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High-Energy Interactions and a Multi-Regge-Pole Hypothesis

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It is assumed that in certain well-defined kinematic regions, production amplitudes may be described as due to the exchange of a number of Regge poles, and formulas are given expressing the amplitude in terms of the permitted trajectories. Experimental tests and consequences of the hypothesis are discussed. Tests should be possible at present accelerator energies. The hypothesis leads to strong damping of production cross sections in the invariant momentum transfers, and therefore could serve as the basis for the assumptions of a previous paper.

IN the previous paper, we have explored the conclusions which can be drawn about high-energy processes from the assumption that certain invariant momentum transfers remain small as the energy in the reaction increases. Here, we should like to investigate the consequences of the more stringent assumption that Regge behavior results whenever any or all of the crossed cosines associated with these momentum transfers becomes large.

We shall use the notation of the previous paper. In terms of the variables defined there, a natural set of crossed cosines can be defined by the equation

$$x_{t_i}=\frac{-2st_i-t_i^2+(s_i+s_i'+m^2+m'^2)t_i-(s_i-m^2)(s_i'-m'^2)}{[t_i^2-2t_i(s_i+m^2)+(s_i-m^2)^2]^{1/2}[t_i^2-2t_i(s_i'+m'^2)+(s_i'-m'^2)^2]^{1/2}}. \tag{1}$$

The grouping of particles associated with this choice is illustrated in Fig. 1(a). Other sets of cosines can also be used. For example, corresponding to the breakup shown in Fig. 1(b), we may define

$$\bar{x}_{t_i}=\frac{-2s_{i,i+1}t_i-t_i^2+(\mu_i^2+\mu_{i+1}^2+t_{i-1}+t_{i+1})t_i-(\mu_i^2-t_{i-1})(\mu_{i+1}^2-t_{i+1})}{[t_i^2-2t_i(\mu_i^2+t_{i-1})+(\mu_i^2-t_{i-1})^2]^{1/2}[t_i^2-2t_i(\mu_{i+1}^2+t_{i+1})+(\mu_{i+1}-t_{i+1})^2]^{1/2}}. \tag{2}$$

Many other choices are evidently possible. However, as we shall make clear below, the Regge asymptotic behavior associated with each leg will turn out to be independent of which choice of cosines we make.

Let us next make our "multi-Regge-pole" assumption more precise. To begin with, we shall discuss the case $n=3$. The amplitude is a function of five variables, which we may choose to be s, t_1, t_2, s_2, and s_1'. Now we may eliminate s_2 and s_1' in favor of x_{t_1} and x_{t_2}. In the physical region for the t_1 channel, we may write a partial-wave expansion:

$$A(s,t_1,t_2,x_{t_1},x_{t_2})=\sum_{l_1} A_{l_1}(s,t_1,t_2,x_{t_2})P_{l_1}(x_{t_1}), \tag{3}$$

and then repeat this for the physical region of the t_2 channel. Thus,

$$A(s,t_1,t_2,x_{t_1},x_{t_2})=\sum_{l_1l_2} A_{l_1l_2}(s,t_1,t_2)P_{l_1}(x_{t_1})P_{l_2}(x_{t_2}). \tag{4}$$

The numbers l_1 and l_2 are the actual total orbital angular momenta of the t_1 and t_2 channels, respectively.

Now we assume that $A_{l_1l_2}$ has an analytic continuation in both l_1 and l_2, and that the only singularities (or at least the singularities furthest to the right) in l_1 and l_2 are the same poles at $l_1=\alpha_1(t_1)$ and $l_2=\alpha_2(t_2)$ that would be allowed in the same channels of two-body processes.

That $A_{l_1l_2}$ should have these poles is plausible through unitarity, which relates the three-particle amplitude to the two-particle one. That the poles should also be the leading singularities is not so plausible; nevertheless, we assume it.

We now let $s\to\infty$ and look in such a kinematic region that $-x_{t_1}$ and $-x_{t_2}\to\infty$ as well. We would then expect

$$A\to\beta(s,t_1,t_2)R_1(t_1)R_2(t_2)(-x_{t_1})^{\alpha_1(t_1)}(-x_{t_2})^{\alpha_2(t_2)}, \tag{5}$$

where $R_i(t_i)=(1\pm e^{-i\pi\alpha_i(t_i)})/\Gamma(1+\alpha_i(t_i))\sin\pi\alpha_i(t_i)$. Our final assumption is that β becomes independent of s for large s. The asymptotic form may then be visualized as in Fig. 2, where two Regge poles are exchanged.[1]

The generalization of these assumptions to the general case of n final particles is obvious. Under the appropriate kinematic conditions, we may expect

$$A\to\beta(t_1\cdots t_{n-1})[\prod_{i=1}^{n-1}R_i(t_i)(-x_{t_i})^{\alpha_i(t_1)}]. \tag{6}$$

What is the kinematic situation which gives rise to all

[1] After completion of this work, we received CERN Report Th 719 (unpublished) by H. Chan, K. Kajantie, and G. Ramft on the Reggeization of the three-particle amplitude. In this paper we found a reference to an earlier work by K. A. Ter-Martirosyan, Nucl. Phys. 68, 591 (1965) on the Reggeization of the n-particle amplitude. There is also a paper by T. W. B. Kibble, Phys. Rev. 131, 2282 (1963) which makes remarks similar to some of those at the beginning of this paper.

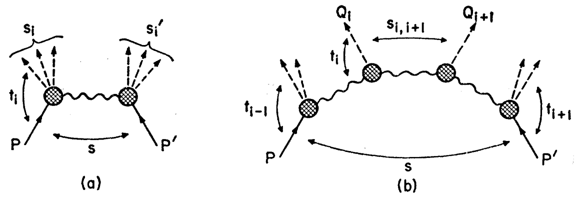
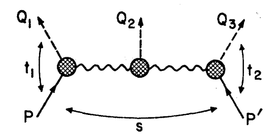


FIG. 1. Two possible groupings of the final-state particles leading to two different sets of crossed cosines.

FIG. 2. Double-Regge-pole exchange.



$-x_{t_i}\to\infty$? It has already been outlined in the previous paper. If we say π_i is finite but $\omega_i\to\infty$, and if[2] $\hat{Q}_i\cdot\hat{P}\to 1$ for $i=1\cdots a$, while $\hat{Q}_i\cdot\hat{P}\to -1$ for $i=a+1\cdots n$, then it is easy to deduce that

$$\omega_1\gg\omega_2\gg\cdots\gg\omega_a\,,\quad \omega_n\gg\omega_{n-1}\gg\cdots\gg\omega_{a+1}\,,$$

and

$$\begin{aligned} -x_{t_i}\sim s/s_is_i'\sim\omega_i/\omega_{i+1}\,,&\quad i=1,\cdots a-1\\ \sim\omega_a\omega_{a+1}\,,&\quad i=a \qquad\qquad (7)\\ \sim\omega_{i+1}/\omega_i\,,&\quad i=a+1,\cdots n-1\,. \end{aligned}$$

Since $\sqrt{s}=\omega_1+\cdots+\omega_n$, we find ω_1, $\omega_n\sim\frac{1}{2}\sqrt{s}$. Consequently, we have

$$\prod_{i=1}^{n-1}(-x_{t_i})\sim s\,. \tag{8}$$

If the kinematics are such that not all $-x_{t_i}$ grow, it is nevertheless true that for those which do become large, we have

$$-x_{t_i}\sim s/s_is_i'\,. \tag{9}$$

A similar Regge-like limit could be obtained by using any of the alternative sets of crossed cosines mentioned previously. However, as long as the t_i are all fixed, the same limit is obtained whichever set is used. For example, under the conditions outlined above,

$$\begin{aligned} s_{i,i+1}&=(Q_i+Q_{i+1})^2\sim\omega_i/\omega_{i+1}+\omega_{i+1}/\omega_i\,,\quad i\neq a\\ s_{a,a+1}&=(Q_a+Q_{a+1})^2\sim\omega_a\omega_{a+1}\,, \end{aligned}$$

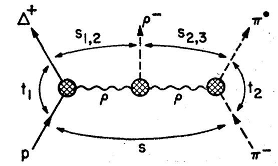
so that we get

$$-\bar{x}_{t_i}\sim\omega_i/\omega_{i+1}\,,\text{ etc.,} \tag{10}$$

exactly as in Eq. (7).

The unambiguous answer for the fully Reggeized amplitude—i.e., the amplitude in the kinematic region

FIG. 3. A simple example of 3-particle Reggeization.



where *all* crossed cosines are large—is that

$$A\to\beta(t_1\cdots t_{n-1})\prod_{i=1}^{n-1}R_i(t_i)(s/s_is_i')^{\alpha_i(t_i)}\,. \tag{11}$$

From the pictorial representation of this form, it is plausible to conjecture that the coefficient β can be factored. We should expect to be able to write

$$\beta(t_1\cdots t_{n-1})=\beta_1(t_1)\beta_2(t_1,t_2)\cdots\beta_{n-1}(t_{n-2},t_{n-1})\beta_n(t_{n-1})\,. \tag{12}$$

Before we discuss tests of the multi-Regge hypothesis, it might be best to review[3] the most characteristic features of two-body Reggeization. There we find

(1) diffraction peak shrinkage, as in $\pi^-p\to\pi^0n$ or $\pi^-p\to\eta n$; (13)
(2) coupling zeros, as in $\pi^-p\to\pi^0n$;
(3) signature zeros, as in $\pi^+p\to p\pi^+$.

As an example of what might be done to observe three-particle Regge behavior [Eq. (5)], consider $\pi^-p\to\pi^0\rho^-\Delta^+$ as shown in Fig. 3. We could look for a coupling zero at the $\Delta^+p\rho$ vertex by changing t_1 while searching for a t_2 diffraction peak shrinkage by varying $s_{2,3}$. For a general n-body process, observation of any of the three characteristics simultaneously operating at each Regge leg would provide a sensitive test of the theory. Note that the particles participating in the reaction should be chosen to minimize the number of allowed trajectory exchanges. In addition, there is enough suspicion that the Pomeranchuk trajectory may not be a Regge pole of the usual type to make it desirable to look for processes with no Pomeranchuk trajectory exchange.

However, if we are willing to accept the Pomeranchuk trajectory as just another Regge pole, with a finite slope and rather normal properties, then not only can we be less finicky about which production processes we study in the laboratory, but we can also say a few things about cosmic-ray experiments. To see what kind of conclusions can be drawn, let us turn next to the calculation of cross sections for n-particle production. Our starting point will be the expression

$$\frac{d\sigma}{dt}=\frac{1}{16\pi^3}\int ds_1\int ds_1'\sigma_1(s_1,t)X\sigma_1'(s_1',t)\frac{[s_1^2-2s_1(m^2+t)+(m^2-t)^2]^{1/2}[s_1'^2-2s_1'(m'^2+t)+(m'^2-t)^2]^{1/2}}{s^2-2s(m^2+m'^2)+(m^2-m'^2)^2}\,. \tag{14}$$

[2] $\hat{V}$ denotes a unit vector in the direction **V**.
[3] S. Frautschi, Phys. Rev. Letters **17**, 722 (1966).

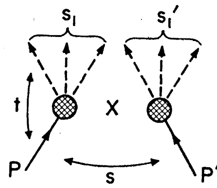


FIG. 4. Factorization of the production process.

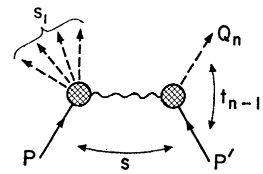


FIG. 5. Recursion diagram used to compute $d\sigma/dt_1\cdots dt_{n-1}$ from $d\sigma/dt_1\cdots dt_{n-2}$.

This equation is represented graphically in Fig. 4. We think of breaking the entire process into two parts, connected by some sort of exchange (denoted X). σ_i is the total cross section for $p+X\to$ the first group of final-state particles and σ_1' in the corresponding cross section for $p'+X$. s_1 and s_1' are the two total c.m. energies squared at which the cross sections σ_1 and σ_1' are evaluated; in addition, σ_1 and σ_1' depend on the mass t of the exchanged object.

The region of integration in s_1 and s_1' is rather complicated, and we shall not go into it further here.

We can use Eq. (14) to relate the differential cross section $d\sigma^{(n)}/dt_1\cdots dt_{n-1}$ for production of n particles to the cross section for $n-1$ particles, by choosing

$$\sigma_1'(s_1',t_{n-1})=g(t_{n-1})\delta(s_1'-\mu_n^2). \tag{15}$$

We find, for large s,

$$\frac{d\sigma^{(n)}}{dt_1\cdots dt_{n-1}}=\int ds_1\frac{[s_1^2-2s_1(m^2+t_{n-1})+(m^2-t_{n-1})^2]^{1/2}}{s^2}\times f(t_{n-1})\frac{d\sigma^{(n-1)}}{dt_1\cdots dt_{n-2}}X. \tag{16}$$

The range of integration in Eq. (16) is, in general, a bit messy. For the case of equal masses for all particles, we have, when s is large, const.$<s_1<s$ and $-s<t_{n-1}-2m^2<0$.

Let us now assume that X represents a Regge exchange. We should then expect

$$X\sim(-x_{t_{n-1}})^{2\alpha_{n-1}(t_{n-1})},$$

where α_{n-1} is the trajectory appropriate to connecting the nth final particle to the first $n-1$ particles. The configuration is shown in Fig. 5.

Now, as $s\to\infty$ and t_{n-1} is held fixed, we have $-x_{t_{n-1}}\sim s/s_1$. Therefore, in Eq. (16), we shall choose

$$X=(s/s_1)^{2\alpha_{n-1}(t_{n-1})}. \tag{17}$$

Any additional factors depending on t_{n-1} can be absorbed in the function f. We assume the only s_1 dependence to be that explicitly written down.

For the case $n=2$ we know the asymptotic behavior as $s\to\infty$, t fixed, is

$$d\sigma^{(2)}/dt_1\to F(t_1)(s/s_0)^{2\alpha_1(t_1)-2} \tag{18}$$

if the Regge-pole approach is assumed. This fact and Eq. (16) allow us to calculate $d\sigma^{(n)}/dt_1\cdots dt_{n-1}$. The integral on s_1 may be broken up into two integrals according as s_1/s_0 is greater than or smaller than s_0. In the first of these we use the (known) asymptotic form of $d\sigma^{(n-1)}/dt_1\cdots dt_{n-2}$ and replace $[s_1^2-2s_1(m^2+t_{n-1})+(m^2-t_{n-1})^2]^{1/2}$ by s_1. In the second, the s dependence comes only from the factors $1/s^2$ and X and is explicitly given. As a result, we find

$$\frac{d\sigma^{(n)}}{dt_1\cdots dt_{n-1}}\to\sum_{i=1}^{n-1}F_i^{(n)}(t_1\cdots t_{n-1})(s/s_0)^{2\alpha_i(t_i)-2} \tag{19}$$

as $s\to\infty$, all t's fixed.

We thus find a sum of contributions from each of the Regge poles which can be exchanged between pairs of final particles. It is important to remember that this is for a particular ordering of the final particles as expressed by the particular set of t_i we use. A different ordering—that is, a different set of t_i—would, of course, allow a different set of Regge poles in general. The contrast between Eq. (19), which gives the cross section as a sum of Regge-pole contributions, and Eq. (11), which gives the amplitude as a product of the same Regge poles, is quite striking.

In the two-body case, the total cross section is usually estimated by integrating (18) over a finite range of t:

$$\sigma^{(2)}(s)=\int_{t_0}^{0}dt_1F(t_1)(s/s_0)^{2\alpha_1(t_1)-2}\approx F(0)(s/s_0)^{2\alpha_1(0)-2}/2\alpha_1'(0)\ln(s/s_0). \tag{20}$$

We may do the same thing for n particles, and we obtain

$$\sigma^{(n)}(s)\approx\sum_{i=1}^{n-1}F_i^{(n)}(0)(s/s_0)^{2\alpha_i(0)-2}/2\alpha_i'(0)\ln(s/s_0). \tag{21}$$

The largest $\alpha_i(0)$ will, of course, dominate this sum. Let us assume this to be the Pomeranchuk trajectory. Then we expect

$$\sigma^{(n)}(s)\sim\text{const}/\ln s. \tag{22}$$

Insofar as we believe the constant to be independent or only weakly dependent on n, we deduce that the total cross sections for producing n particles all behave comparably, and all behave like $1/\ln s$. Furthermore, as we saw in the previous paper, the condition t_i bounded forced the maximum allowable number of particles at a given s to be like $\ln s$. The total cross section is therefore a constant,

$$\sigma_T(s)=\sum_{n=2}^{\sim\ln s}\sigma^{(n)}(s)\sim\text{const}, \tag{23}$$

which, of course, is consistent with the optical theorem.

In the same way, the average number of particles is

$$\langle n\rangle=\sum_{n=2}^{\sim\ln s} n\sigma^{(n)}(s)/\sigma_T(s)\sim\ln s. \tag{24}$$

We also deduce that the ratio $\sigma^{(n)}(s)/\sigma^{(n')}(s)$, that is, the ratio of the probability to produce n particles to that to produce n' particles, is independent of s.

The basic ingredient in both (23) and (24) is the assumption that we can neglect all kinematic regions in which any t_i is large. Without this restriction, the maximum number of particles allowed at a given s might be expected to be as much as

$$n=(\sqrt{s})/\mu,$$

where μ is the average rest mass of the particle in the final state.

The principal conclusions to be drawn about the multi-Regge-pole hypothesis then are (1) It is internally consistent. (2) It is best tested by looking for diffraction-peak shrinkage and dips in the differential cross section. (3) If the Pomeranchuk trajectory is a Regge pole, then at ultra-high energies, the particle multiplicity grows like $\ln s$, and the cross section to produce n particles falls like $1/\ln a$.

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High-Energy Limit of Photon Scattering on Hadrons*†

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We show that the presence of a fixed pole at $J=1$ in the process $\gamma+\gamma\rightarrow h+\bar{h}$ reinstates the coupling of the γ-γ state to the vacuum trajectory, and hence permits a finite total photon cross section as $s\rightarrow\infty$.

I. INTRODUCTION

IT has been observed by Abarbanel and Nussinov[1] and Mur[2] that in a naive Regge-pole model, the Pomeranchuk Regge trajectory with $\alpha(0)=1$ does not contribute to the forward nonhelicity-flip Compton amplitude. Hence, by the optical theorem the total photoabsorption cross section will go to zero as $E\rightarrow\infty$. We will show that as a consequence of the linear unitarity (more precisely, the absence of bilinear unitarity) for processes to low order in the weak and electromagnetic interactions there exist fixed poles at nonsense values of the angular momentum. The fixed pole in the angular-momentum variable J will exist together with the Regge pole (if it exists) in a multiplicative fashion. In particular, there is a fixed pole in the Compton amplitude at the point $J=1$, the first nonsense wrong signature point for the relevant partial-wave amplitude for the crossed channel. The fixed pole at $J=1$ will not contribute to the physical scattering amplitude, but will contribute in such a manner as to restore the contribution of the Pomeranchuk trajectory to the forward nonhelicity-flip Compton amplitude.

Section II is devoted to some kinematic preliminaries for the Compton amplitude. In Sec. III the fixed pole in the Compton case will be derived for the same model which produces a fixed pole in the current commutator case.[3] In Sec. IV we will show the existence of fixed poles in general as a consequence of the unitarity of lowest-order weak and electromagnetic processes. Finally, Sec. V will be devoted to conclusions and speculations.

All of our remarks on Compton scattering will be

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[3] J. B. Bronzan, I. S. Gerstein. B. W. Lee, and F. E. Low, Phys. Rev. Letters 18, 32 (1967); and V. Singh, *ibid.* 18, 36 (1967). See also J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. (to be published).