t)[(J+2)(J)(J+1)(J-1)]^{-1/2}}{(2p_iq)^J},\qquad z_t=\cos\theta_t$$

and $q=(\frac{1}{4}t)^{1/2}$, q being the photon momentum. The physical amplitude is given by $T(s,t)=T^+(s,t)+T^+(u,t)$.

For our purposes the amplitude $b^{(+)}(J,t)$ is defined for large real J by the Froissart-Gribov formula.

$$b_i{}^{(+)}(J,t)=\frac{1}{(2p_iq)^J}\int^{\infty}dz_t{}'Q_{J-2}(z_t{}')\operatorname{Im}T_{1i}(z_t{}',t). \tag{4.3'}$$

The analytic structure of the amplitude $b_i{}^{(+)}(J,t)$ in the t plane is the normal right- and left-hand cut.

In our approximation the unitarity condition for the amplitude $b_i{}^{(+)}(J,t)$, is for real J, and t above the threshold for all open channels

$$\operatorname{Im}b_i{}^{(+)}(J,t)=\sum_k[m_{ki}{}^{(+)}(J,t)]^*\rho_k(t)b_k{}^{(+)}(J,t), \tag{4.4}$$

where $m_{ik}{}^{(+)}(J,t)$ is the even-signature partial-wave amplitude for the many-channel hadron processes. In Eq. (4.4), k runs over all open channels.

The corresponding quadratic unitarity condition for $\mathbf{m}^+(J,t)$ would be given by

$$\operatorname{Im}m_{ij}{}^{(+)}(J,t)=\sum_k[m_{ki}{}^{(+)}]^*\rho_k(t)m_{kj}{}^{(+)}(J,t), \tag{4.5}$$

where

$$\rho_k(t)=\frac{p_k}{(2p_k{}^2)^J\sqrt{t}}.$$

Again the analytic structure of the partial-wave amplitude $m^{(+)}(J,t)$ is the usual right- and left-hand cut. A solution for the unitarity condition Eq. (4.5) can be written

$$m_{ij}{}^{(+)}(J,t)=\sum_k[\mathbf{D}^{-1}(J,t)]_{ik}N_{kj}{}^{(+)}(J,t), \tag{4.6}$$

where $\mathbf{N}$ has the left-hand cut and $\mathbf{D}$ the right-hand cut. The phase of the determinant of $\mathbf{D}$ in the region of one open channel $t_{\text{inel}}\geq t\geq t_{\text{el}}$ is δ_{el}, the elastic-scattering phase shift. The $\mathbf{D}$ matrix which contains only the

[5] See Ref. 8, where it is shown that it is possible for fixed poles to exist in hadron processes in the presence of third double-spectral functions.

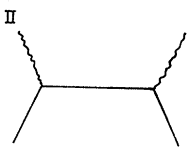

FIG. 2. Born diagrams.

right-hand cut can be written

$$D_{ij}(J,t)=\delta_{ij}-\frac{1}{\pi}\int\frac{dt'N_{ij}^{(+)}(J,t')\rho_j(t')}{t'-t}. \qquad (4.7)$$

We have chosen to normalize the D matrix to 1 at $t\to\infty$ in analogy with potential theory. Similarly, a solution of the weak unitarity Eq. (4.4) can be written

$$b_i^{(+)}(J,t)=\sum_k [D^{-1}(J,t)]_{ik}\eta_k(J,t), \qquad (4.8)$$

where again η is the corresponding numerator function for the weak amplitude having a left-hand cut only. That Eq. (4.8) is the solution of Eq. (4.4) can be checked immediately. We take the imaginary part of Eq. (4.8):

$$\text{Im}\mathbf{b}^{(+)}=(\text{Im}\mathbf{D}^{-1})\boldsymbol{\eta}. \qquad (4.9)$$

Using Eqs. (4.6) and (4.5) in the matrix form of Eq. (4.9), we have

$$\text{Im}\mathbf{b}^{(+)}=(\text{Im}\mathbf{m})\mathbf{N}^{-1}\boldsymbol{\eta}=\mathbf{m}^{+}\boldsymbol{\rho}\mathbf{m}\mathbf{N}^{-1}\boldsymbol{\eta}.$$

Again using Eq. (4.6) we obtain finally

$$\text{Im}\mathbf{b}^{(+)}=\mathbf{m}^{+}\boldsymbol{\rho}\mathbf{b}, \qquad (4.10)$$

which is the result to be proved. Next we use the fact that the Regge poles emerge through the right-hand cut, and the positions of the Regge trajectories are given by the condition $\text{Det}(\mathbf{D})=0$.

In Eqs. (4.6) and (4.7) if we encounter any fixed poles in the N function (which we in general do, since N has the fixed poles of the potential at nonsense values of J), the D matrix turns the fixed poles into moving Regge poles. As is well known, the Regge poles are asymptotic to the fixed poles at $t=\infty$. The strong unitarity does not permit any fixed poles in the amplitude.

In contrast, if η_i develops a fixed pole at a nonsense value of the angular momentum, the D matrix, which is determined entirely by the strong interactions, will not remove the fixed poles of η_i. The η function will, in general, have the fixed poles of the Born diagrams (Fig. 2). In particular, if we ask that the amplitude b_i have the left-hand discontinuity of the Born term v_i, the η_i function is then given by

$$\eta_i(J,t)=v_i(J,t)+\frac{1}{\pi}\int\frac{dt'N_{ij}(J,t')\rho_i(t')[v_j(t')-v_j(t)]}{t'-t}. \qquad (4.11)$$

The Born term v_i will have fixed poles at the nonsense values of angular momentum through the Q function in Eq. (4.3). Note that the fixed poles will generally occur in the amplitude in a multiplicative way, since they are poles of the η_i function. In the vicinity of the fixed pole for the Compton case, where the first nonsense value is $J=1$,

$$b_i^{(+)}(J,t)\underset{J\to1}{\simeq}\frac{r_i}{(J-1)D(J,t)}. \qquad (4.12)$$

The arguments produced here will give fixed J poles at nonsense values in all weak amplitudes at both correct and wrong signature points. The current algebra case is an example of an amplitude with a fixed pole at a correct signature nonsense point.[3] The main reason that the fixed poles are possible is the linear unitarity relation Eq. (4.4); the argument does not depend upon any results from current algebra. Note that the residue of the *Regge* pole in Eq. (4.12) will be singular. Such a singularity will not affect the factorization theorem of Regge residues.

Some Particular Features of Compton Scattering

We remind the reader that the even-signature amplitude $T_{1i}^{(+)}(s,t)$ may have a kinematic pole at $t=0$ because of the one-particle intermediate state [Eq. (3.4)]:

$$T_{1i}^{(+)}(s,t)=\frac{8e_i^2}{t}\frac{1}{m_i^2-s}+\frac{1}{\pi}\int\frac{\text{Im}T_{1i}(s',t)ds'}{s'-s}. \qquad (4.13)$$

The second term in Eq. (4.13) is the nonsingular continuum contribution, and e_i is the charge of the particular hadron in question $\pm e$ and 0. The partial-wave amplitude v_i corresponding to the Born term [Eq. (4.13)] is given by projecting into the helicity amplitudes, Eqs. (4.1) and (4.2).

$$v_i^{(+)}(J,t)=\frac{1}{(2p_iq)^{J-1}}\left(\frac{1}{\pi}\right)Q_{J-2}\left(\frac{t}{t-4m_i^2}\right)^{1/2}\frac{4e_i^2}{t}. \qquad (4.14)$$

All other contributions are less singular at $t=0$.

We can now construct a solution with the following properties.

(a) $b_i(J,t)$ is analytic in the t plane with the right-hand cut starting at the lowest threshold and the left-hand cut of the Born term.[6]

(b) The discontinuity across the right-hand cut is given by Eq. (4.4).

(c) $b_i(J,t)\to v_i(J,t)$ at $t=0$.

[6] The Born term can be rewritten

$$v_i(J,t)=\frac{\sqrt{\pi}}{2^{J-1}}\frac{\Gamma(J-1)}{\Gamma(J+\frac{1}{2})}t^{1-J}\frac{1}{t}F_{21}\left(\frac{J}{2},\frac{J-1}{2},J+\frac{1}{2},\frac{t-4m^2}{t}\right),$$

where F_{21} is the usual hypergeometric function since the hypergeometric function analytic in the cut plane between -1 and $-\infty$, the Born term has only the left-hand cut between 0 and $-\infty$.

The solution can be written in general

$$b_i^{(+)}(J,t)=\sum_{kj}[\mathbf{D}^{-1}(J,t)]_{ik}\times[D_{kj}(J,0)v_j^{(+)}(J,t)+l_j^{(+)}(J,t)], \quad (4.15)$$

where $l_j^{(+)}(J,t)$ contains additional contributions from the left-hand cut. These additional contributions will also be singular at $J=1$ but without the pole at $t=0$ of the Born term.

[If by some miracle it should suffice to consider only one strongly interacting channel and l_j were negligible, the solution would be

$$b^{(+)}(J,t)=v^{(+)}(J,t)D(J,0)/D(J,t), \quad (4.16)$$

where $D(J,t)$ is the single-channel D function.[7] Equation (4.16) can be written in the neighborhood of $\alpha(t)=1$ and $J=1$ as

$$b^{(+)}(J,t)\simeq\frac{1}{\pi}\frac{4e^2}{(J-1)t}\frac{[J-\alpha(0)](\partial D/\partial J)_{J=\alpha(0)}}{[J-\alpha(t)](\partial D/\partial J)_{J=\alpha(t)}}. \quad (4.17)$$

An inspection of Eq. (4.17) shows that the fixed pole will be cancelled for $\alpha(0)=1$, $I=0$ and will be present for $\alpha(0)\neq1$ $I=1$ and 2. In this approximation the $I=0$ even signature Compton amplitude will satisfy a superconvergence relation for $t<0$. By the standard arguments of Regge theory, Eqs. (4.17), (4.3), and (2.3) yield the following result for the contribution of the Pomeranchuk Regge trajectory to the total photoabsorption cross section:

$$\sigma_T=\frac{8\pi^2}{137}\alpha'(0)[\tfrac{1}{4}Y^2+\tfrac{1}{3}I(I+1)] \quad (4.18)$$

for a target of hypercharge Y and isotopic spin I. The nonsense factor of P_J'' [Eq. (4.3)] in this approximation is cancelled by the $1/t$ in the residue Eq. (4.17). Either additional right-hand- or left-hand-cut contributions will invalidate Eq. (4.18).

In general, the Pomeranchuk Regge trajectory with $\alpha(0)=1$ will be restored in Eq. (4.15) by the multiplicative fixed pole which produces a singular residue at $t=0$. This mechanism is indistinguishable from the singular residue arising directly from the $1/t$ in the Born term. We may see this in the following way. We write for $b_i^{(+)}(J,t)$

$$b_i^{(+)}(J,t)=\frac{4e_i^2}{\pi(J-1)t}\frac{R_i(J,t)}{D(J,t)}, \quad (4.19)$$

where

$$R_i(J,0)=D(J,0),$$

and therefore, to first order in t

$$b_i^{(+)}(J,t)=\frac{4e_i^2}{\pi(J-1)t}\frac{D(J,0)+tR_i'(J,0)}{D(J,t)}, \quad (4.20)$$

If $\alpha(0)=1$, then, since $D(J,t)\sim J-\alpha(t)$, $D(J,0)\sim(J-1)$ and the fixed pole is cancelled in the first term of Eq. (4.20). If $R_i'(J,0)$ happens to be zero, we then obtain the magic result of Eq. (4.18). If $\alpha(0)\neq1$, i.e., for an $I=2$ channel, then the fixed pole cannot be cancelled. Finally, it is clear that both terms in Eq. (4.20) contribute indistinguishably to the forward Compton amplitude; the $1/t$ in the first term producing a singular Regge residue without the help of the fixed pole, and the second term producing the singular residue by means of the fixed pole.]

Although we have used unitarity including only two-particle channels with the neglect of three-particle and higher states, we do not feel that the existence of the fixed pole depends upon this approximation.

It has been proven independently that inelastic unitarity[8] allows for fixed poles in hadronic amplitudes at nonsense wrong-signature points. These fixed poles are a consequence of the Mandelstam third double-spectral functions. Of course, they will also be present in the η function. Eq. (4.11) In the presence of third double-spectral functions both fixed poles will be present, and without more detailed knowledge of the relative size of these contributions, it is difficult to estimate their cumulative effects. The fixed poles found in this section will remain even in the absence of third double-spectral functions, at both correct and incorrect signature points.

V. CONCLUSIONS

We have shown that there will be fixed poles in weak amplitudes at nonsense points of both signatures. The fixed poles were proven first in a model that related this process to the scattering of virtual charged photons, using the current algebra. Next it was shown that the linear unitarity relation satisfied by weak amplitudes does not provide a mechanism for the poles found in lowest-order perturbation theory to disappear.

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[7] Equation (4.16) is a solution of a well-known integral equation of the Omnes-Muskhelishvilli type. R. Omnes, Nuovo Cimento **8**, 316 (1958); N. I. Muskhelishvilli, *Singular Integral Equations* (Erwin P. Noordhoff, N. V. Groningen, Netherlands, 1953).

[8] C. E. Jones and V. L. Teplitz, Phys. Rev. (to be published); S. Mandelstam and L. Wang, Phys. Rev. (to be published).