Bounded-Momentum Transfer Restrictions on High-Energy Interactions

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A discussion is given of experimental implications of the hypothesis that production cross sections die off rapidly with an increase in any of the independent invariant momentum transfers in the process. Consequences are presented for both high-energy laboratory and cosmic-ray conditions. The results are consistent with existing evidence.

I. INTRODUCTION

IN the near future we expect to obtain accurate data on proton-proton interactions up to center-of-mass energies $\sqrt{s}$ equal to 50 BeV.[1] We already have crude experimental information available from a variety of cosmic-ray experiments.[2] In this paper we explore some restrictions placed on the reaction

$$P+P' \to Q_1+\cdots+Q_n, \tag{1}$$

which follow from kinematics and the dynamical requirement that certain momentum transfers be kept small as the energy of the reaction or the number of particles in the final state is allowed to increase. Here, $P, P', Q_1, \cdots, Q_n$ stand for the initial and final particles as well as their four-momenta. A natural set of variables for the description of this process will be presented, and we will obtain information concerning the transverse momentum distribution, energy distribution, multiplicity, and inelasticity of the event. We then arrive at a theoretical description of a phenomenon that is loosely referred to by cosmic-ray experimentalists as "fireballs." In the following paper we explore additional consequences of more detailed dynamical assumptions. In both this and the following papers we essentially assume that the high-energy n-particle production amplitude may be factored, i.e., may be written as a combination of simpler scattering amplitudes. Present accelerator energies should be sufficient to test our hypotheses.

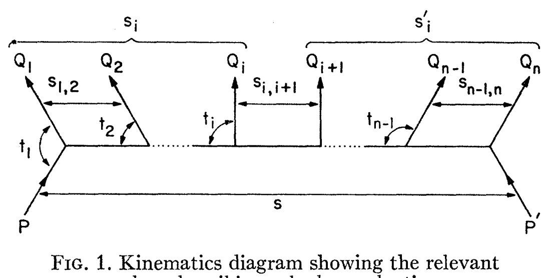

FIG. 1. Kinematics diagram showing the relevant scalars describing n-body production.

[1] Two cosmic-ray experiments employing hydrogen targets, magnetic fields, and spark chambers have been proposed by L. Alvarez and L. Jones. In addition, CERN is constructing a proton colliding-beam facility.

[2] The most up-to-date summary is given by Y. Pal, "Cosmic Rays and Their Interactions," to be published in the 2nd edition of *Handbook of Physics*, edited by E. U. Condon and H. Odishaw.

II. KINEMATICS

We shall always work in the c.m. system, where we decompose the space part of each Q_i into a component $\mathbf{q}_i$ parallel to the incident particle direction and another component $\boldsymbol{\pi}_i$ perpendicular to this direction. Furthermore, we assign the indices i so that particles 1 through a travel in the positive direction of P while particles $a+1$ to n travel in the opposite direction.

$$\begin{gathered} p=(E,\mathbf{p}), \quad p'=(E',\mathbf{p}')=(E',-\mathbf{p}), \\ Q_i=(\omega_i,\mathbf{q}_i,\boldsymbol{\pi}_i), \quad i=1,\cdots,n, \\ Q_i^2=\mu_i^2, \quad P^2=m^2, \quad P'^2=m'^2. \end{gathered} \tag{2}$$

There are $n-1$ independent momentum transfers that appear in the reaction. We take these to be

$$t_i=\left(\sum_1^i Q-P\right)^2, \quad i=1,\cdots,n-1. \tag{3}$$

The energy variables that we will need are

$$\begin{aligned} s&\equiv(P+P')^2, \\ s_{i,i+1}&\equiv(Q_i+Q_{i+1})^2, \quad i=1,\cdots,n-1, \\ s_i&=(Q_i+\cdots+Q_i)^2, \\ s_i'&\equiv(Q_{i+1}+\cdots+Q_n)^2. \end{aligned} \tag{4}$$

The relevant kinematical quantities are summarized in Fig. 1.

III. DYNAMICS

A striking characteristic of *all* two-body reactions at high energies is the strong damping that occurs at large momentum transfers t,[3] i.e., differential cross sections have a t dependence that goes something like

$$d\sigma/dt \sim e^{t/\tau}. \tag{5}$$

Possible slowly varying functions of t have been suppressed. Here, τ varies with s and the reaction under consideration, but is usually within a factor of 2 of $\frac{1}{10}$ BeV^2. The energy s_0 at which this exponential form sets in also changes with the reaction but, roughly speaking, we may expect exponential damping whenever s is greater than about 2 BeV^2.

[3] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 423 (1963) and 11, 503 (1963).

We conjecture that this exponential form is valid even if we have more than 2 final-state particles produced, just as long as these particles may be clustered into two groups whose invariant masses are small, say, less than s_1. Then t would stand for the momentum transferred between the incoming particle and outgoing cluster. That is, we believe that Eq. (5) is not a relationship peculiar to two-particle scattering. The number or type of particle entering the reaction either in the initial or final state is really irrelevant. *Equation (5) should be viewed as a statement about the high-energy scattering of invariant masses.*

If the final-state particles cannot be clustered into two groups with invariant masses $\lesssim s_0$, then (5) is still correct, but we expect additional momentum transfer damping. For example, when $n=3$ and both final-state invariant masses $s_{1,2}$ and $s_{2,3}$ are large ($s_{1,2}\gtrsim s_{01}$, $s_{2,3}\gtrsim s_{02}$), we believe

$$d\sigma/dt_1dt_2ds_{1,2}ds_{2,3}\sim e^{t_1/\tau_1}e^{t_2/\tau_2}=\exp\left(\sum_1^2 t/\tau\right). \quad (6)$$

In making this conjecture, we argue that the amplitude to produce three particles behaves like the product of two amplitudes to produce two particles, as shown in Fig. 2. τ_1 and τ_2 may be slowly varying functions of $s_{1,2}$ and $s_{2,3}$, respectively.

Presuming this factor property of the three-particle amplitude to be correct, we may reasonably expect that events with large momentum transfers are unlikely to occur; that is, we may expect

$$-t_i/\tau_i\lesssim 1, \quad i=1,2. \quad (7)$$

This leads us to conjecture that for the n-body reaction, if $s_{i,i+1}\gtrsim s_{0i}$, $i=1,\cdots,n-1$:

$$(1) \quad d\sigma/dt_1\cdots dt_{n-1}ds_{1,2}\cdots ds_{n-1,n}\sim\exp\left(\sum_1^{n-1} t/\tau\right). \quad (8)$$

We shall also assume that it follows from conjecture (1) that a given event is probable only if

$$(2) \quad -t_i/\tau_i\lesssim 1, \quad i=1\cdots n-1, \quad (9)$$

where the τ_i are of the same order of magnitude as the τ's obtained in two-body scattering reactions.[4]

[4] There are two papers that also arrive at some of the conclusions described here. They are by S. C. Frautschi, Nuovo Cimento **28**, 409 (1963) and by S. Fubini in *High Energy Physics Strong Interactions*, edited by R. Moorhouse (Oliver and Boyd, Edinburgh, 1964). [Fubini's article contains references to other work on the multiperipheral model. For example, see Amati *et al.*, Nuovo Cimento **26**, 896 (1962)]. Each of them makes the assumption that production processes are damped for large invariant momentum transfers, in one case exponentially as here, in the other by the "one-pion exchange" mechanism, and they are thus able to understand the qualitative fireball picture and the bound on the transverse momentum as well as the multiplicity. The remaining results obtained here are consequences of the explicit assumption of a *product* of exponentials in the momentum transfers for the differential cross section when *all* "two-body scattering energies" are large, which leads us to the strong ordering of cluster energies and for which experimental tests are suggested. This strong ordering is also what permits us to make the multi-Regge-pole hypothesis discussed in the following paper.

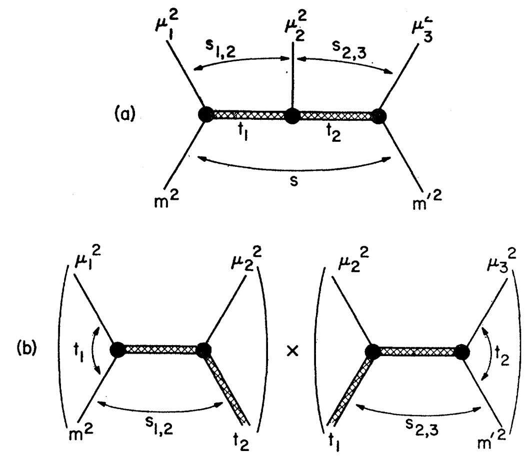

FIG. 2. The 3-body production amplitude (a) is assumed to behave like the product of two 2-body amplitudes (b).

Conjectures (1) and (2) deal with the case where all $s_{i,i+1}$ are large. Suppose, however, that several particles, say, $k+1,\cdots,k+l$ move off together so that $(Q_{k+1}+\cdots+Q_{k+l})^2<s_0$. By our earlier arguments, we expect that these particles should be treated collectively as an indivisible cluster. This cluster is then to be viewed as entering the reaction as a single unit of invariant energy $=(Q_{k+1}+\cdots+Q_{k+l})^2$. Consequently, although the particles in this cluster generate $l-1$ momentum transfers $t_{k+1},\cdots,t_{k+l-1}$, these t_i are not required to be small. Only if $(Q_{k+1}+\cdots+Q_{k+l})^2\gtrsim s_0$ is the cluster divisible. Then we require the existence of some t_r, $k+1<r<k+l$ with $-t_r\lesssim\tau_r$. From now on, Q_i will stand for the four-momentum of the ith cluster with conjectures (1) and (2) applying to these clusters. We expect the particles within each cluster to behave in much the same way as they would if produced in a two-body reaction at an energy corresponding to the invariant mass of the cluster. Our main concern will be with the behavior of the clusters and not with the detailed low-energy dynamics within each cluster.

A given number n of particles produced at energy s will group themselves into N clusters subject only to the constraint that $Q_i^2\lesssim s_{0i}$ and $-t_i/\tau_i\lesssim 1$. The possible values of N will of course,, depend upon s. Note that we are not implying the exchange of any particular particle between clusters (Fig. 2). The type or complexity of exchange is essentially irrelevant (other than in determining τ or s_0.) What counts is the idea that the invariant mass of the exchange is to be thought of as scattering off other invariant masses in the problem. Crudely speaking, we are trying to understand that aspect of the high-energy event that does not depend on the concept of particle.

The remainder of this paper now breaks into two parts. First, we discuss the simplest laboratory test of the conjectures. Then we describe in some detail the structure of the high-energy multiparticle event.

IV. A LABORATORY TEST

As an example of what must be done to check the validity of conjecture (1), we consider the reaction $\pi^-+p\rightarrow\pi^-+\pi^0+p$. In terms of our earlier notation, the initial particles π^- and p are just P and P', while the final particles π^-, π^0, p are matched with Q_1, Q_2, Q_3, respectively. We select only events where the invariant masses $s_{1,2}$ and $s_{2,3}$ of the $\pi^-\pi^0$ and $\pi^0 p$ are greater than about 2 BeV^2 and for which the energy is sufficiently large that the momentum transfers can be kept small. For these events we make a scatter plot of t_1 versus t_2 and check to see if the resulting distribution is of the form $\exp(t_1/\tau_1+t_2/\tau_2)$ for some constants τ_1 and τ_2. The precise values of τ_1 and τ_2 are presently uncertain and should be determined experimentally. They may depend weakly on $s_{1,2}$ and $s_{2,3}$. It will be interesting to compare these values of τ with those determined in two-body scattering.

Note that $s_{1,2}$ and $s_{2,3}$ large, with t_1 and t_2 small, corresponds in the cm system to the configuration where the π^- and p maintain their energy and direction, while the π^0 gets relatively little energy and may come off in any direction.

V. TRANSVERSE MOMENTUM DISTRIBUTION

Let us first consider the two-body scattering $P+P'\rightarrow Q_1+Q_2$. In the c.m. system, at high energy and small momentum transfer, we may neglect m^2, m'^2, and t compared to s. This yields

$$(\boldsymbol{\pi}_1)^2\approx -t+(m^2\mu_2^2+m'^2\mu_1^2)/s+t(\mu_1^2+\mu_2^2)/s \\ -m'^2(\mu_1^2/s)^2-m^2(\mu_2^2/s)^2-\mu_1^2\mu_2^2/s. \quad (10)$$

The positive terms on the right-hand side of Eq. (10) are all less than or the order of m^2; since $(\boldsymbol{\pi}_1)^2$ is positive, the negative terms on the right-hand side of (10) are also less than or the order of m^2. In particular, $\mu_1^2\mu_2^2/s\leq O(m^2)$. The entire right-hand side is therefore less than or the order of m^2, and hence so is $(\boldsymbol{\pi}_1)^2$.

Replacing Q_1, Q_2 by

$$\sum_1^i Q,\ \sum_{i+1}^n Q,$$

respectively, we obtain

$$\left|\sum_1^i \boldsymbol{\pi}\right| \quad \text{or} \quad |\boldsymbol{\pi}_i|\lesssim \text{several hundred MeV}/c\,, \quad (11)$$

and

$$s_is_i'\leq O(m^2s),\quad i=1,\cdots,n-1.$$

The transverse momentum of an individual particle in cluster i will be composed of two contributions, one coming from $\boldsymbol{\pi}_i$, the transverse momentum of the cluster, and another coming from the motion of the particle relative to the center of mass of the cluster. Since the invariant mass of the cluster is small, internal cluster motion is of relatively low energy. Consequently, we expect the particle transverse momentum distribution to be independent of incident energy and of the order of several hundred MeV/c. This is in good agreement with cosmic-ray experiments.

VI. ENERGY DISTRIBUTIONS AND CORRELATIONS BETWEEN TRANSVERSE MOMENTA

The type of event we are interested in has all "two-body scattering energies" $s_{i,i+1}$ large. To understand what this implies about the energies ω_i, we suppose that all clusters have large momenta so that

$$\omega-q\approx(\pi^2+\mu^2)/2\omega\equiv M^2/2\omega, \quad (12)$$

where μ is a mean cluster mass. Then it is easy to obtain

$$s_{i,i+1}\approx(\omega_i+\omega_{i+1})(M_i^2/\omega_i+M_{i+1}^2/\omega_{i+1}),\quad i\neq a \\ s_{a,a+1}\approx 4\omega_a\omega_{a+1}. \quad (13)$$

Hence, if $s_{i,i+1}$ is to be large, the energies must be strongly ordered, so that if the indices are appropriately assigned,

$$\omega_1\gg\omega_2\cdots\gg\omega_a,\quad \omega_n\gg\omega_{n-1}\cdots\gg\omega_{a+1} \quad (14)$$

yielding

$$s_{i,i+1}=M_{i+1}^2\omega_i/\omega_{i+1},\quad i=1,\cdots,a-1 \\ =M_i^2\omega_{i+1}/\omega_i,\quad i=a+1,\cdots,n-1. \quad (15)$$

This result may be quickly understood by thinking of particles k and $k+1$ as resulting from the scattering of two objects with invariant masses t_{k-1} and t_{k+1} with momentum transfer t_k. If particles k and $k+1$ come off together in the c.m. system of the over-all reaction, i.e., if $\omega_k\approx\omega_{k+1}$, then in the c.m. system of the scattering $t_{k-1}+t_{k+1}\rightarrow\mu_k^2+\mu_{k+1}^2$, and particles k and $k+1$ will be formed at rest with a correspondingly low value of $s_{i,i+1}$.

The separation of clusters with respect to their energies, (14), makes the factor hypothesis contained in conjecture (1) more plausible. We believe that the strong-interaction dynamics within each cluster occurs in a characteristic time which is long enough to ensure that clusters become widely separated in space so that interactions between particles of different clusters are unlikely.

In order to explore the kinematic consequences of small momentum transfers, we have derived the following expression for t_i, valid when (12) holds:

$$-t_i\approx(1-\delta_{1i}m^2/M_1^2-\delta_{n,i+1}m'^2/M_n^2)M_i^2M_{i+1}^2/s_{i,i+1} \\ +\left(\sum_1^i\boldsymbol{\pi}\right)^2,\quad i=1,\cdots,n-1. \quad (16)$$

The lack of symmetry, with particles 1 and n receiving special attention, is an indication that particle 1, for example, is produced via the "two-body reaction $m^2+t_2\rightarrow\mu_1^2+\mu_2^2$, while particle 2 proceeds through $t_1+t_3\rightarrow\mu_2^2+\mu_3^2$ with $|t_1|\ll m^2$.

Conjecture (2), $-t_i/\tau_i\lesssim 1$ must now imply both

$$(1-\delta_{1i}m^2/M_1^2-\delta_{n,i+1}m'^2/M_n^2)M_i^2M_{i+1}^2/\tau_i s_{i,i+1}\lesssim 1,\quad i=1,\cdots,n-1, \tag{17}$$

and

$$\left(\sum_1^i \pi\right)^2/\tau_i\lesssim 1.$$

The cluster configuration that minimizes

$$\sum_1^{n-1}(-t/\tau)$$

turns out to be

$$|\pi_i|=0,\quad i=1,\cdots,a$$

and

$$M_i^2M_{i+1}^2(1-\delta_{1i}m^2/M_1^2)/s_{i,i+1}=\epsilon\tau_i, \tag{18}$$

where

$$\epsilon=M_a^2M_{a+1}^2/\tau_a s_{a,a+1}, \tag{19}$$

so that

$$-t_i/\tau_i=\epsilon,\quad i=1,\cdots,a \tag{20}$$

and similarly for $i=a+1,\cdots,n$.

VII. MULTIPLICITY

We have found that the energies of the clusters must be "strongly" ordered:

$$\omega_1\gg\omega_2\gg\cdots\gg\omega_a. \tag{21}$$

This places a restriction on the maximum number of particles we may expect at any given energy s. For example, if we take the most probable configuration in a reaction where all τ's and cluster masses are the same, then

$$\tfrac{1}{2}\sqrt{s}\approx\omega_1\approx\Lambda\omega_2\approx\cdots\approx\Lambda^{a-1}\omega_a, \tag{22}$$

$$\Lambda\approx s_{a,a+1}/\mu^2\gg 1, \tag{23}$$

and we obtain:

$$\text{number of final state particles}\approx\bar{n}_c n\sim\bar{n}_c\times[2+\ln(s/s_{a,a+1})/\ln\Lambda], \tag{24}$$

where $\bar{n}_c$ is the average number of particles in each cluster. Consequently, the multiplicity can increase at most as fast as ln$s$. This upper limit is quite consistent with cosmic-ray data.

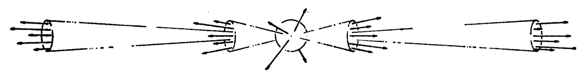

FIG. 3. Schematic representation of a five-fireball event.

VIII. INELASTICITY

Suppose that there are l particles in the first cluster. Let $\omega_{11}\geq\omega_{12}\geq\cdots\geq\omega_{1l}$ be the energies of these particles. Then, the inelasticities I and I_c are defined by

$$I_c\equiv\sum_{j=2}^{l}\omega_{1j}/E,\quad I\equiv I_c+\sum_{j=2}^{a}\omega_j/E.$$

This allows us to write

$$-t_1\approx(1-m^2/M_1^2)M_1^2(I-I_c)/(1+I_c-I)+\pi_1^2\lesssim\tau_1, \tag{25}$$

which places restrictions on $I-I_c$. Usually, we would expect that $M_1^2-m^2>0$ and $\tau_1-\pi_1^2>0$, so that we find

$$0\leq I-I_c\lesssim\frac{\tau_1-\pi_1^2}{\tau_1-\pi_1^2+M_1^2-m^2}=O\left(\frac{\tau_1}{M_1^2-m^2}\right).$$

Note that the inelasticities are expected to be independent of s, as observed in cosmic-ray experiments.

IX. FIREBALLS

The strong ordering (14) in conjunction with the clustering that is required for high-energy, high-multiplicity events may be responsible for the fireball phenomena observed by cosmic-ray experimentalists. A schematic representation of a typical event is given in Fig. 3. under the additional assumption that the transverse momenta of the clusters are approximately the same. The directions of the transverse momenta must then be strongly correlated to keep $(\sum_1^i\pi)^2/\tau_i$ small. The diagram is shown for the case $n=5$, with the energy of the center cluster small. The two outside clusters, $i=1$ and $i=5$, have $\omega_5\approx\omega_1\approx\frac{1}{2}\sqrt{s}$. The remaining clusters, $i=2$ and $i=4$, while of high energy, have ω_2 and $\omega_4\ll\sqrt{s}$. Since the transverse momenta of all clusters are expected to be comparable, the faster clusters have a correspondingly smaller angular spread.

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