apparent to some extent in our data, and is even more evident in the wider energy range covered by the bubble-chamber data. Both DESY and CEA bubble-chamber collaborations have found that the model does not correctly describe the distribution of the N^* decay angles.

From a theoretical point of view, the OPE model alone is not acceptable because it is not gauge-invariant. A gauge-invariant Born approximation has been calculated using the four diagrams shown in Fig. 11,[18] treating the N^* as a stable particle. Even when the coupling of the photon to the anomalous moments of the nucleon and the N^* is neglected, the calculated cross section is in serious disagreement with the observed π^- angular distributions in N^* production, being generally too large and increasing to very large values in the backward direction. Stichel and Scholz also performed a gauge-invariant calculation, but included only parts of the amplitudes from diagrams 11(b) and 11(d).[19] Angular distributions calculated from the Stichel-Scholz amplitude using $\lambda^2/4\pi=19.1$ GeV^{-2} are shown in Fig. 8. These curves do not drop off as rapidly as the data on either side of the peak at small angles, and are generally too large in magnitude.

[18] J. Mathews (private communication).

[19] P. Stichel and M. Scholz, Nuovo Cimento 34, 1381 (1964).

We conclude that the OPE model predicts many of the qualitative features of pion-pair photoproduction quite well, particularly in the pseudo-two-body channel $\gamma+p\rightarrow\pi^-+N^*(1238)^{++}$. However, a considerable amount of analysis remains to be done if we are to have a quantitative understanding of pion-pair production within the framework of a gauge-invariant theory.

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Multiple Particle Production in 22.8-GeV/*c* Proton-Nucleon Interactions*

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The aim of the present work is to study the contribution of baryon excited states to multiple particle production. The interactions have been divided into two categories, viz., those having shower multiplicity less than or equal to four and those having multiplicity greater than four. The features that have been studied are the angular distribution, inelasticity, transverse momentum, and center-of-mass momentum of protons and pions. It has been found that the characteristics of high-multiplicity events are well accounted for on the basis of the statistical theory, whereas those of low-multiplicity events can be well explained by considering the final-state particles as decay products of isobars. The results indicate the probable dominance of isobars having isospin $\frac{1}{2}$.

I. INTRODUCTION

The study of high-energy proton-nucleon interactions has been carried out by a large number of investigators at primary momenta of 1–25 GeV/*c*. The first theoretical attempt to explain the observed features of particle production was advanced by Fermi,[1] who suggested that statistical ideas could be applied to describe the multiple-particle production processes at high energies. The theory was modified by Kovacs[2] to include the final-state interactions between the particles, and in particular the pion-nucleon interaction in the $T=J=\frac{3}{2}$ state and the nucleon-nucleon interaction. Although the modified form improved the predictions of the statistical theory, it could not explain satisfactorily all the observed features of the particle production. The observation of a strong pion-nucleon interaction in π-p scattering led Lindenbaum and Sternheimer[3] to conclude that the pion proceeds via the

* A preliminary report of this work was presented at the Oxford International Conference on Elementary Particles, 1965 (unpublished).

[1] E. Fermi, Progr. Theoret. Phys. (Kyoto) 5, 570 (1950); Phys. Rev. 92, 452 (1953); 93, 1435 (1954).

[2] J. S. Kovacs, Phys. Rev. 101, 397 (1956).

[3] S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 105, 1874 (1957); 123, 333 (1961).

excitation of one or both nucleons to an isobaric state. Considering the formation of the $N^*(\frac{3}{2},\frac{3}{2})$ state, they were able to obtain a good fit to the experimental results from p-p collisions at 1.0–3.0 GeV/c. The statistical theory has been modified[4] to a large extent, and its predictions have been applied to p-p collisions at 25 GeV/c. However, the agreement of the theory with the experimental data at this energy is not satisfactory.

The formation of nucleon excited states at high primary momenta between 3.6–11.8 GeV/c has been evidenced[5] in quasielastic p-p collisions. The inelastic reactions at high energies were found to be characterized by a low-momentum transfer between the colliding particles and by the tendency of the secondary particles to be collimated along the direction of the incident nucleons. Experiments[6] have indicated the presence of peaks in inelastic p-p scattering at 22 GeV/c which correspond to the resonances occurring in π-p scattering. These observations led some investigators[7,8] to conclude that isobar formation might play a dominant role in particle production at high energies. The predictions of the isobar model have been applied to p-p collisions at 22 GeV/c, and there are divergent views on the applicability of this model. Whereas some authors[8] believe that all the pions and kaons could be considered as decay products of excited nucleon states, others[9] do not find any evidence of fast-excited baryons in the forward direction; a few observed fast pions which they believe could come from the tail of the statistical distribution.

The present investigation has been aimed at determining the contribution of excited nucleons to multiple particle production. The momentum spectra and the angular distribution of the secondary particles produced by the interaction of 22.8-GeV/c protons with nucleons in emulsion have been determined. The contribution of various errors in ionization and scattering measurements has been carefully ascertained in order to arrive at a correct estimate of the energy and identity of a particle. The results have been compared with the predictions of the isobar model in which it is assumed that the isobars are produced in direct collisions and not in final-state interactions, which are taken account of in the modified form of the statistical theory. The isobars are assumed to decay isotropically in their rest systems. From the analysis of pure p-N collisions obtained by applying rigid selection criteria, it is found that in low-multiplicity events ($n_s \leq 4$), the major contribution to particle production comes from the decay of excited baryons, produced in peripheral collisions. The results indicate that the isobars with isospin$=\frac{1}{2}$ contribute significantly to particle production, and there is no evidence of the formation of $T=\frac{3}{2}$ states. This view is in agreement with the results of other investigations.[10] The features of high-multiplicity events ($n_s > 4$) resulting mainly from central collisions are well accounted for on the basis of statistical theory.

II. EXPERIMENTAL DETAILS

A. Exposure

A stack, consisting of 51 30 cm×15 cm×600 μ Ilford G_5 and 81 30 cm×16 cm×400 μ Ilford L_4 hypersensitized nuclear-emulsion pellicles, was exposed to a scattered-out proton beam of momentum 22.8 GeV/c from the CERN proton synchrotron. The flux of beam protons in the emulsion was 10^4/cm^2.

B. Scanning and Selection Criteria

Scanning was carried out by the along-the-track following method in central plates starting with 1 cm from the edge facing the incident beam. Neglecting the small-angle ($\leq 5°$) scatterings and electromagnetic events, a total of 738 interactions was collected by following 276.34 m of proton track length. The mean free path for inelastic interactions was found to be 37.4±1.4 cm. This agrees well with the values given by other authors[11] for similar incident momenta.

The usual nomenclature[12] was adopted for classifying the tracks due to secondary particles.

In order to select proton-nucleon events from the total sample, the following criteria were adopted:

1. There should be no black track associated with the interaction.
2. The event should be clean and no recoil or blob should be present at the point of interaction.

[4] R. Hagedorn, Nuovo Cimento **15**, 434 (1960).

[5] G. Cocconi, E. Lillethun, J. P. Scanlon, C. A. Stahlbrandt, C. C. Ting, J. Walters, and A. M. Wetherell, Phys. Letters **8**, 134 (1964).

[6] G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **7**, 450 (1961); A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 576.

[7] B. Peters, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 623; Yash Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **33**, No. 15 (1964).

[8] G. Damgard and K. H. Hansen, reported at the *Siena International Conference on Elementary Particles, 1963*, edited by G. Bernardini and G. P. Puppi (Societa Italiana di Fisica, Bologna, 1963).

[9] A. De Marco-Trabucco, L. Montanet, and S. Nilsson, Nucl. Phys. **60**, 209 (1964).

[10] D. Dekkers, J. A. Geibel, R. Mermod, G. Weber, T. R. Willitts, K. Winter, B. Jordan, M. Vivargent, N. M. King, and E. J. N. Wilson, Phys. Rev. **137**, 962 (1965).

[11] (a) G. Civijanovich, B. Dayton, P. Egli, B. Klaiber, W. Koch, M. Nikolic, R. Schneeberger, H. Winzeler, J. C. Combe, W. M. Gibson, W. O. Lock, M. Schneeberger, and G. Vanderhaeghe, Nuovo Cimento **20**, 1012 (1961); (b) A. Barbaro-Galtieri, A. Manfredini, B. Quassiati, C. Castagnoli, A. Gainotti, and I. Ortalli, *ibid.* **21**, 469 (1961); (c) Y. Baudinet-Robinet, M. Morand, Tsai-Chu, C. Castagnoli, G. Dascola, S. Mora, A. Barbaro-Galtieri, G. Baroni, and A. Manfredini, Nucl. Phys. **32**, 452 (1962); (d) P. G. Bizzetti, A. M. Cartacci, M. G. Dagliana, M. Dellacorte, L. Tocci, G. Bobel, G. Tomasini, and A. Marzari-Chiesa, Nuovo Cimento **27**, 6 (1963).

[12] R. H. Brown, U. Camerini, P. H. Fowler, H. Heitler, D. T. King, and C. F. Powell, Phil. Mag. **40**, 862 (1949).

3. The number of grey tracks associated with the interaction must not be more than one, and that one must be in the forward direction.

Adopting these criteria, 94 events were classified as probable proton-nucleon interactions. Two of them were found to be kinematically consistent with p-p elastic scattering.

C. Measurements

Ionization and multiple Coulomb scattering measurements were performed on all secondary shower tracks having dip angles $\leq 5°$ in the unprocessed emulsion. On an average, 1000 blobs were counted on each track. Depth variation was determined by performing a blob count on the tracks of primary protons at various depths. All measurements on ionization were normalized to correspond to the value of the blob density in the central region of the pellicles. Extensive measurements were carried out on the primary beam of protons to determine the contribution of spurious scattering to the observed signals of secondary tracks. Each plate was divided into two regions along its length. For each region, the amount of spurious scattering was determined using cell lengths of 500 μ and 1000 μ in all the plates in which scanning was done. In order to reduce the statistical error in the value of $p\beta$, a large number of cells were taken for scattering measurements, by following the tracks in several successive plates. The low flux of beam protons permitted an unambiguous follow through of the secondaries. All the grey particles were followed till they stopped in the emulsion or left the stack. In the former case, the energies of the particles were determined from their ranges, in the latter case, from their blob densities, assuming them to be protons.

D. Identification of the Particles

The results of measurements of multiple Coulomb scattering ($p\beta$) and blob density (b^*) of the shower particles are plotted in Fig. 1. The b^*-$\log_{10}(\gamma-1)$ curve given by O'Ceallaigh[13] was used for identification of the particles. The average statistical errors in the values of $p\beta$ and b^* were 12% and 3%, the maximum allowed errors being 20% and 4%, respectively. It is clear from Fig. 1 that for particles having $p\beta$ lying between 1.5 and 2.5 GeV/c, the identification is not straightforward.

TABLE I. Values of mean charged-particle multiplicity in p-p collisions around 24 GeV/c.

Detector used	$\langle n\rangle$ for p-p collisions	Reference
Emulsion	4.1±0.6	Civijanovich *et al.*[a]
Bubble chamber	4.2±0.1	Dodd *et al.*[b]
Emulsion	4.7±0.3	Baudinet-Robinet *et al.*[c]
Emulsion	4.2±0.2	Present work

[a] See Ref. 11(a). [b] See Ref. 15. [c] See Ref. 11(c).

[13] C. O'Ceallaigh (private communication).

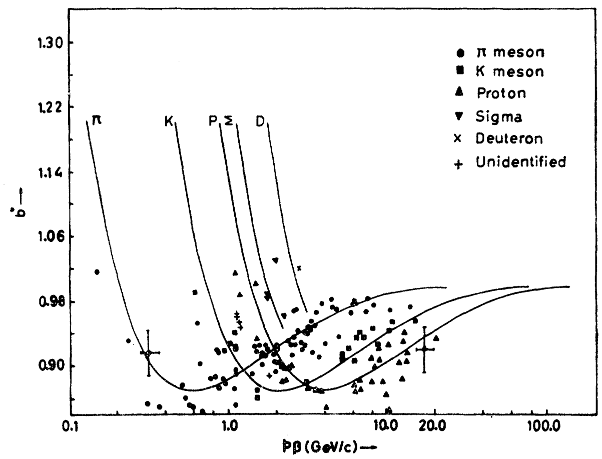

FIG. 1. Plot of blob density as a function of momentum times velocity of shower tracks.

In a majority of cases, the ambiguity in the identification was resolved by adopting the following procedure. The energies and angles of emission of these particles were calculated, assuming them to be protons, kaons, and pions. These values were compared with the corresponding values for well-identified particles, and from consistency arguments their identities could be determined. The identities of 159 secondary shower tracks could be established. The grey particles were identified from their terminal behavior. Six steep grey tracks did not stop within the stack, and they were assumed to be protons. It is observed that of the total number of identified charged particles—protons, π mesons, and K mesons—constituted 28%, 58%, and 10%, respectively. To take account of the measurement bias, each track was assigned a geometric factor which depended on its dip and space angles. The number of the particles per star whose identity and energy have been determined is equal to 1.9, which is higher than the value given by other authors ($\sim$1.4).[14] The larger number of identified particles per star has meant a reduction in the correction factor.

III. RESULTS

A. Multiplicity

The mean multiplicity of secondary shower particles is found to be $\langle n_s\rangle = 4.3\pm0.2$. Table I shows the comparative values of charged-particle multiplicity in p-p collisions as obtained by investigators using emulsion and bubble-chamber techniques. It is clear that our value of the multiplicity for inelastic p-p collisions (even prong events) is in agreement with the results of other authors.[11a,11c,15] The value for $\langle n_s\rangle$ can be com-

[14] M. Csejthey-Barth, Nuovo Cimento **32**, 545 (1964).

[15] P. Dodd, M. Jobes, J. Kinson, B. Tallini, B. R. French, H. J. Sherman, I. O. Skillicorn, W. T. Davies, M. Derrick, and D. Radojicic, in *The Aix-en-Provence International Conference on Elementary Particles, 1961*, edited by E. Cremieu-Alcan, P. Falk-Vairant, and O. Lebey (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, France, 1961), Vol. 1, p. 433.

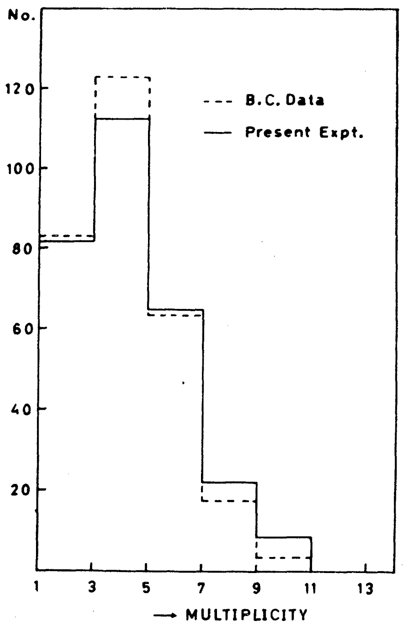

FIG. 2. Comparison of multiplicity distribution of shower particles from the present experiment (solid line) with the results of Dodd *et al.* (Ref. 15) (dashed line).

pared with 4.8±0.2 predicted by Hagedorn's statistical theory. The comparison would be more meaningful if the contribution of fast isobars were excluded from the value of the mean multiplicity, since such isobars cannot be produced in statistical processes.

Comparing the multiplicity distribution with that obtained from bubble-chamber results as shown in Fig. 2, we find that the two distributions are similar. This implies that the multiplicity distribution from our results is not distorted by the probable occurrence of cascading effect in proton–bound-nucleon events. The mean number of charged particles created is equal to 3.06±0.24, which is obtained by subtracting the contribution of fast-charged baryons from the mean-shower multiplicity.

The total number of events were divided into two categories, viz., those having shower multiplicity less than or equal to 4 and those having shower multiplicity greater than 4, a majority of the former being identified as due to peripheral collisions, and of the latter, as central collisions. The justification for this classification is suggested by the different behavior of various characteristics such as the angular distribution, inelasticity, transverse momentum, and center-of-mass momentum of the secondary particles for the two classes of events, as is detailed below.

B. Angular Distribution of Secondary Particles

The angular distributions of protons and pions in the center-of-mass system were plotted for events having $n_s \leq 4$ and $n_s > 4$. It is observed that for $n_s \leq 4$ events, the particles are collimated backwards and forwards about the collision axis, the protons exhibiting greater collimation than the pions. For events having $n_s > 4$, the pions are distributed in an isotropic manner. This is in agreement with the results of Dodd *et al.*[15] The strong collimation of protons suggests that the incident protons underwent a peripheral collision. The anisotropy of the particles is found to decrease with increase in multiplicity.

The integral angular distributions of the secondary particles for events having $n_s \leq 4$ and for those having $n_s > 4$ are displayed in the form of a Duller-Walker[16] plot in Fig. 3. Values of $\log_{10}[F/(1-F)]$ are plotted against $\log_{10} \tan\theta$, the intercept on the abscissa gives the value of $\log_{10}\gamma_c$. The values of γ_c obtained from $n_s \leq 4$ and $n_s > 4$ plots are 6.0 and 3.89, respectively, as against the actual value of 3.56. The disagreement of the value of γ_c for $n_s \leq 4$ events with the actual value arises from the higher velocity of the secondary particles as compared to that of the center of mass.

The center-of-mass angular distribution of the secondaries for events having $n_s \leq 4$ and $n_s > 4$ was plotted for each individual event, as suggested by Dobrotin *et al.*[17] For some events, the plot is displayed in Fig. 4. It is observed that the secondaries for $n_s \leq 4$ events

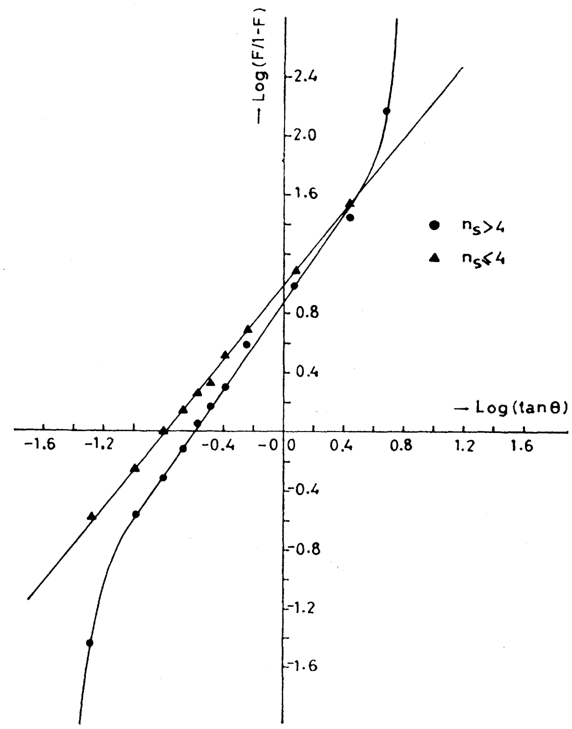

FIG. 3. Duller-Walker plot for $n_s \leq 4$ (▲) and $n_s > 4$ (●) events.

[16] N. M. Duller and W. D. Walker, Phys. Rev. **93**, 215 (1954).
[17] N. A. Dobrotin and S. A. Slavatinsky, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 819; N. A. Dobrotin, V. V. Guseva, K. A. Kotelnikov, A. M. Lebedev, S. V. Ryabikov, S. A. Slavatinsky, and N. G. Zelevinskaya, Nucl. Phys. **35**, 152 (1962).

reveal sharp forward-backward asymmetry and for events having $n_s>4$, the secondaries are distributed in a symmetric manner. The different characteristics of angular distribution for secondaries from $n_s \leq 4$ and $n_s>4$ events suggest two different mechanisms of particle production for the two classes of events.

C. Inelasticity

A characteristic feature of high-energy interactions is that only a minor fraction of the available energy is used in the creation of new particles, while most of it is being retained by the incident nucleon. This property is expressed by a parameter called inelasticity, K, which can be defined as the fraction of energy available in the center-of-mass system expended in the creation of pions and other particles. The majority of the produced particles being pions, the fraction of energy expanded in their production is given by

$$K_\pi = \langle n_\pi \rangle \langle E_\pi^* \rangle / 2(\gamma_c - 1) M_p .$$

Substituting the values, we get $K_\pi = 0.45 \pm 0.05$. This value agrees with the values of 0.41 ± 0.06 and 0.38 ± 0.04 for p-n and p-p interactions at 27 GeV, respectively, as determined by Baudinet-Robinet *et al.*[11c]

The values of pion inelasticity for events having $n_s \leq 4$ and for those having $n_s > 4$ have been found to be

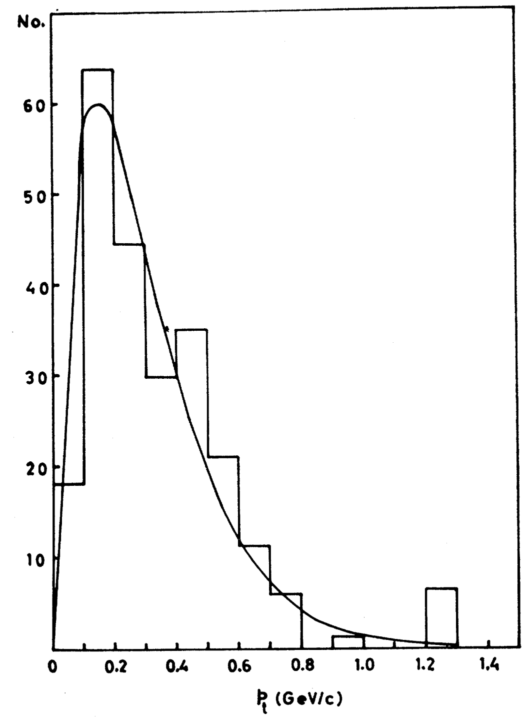

FIG. 5. Transverse-momentum distribution of pions for all events. The solid curve drawn denotes $cp_t e^{-p_t/p_0} dp_t$.

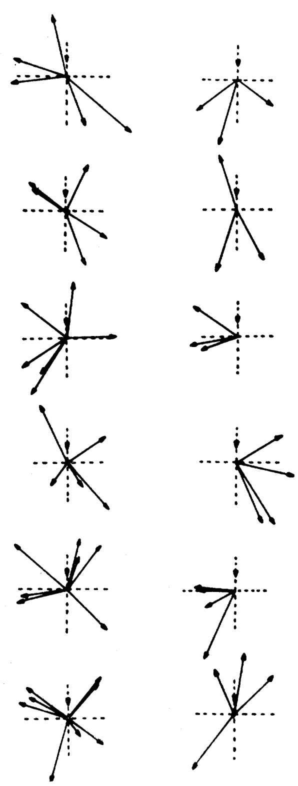

FIG. 4. Angular distribution of secondary particles in the center-of-mass system. The direction of primary is indicated by the dashed line (with arrow).

0.36 ± 0.05 and 0.62 ± 0.09, respectively. The statistical theory predicts a high value of K since the whole energy in the center-of-mass system is available for particle production. The higher value of K_π for $n_s>4$ events is in better agreement with the predictions of statistical theory than that for $n_s \leq 4$ events. The low value of K_π for $n_s \leq 4$ events can be explained on the basis of excitation of baryon isobars as follows. A nucleon isobar of mass M^* decaying in its rest system to its ground state will transfer an amount of energy equal to $(M^{*2} - M_\pi^2 + M_p^2)/2M^*$ to the single decay pion. Considering nucleon isobars having masses 1512 MeV and 1688 MeV, it is observed that 32% and 29% of the total available energy is transferred to the decay pion, respectively, the rest of the energy being retained by the nucleons. For isobars moving with a certain velocity, the decay pions will share the isobar momentum. The observation of fast pions and a single nucleon retaining a large fraction of the primary energy for $n_s \leq 4$ events is understood on the basis of isobar model.

D. Transverse-Momentum Distribution of Secondary-Particle Pions

In order to understand the behavior of transverse momentum at different primary energies, a comparative study of the p_t distribution of pions from the present investigation with the distributions obtained in p-N collisions at 24, 9, 6, and 3.5 GeV has been made. The values are given in Table II. It is obvious that p_t

TABLE II. Average and most probable values of transverse momentum of pions at various primary energies.

Detector	Incident energy (GeV)	p_t (MeV c) Average	Most probable	Reference
Emulsion	3.5	111± 9	0–50	Piserchio *et al.*[a]
Emulsion	6.2	225±33	150	Daniel *et al.*[b]
Emulsion	9.0	240±20	···	Wang-Shu-Fen *et al.*[c]
Bubble chamber	24.0	330±20	···	P. Dodd *et al.*[d]
Emulsion	22.8	332±23	150	Present work
Ionization calorimeter	>100	350±45	210	Dobrotin *et al.*[e]

[a] R. S. Piserchio and R. M. Kalbach, Nuovo Cimento 26, 729 (1962).
[b] R. R. Daniel, N. Kameswara Rao, P. K. Malhotra, and Y. Tsuzuk, Nuovo Cimento 16, 1 (1960).
[c] Wang Shu-Fen, T. Visky, I. M. Gramenitskii, V. G. Grishin, N. Dalkhazhav, R. M. Lebedev, A. M. Nomofilov, M. I. Podgoretskii, and V. N. Steltsov, Zh. Eksperim. i Teor. Fiz. 12, 663 (1961) [English transl.: Soviet Phys.—JETP 39, 957 (1960)].
[d] See Ref. 15.
[e] See Ref. 17.

depends on the primary energy. As the energy increases, the most probable value as well as the average value of p_t shifts to higher values. The increase of transverse momentum of pions with primary energies has been qualitatively explained on the basis of the excitation of progressively increasing heavier isobars. It is clear that as the mass of the isobar increases, the transverse momentum of the corresponding decay pion also increases. Since the threshold for the double excitation of the heaviest known isobars is reached at 10 GeV in the laboratory system, the variation of p_t of pions from 10-GeV primary energy to cosmic-ray energies is expected to be slow, arising chiefly because of the increase in the probability of excitation of heavy isobars. With increase in mass of an isobar, the probability of multipion decay will also increase, so that the heavy isobars will reach the ground state through a set of successive channels. Hence no appreciable variation of transverse momentum is expected in such cases.

The transverse-momentum distribution of pions for all events is shown in Fig. 5 and the distribution can be well represented by

$$dN = cp_t e^{-p_t/p_0} dp_t\,, \tag{1}$$

where p_0 is the most probable value of the distribution and is equal to 150 MeV/c. A number of authors[18,19] have reported such an exponential distribution of trans-

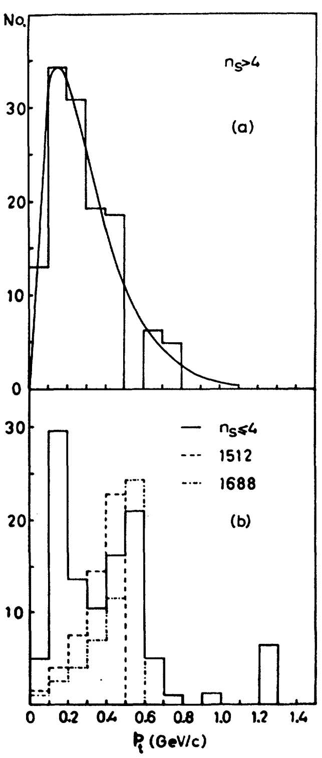

FIG. 6. (a) Transverse-momentum distribution of pions for $n_s>4$ events. The solid curve drawn denotes $cp_te^{-p_t/p_0}dp_t$. (b) Transverse-momentum distribution of pions for $n_s\leq4$ events. The contributions from decays of N^*(1512 MeV) (dashed line) and N^*(1688 MeV) (dot-dash line) are shown, the number being given in arbitrary units.

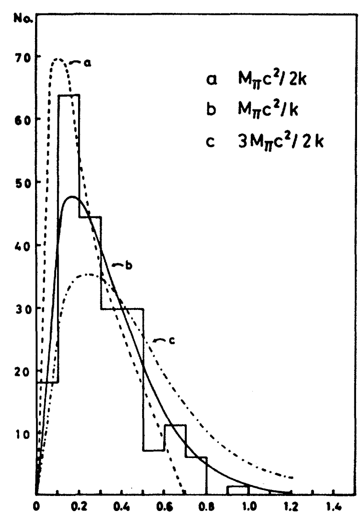

FIG. 7. Transverse-momentum distribution of all pions after subtracting the contribution of isobars. The curves drawn denote the distributions calculated by Milećin and Rozental' (Ref. 20) for temperatures (a) $M_\pi c^2/k$, (b) $M_\pi c^2/2k$, and (c) $3M_\pi c^2/2k$, respectively.

[18] G. Cocconi, L. J. Koestler, and D. H. Perkins, Lawrence Radiation Laboratory Report No. UCID-1444, 1961 (unpublished).
[19] P. L. Jain, H. C. Glahe, J. D. Rinaldo, and P. D. Bhardwaj, Nuovo Cimento 32, 873 (1964).

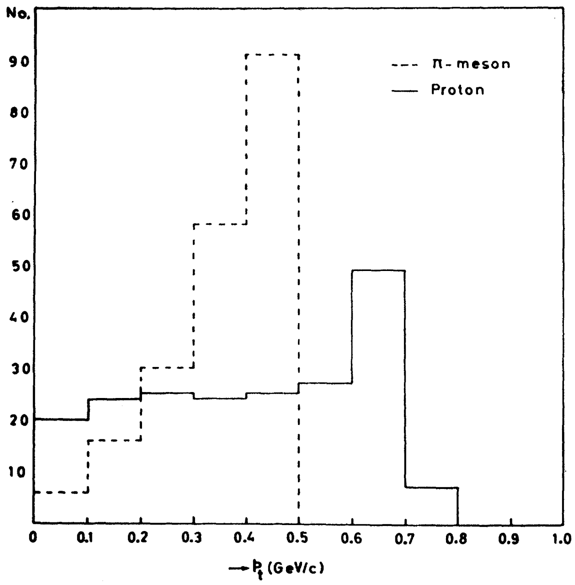

FIG. 8. Transverse-momentum distributions of pions and protons in the center-of-mass system as decay products of N^*(1512 MeV).

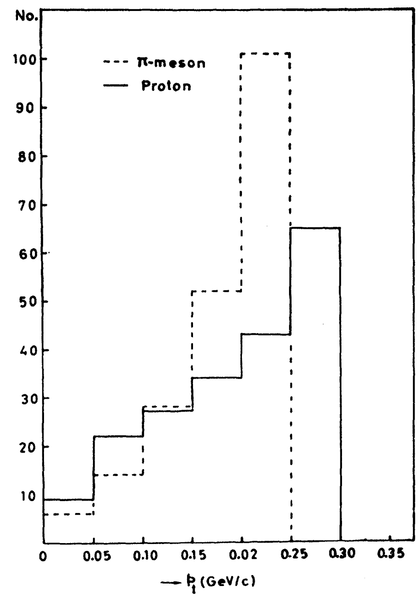

FIG. 9. Transverse-momentum distributions of pions and protons in the center-of-mass sytem as decay products of N^*(1238 MeV).

verse momentum for pions. The peak and the average value are in agreement with the results of bubble-chamber experiment.[15] In 1961, Cocconi, Koestler, and Perkins[18] pointed out that the distribution function $dN/dp_t = p_t e^{-ap_t}$, where $a = 1/165$ MeV/c, fits the transverse-momentum distribution of pions from 10 to 30 GeV/c incident momentum.

In the following, the p_t distribution of pions was studied for events having $n_s > 4$ and $n_s \leq 4$, the mean values for the two classes of events being (274 ± 41) MeV/c and (460 ± 41) MeV/c, respectively. In Fig. 6(a) is shown the p_t distribution of pions for events having $n_s > 4$. The curve drawn shows the distribution expected from Eq. (1) and is in agreement with the experimental distribution. The curve given by Eq. (1) does not fit the p_t distribution of pions for $n_s \leq 4$ events and there is an indication of two peaks in the distribution, as is shown by the solid line in Fig. 6(b). The first peak which occurs at 0.15 GeV/c is present in $n_s > 4$ events also, and this might occur as a result of the same processes as those responsible for $n_s > 4$ events. The second peak may be caused by the contribution of nucleon excited states. This can be seen from Fig. 6(b), in which the contributions from the decays of N^*(1512 MeV) and N^*(1688 MeV) are shown. As a first-order approximation, Eq. (1) can be used to estimate the contribution of isobars to pion production from $n_s \leq 4$ plot. This amount is subtracted from the combined plot (Fig. 5), and the resultant distribution is shown in Fig. 7. The curves obtained by Milecin and Rozental'[20] on the basis of hydrodynamical theory[21] have been plotted for different temperatures, such as $M_\pi c^2/2k$, $M_\pi c^2/k$, and $3M_\pi c^2/2k$, where M_π is the mass of a pion and k is the Boltzmann constant. It is seen that the best agreement of the theory with the experimental distribution occurs at the temperature $T = M_\pi c^2/k$.

The variation of p_t with mass has been observed by a number of investigators in p-p and π-p collisions. The mean values of p_t for pions and protons from our results are 332 ± 23 MeV/c and 425 ± 35 MeV/c, respectively, and for kaon, lambda, and sigma particles obtained from the results of De Marco-Trabucco *et al.*[9] are 400 ± 30, 390 ± 40, and 590 ± 80 MeV/c, respectively. It is clear that $\langle p_t \rangle$ increases with mass of the particle. This can be explained if we assume that the isobars are moving at an angle to the collision axis. Considering N^*(1512 MeV) moving with a velocity of $0.75c$ at an angle of 3° to the collision axis, the plot of p_t distributions for pions and protons in the center-of-mass system is shown in Fig. 8. It is observed that the p_t distribution of pions is inappreciably affected by the Lorentz transformation from the isobar to the center-of-mass system, whereas that of protons is substantially shifted to higher values. The mean values of transverse momentum for pions and protons are found to be 352 and 411 MeV/c, respectively, and they compare well with the experimental values. Similar calculations for N^*(1238 MeV) moving with a velocity of $0.70c$ at an angle of 3° to the collision axis yield the distributions shown in Fig. 9. The p_t distribution for pions is not changed from the isobar to the center-of-mass system. It is clear that the p_t distribution for protons is slightly shifted towards higher values, the average values for pions and protons being 181 and 192 MeV/c, respectively. The observed

[20] G. A. Milecin and I. L. Rozental', Nuovo Cimento Suppl. 8, 770 (1958).

[21] L. D. Landau, Izv. Akad. Nauk. SSSR, Ser. Fiz. 17, 51 (1953).

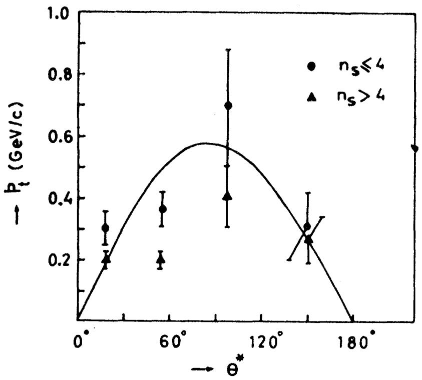

FIG. 10. Variation of average value of transverse momentum of pions as a function of angle in the c.m. system for $n_s \leq 4$ (●) and $n_s > 4$ (▲) events. The curve shows the values of p_t from the decay of N^*(1688 MeV).

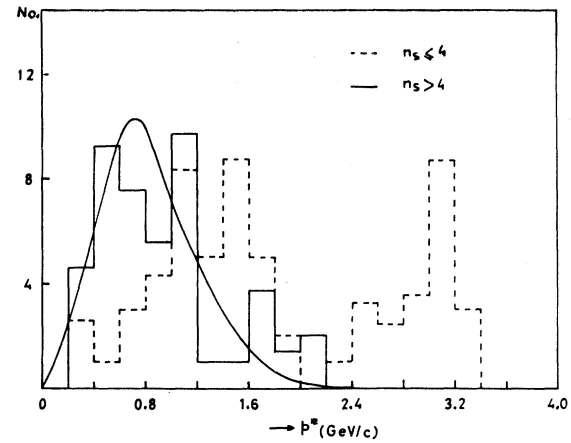

FIG. 12. Momentum distribution of protons in the center-of-mass system for $n_s \leq 4$ (dashed line) and $n_s > 4$ (solid line) events. The solid curve shows the predictions of statistical theory.

higher value of p_t for protons compared to that of pions cannot be accounted for if we assume the pions and protons to proceed from the decay of N^*(1238 MeV).

The increase of p_t with angle was observed by Ciurlo[22] in 16.1-GeV/c π-N interactions. From our results, the mean values of p_t for different angular intervals in the center-of-mass system are plotted in Fig. 10 for events having $n_s \leq 4$ and for those having $n_s > 4$. It is clear that there is a greater increase in the value of $\langle p_t \rangle$ with angle up to 90° for $n_s \leq 4$ events than that for $n_s > 4$ events. The solid line in the figure denotes the calculated values of p_t from the decay of N^*(1688 MeV) moving with a velocity of 0.09c in the center-of-mass system.

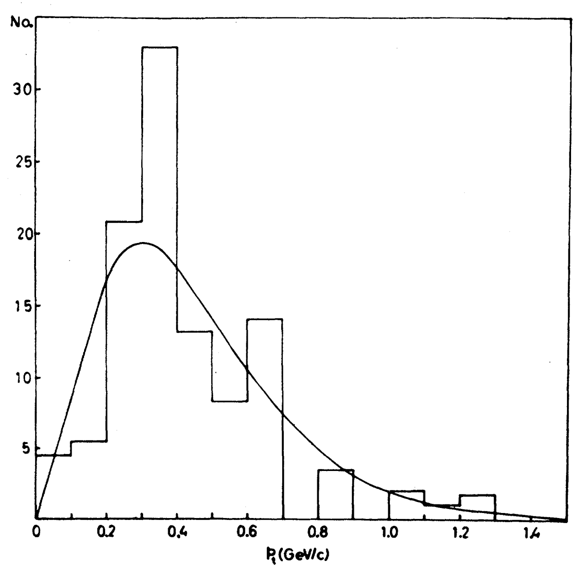

FIG. 11. Transverse-momentum distributions of protons. The curve drawn denotes $cp_t^2 e^{-p_t/p_0} dp_t$.

[22] S. Ciurlo, E. Picasso, G. Tomasini, A. Gainotti, C. Lamborizio, and S. Mora. Nuovo Cimento **27**, 791 (1963).

1. *Protons*

The observed distribution of transverse momentum of protons in p-N collisions is shown in Fig. 11. The curve drawn to fit the experimental distribution is of the form $dN = cp_t^2 e^{-p_t/p_0} dp_t$. The distribution has a peak value of $\sim$350 MeV/c ($=2p_0$) and a mean value of 425±35 MeV/c, which is in agreement with the results of Dodd *et al.*[15]

Hagedorn[23] has explained the Maxwell-Boltzmann distribution of p_t for protons by assuming that in a high-energy collision a thermodynamical equilibrium is established corresponding to the highest possible temperature, T_0. Excited hadrons leave the region of interaction and decay strongly through a number of steps into their stable forms K, π, N, γ, etc. He has concluded that in all high-energy collisions with sufficient total energy and momentum transfer, the p_t distribution will be of the Boltzmann type:

$$W(p_t) = cp_t^{3/2} e^{-p_t/T_0}, \quad \text{where} \quad T_0 = 158 \text{ MeV},$$

which is independent of the primary energy, of the colliding partners, and of the multiplicity.

The p_t distributions of protons for events having $n_s \leq 4$ and $n_s > 4$ exhibits the same characteristics, the mean values of p_t being 433±60 and 418±41 MeV/c for the two classes of events, respectively.

E. Momentum Spectra of the Secondary Particles

1. *Protons*

The center-of-mass momentum (p^*) distribution of protons is shown in Fig. 12; the solid line stands for $n_s > 4$ events and the dashed line for $n_s \leq 4$ events. The mean values of center-of-mass momentum for $n_s \leq 4$ and $n_s > 4$ events are found to be 1.837±0.153 and 0.943 ±0.114 GeV/c, respectively. The curve drawn shows

[23] R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965).

the momentum spectrum predicted by the statistical theory. Fair agreement is found for small momenta where mainly high-multiplicity events are contributing. It is seen that a majority of the low-multiplicity events have center-of-mass momenta beyond 1.3 GeV/c, where very little contribution is expected from statistical theory. The results of Dodd *et al.*[15] also show a large number of protons contributing in the higher region of the center-of-mass momentum distribution. Such high-momentum protons may result from excitations of nucleon isobars.

2. *Pions*

The momentum distributions of pions having $n_s > 4$ (dashed line) and $n_s \leq 4$ (solid line) are compared in Fig. 13, the average values of center-of-mass momenta being 589 ± 74 and 424 ± 35 MeV/c, respectively. The two distributions are not distinctly different from each other, although there are some high-momentum pions present in the former distribution. These fast pions could come from the decay of excited nucleons since they share the primary momentum of the isobar. The momentum distribution of all the pions in the center-of-mass system is shown in Fig. 14, and it is in agreement with the results of Dodd *et al.*[15] The solid curve represents the prediction of statistical theory which seems to agree with the experimental distribution. The dotted curve shows the momentum distribution of pions from the decay of N^*(1512 MeV) moving with a velocity of $0.70c$ in the center-of-mass system. Calculations show that the nature of the momentum distribution is not substantially altered if we consider the pions from the decay of N^*(1688 MeV). Also the increase in the velocity of an isobar only leads to a broadening of the momentum distribution. For heavy isobars which decay predominantly via successive channels, the contribution to pion momentum distribution is again confined to the lower side of the distribution. In the center-of-mass momentum distribution of pions, the contributions from the statistical theory and that from the isobar model overlap in the low-momentum region of the spectrum to a large extent. Thus it is not possible to isolate the contributions from the two processes only from a study of the momentum distribution. However, a better picture seems to emerge if we combine the information on angular and momentum distribution by plotting the values of transverse momentum against center-of-mass longitudinal momentum (p_l^*). In this diagram, each point is a terminal point of the momentum vector of the particle starting from the origin of coordinates. The kinematical limit of this plot is a semicircle on which prongs from elastic events would lie.

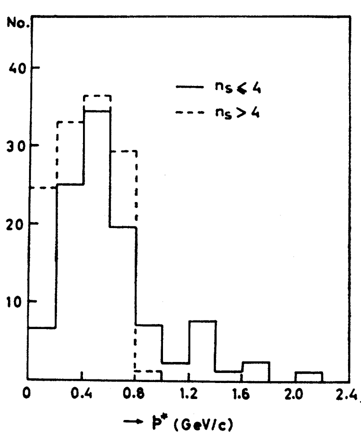

FIG. 13. Momentum distributions of pions in the center-of-mass system for $n_s \leq 4$ (solid line) and $n_s > 4$ events (dotted line).

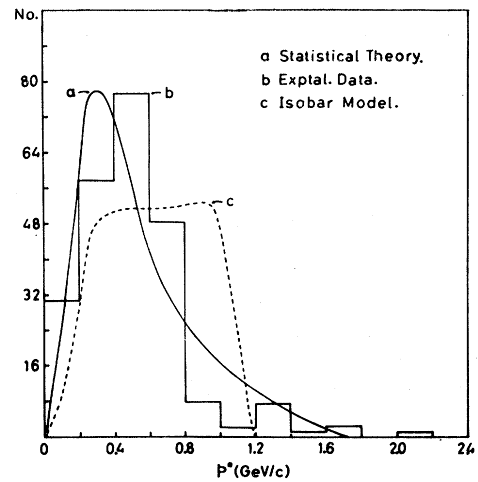

FIG. 14. Momentum distribution of all pions in the center-of-mass system. The solid curve denotes the predictions of statistical theory and the dashed curve shows the momentum distribution of pions from the decay of N^*(1512 MeV).

IV. THE ISOBAR MODEL

In this section we calculate the angular and momentum distributions of protons and pions formed as the decay products of excited nucleon states in reactions of the type

$$p+N \to N^*+N+\pi, \qquad N^* \to N+\pi. \tag{2}$$

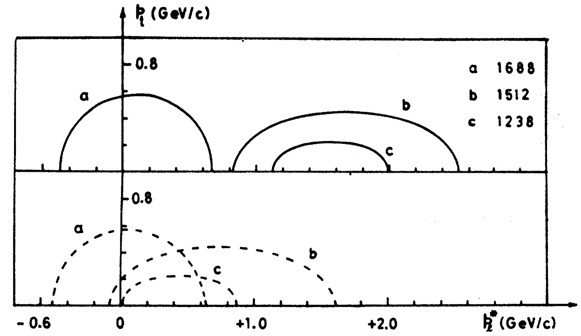

FIG. 15. Longitudinal c.m. momentum versus transverse momentum of protons (solid curves) and pions (dashed curves) from the decays of N^*(1238 MeV), N^*(1512 MeV), and N^*(1688 MeV).

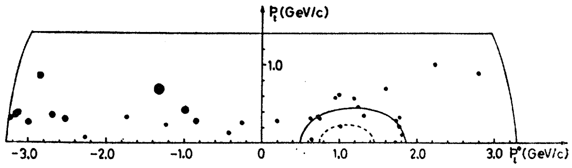

FIG. 16. Longitudinal c.m. momentum versus transverse momentum of protons for $n_s \leq 4$ events. The solid curve and the dashed curve show the p_l^*–p_t distributions of protons as decay products of N^*(1512 MeV) and N^*(1238 MeV), respectively.

It is assumed that the isobars are moving with zero transverse momentum along the axis of collision and decay isotropically in their rest systems. The momenta and angles of emission of the particles are transformed from the isobar system to the N-N center-of-mass system in order to compare with the experimental data. Values of center-of-mass longitudinal momenta for protons produced from the decays of N^*(1238 MeV), N^*(1512 MeV), and N^*(1688 MeV) are plotted in Fig. 13 (solid curves) against p_t values, the velocities of the isobars in the N-N c.m. system being $0.85c$, $0.85c$, and $0.09c$, respectively. Values of p_l^*, p_t have been calculated for pions produced as the decay signals from the above-mentioned nucleon isobars moving with the same velocities as detailed above. The curves thus obtained are shown in Fig. 15 (dashed curves), which show the general pattern of the distributions for given velocities. Let us now consider the double isobar formation and their subsequent decay into protons and pions in reactions of the type

$$p+N \rightarrow N_1^*+N_2^*,$$
$$N_1^* \rightarrow N+\pi, \quad \text{and} \quad N_2^* \rightarrow N+\pi.$$

For a given isobar, the mass of its partner uniquely determines its velocity in two-body processes. For example, the velocity of N^*(1512 MeV) would range from $0.895c$ to $0.855c$ as the mass of the other isobar moving in the opposite direction varies between 1268 and 2817 MeV. From the above consideration, it appears that a knowledge of the isobar velocity would allow one to verify double isobar formation and to determine the mass of its partner.

V. COMPARISON WITH EXPERIMENTAL RESULTS

The p_l^*-p_t distribution of protons for events having $n_s \leq 4$ and $n_s > 4$ are plotted in Figs. 16 and 17, respectively. It is observed that the events in the latter diagram are confined to lower p_l^*-p_t values as compared to those in the former plot.

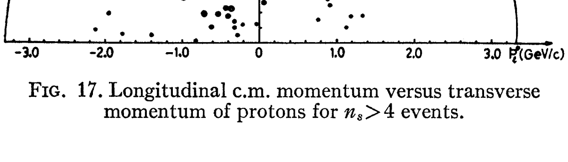

FIG. 17. Longitudinal c.m. momentum versus transverse momentum of protons for $n_s > 4$ events.

Furthermore, the features of the p_l^*-p_t plot in Fig. 16 are not the same in the forward and backward hemispheres. This is due to bias involved in our measurements. The selection criteria for the tracks on which measurements could be made favored the particles emitted at small angles in the laboratory system, since their dip angles were small. Hence, mass and energy determinations for the particles emitted in the forward direction in the center-of-mass system could be easily carried out. The particles in the backward direction in the center-of-mass system being the ones that go at wide angles in the laboratory system in a majority of cases, their dip angles are larger and consequently scattering and ionization measurements were not made on them. To account for this bias, a large correction factor was assigned to the few measured tracks in the backward direction as is evidenced by the large size of the points.

Let us now consider the distribution in the forward hemisphere in Fig. 16. It is seen that very few fast protons lie near the collision axis corresponding to small p_t values and a majority of the protons have p_t around 0.3 GeV/c. This suggests them to be produced as decay products of N^*(1512 MeV). The solid curve in the figure shows the calculated distribution for N^*(1512 MeV) having velocity $0.75c$ and the dashed curve shows the distribution for N^*(1238 MeV) moving with velocity $0.70c$. It is evident that a good number of experimental points lie around the former curve and practically none on the latter curve. Assuming N^*(1512 MeV) to be moving at small angle ($\sim 3°$) with respect to the collision axis, a spread of the calculated curve for protons is obtained which can envelope a majority of the points. This can be understood from Fig. 8 in which the p_t distribution for protons extends up to 0.8 GeV/c. If we assume N^*(1238 MeV) moving with a velocity of $0.70c$, at an angle of 3°, the spread of the dashed curve will not extend (along p_t axis) beyond 0.3 GeV/c, as is evidenced from Fig. 9. This can be interpreted to mean that N^*(1512 MeV) is produced at this energy, whereas N^*(1238 MeV) is not excited. The velocity of $0.75c$ for N^*(1512 MeV), which agrees with the experimental data, lies outside the velocity spectrum for isobar partner having mass lying between 1238 and 2828 MeV. This suggests that the observed isobar N^*(1512 MeV) is not produced in association with any of the known isobars in two-body processes. It may be produced in collisions of the type denoted by Eq. (2). If, however, one assumes N^*(1512 MeV) having a velocity of $0.75c$ in the center-of-mass system, to be produced in association with a single partner, the mass of the latter turns out to be $\sim$4000 MeV.

The protons in the backward hemisphere near 3 GeV/c longitudinal momentum are mostly from two-prong events. Our observations for two-prong events are in agreement with the results of Morrison,[24] who also gets a large contribution of two-prong events around 3 GeV/c center-of-mass longitudinal momentum. They could be quasielastic events of the type

$$p+p \rightarrow p+p+\pi^0,$$
$$p+p \rightarrow p+n+\pi^+.$$

The fast protons in the backward direction could be the decay products of a fast-excited nucleon. Assuming them to be produced via the decay of a single excited nucleon, i.e.,

$$p+p \rightarrow p+N^*,$$
$$N^* \rightarrow N+\pi^{0+}.$$

Our calculations agree with the production of protons from the decay of an isobar of mass 1512 MeV, having a velocity of 0.89c in the center-of-mass system.

The p_l^*-p_t distribution of pions for events having $n_s \leq 4$ is shown in Fig. 18. It is seen from the figure that a large number of fast pions are observed, the solid curves in the figure stand for p_l^*-p_t distributions of pions emitted from N^*(1512 MeV) and N^*(1688 MeV) isobars moving with the velocities 0.75c and 0.08c, respectively. The higher value of p_t for some pions around $p_l^*=0$, is well accounted for, from the decay of N^*(1688 MeV). The dashed curve stands for the momentum distribution of pions from the decay of N^*(1238 MeV), having a velocity of 0.70c in the N-N center-of-mass system. Although some of the pions having low p_l^*, p_t values lie around this curve, such low-momentum pions could come from statistical processes also, since their contributions to $n_s \leq 4$ events cannot be ruled out. Further, there are no corresponding protons which could be traced to the decay from N^*(1238 MeV). This can be interpreted to mean that N^*(1238 MeV) is not excited at this energy. Figure 19 shows the p_l^*-p_t distributions of pions for $n_s>4$ events. It is seen from the figure that the pions are somewhat lumped together around low p_l^*, p_t values, and their spectrum does not extend very far in the region of high p_l^*. If the high value of p_l^* for pions is attributed to the high velocity of the emitting centers, which is clear from our calculations for isobar decay, then the very low values of p_l^* for pions can be attributed to the stationary condition of the pion source. This implies that the pion source is at rest in the center-of-mass system, and that the pions are emitted in all possible directions. In such collisions, the observed features are well reproduced on the basis of statistical theory.

VI. DISCUSSION

The classification of events on the basis of low and high multiplicity seems to coincide with the change in characteristics of the mechanism of interactions. The

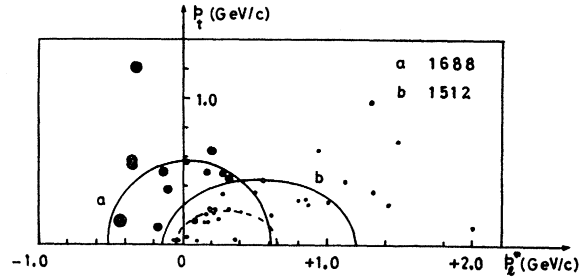

FIG. 18. Longitudinal c.m. momentum versus transverse momentum of pions for $n_s \leq 4$ events. The two solid curves and the dashed curve denote the p_l^*–p_t distributions of pions as decay products of N^*(1512 MeV), N^*(1688 MeV), and N^*(1238 MeV), respectively.

angular distributions for protons and pions show forward-backward collimation about the collision axis for $n_s \leq 4$ events and isotropy for $n_s>4$ events. The strong forward-backward collimation for events having $n_s \leq 4$ suggests them to be produced in peripheral processes in which there is a low-momentum transfer to the target nucleon. The isotropic angular distributions of the secondaries for $n_s>4$ events suggests that they are produced in the statistical processes in which there is no preferential direction of motion of the particles coming out of the interaction volume. The high value of the inelasticity coefficient K for $n_s>4$ events is consistent with the predictions of the statistical theory, in which it is assumed that the whole energy in the center-of-mass system is expended in particle production. The low value of K for $n_s \leq 4$ events suggests that these events are dominated by isobar excitation.

Fermi's theory predicts a high value of $\langle p_t \rangle$ (several GeV/c) which is not borne out from our experimental results. It has been attempted to explain the different features of transverse momentum on the basis of excitation of the colliding nucleons. The increasing value of p_t for the increasing mass of the secondary particles such as π, K, p, Λ, and Σ could be explained by assuming them to be formed from the decay of an isobar. The variation of p_t of pions with angle has been explained from our isobar model calculations. The statistical theory fits well with the experimental center-of-mass momentum distribution of protons for $n_s>4$ events. For events having $n_s \leq 4$, a large number of high-momentum protons are observed, the disagreement with the statistical theory is obvious and hence the isobar

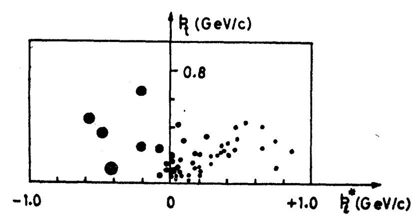

FIG. 19. Longitudinal c.m. momentum versus transverse momentum of pions for $n_s>4$ events.

[24] D. R. O. Morrison, in *The Aix-En-Provence International Conference on Elementary Particles, 1961*, edited by E. Cremieu-Alcan, P. Falk-Vairant, and O. Lebey (Centre d'Etudes Nucléaires des Saclay, Siene et Oise, France, 1961), Vol. I, p. 407.

model was invoked to explain their momentum distribution. The features of pion production for events having $n_s \leq 4$, such as low value of inelasticity, increase of transverse momentum with angle and mass, and the observation of high-momentum pions seem to be well explained on the basis of formation of nucleon excited states. From the momentum distribution of pions, Fig. 14, it is clear that no resolution is obtained between the contribution of the statistical theory and that from the decay of an isobar. If the pions are quite fast, then they can be easily identified with one of the above two processes, and this can be achieved by using much higher primary energies than the present one. The p_l^*-p_t distribution of protons calculated from the decay of an isobar of mass 1512 MeV accounts well for some of the high-momentum protons in the forward direction. The decay protons from N^*(1512 MeV) also explain the production of fast protons in the backward direction which could arise from quasielastic collisions. None of the protons seem to be produced from the decay of N^*(1238 MeV). Our results indicate that the main contribution comes from isobars having isospin $\frac{1}{2}$. This has been amply justified from the results of Cocconi *et al.*[6] and Dekkers *et al.*[10] Cocconi *et al.* have measured the inelastic spectra for 22-GeV/c p-p scattering at 56, 20, and 110 mrad. A double bump structure has been observed which corresponds to the missing mass at 1.15 ± 0.01 and 1.69 ± 0.01 GeV. These bumps have been identified with the second and third resonances ($T=\frac{1}{2}$) found in π-p scattering. Their results have shown no evidence for the production of a peak corresponding to the $(\frac{3}{2},\frac{3}{2})$ π-p resonance. According to the model of Drell and Hilda,[25] the excited states are produced by an inelastic diffraction scattering process, suppression of $(\frac{3}{2},\frac{3}{2})$ state is established since the isotopic spin does not change in diffraction process. The observed ratio[10] of positive to negative pions lends further support to the above conclusions. Since in p-p interaction the initial state has a positive charge of two, the same may be retained by the final state. Plotting the ratio of $N(\pi^+)/N(\pi^-)$ as a function of secondary momentum, it has been observed[10] that at low secondary momenta the ratio is close to one, as the momentum increases a rise is observed, and finally a constant value is reached, varying only with the angle. The charge excess is carried off mainly by high-momentum pions which can arise only through the decay of fast excited baryons. The observation of few negative pions from isobar decay implies that the incident nucleon does not change its charge when it is excited, meaning thereby that the isobars are excited without exchange of isospin. From the conservation of isospin it follows that the decay probabilities of a positively charged isobar into a π^+ or π^0 are in the ratio $N(\pi^+)/N(\pi^0)=2$ for isospin $\frac{1}{2}$ and $\frac{1}{2}$ for isospin $\frac{3}{2}$. From the data of Dekkers *et al.*[10] at 18.8-GeV/c N-N collisions, we have the value of $N(\pi^+)/N(\pi^0)=1.6\pm0.3$, which is compatible with isospin $\frac{1}{2}$ but not with $\frac{3}{2}$.

The statistical model predicts the formation of all isospin and spin states according to their statistical weights. The preferential production of certain isospin states could hardly be accounted for on the basis of this model. It is concluded that the isobar model explains to a large extent the features of high-energy interactions which were hitherto unaccounted for on the basis of statistical theory.

[25] S. D. Drell and K. Hilda, Phys. Rev. Letters **7**, 199 (1961).

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