Model for $p\bar{p}$ Annihilation in Flight with Multipion Production*

VITTORIO BERZI
Istituto di Scienze Fisiche dell'Università di Milano, Milano, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Italy

AND

ERASMO RECAMI
Istituto di Scienze Fisiche dell'Università di Milano, Milano, Italy
and
Scuola di Perfezionamento in Fisica Atomica e Nucleare dell'Università di Milano, Milano, Italy

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In this work we deal with a "nucleon-exchange" model for $p\bar{p}$ annihilation in flight into pions. The aim of the model is to explain the observed fact that, in the c.m. system, charged mesons seem to prefer the direction of the nucleon of equal charge. As suggested by Minami, it is assumed that a virtual annihilation with multipion production (treated in the spirit of the statistical model) is preceded by a peripheral emission of one pion by both nucleon and antinucleon. We have taken into proper account the conditions that Lorentz and isospin invariances impose on the structure both of the contributions of the peripheral-emission vertices, and of the virtual-annihilation amplitude. A final formula for the π^+ (or π^-) angular distribution is given. With the help of some physical simplifying considerations, this formula is reduced to a numerically evaluable one (by using some phase-space techniques), and the results are compared with the available experimental data at the entering laboratory momenta of 1.6 GeV/c, 3.3 GeV/c, and 5.7 GeV/c. Despite the fact that our model neglects resonance production, a satisfactory enough accord has been found.

I. INTRODUCTION

RECENTLY it has been observed that the charged mesons emitted from annihilation in flight of antiprotons with a laboratory momentum of a few GeV/c have a rather definite orientation with respect to the incoming particles[1,2]: Negatively charged mesons prefer small c.m. angles with the antiproton momentum direction; positively charged mesons prefer large ones.[3,4] That is, charged mesons from $p\bar{p}$ annihilation in flight seem to prefer the direction of the nucleon of equal charge, in contrast with a purely statistical model.

A mechanism for producing angular asymmetries of annihilation mesons is easily established in the Koba-Takeda model.[5,6] In this model, the $p\bar{p}$ annihilation is considered to proceed via a "core" annihilation (treated for instance according to the statistical theory), coupled to the dispersion of the pion clouds without further interactions.

Thus, in $p\bar{p}$ annihilation, the proton cloud, in which positive charge dominates,[7] continues its forward flight in the global center-of-mass system as the antiproton cloud does, in which negative charge dominates.

For a quantitative treatment, one might assume that cloud mesons are emitted isotropically in the rest frame of their mother nucleon. But Pilkuhn[8] observed that the statistical core-annihilation probability then becomes a complicated function of the c.m. angles and momenta of the cloud mesons which is difficult to calculate numerically.

Therefore Pilkuhn tried[8] to obtain the asymmetries by working with a "pole model," one pole being associated with a *peripheral* emission of one meson. (See Fig. 1, where the case with *one* "peripheral" meson is illustrated.) His conclusions were contrary to the assumption of exactly one pole for all π multiplicities, and they were doubtful about the case of postulating exactly two poles for all multiplicities.

Recently Minami[9] proposed that the $p\bar{p}$ annihilation is dominated by a graph with two internal nucleon lines (as in Fig. 2). Thus, here we consider a model in which a virtual annihilation with statistical[10] *multipion* pro-

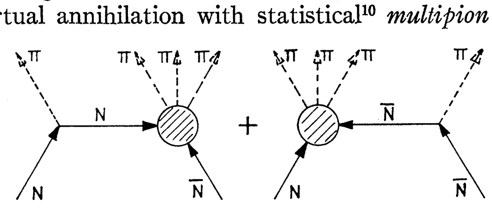

FIG. 1. Graphs of the pole model, as considered by H. Pilkuhn. (See Ref. 8.)

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[1] For the first experimental evidence of this fact, see B. Maglić, G. Kalbfleisch, and M. Stevenson, Phys. Rev. Letters **7**, 137 (1961).

[2] For subsequent experimental work related to four-pronged annihilations, see e.g. Refs. 12–15.

[3] As pointed out by Pais (see Ref. 4), the charge-conjugation invariance implies that the π^+ and π^- c.m. angular distributions, with respect to the direction of the nucleon of equal charge, are the same if both p and $\bar{p}$ are unpolarized.

[4] A. Pais, Phys. Rev. Letters **3**, 242 (1959).

[5] Z. Koba and G. Takeda, Progr. Theoret. Phys. (Kyoto) **19**, 269 (1958).

[6] A. Stajano, Nuovo Cimento **28**, 197 (1963); **24**, 774 (1962).

[7] H. Miyazawa, Phys. Rev. **101**, 1564 (1956); S. Fubini and W. E. Thirring, *ibid.* **105**, 1382 (1957).

[8] H. Pilkuhn, Arkiv Fysik **23**, 259 (1963).

[9] S. Minami (unpublished). Previously the model was suggested to us by V. Pelosi.

[10] By the word "statistical" we mean that the off-mass-shell amplitude for the "core" annihilation is assumed to depend in the simplest possible way on all the variables on which it *a priori* must depend.

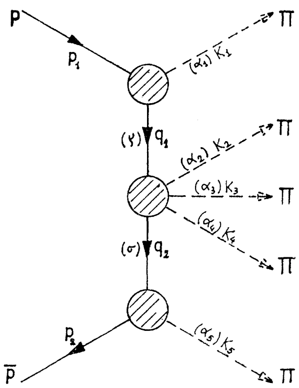

FIG. 2. Our model for $p\bar{p} \to 5\pi$.

duction is preceded by a peripheral symmetric emission of one pion by both nucleon and antinucleon. Roughly, one expects *a priori* that this "peripheroidal" model will explain qualitatively the main physical characteristics of the pionic c.m. angular distributions in $p\bar{p}$ annihilation, expecially if one bears in mind the experimental observation that the F/B asymmetry *increases* as the total energy increases and as the pion multiplicity decreases. Moreover, for every multiplicity, both the anisotropy and the asymmetry in the angular distribution of the charged pions are experimentally due mainly to the pions emitted with greater impulse. In particular,[11] for $pp \to 5\pi$, the model assumes the diagram of Fig. 2.

We were led to analyze carefully the consequences of the model, taking into account all kinematic coefficients and spin and isospin factors. Obviously our model, in this version, does not consider resonance production.

II. GENERAL FORMULATION OF THE MODEL

We consider in this work the particular process

$$p\bar{p} \to \pi^+\pi^+\pi^-\pi^-\pi^0, \tag{1}$$

for which good experimental information (i.e., with good statistics) is available.[1,12–15] We want to study the contributions to the transition matrix elements ($T \equiv S-1$) for this process, arising from graphs with the same structure as the one of Fig. 2.

In Fig. 2 the four-vectors p_1, p_2, K_1, K_2, K_3, K_4, and K_5 are the four-momenta of the corresponding (entering or outgoing) particles, while the indices $\alpha_i (i=1, \cdots, 5)$, which can assume the values ± 1, 0, determine the charge state of the outgoing pions. Obviously

$$\sum_{i=1}^{5} \alpha_i = 0. \tag{2}$$

The two internal lines of the graphs refer to virtual nucleons with four-momenta $q_1 = p_1 - K_1$ and $q_2 = K_5 - p_2$, and with the third component of isospin equal to ρ and σ. The ρ and σ may assume the values $\pm\frac{1}{2}$, and they are univocally determined when we fix α_1 and α_5, owing to charge conservation. With this notation, the contribution to the T-matrix element from the graph of Fig. 2 can be written

$$\langle f|T|i\rangle = \delta^{(4)}(P_f - P_i) \times M_{fi}, \tag{3}$$

where, applying the standard rules, one obtains[16]

$$M_{fi} = -\frac{m}{(2\pi)^{21/2}} \frac{G_{\alpha_1} G_{\alpha_5}}{[32 p_{10} p_{20} k_{10} k_{20} k_{30} k_{40} k_{50}]^{1/2}} \bar{v}(\mathbf{p}_2)\gamma_5 \times \frac{\gamma\cdot q_2 + m}{q_2^2 - m^2} \mathcal{Q}^{\alpha|\rho\sigma}(q_1 q_2 K) \frac{\gamma\cdot q_1 + m}{q_1^2 - m^2} \gamma_5 w(\mathbf{p}_1). \tag{4}$$

In Eq. (4):

(a) $G_{\pm 1} = \sqrt{2}G$; $G_0 = G$, G being the $(pp\pi^0)$ coupling constant.[17]

(b) m is the nucleon mass.

(c) $w(\mathbf{p}_1)$ is a positive-energy spinor with momentum $\mathbf{p}_1$; $v(\mathbf{p}_2)$ is a negative-energy spinor with momentum $-\mathbf{p}_2$, satisfying the equations $(\gamma\cdot p_1 - m)w(\mathbf{p}_1) = (\gamma\cdot p_2 + m)v(\mathbf{p}_2) = 0$. The adopted normalization is $\bar{w}(\mathbf{p}_1)w(\mathbf{p}_1) = 1$ and $\bar{v}(\mathbf{p}_2)v(\mathbf{p}_2) = -1$; the helicity indices are understood.

(d) $\alpha \equiv (\alpha_2, \alpha_3, \alpha_4)$ and $K \equiv (K_2, K_3, K_4)$.

(e) $\mathcal{Q}^{\alpha|\rho\sigma}(q_1 q_2 K)$ is a 4×4 matrix in the Dirac-

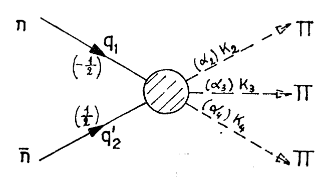

FIG. 3. The statistical "core" annihilation here considered. Note that with our conventions $\sigma' = -\sigma$; $q_2' = -q_2$.

[11] Actually, for momenta up to a few GeV/c, the mean π multiplicity is about 5, and the four-prong annihilations in $(5\pi)^0$ constitute a large number of the annihilation processes.

[12] K. Bockmann, B. Nellen, E. Paul, B. Wagini, I. Borecka, J. Diaz, V. Wolff, J. Kidd, L. Mandelli, L. Mosca, V. Pelosi, S. Ratti, and L. Tallone, Nuovo Cimento 42, 954 (1966), and references therein; V. Russo (private communication); A. M. Rusconi, thesis, Universitá di Milano, 1965 (unpublished).

[13] T. Ferbel, A. Firestone, J. Sandweiss, H. D. Taft, M.Galloud, T. W. Morris, W. J. Willis, A. H. Bachman, P. Baumel, and R. M. Lea, Phys. Rev. **143**, 1096 (1966).

[14] V. Alles-Borelli, B. French, Å. Frisk, L. Michejda, and E. Paul, Nuovo Cimento (to be published).

[15] See also, e.g., T. Ferbel, J. Sandweiss, H. D. Taft, M. Gailloud, T. W. Morris, R. M. Lea, and T. E. Kalogeropulos, in *Proceedings of the 1962 International Conference on High-Energy Nuclear Physics at CERN, 1962*, edited by J. Prentke (CERN, Geneva, 1962); C. Baltay, T. Ferbel, J. Sandweiss, H. D. Taft, B. B. Culwick, W. B. Fowler, M. Gailloud, J. K. Kopp, R. I. Louttit, T. W. Morris, J. R. Sanford, R. P. Shutt, D. L. Stonehill, R. Stump, A. M. Thorndike, M. S. Webster, W. J. Willis, A. H. Bachman, P. Baumel, and R. M. Lea, in *Nucleon Structure*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1964), p. 267; T. Ferbel (unpublished).

[16] With the metric (+−−−), and in natural units.

[17] The experimental value of G is $G^2/4\pi = 14.6$.

spinor space, and has the proper Lorentz and isospace transformation properties.

Bearing in mind that the intrinsic parity of a π is -1, the Lorentz structure of $\mathcal{A}$ is assumed to be the following:

$$\mathcal{A}^{\alpha|\rho\sigma}(q_1q_2K)=\gamma_5 A^{\alpha|\rho\sigma}(q_1q_2K)\,, \tag{5}$$

where $A^{\alpha|\rho\sigma}(q_1q_2K)$ is a Lorentz scalar.

Taking into account its transformation properties for isorotation, we can write (see Fig. 3)

$$A^{\alpha|\rho\sigma}(q_1q_2K)=\sum_{\nu,T}\langle\alpha|T,\nu\rangle A^{T,\nu}(q_1q_2K)\langle T|\rho,-\sigma\rangle. \tag{6}$$

The $\langle\rho,\sigma|T\rangle$ and $\langle\alpha|T,\nu\rangle$ are the coefficients of the decomposition into eigenstates of total isospin T and its third component (which is not explicitly written), respectively, for a state formed by two particles with isospin $\frac{1}{2}$ and third components ρ, $-\sigma$, and for a state formed by three particles with isospin 1 and third components α_2, α_3, α_4. It is well known that in the second case the total isotopic spin and its 3rd component are not enough to single out the decomposition terms, and it is necessary to introduce a third quantum number ν, which appears in formula (6).

At this point, since more detailed dynamic information is lacking, we make *the "statistical"*[18,19] *hypothesis* that $A^{T,\nu}(q_1q_2K)$ is independent of all those variables on which *a priori* it should depend, and write

$$|A^{T,\nu}(q_1q_2K)|^2=\Lambda^4, \tag{7}$$

where Λ is a constant with the dimensions of a length, which—if one takes the model seriously—will turn out to be, e.g., about 8 F for an entering laboratory momentum of 5.7 GeV/c. (Actually, as we do not concern ourselves with different-multiplicity processes, the introduction of the Λ parameter is not strictly necessary.) With our assumptions, we get

$$|A^{\alpha|\rho\sigma}(q_1q_2K)|^2=\Lambda^4 I^{\alpha}{}_{\rho,-\sigma}+\text{interference terms}, \tag{8}$$

having set

$$I^{\alpha}{}_{\rho,-\sigma}=\sum_{\nu,T}|\langle\alpha|T,\nu\rangle|^2|\langle T|\rho,-\sigma\rangle|^2. \tag{9}$$

Let us consider the reaction

$$p(p_1)+\bar{p}(p_2)\rightarrow\pi(K_1\alpha_1)+\pi(K_2\alpha_2)+\pi(K_3\alpha_3)+\pi(K_4\alpha_4)+\pi(K_5\alpha_5). \tag{10}$$

The contribution of the graph of Fig. 2 to the differential cross section, averaged on the entering nucleon helicities, is given, if we neglect the interference terms in Eq. (8), by

$$d\sigma=\Lambda^4\frac{G_{\alpha_1}{}^2G_{\alpha_5}{}^2}{(2\pi)^{19}}\frac{m^2\delta^{(4)}(p_1+p_2-\sum_{i=1}^{5}K_i)}{[(p_1\cdot p_2)^2-m^4]^{1/2}}\frac{I^{\alpha}{}_{\rho,-\sigma}}{(q_1{}^2-m^2)^2(q_2{}^2-m^2)^2}\times\tfrac{1}{4}\sum_{\text{(helicities)}}|\bar{v}(\mathbf{p}_2)\gamma_5(\gamma\cdot q_2+m)\gamma_5(\gamma\cdot q_1+m)\gamma_5 w(\mathbf{p}_1)|^2\frac{d\mathbf{K}_1}{2K_{1_0}}\frac{d\mathbf{K}_2}{2K_{2_0}}\frac{d\mathbf{K}_3}{2K_{3_0}}\frac{d\mathbf{K}_4}{2K_{4_0}}\frac{d\mathbf{K}_5}{2K_{5_0}}. \tag{11}$$

We find easily[20]

$$\tfrac{1}{4}\sum_{\text{(helicities)}}|\bar{v}(\mathbf{p}_2)\gamma_5(\gamma\cdot q_2+m)\gamma_5(\gamma\cdot q_1+m)\gamma_5 w(\mathbf{p}_1)|^2=\frac{1}{16m^2}\operatorname{Tr}\{(\gamma\cdot q_2-m)(\gamma\cdot q_1+m)(\gamma\cdot p_2+m)(\gamma\cdot q_2+m)\times(\gamma\cdot q_1-m)(\gamma\cdot p_1+m)\}\equiv\frac{1}{16m^2}F(p_1p_2K_1K_5). \tag{12}$$

The explicit trace expression is

$$F(p_1p_2K_1K_5)=4\{m^6+m^4(p_1\cdot p_2-q_1{}^2-q_2{}^2)+m^2[2(q_1\cdot q_2)(q_1\cdot q_2-2p_1\cdot p_2-p_1\cdot q_1-p_1\cdot q_2+p_2\cdot q_1+p_2\cdot q_2)-2(p_1\cdot q_1-p_1\cdot q_2)(p_2\cdot q_1-p_2\cdot q_2)+q_1{}^2(p_1\cdot p_2-2p_2\cdot q_2)+q_2{}^2(p_1\cdot p_2+2p_1\cdot q_1-2p_2\cdot q_1-q_1{}^2)]-2(p_1\cdot q_1)(q_1\cdot q_2)(p_1\cdot q_2+p_2\cdot q_2)+2q_1{}^2(p_1\cdot q_2)(p_2\cdot q_2)+2q_2{}^2(p_1\cdot q_1)(p_2\cdot q_1)+2(p_1\cdot p_2)(q_1\cdot q_2)^2-q_1{}^2q_2{}^2(p_1\cdot p_2)\}.$$

Some aspects of this evaluation are given in Appendix B, while in Appendix A we give the meaning of the invariants one meets with in this trace calculation.

If we put

$$G(1,5)\equiv G(p_1p_2K_1K_5)\equiv\frac{F(p_1p_2K_1K_5)}{(q_1{}^2-m^2)^2(q_2{}^2-m^2)^2},$$

where $q_1=p_1-K_1$ and $q_2=K_5-p_2$, we get

$$d\sigma=\Lambda^4\frac{G_{\alpha_1}{}^2G_{\alpha_5}{}^2}{16(2\pi)^{19}}[(\mathbf{p}_1\cdot\mathbf{p}_2)^2-m^4]^{-1/2}G(1,5)I^{\alpha}{}_{\rho,-\sigma}\delta^{(4)}(p_1+p_2-\sum_{i=1}^{5}K_i)\frac{d\mathbf{K}_1}{2K_{1_0}}\cdots\frac{d\mathbf{K}_5}{2K_{5_0}}. \tag{13}$$

[18] M. Kretzschmar, Ann. Rev. Nucl. Sci. **11**, 1 (1961); R. Hagedorn, Nuovo Cimento **15**, 434 (1960).

[19] P. Srivastava and G. Sudarshan, Phys. Rev. **110**, 765 (1958).

[20] See, e.g., S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson and Company, Evanston, Illinois, 1961).

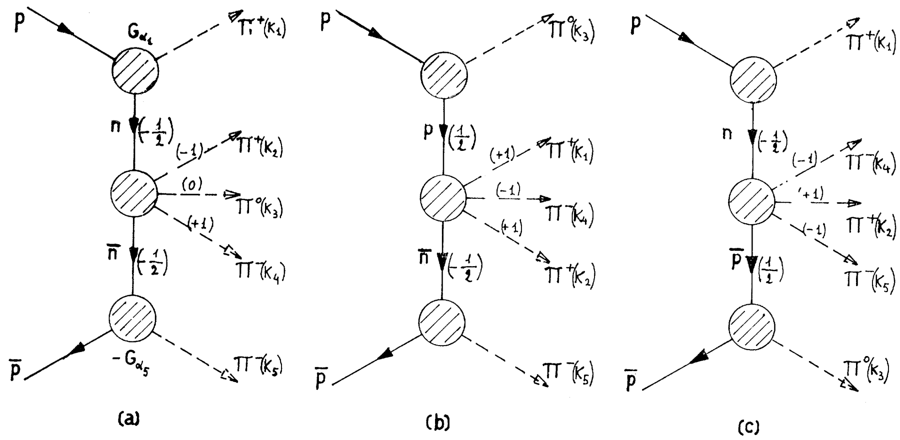

FIG. 4. The possible final states for reaction (14). From each diagram we can get *four* graphs by exchanging the momenta of the identical particles with one another in all possible ways.

Let us now restrict ourselves to reaction (1), that is

$$p(p_1)+\bar{p}(p_2)\to \pi^+(K_1)+\pi^+(K_2)+\pi^0(K_3)+\pi^-(K_4)+\pi^-(K_5)\,, \tag{14}$$

and consider for that process the contributions of the twelve graphs that one can obtain by exchanging the identical-particle momenta with one another in the three diagrams of Fig. 4.

With the same procedure used for the diagram of Fig. 2 (and with the same approximations), we will evaluate the contribution of those graphs to the transition rate and to the cross section.

Then, if we neglect the interference terms between the various graph contributions, we get, for the differential cross section of reaction (14), the expression

$$d\sigma=\frac{\Lambda^4G^4}{16(2\pi)^{19}}\{4I_{-1/2,1/2}{}^{1,0,-1}[G(1,5)+G(2,5)+G(1,4)+G(2,4)]+2I_{-1/2,-1/2}{}^{1,-1,-1}[2G(1,3)+2G(2,3)]+2I_{1/2,1/2}{}^{1,1,-1}$$
$$\times[2G(3,5)+2G(3,4)]\}[(\mathbf{p}_1\cdot\mathbf{p}_2)^2-m^4]^{-1/2}\,\delta^{(4)}(p_1+p_2-\sum_{i=1}^{5}K_i)\frac{d\mathbf{K}_1}{2K_{1_0}}\cdots\frac{d\mathbf{K}_5}{2K_{5_0}}, \tag{15}$$

with, in general,

$$G(j,h)\equiv G(p_1p_2K_jK_h)\equiv\frac{F(p_1p_2K_jK_h)}{(q_1{}^2-m^2)(q_2{}^2-m^2)}, \tag{16}$$

where now $q_1=p_1-K_j$ and $q_2=K_h-p_2$, and with[21]

$$I_{-1/2,1/2}{}^{1,0,-1}=\tfrac{1}{6}\times\tfrac{1}{2}+\tfrac{2}{5}\times\tfrac{1}{2}=17/60;\quad I_{-1/2,-1/2}{}^{1,-1,-1}=I_{1/2,1/2}{}^{1,1,-1}=\tfrac{3}{5}. \tag{16'}$$

Therefore, with our assumptions, if we define

$$H(p_1p_2K_1\cdots K_5)\equiv(17/15)[G(1,5)+G(2,5)+G(1,4)+G(2,4)]+(12/5)[G(1,3)+G(2,3)+G(3,5)+G(3,4)], \tag{17}$$

then the c.m. angular distribution of π^+ (or π^-) from reaction (14), as a function of the scattering-angle cosine, will be

$$\frac{1}{c}\frac{d\sigma^+}{d\cos\theta_1}=\frac{\Lambda^4G^4}{8(2\pi)^{18}}\frac{1}{[s(\frac{1}{4}s-m^2)]^{1/2}}\int_{-\infty}^{\infty}\frac{d\mathbf{K}_2}{2K_{2_0}}\int_{-\infty}^{\infty}\frac{d\mathbf{K}_4}{2K_{4_0}}\int_{-\infty}^{\infty}\frac{d\mathbf{K}_5}{2K_{5_0}}$$
$$\times\int_0^{\infty}\frac{d|\mathbf{K}_1||\mathbf{K}_1|^2}{2K_{1_0}}\frac{\delta(2p_0-\sum_{i=1}^{5}K_{i_0})}{2K_{3_0}}H(p_1p_2K_1\cdots K_5)\Big|_{\mathbf{K}_3=-\mathbf{K}_1-\mathbf{K}_2-\mathbf{K}_4-\mathbf{K}_5}. \tag{18}$$

III. NUMERICAL EVALUATION AND COMPARISON WITH EXPERIENCE

If we put, for every function $G(j,h)$ entering in formula (17),

$$g_{j,h}=\int\frac{d\mathbf{K}_2}{2K_{2_0}}\int\frac{d\mathbf{K}_3}{2K_{3_0}}\int\frac{d\mathbf{K}_4}{2K_{4_0}}\int\frac{d\mathbf{K}_5}{2K_{5_0}}G(j,h)\delta(\sqrt{s}-\sum_{i=1}^{5}K_{i_0})\delta^{(3)}(\sum_{i=1}^{5}\mathbf{K}_i)\,, \tag{19}$$

[21] See F. Cerulus, Nuovo Cimento Suppl. 15, 402 (1960).

we immediately see that

$$\mathcal{J}_{1,5}=\mathcal{J}_{1,4}=\mathcal{J}_{1,3}\equiv\mathcal{J}_1(s,|\mathbf{K}_1|,\cos\theta_1),$$

and

$$\mathcal{J}_{2,5}=\mathcal{J}_{2,4}=\mathcal{J}_{2,3}=\mathcal{J}_{3,5}=\mathcal{J}_{3,4}\equiv\mathcal{J}_2(s,|\mathbf{K}_1|,\cos\theta_1).$$

Therefore, formula (18) may be rewritten as follows:

$$\frac{d\sigma^+}{d\cos\theta_1}=\Lambda^4\alpha(s)\int_0^\infty\frac{d|\mathbf{K}_1||\mathbf{K}_1|^2}{2x}\mathcal{J}(s,|\mathbf{K}_1|,\cos\theta),\tag{20}$$

where

$$x\equiv K_{1_0},\quad \cos\theta\equiv\cos\theta_1,\quad \alpha(s)\equiv\frac{G^4}{8(2\pi)^{18}}[s(\tfrac{1}{4}s-m^2)]^{-1/2},\quad \mathcal{J}(s,|\mathbf{K}_1|,\cos\theta)=(14/3)\mathcal{J}_1+(142/15)\mathcal{J}_2.\tag{21}$$

Thus the problem has been reduced to the evaluation of the two integrals $\mathcal{J}_j(j=1,2)$. One may write, applying the *generalized mean-value theorem*, that

$$\mathcal{J}_j=\int\frac{d\mathbf{K}_2}{2K_{2_0}}\int\frac{d\mathbf{K}_3}{2K_{3_0}}\int\frac{d\mathbf{K}_4}{2K_{4_0}}[G(j,5)]\Bigg|_{\mathbf{K}_5=\mathbf{K}_5^{(m)}}\int\frac{d\mathbf{K}_5}{2K_{5_0}}\times\delta(\sqrt{s}-\sum_{i=1}^{5}K_{i_0})\delta^{(3)}(\sum_{i=1}^{5}\mathbf{K}_i),\quad(j=1,2)\tag{22}$$

where (m) stands for *mean.*

Let us consider first the integral $\mathcal{J}_1$. In order to be able to evaluate it numerically, *we make the assumption*, apparently reasonable on a physical basis, that (for every fixed K_1)

$$\mathbf{K}_5^{(m)}=-\mathbf{K}_1,\tag{23}$$

which is equivalent, more generally, to the substitution of

$$\mathbf{q}_2=\mathbf{q}_2^{(m)}=\mathbf{q}_1,\quad q_{2_0}=q_{2_0}^{(m)}=-q_{1_0}\tag{24}$$

into the function $G(1,5)$ [see also Eq. (A1) of Appendix A]. If we put

$$\begin{gathered}G(s,x,\cos\theta)\equiv G(1,5)\Bigg|_{\mathbf{K}_5=-\mathbf{K}_1},\\ \xi=(\sqrt{s})-x,\\ R_4(\mathbf{Q},\xi)=\int\frac{d\mathbf{K}_2}{2K_{2_0}}\int\frac{d\mathbf{K}_3}{2K_{3_0}}\int\frac{d\mathbf{K}_4}{2K_{4_0}}\int\frac{d\mathbf{K}_5}{2K_{5_0}}\delta(\sum_{i=2}^{5}K_{i_0}-\xi)\delta^{(3)}(\sum_{i=2}^{5}\mathbf{K}_i-\mathbf{Q}),\end{gathered}\tag{25}$$

then we have

$$\mathcal{J}_1=G(s,x,\cos\theta)R_4(-\mathbf{K}_1,\xi).\tag{26}$$

The four-body "phase space" (for equal-mass particles) R_4 can be easily calculated.[22,23] Let us set

$$\begin{aligned}&\xi_5\equiv\sqrt{s}, &&\xi_n=(\xi_{n+1}{}^2-2x_{n+1}\xi_{n+1}+\mu^2)^{1/2},\\ &x_5\equiv x\equiv K_{1_0}, &&X_{n+1}=[\xi_{n+1}{}^2-(n^2-1)\mu^2]/2\xi_{n+1}\quad(n=2,3,4).\end{aligned}\tag{27}$$

As R_4 is *Lorentz-invariant,* we shall have

$$R_4(-\mathbf{K}_1,\xi)=R_4(\mathbf{0},\xi_4).\tag{28}$$

And, using a simple recurrence relation,[19] we have (μ=pion mass)

$$R_4(\mathbf{0},\xi_4)=\int\frac{d\mathbf{K}}{2K_0}\Theta(\xi_3-3\mu)R_3(\mathbf{0},\xi_3)=2\pi\int_\mu^{X_4}dx_4(x_4{}^2-\mu^2)^{1/2}R_3(\mathbf{0},\xi_3),\tag{29}$$

where the explicit expression of the Lorentz-invariant three-body "phase space" (for equal-mass particles);

[22] See, e.g., G. Kalbfleish, University of California Lawrence Radiation Laboratory, Phys. Notes, Memo 150, 1960 (unpublished); O. Skjeggestad, in CERN Report No. 64-13, Vol. II (unpublished); see also M. Block, Phys. Rev. **101**, 796 (1956).

[23] While considering the kinematics connected with our process with five final bodies, we evaluated also the c.m. volume of the allowed kinematical region for the three-momentum of a final particle, at fixed momentum of *one* of the other four final bodies. Owing to its intrinsic interest, we have reported this evaluation in Appendix C.

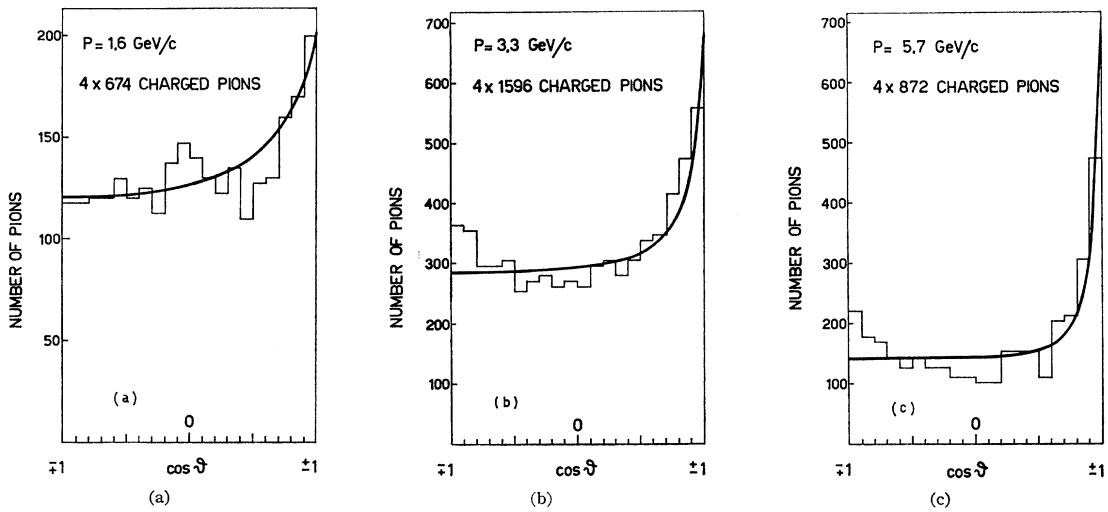

FIG. 5. The c.m. distributions of the (charged) $\pi^{\mp}$ from the reaction (14), with respect to the direction of the incoming antiproton, at the three experimentally available laboratory momenta. The continuous lines are the theoretical curves yielded by our model. The experimental data are respectively taken: (a) from Ref. 1, for 1.6 GeV/c; (b) from Ref. 13, for 3.3 GeV/c; (c) from Ref. 14, for 5.7 GeV/c.

R_3 is well known[22]:

$$R_3(0,\xi_3)=\pi^2\int_\mu^{X_3} dx_3(x_3^2-\mu^2)^{1/2}\left[\frac{\xi_2^2-4\mu^2}{\xi_2^2}\right]^{1/2}. \tag{30}$$

Therefore, we may rewrite Eq. (26) as follows:

$$\mathcal{I}_1=2\pi^3 G(s,x,\cos\theta)\int_\mu^{X_4} dx_4(x_4^2-\mu^2)^{1/2}\int_\mu^{X_3} dx_3(x_3^2-\mu^2)^{1/2}\left[\frac{\xi_2^2-4\mu^2}{\xi_2^2}\right]^{1/2}, \tag{31}$$

and we get in conclusion

$$I_1(s,\cos\theta)=\frac{7}{3}\alpha(s)\int_0^\infty dx_5(x_5^2-\mu^2)^{1/2}\Theta(x_5-\mu)\Theta(\xi_4-4\mu)\mathcal{I}_1(s,x,\cos\theta)=\frac{7}{3}\alpha(s)\int_\mu^{X_5} dx_5(x_5^2-\mu^2)^{1/2}\mathcal{I}_1(s,x,\cos\theta). \tag{32}$$

Considering now the integral $\mathcal{I}_2$ of formula (22), we could proceed as we did for $\mathcal{I}_1$, assuming in this case

$$\mathbf{K}_5^{(m)}=-\mathbf{K}_2, \tag{33}$$

that is to say, more generally, effecting the substitution (24), for every fixed q_1, in the function $G(2,5)$. But, as $\mathcal{I}_2$ relates to the charged pions emitted in the virtual "core" annihilation, one may reasonably assume that it depend only weakly on the direction of $\mathbf{K}_1$, thus supplying a quasi-isotropic contribution to the charged-pion distribution. We do not make any attempt to evaluate such a "background," but we keep it as an *additive fitting parameter*, depending only on the total energy $\sqrt{s}$.

In conclusion, one obtains the following final formula for the charged-pion distribution from reaction (14):

$$\frac{d\sigma}{d\cos\theta}=\Lambda^4 I_1(s,\cos\theta)+Z(s). \tag{34}$$

The comparison with experimental data has been done for 1.6 GeV/c,[1] 3.3 GeV/c,[13] and 5.7 GeV/c[14] laboratory momenta, using an IBM-7040 computer. It is shown in Fig. 5. The best fit has been obtained with quite reasonable[18,19,24] Λ values: namely, e.g., $\Lambda=(13.9\pm0.5)$ F for 3.3 GeV/c, and $\Lambda=(7.8\pm0.3)$ F for 5.7 GeV/c. The accord between the theoretical lines, nor-

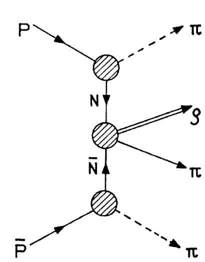

FIG. 6. A natural modification of the model. We believe that the c.m. charged-pion distributions would not be affected substantially by this change. Here N means nucleon.

[24] B. Desai, University of California Lawrence Radiation Laboratory Report No. UCRL-9024 (unpublished).

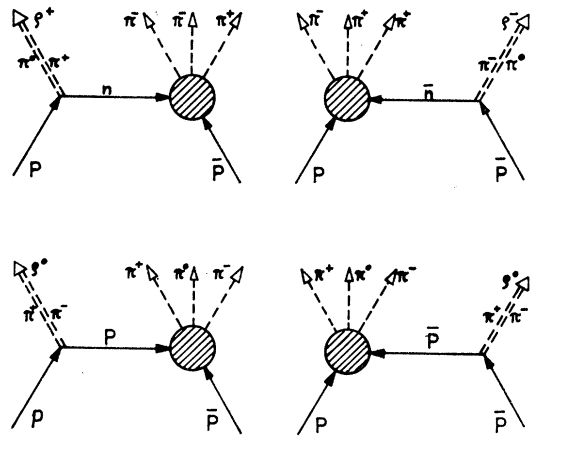

FIG. 7. Another proposed "model," whose contribution *at high energies* could possibly explain the "backward tail" we can observe in the charged pion distributions (see in particular Fig. 5).

malized to the charged-pion numbers, and the experimental histograms[1,13,14] is satisfactory enough, *except* for the backward "tail," which appears at the higher momenta, i.e., at 3.3 and 5.7 GeV/c.

Our model does not take into account the production of resonances, which seem to appear largely in the more recent data, for the pion multiplicity here considered (especially the ρ, which enters very abundantly). A natural modification of the model would be the one represented in Fig. 6. But we believe—as it may be argued also *a priori*—that the c.m. distributions of the charged pions would not be substantially affected by this change. On the other hand, the aforementioned "backward tail" could possibly be obtained considering also graphs of the type one gets from Fig. 1, substituting a peripheral ρ-emission vertex to the one-pion vertex (see Fig. 7). Finally, another model, similar to the one shown in Fig. 6 but with only one "peripheral" vertex, has been proposed very recently in Ref. 14.

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APPENDIX A

The kinematics of interest for reaction (14), in the global c.m. system, is the following:

$$\mathbf{p}_1=-\mathbf{p}_2\,,\quad p_{1_0}=p_{2_0}\equiv p_0\,,\quad p_1^2=p_2^2\equiv p^2=m^2\,,$$
$$p_1+p_2=K_1+K_2+K_3+K_4+K_5\,.$$

By definition (see Fig. 2)

$$q_1=p_1-K_1\,,\quad q_2=K_5-p_2\,,\quad |\mathbf{p}_1|=|\mathbf{p}_2|\equiv|\mathbf{p}|\equiv P\,.$$

Limiting ourselves to the diagram of Fig. 4(a), we can choose the variables

$$s\equiv(p_1+p_2)^2=4p_0^2\,,$$
$$E_1\equiv x\equiv K_{1_0}\,,\quad E_5\equiv K_{5_0}\,,$$
$$Q^2=(K_1+K_5)^2$$

(θ_1,θ_5: scattering angles of pions 1 and 5 relative to the entering antiproton direction).

Then we get

$$(s-4m^2)^{1/2}=2P\,,\quad 2(\mathbf{p}^2+m^2)^{1/2}=\sqrt{s}\,,$$
$$|\mathbf{K}_1|=(E_1^2-\mu^2)^{1/2}\,,\quad |\mathbf{K}_5|=(E_5^2-\mu^2)^{1/2}\,,$$
$$p_1\cdot p_2=\tfrac{1}{2}s-m^2\,,$$
$$K_1\cdot p_1=\tfrac{1}{2}E_1\sqrt{s}+P|\mathbf{K}_1|\cos\theta_1\,,$$
$$K_1\cdot p_2=\tfrac{1}{2}E_1\sqrt{s}-P|\mathbf{K}_1|\cos\theta_1\,,$$
$$K_5\cdot p_1=\tfrac{1}{2}E_5\sqrt{s}+P|\mathbf{K}_5|\cos\theta_5\,,$$
$$K_5\cdot p_2=\tfrac{1}{2}E_5\sqrt{s}-P|\mathbf{K}_5|\cos\theta_5\,,$$
$$p_1\cdot q_1=m^2-K_1\cdot p_1\,,$$
$$p_2\cdot q_2=K_5\cdot p_2-m^2\,,$$
$$p_1\cdot q_2=K_5\cdot p_1-p_1\cdot p_2\,,$$
$$p_2\cdot q_1=p_1\cdot p_2-K_1\cdot p_2\,,$$
$$q_1^2=m^2+\mu^2-E_1\sqrt{s}-2P|\mathbf{K}_1|\cos\theta_1\,,$$
$$q_2^2=m^2+\mu^2-E_5\sqrt{s}+2P|\mathbf{K}_5|\cos\theta_5\,,$$
$$q_1\cdot q_2=\mu^2-\tfrac{1}{2}Q^2-p_1\cdot p_2+K_5\cdot p_1+K_1\cdot p_2\,.$$

The assumption

$$\mathbf{q}_1=\mathbf{q}_2\,,\quad q_{1_0}=-q_{2_0}\,,\qquad\text{(A1)}$$

brings many simplifications. It is equivalent *in the present case* [see Eq. (23) of the text] to setting

$$\mathbf{K}_5=-\mathbf{K}_1\,,\quad E_5=E_1\,,\quad \cos\theta_5=-\cos\theta_1\,,\quad Q^2=4E_1^2\equiv 4x^2.$$

APPENDIX B

We want evaluate the spin factor *for the first graph* [*Fig. 4(a)*], i.e.,

$$\tfrac{1}{4}\sum_{r,s}^{1,2}|\bar{v}^r(p_2)Ow^s(p_1)|^2\equiv\frac{1}{16m^2}F(p_1p_2K_1K_5)\,,$$

with

$$O=\gamma_5(q_2+m)\gamma_5(q_1+m)\gamma_5\,.$$

The procedure is "standard." The explicit expression of the function F of formula (12) will be

$$F(p_1p_2K_1K_5)=\mathrm{Tr}\{(q_2-m)(q_1+m)(p_2+m)\times(q_2+m)(q_1-m)(p_1+m)\}$$
$$=4(m^6+m^4T_2+m^2T_4+T_6)\,,$$

where

$$T_2\equiv p_1\cdot p_2-q_1^2-q_2^2\,,$$
$$T_4\equiv 2(q_1\cdot q_2)(q_1\cdot q_2-2p_1\cdot p_2-p_1\cdot q_1-p_1\cdot q_2+p_2\cdot q_1+p_2\cdot q_2)+2(q_1\cdot p_2)(p_1\cdot q_2-p_1\cdot q_1)+2(q_2\cdot p_2)(p_1\cdot q_1-p_1\cdot q_2)+q_1^2(p_1\cdot p_2-2p_2\cdot q_2)+q_2^2(p_1\cdot p_2+2p_1\cdot q_1-2p_2\cdot q_1-q_1^2)\,,$$

and

$$T_6 \equiv 2(q_1\cdot q_2)[(p_1\cdot p_2)(q_1\cdot q_2)-(p_1\cdot q_1)(p_2\cdot q_2) -(p_1\cdot q_2)(p_2\cdot q_1)]+2q_1^2(p_1\cdot q_2)(p_2\cdot q_2) +2q_2^2(p_1\cdot q_1)(p_2\cdot q_1)-q_1^2q_2^2(p_1\cdot p_2).$$

From a computational point of view, *with the simplifying assumption* (A1), one gets in the c.m. system [see Eq. (23) of the text]

$$\tfrac{1}{4}F(p_1p_2K_1K_5)\equiv\tfrac{1}{4}F(s,x) =\mathcal{A}+\mathcal{B}\cos\theta_1+\mathcal{C}\cos^2\theta_1+\mathcal{D}\cos^3\theta_1,$$

where $x\equiv E_1\equiv K_{1_0}$, and where

$$\mathcal{A}\equiv m^6+m^4(t-2d)+m^2(2w^2-2a^2-2c^2-d^2+2cd +2dt+4ac+4ad-4aw-4cw-4wt)+2a^2w +2c^2w+2w^2t-d^2t-4acd,$$

$$\mathcal{B}\equiv b[4m^4+m^2(8w-8a-4c-4t-2d)+4ad+4cd +4dt-4aw-4cw+8ac],$$

$$\mathcal{C}\equiv 4b^2[2m^2+w-2a-2c-d-t),$$

$$\mathcal{D}\equiv 8b^3,$$

in which

$$a\equiv m^2-\tfrac{1}{2}x\sqrt{s},$$
$$b\equiv[(\tfrac{1}{4}s-m^2)(x^2-\mu^2)]^{1/2},$$
$$c\equiv\tfrac{1}{2}\sqrt{s}(x-\sqrt{s})+m^2,$$
$$d\equiv m^2-e,$$
$$e\equiv x\sqrt{s}-\mu^2,$$
$$t\equiv p_1\cdot p_2=\tfrac{1}{2}s-m^2,$$
$$w\equiv\mu^2-2x^2-t+x\sqrt{s}-2b\cos\theta_1.$$

Besides, in the adopted approximation,

$$(q_1^2-m^2)(q_2^2-m^2)=(2b\cos\theta_1+e)^2$$

APPENDIX C

While considering the kinematics of our reaction with five equal-mass final bodies, we evaluated also the c.m. 'volume', in the impulse space, of the allowed kinematical region for the three-momentum $\mathbf{K}_5$ of a final particle, at fixed three-momentum $\mathbf{K}_1$ of *one* of the other four final particles.

Owing to its intrinsic interest, we report that evaluation here. We propose to calculate the integral

$$g(s,\mathbf{K}_1)=\int_{C_1} d\mathbf{K}_5, \qquad \text{(C1)}$$

where $C_1=C_1(\mathbf{K}_1)$ is the set of values of $\mathbf{K}_5$ for which, at fixed $\mathbf{K}_1$, the following system $[K_{i_0}=(\mathbf{K}_i^2+\mu^2)^{1/2}]$

$$\sqrt{s}-\sum_{i=1}^{5}K_{i_0}=0, \qquad \sum_{i=1}^{5}\mathbf{K}_i=0, \qquad \text{(C2)}$$

can be satisfied. That is to say, we have to determine, for each fixed $\mathbf{K}_1$, the set of the values of $\mathbf{K}_5$ in correspondence to which there exist vectors $\mathbf{K}_2$, $\mathbf{K}_3$, and $\mathbf{K}_4$ that satisfy the system (C2). As before, the dependence on s is often simply understood.

Let us first notice that, whatever $\mathbf{K}_5$ may be, the second equation (C2) can be satisfied provided that one chooses $\mathbf{K}_4=-(\mathbf{K}_1+\mathbf{K}_2+\mathbf{K}_3+\mathbf{K}_5)$. Thus one is driven to look for the values of $\mathbf{K}_5$, in correspondence to which there exist some $\mathbf{K}_2$ and $\mathbf{K}_3$ that satisfy

$$\sqrt{s}-K_{1_0}-K_{2_0}-K_{3_0} -[(K_1+K_5+K_2+K_3)^2+\mu^2]^{1/2}-K_{5_0}=0. \qquad \text{(C3)}$$

We may try to solve the problem in steps. First, one asks what conditions one has to impose upon $\mathbf{K}_1$, $\mathbf{K}_5$, and $\mathbf{K}_2$ in order that (C3) may be satisfied by some values of $\mathbf{K}_3$. Those conditions single out a certain region $C(\mathbf{K}_1\mathbf{K}_5\mathbf{K}_2)$. Next, one asks what conditions upon $\mathbf{K}_1$ and $\mathbf{K}_5$ are necessary to the existence of values of $\mathbf{K}_3$ for which $C(\mathbf{K}_1\mathbf{K}_5\mathbf{K}_2)$ is not empty. Thus one obtains a new region $C(\mathbf{K}_1\mathbf{K}_5)$; the set of the values of $\mathbf{K}_5$ for which $C(\mathbf{K}_1\mathbf{K}_5)$ is not empty will be the integration domain $C_1(\mathbf{K}_1)$ we are looking for.

To make this program progress, let us put $(x\equiv K_{1_0})$

$$\xi\equiv\sqrt{s}-K_{1_0}\equiv\sqrt{s}-x,$$
$$A\equiv\xi-K_{5_0},$$
$$B\equiv A-K_{2_0},$$
$$\mathbf{v}\equiv\mathbf{K}_1+\mathbf{K}_5,\quad v\equiv|\mathbf{v}|, \qquad \text{(C4)}$$
$$\mathbf{u}\equiv\mathbf{v}+\mathbf{K}_2,\quad u\equiv|\mathbf{u}|,$$
$$k_i\equiv|\mathbf{K}_i|,\quad (i=1,\cdots,5).$$

Equation (C3) may be rewritten, setting $z''=\hat{K}_3\cdot\hat{u}$,

$$B-(k_3^2+\mu^2)^{1/2}+(u^2+k_3^2+2uk_3z''+\mu^2)^{1/2}=0. \qquad \text{(C5)}$$

The above-defined region $C(\mathbf{K}_1\mathbf{K}_5\mathbf{K}_2)$ is determined by the condition that Eq. (C5) may be satisfied by some values of k_3 and z'', with $k_3\geqslant 0$, $|z''|\leqslant 1$. One obtains

$$C(\mathbf{K}_1\mathbf{K}_5\mathbf{K}_2):\quad B\geqslant(4\mu^2+\mu^2)^{1/2}. \qquad \text{(C6)}$$

More explicitly, if one sets $z'=\hat{K}_2\cdot\hat{v}$, one has

$$C(\mathbf{K}_1\mathbf{K}_5\mathbf{K}_2):\quad A-(k_2^2-\mu^2)^{1/2} -(4\mu^2+v^2+k_2^2+2vk_2z')^{1/2}\geqslant 0. \qquad \text{(C7)}$$

The condition that Eq. (C7) be satisfied by some values of k_2 and z', with $k_2\geqslant 0$, $|z'|\leqslant 1$, picks out the region $C(\mathbf{K}_1\mathbf{K}_5)$:

$$[A\geqslant 0 \text{ and } A^2-v^2-3\mu^2\geqslant 0]$$

or

$$[A^2-v^2-3\mu^2\geqslant 2\mu(A^2-v^2)^{1/2}]. \qquad \text{(C8)}$$

This condition (at fixed $\mathbf{K}_1$) depends only on k_5 and $z=\hat{K}_1\cdot\hat{K}_5$. Let us now identify $\mathbf{K}_5$ by means of its polar coordinates k_5, z, ϕ, the last being the azimuthal angle with respect to a reference polar plane passing through $\mathbf{K}_1$. It is then clear that, for every allowed pair of values of k_5 and z, *all* the ϕ values are allowed too.

Consequently, $C_1(\mathbf{K}_1)$ is the direct product of the itnerval $(0,2\pi)$ and the set $\bar{C}_1(\mathbf{K}_1)$, consisting of all the pairs k_5 and z for which at least one inequality (C8) may hold.

Thus one reaches the following result: $\bar{C}_1(\mathbf{K}_1)$ is empty, unless (for $\sqrt{s}>5\mu$)

$$x\leqslant (s-15\mu^2)/2\sqrt{s}. \tag{C9}$$

If (C9) is satisfied, the results for $\bar{C}_1(\mathbf{K}_1)$ are as follows ($x\geqslant\mu$):

(1) If $x\leq\dfrac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)}$, then

$$\bar{C}_1(\mathbf{K}_1):\quad -1\leq z\leq z_1(s,x,K_{50}),\quad \mu\leq K_{50}\leq \epsilon_2(s,x), \tag{C10}$$

where

$$z_1(s,x,K_{5,})\equiv 1 \quad \text{when } \epsilon_1(s,x)\leq K_{50}\leq\epsilon_2(s,x)$$
$$\equiv\frac{\xi^2-k_1^2-8\mu^2-2\xi K_{50}}{2k_1(K_{50}^2-\mu^2)^{1/2}} \quad \text{when } \mu\leq K_{50}\leq\epsilon_1(s,x).$$

(2) If $x\geq\dfrac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)}$, then

$$\bar{C}_1(\mathbf{K}_1):\quad -1\leq z\leq z_2(s,x,K_{50});\quad \epsilon_1(s,x)\leq K_{50}\leq\epsilon_2(s,x), \tag{C11}$$

where

$$z_2(s,x,K_{50})\equiv\frac{\xi^2-k_1^2-8\mu^2-2\xi K_{50}}{2k_1k_5}.$$

$\epsilon_1(s,x)$ and $\epsilon_2(s,x)$ are the two solutions of the equation:

$$4[(\xi-\mu)^2-k_1^2]\epsilon^2-4(\xi-\mu)[(\xi-\mu)^2-k_1^2-3\mu^2]\epsilon+\{[(\xi-\mu)^2-k_1^2-3\mu^2]+4k_1^2\mu^2\}=0. \tag{C12}$$

That is to say

$$\begin{pmatrix}\epsilon_1(s,x)\\ \epsilon_2(s,x)\end{pmatrix}=\frac{\xi(\xi^2-k_1^2-8\mu^2)}{2(\xi^2-k_1^2)}\pm\frac{k_1}{2(\xi^2-k_1^2)}[(\xi^2-k_1^2-8\mu^2)^2-4\mu^2(\xi^2-k_1^2)]^{1/2}. \tag{C13}$$

According to these results, except for the total-energy dependence, $g(s,\mathbf{K}_1)$ depends only on k_1:

$$g(s,\mathbf{K}_1)=\bar{g}(s,x), \tag{C14}$$

and finally we have

$$\bar{g}(s,x)=2\pi\,\Theta\left(\frac{s-15\mu^2}{2\sqrt{s}}-x\right)\left\{\Theta\left(\frac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)}-x\right)\bar{g}_1(s,x)+\Theta\left(x-\frac{(\sqrt{s}-\mu)^2-8\mu^2}{2(\sqrt{s}-\mu)}\right)\bar{g}_2(s,x)\right\}, \tag{C15}$$

with

$$\bar{g}_1(s,x)=\int_{\mu}^{\epsilon_2(s,x)}dK_{50}K_{50}(K_{50}^2-\mu^2)^{1/2}[z_1(s,x,K_{50})+1]=\tfrac{3}{2}(\epsilon_2^2-\mu^2)^{3/2}-\tfrac{3}{4}(\epsilon_1^2-\mu^2)^{3/2}$$
$$+\frac{1}{4k_1}[(\xi^2-x^2-7\mu^2)(\epsilon_1^2-\mu^2)-\xi(\epsilon_1^3-\mu^3)], \tag{C16}$$

$$\bar{g}_2(s,x)=\int_{\epsilon_1(s,x)}^{\epsilon_2(s,x)}dK_{50}K_{50}(K_{50}^2-\mu^2)^{1/2}[z_2(s,x,K_{5,})+1]=\tfrac{3}{4}(\epsilon_2^2-\mu^2)^{3/2}-\tfrac{3}{4}(\epsilon_1^2-\mu^2)^{3/2}$$
$$+\frac{1}{4k_1}[(\xi^2-x^2-7\mu^2)(\epsilon_2^2-\epsilon_1^2)-\xi(\epsilon_2^3-\epsilon_1^3)]. \tag{C16$'$}$$