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Energy Dependence of the Form Factor in K_{e3}^+ Decay*

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In a spark chamber and counter experiment at the Princeton-Pennsylvania Accelerator we have investigated the decay $K^+ \rightarrow e^+ + \pi^0 + \nu$. A Dalitz plot of 1393 K_{e3}^+ events agrees well with vector coupling. The π^0 energy spectrum was investigated to determine the energy dependence of the strong-interaction form factor f_+ which is a function of $q^2 = M_K^2 + M_\pi^2 - 2M_K E_\pi$ for K^+ decays at rest. For a linear expansion $f_+(q^2) = f_+(0)(1 + \lambda q^2/M_\pi^2)$, we obtain $\lambda = 0.016 \pm 0.016$, which agrees well with previous measurements of λ for K_{e3}^+. Assuming the interaction is dominated by a $J=1^-$, $I=\frac{1}{2}$ intermediate state of mass M, such as the K^* resonance at 890 MeV, we obtain $M = (1180_{-387}^{+\infty})$ MeV. This result is consistent with K^* dominance but also with no energy dependence for f_+. The value of λ for K_{e3}^+ obtained in this experiment agrees with the weighted mean value $\langle\lambda^0\rangle = 0.010 \pm 0.018$ obtained from four K_{e3}^0 experiments and thus offers no evidence for violation of the leptonic $|\Delta\mathbf{I}| = \frac{1}{2}$ rule.

INTRODUCTION

PRESENT experimental results on β decay, π decay, and μ capture and decay show remarkable agreement with the predictions of the V–A theory of the strangeness-conserving part of the weak interaction. It is natural also to try to treat the strangeness-nonconserving weak interactions within the framework of the same theory. A recent measurement of the $K^+ \rightarrow e^+ + \nu$ decay rate[1] relative to that of the $K^+ \rightarrow \mu^+ + \nu$ mode is in good agreement with the V–A prediction and sets an upper limit of 3×10^{-3} on the ratio of the pseudoscalar and axial-vector coupling constants. Additional support for the polar and axial-vector covariant forms has also come from several experiments[2–6] on the three-body leptonic decays of the K mesons. Measurements of decay spectra and angular correlations in the decays $K^+ \rightarrow l^+ + \pi^0 + \nu$ and $K_2^0 \rightarrow l^\pm + \pi^\mp + \nu$, where l is a lepton, agree well with pure vector coupling but disagree completely with pure scalar or tensor coupling, the other interactions permitted by a theory with local bilinear coupling. Mixtures of vector, tensor, and scalar couplings are possible. For K_{e3}^+ decay, limits[2] of 0.3 on the ratio of each of the scalar and tensor coupling constants to the vector coupling constant have been obtained under the assumption that all form factors have the same energy dependence. Similar limits[3] from K_{e3}^+ also have been obtained under the assumption of constant form factors.

Recently, attention has turned to a closer examination of the two strong interaction form factors involved in K_{e3} and $K_{\mu3}$ decays. The form factors are of interest because they offer a description of the virtual strong interactions which accompany K decay. Dispersion relations[7] have been used to calculate the energy dependence of the form factors that would result from various hypotheses, for example, the hypothesis that the strong part of the interaction is dominated by a $I=\frac{1}{2}$, $J=1^-$ intermediate state such as the K^* resonance at 890 MeV. Secondly, the form factors are of interest because they provide a test of the $|\Delta\mathbf{I}| = \frac{1}{2}$ rule in leptonic K decays. The isotopic spin of the strongly interacting particles in these decays may, in general, change by $\frac{1}{2}$ or $\frac{3}{2}$ with the amplitudes for $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ for the K^+ and K^0 decays related by known Clebsch-Gordan coefficients. If only the amplitude for $\Delta I = \frac{1}{2}$ contributes, then the form factors in K_{e3}^+ and $K_{\mu3}^+$ are identical, within a factor of $\sqrt{2}$, with the corresponding form factors in K_{e3}^0 and $K_{\mu3}^0$, respectively. Thirdly, the form factors are of interest because a comparison of their values in K_{e3} and $K_{\mu3}$ provides a test of muon-electron universality in weak interactions.

For pseudoscalar K mesons, vector coupling yields for the differential decay probability for K_{e3} decay in the K-meson rest frame[8]

$$W(E_e,E_\pi) = \frac{G^2}{16\pi^3}|f_+|^2[M_K[2E_eE_\nu - M_K(W-E_\pi)] + |f_2|^2 M_e^2(W-E_\pi) - (f_2^* f_+ + f_2 f_+^*) M_e^2 E_\nu], \quad (1)$$

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TABLE I. Summary of measurements of λ for K_{e3}.

Decay	Experimenter	λ	Number of useful events	Technique
K_{e3}^+	Brown *et al.*[a]	$+0.038\pm0.045$		Xenon bubble chanber
	Jensen *et al.*[b]	-0.010 ± 0.029	407	Xenon bubble chamber
	Borreani *et al.*[c]	$-0.04\ \pm0.05$	230	Hydrogen bubble chamber
	Kalmus *et al.*[d]	$0.028_{-0.014}^{+0.013}$		Freon bubble chamber
	Bellotti *et al.*[e]	$+0.025\pm0.018$	457	Heavy liquid bubble chamber
	Present experiment	$+0.016\pm0.016$	1393	Spark chamber
K_{e3}^0	Luers *et al.*[f]	$+0.07\ \pm0.06$	153	Hydrogen bubble chamber
	Fisher *et al.*[g]	$+0.15\ \pm0.08$		Spark chamber
	Firestone *et al.*[h]	$-0.01\ \pm0.02$	762	Hydrogen bubble chamber
	Lowys *et al.*[i]	$+0.08_{-0.08}^{+0.10}$	240	Heavy liquid bubble chamber

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[b] See Ref. 2.
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[e] See Ref. 10.
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[g] See Ref. 11.
[h] See Ref. 6.
[i] J. P. Lowys, B. Aubert, L. M. Chounet, C. Pascaud, and L. Behr, Phys. Letters **24**, 75 (1967).

where

$$E_\nu = M_K - E_\pi - E_e\,,$$
$$W = (M_K^2 + M_\pi^2 - M_e^2)/2M_K\,,$$

and

$$f_2 = \tfrac{1}{2}(f_+ - f_-).$$

We have chosen to write the decay probability in terms of the electron and pion energies E_e and E_π. M_K, M_π, and M_e are the masses of the K, π, and electron, respectively, and f_+ and f_- are the strong interaction form factors which are functions only of q^2, the square of the four-momentum transfer to the leptons, given by $q^2 = (P_K - P_\pi)_\alpha{}^2 = M_K^2 + M_\pi^2 - 2M_K E_\pi$ for K^+ decays at rest.

The last two terms in Eq. (1) are proportional to M_e^2 and can be ignored. Hence the decay probability in K_{e3} decay is determined entirely by the first term in (1) which involves only the absolute magnitude of the form factor f_+. If the pion energy dependence of f_+ is small, it can be approximated by a linear variation of f_+ with q^2, i.e., $f_+(q^2) = f_+(0)(1+\lambda q^2/M_\pi^2)$. The measured values of λ for both K_{e3}^+ and K_{e3}^0 decays are shown in Table I. The values of λ from K^+ decay, are consistent with $\lambda = 0.0$, but only the experiments of Jensen *et al.*,[2] Kalmus and Kernan,[9] and Bellotti *et al.*,[10] are of sufficient accuracy to rule out a large value of λ such as $\lambda = 0.10$. The experimental situation in K_{e3}^0 decays is not clear. The measurement of Fisher *et al.*,[11] although having a large error, indicates that the energy dependence of the form factor is large and is inconsistent with $\lambda = 0.0$, while the most accurate measurement, that of Firestone *et al.*,[6] is consistent with $\lambda = 0.0$ and rules out a value of λ as large as 0.10. New, more accurate measurements of λ for both K_{e3}^+ and K_{e3}^0 are necessary to settle this question. Further interest is added by the fact that recent polarization measurements[12–15] for both $K_{\mu3}^+$ and $K_{\mu3}^0$ give values of $\xi = f_-/f_+$ which appear to be inconsistent with the values of ξ obtained from $K_{\mu3}$ and K_{e3} branching ratios under the assumption that f_+ and f_- have weak pion energy dependence.

In this paper we report a measurement of λ in K_{e3}^+ which is based on a sample of 1393 events obtained in a scintillation counter-spark chamber experiment. All previous values of λ in K_{e3}^+ were obtained from bubble chamber experiments with somewhat lower statistics. In this experiment a value of $\lambda = 0.016\pm0.016$ is obtained, with the total error arising from a statistical error of 0.013 and an error due to backgrounds of 0.010. Detailed checks on various sources of background and on the measurement technique were obtained from analyses of samples of $K_{\mu2}^+$ and $K_{\pi2}^+$ events.

EXPERIMENTAL APPARATUS

A focused 525-MeV/c positive beam was obtained at the Princeton-Pennsylvania Accelerator (PPA) with the beam transport system shown in Fig. 1. The experimental apparatus is shown in Fig. 2. With the exception of the shower spark chambers this apparatus and the way it was employed were very similar to that

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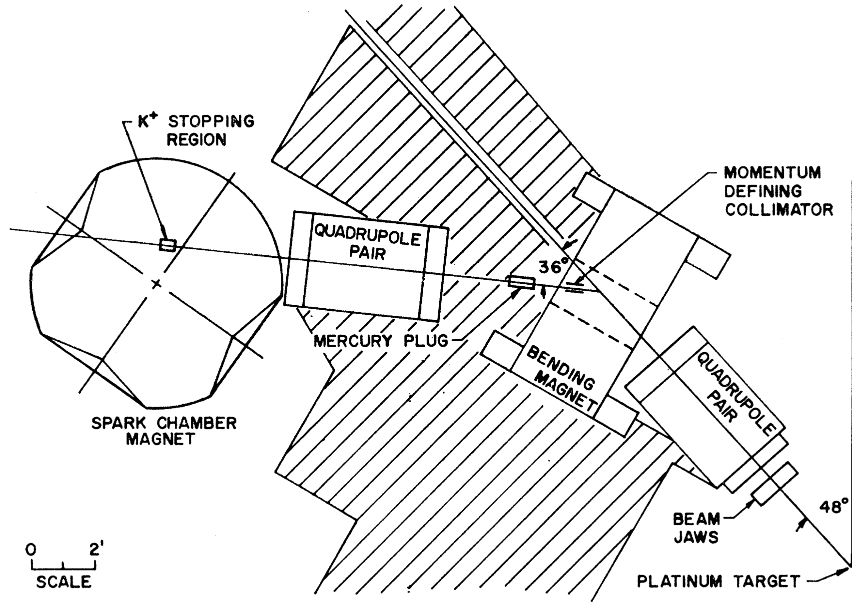

FIG. 1. The beam-transport system.

of three previous experiments[1,3,16] on K^+ decays at the PPA.

Briefly, the K^+ mesons were identified by range and by time of flight and were stopped in a beryllium target. Six thin-plate optical spark chambers, located in a 3-kG magnetic field, were used to determine the momentum of the charged secondaries from K^+ decay which emerged from the stopping region at an angle of approximately 90° to the direction of the incident K^+ beam. The momentum resolution for positrons in the momentum region of interest, 80–230 MeV/c, was determined to be 3.1% from observation of the charged secondaries in $K^+ \rightarrow \mu^+ + \nu$ and $K^+ \rightarrow \pi^+ + \pi^0$ decays. This uncertainty arose from energy loss in the beryllium stopping region, multiple scattering in the 0.18 g cm^{-2} of aluminum in the spark chambers, and from individual spark scatter. After traversing the momentum chambers the decay particle was detected by scintillation counters $C4$ and $C5$ and then by a threshold gas Čerenkov counter[17] which was used for positron identification. The apparatus was designed to provide a momentum-independent detection efficiency for posi-

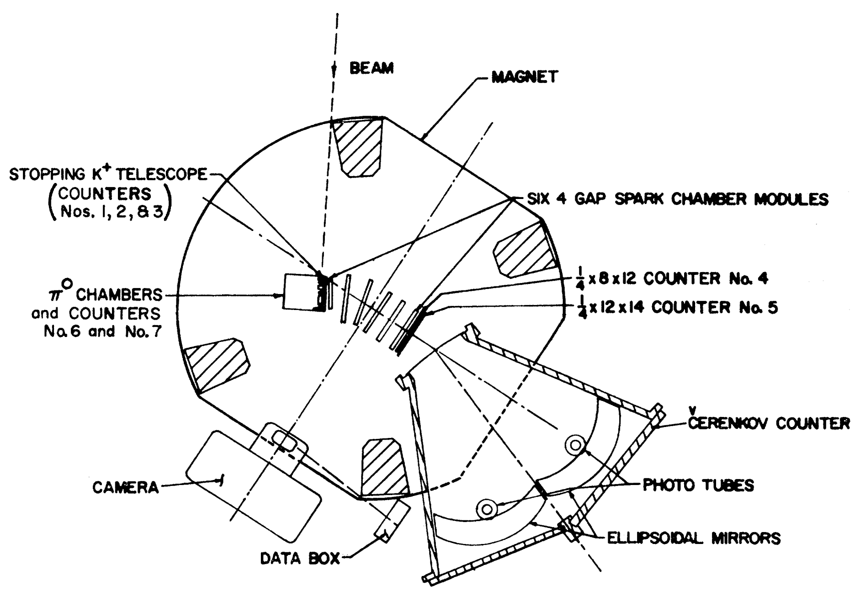

FIG. 2. The experimental apparatus. The two shower spark chambers used to detect the π^0 were located above and below the stopping K^+ telescope.

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[17] J. MacG. Dobbs, W. K. McFarlane, and D. Yount, Princeton- Pennsylvania Accelerator Report No. PPAD-586E (unpublished).

trons of momentum greater than 80 MeV/c which came from the stopping region. The Čerenkov counter was initially tested in a 150-MeV/c electron beam and found to have an efficiency of $(97\pm1)\%$ throughout a large selected fiducial region in the phase space of the electron coordinates and direction at the counter entrance. The Čerenkov counter was subsequently used in two experiments,[1,3] in one of which[3] the uniformity of detection efficiency was carefully confirmed.

The γ rays from the decay of the π^0 were detected in two shower spark chambers located above and below the K^+ stopping region. Because of limited space each chamber was restricted to ten spark gaps. Each chamber had four lead-alloy plates containing 7% antimony for mechanical strength. The plate thickness of 0.2 cm (0.38 radiation lengths) was chosen to provide as large a γ-ray conversion probability as possible consistent with a high-detection efficiency for the showers from the lowest-energy γ rays from K_{e3}^+ decay. The γ rays from K_{e3}^+ decay which entered the two shower spark chambers varied in energy from 23 to 235 MeV; over this energy range the mean γ-ray conversion length varied from 1.37 to 0.90 cm. The first two spark gaps, located in front of the first lead plate, were used to distinguish γ rays from charged particles; any track starting in either of these two gaps was assumed to arise from a charged particle rather than from a γ ray.

Scintillation counters $C6$ and $C7$ were located behind the shower chambers in order to ensure, where possible, that the showers observed in these chambers occurred at the time of the K^+ decay. When the showers appeared to stop before entering $C6$ and $C7$, it was of course impossible to impose this timing requirement. The spark chambers were triggered by a coincidence between a stopping K^+ ($C1$, $C2$, $\bar{C}3$, RF)[1] and a decay positron ($C4$, $C5$, Čerenkov counter). The RF refers to a time of flight signal obtained from the accelerator. Approximately 75% of the data were obtained with the additional requirement of a pulse from either $C6$ or $C7$ in the triggering logic. The signals from counters $C6$ and $C7$, together with pulses indicating the stopping K^+ meson and the decay positron for each event were displayed on an oscilloscope and photographed. The spark chambers were photographed in 90° stereo.

DATA ANALYSIS

Selection and Reconstruction of Events

Events were first required to satisfy the following three selection criteria. (1) A track from a positive charged particle was observed in the six thin-plate momentum spark chambers. This track was required to fit well to some momentum value, to extrapolate back to the K^+ stopping region, and to extrapolate into counter $C4$ and the Čerenkov counter. These requirements removed 17.2% of the nominal K_{e3}^+ events. As previously stated, the detection efficiency for positrons was uniform only above 80 MeV/c. Hence all K_{e3}^+ events below 80 MeV/c were rejected. Finally, 2.5% of the events were rejected because by visual inspection, the positron belonged to an electron-positron pair. (2) One and only one γ ray was identified in each of the two shower spark chambers. For about one third of the γ rays a definite shower could be recognized, but for the remainder only a single track could be clearly seen. The scanning criteria for identification of a γ ray required three or more sparks including two consecutive sparks following a lead plate. To prevent spurious association of sparks, no more than two consecutive gaps between sparks were allowed. Any track which started in either of the first two gaps was assumed to arise from a charged particle rather than from a γ ray. (3) The counter behind each spark chamber gave a pulse at the time of the K^+ decay if the shower in that chamber contained sparks in either of its last two gaps. In any case it was required that at least one of the two counters $C6$ and $C7$ behind the two shower chambers give a pulse at the time of the K^+ decay.

Each event was reconstructed using the charged-particle momentum, the conversion vertices of the two γ rays, and the position of the stopped K^+. The direction of each γ ray was determined from the conversion point of that γ ray and from the position of the stopped K^+. Because of multiple scattering in the lead plates no attempt was made to determine the directions of the two γ rays from their showers. Also, due to the limited number of sparks per shower, it was not possible to obtain information on the γ-ray energies from their showers. The position of the stopped K^+ was known to lie along the positron trajectory, but was uncertain to ±0.95 cm, one half the transverse thickness of the beryllium block which was used to stop K^+ mesons. The K^+ was first assumed to have stopped at the intersection of the positron trajectory with the center of the beryllium block. A small number ($\sim10\%$) of the nominal K_{e3}^+ events failed to reconstruct with this assumed stopped K^+ position. For these events reconstruction was attempted with other assumed positions for the stopped K^+ along the positron trajectory and within the beryllium block. If the event failed to reconstruct with any of these positions, it was rejected. An error of 0.95 cm in the position of the stopped K^+ typically introduced an error of 5 MeV in the calculated energy of the π^0 from K_{e3}^+. As a result of a kinematic ambiguity, there were two solutions for the energy of the π^0 and both solutions were used in the subsequent analysis.

Checks on Measurement Technique and Backgrounds

Detailed checks on various sources of background and on the measurement technique were obtained from analyses of samples of $K_{\pi2}^+$ and $K_{\mu2}^+$ events. From these checks and from analysis of the K_{e3}^+ sample, the magnitude of the background in the K_{e3}^+ sample was determined to be $(23\pm2)\%$. The energy distribution of the background was also determined with high accuracy so that the error in λ arising from uncertainties in the background was less than the statistical error.

We analyzed a sample of 738 $K_{\pi2}^+$ events, obtained with the Čerenkov counter removed from the logical requirement for an event. These events were subjected to the same initial selection criteria as the nominal K_{e3}^+ events but in addition were required to have a charged decay particle momentum between 192 and 220 MeV/c. The $K_{\pi2}^+$ events were useful because they were overdetermined by two variables; in particular, the decay products had to be coplanar and, as we used them, the events had to show a calculated K^+ stopping position within the K^+ stopping region. We took as a measure of coplanarity the cosine of the angle θ be-

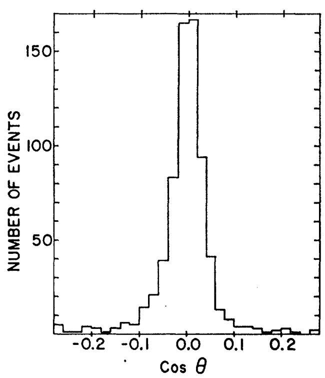

FIG. 3. Coplanarity distribution of the 738 events in the $K_{\pi2}^+$ sample. Coplanarity is measured by the cosine of the angle θ between the π^+ and the normal to the plane of the two γ rays. Of the 738 events, 56 had a coplanarity larger than 0.24 in magnitude.

tween the π^+ and the normal to the plane of the two γ rays. Figure 3 shows the coplanarity of events in the $K_{\pi2}^+$ sample. Of these events, 89.5% had a coplanarity smaller in magnitude than 0.14. Further evidence that most of these events were $K_{\pi2}^+$ was provided by the distribution in the calculated position of the stopped K^+ shown in Fig. 4. This K^+ stopping position was calculated using the known π^+ and π^0 momenta (205.3 MeV/c), the conversion points of the two γ rays, and the measured π^+ direction. 89.4% of the events had a calculated K^+ stopping position within ±3 cm of the center of the target.

Of the total $K_{\pi2}^+$ sample, 623 events or 84.4%, were identified as $K_{\pi2}^+$ because they satisfied both the coplanarity requirement and the condition on the calculated K^+ stopping position. In addition, the 623 events were required to exhibit a π^0 opening angle greater than 61.3 degrees. The distribution for these $K_{\pi2}^+$ events of the π^0 opening angle, the angle between the two γ rays, is shown in Fig. 5; it provided a check

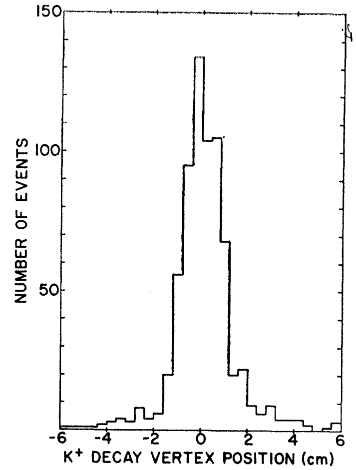

FIG. 4. Distribution of the calculated K^+ stopping position of the 738 events in the $K_{\pi2}^+$ sample. The physical limits of the target are ±0.95 cm. Of the 738 events, 49 had a calculated K^+ stopping position larger in magnitude than 5.2 cm.

on possible experimental biases. The agreement of the experimental distribution with the Monte Carlo calculation indicates that there was no serious bias (e.g., against events with large opening angles) which would lead to an erroneous value of λ in the K_{e3}^+ sample.

On the basis of a Monte Carlo calculation described below, 14 of the 115 rejected events originally identified as $K_{\pi2}^+$ were expected to arise from K_{e3}^+ and $K_{\mu3}^+$ decays with charged decay particle momentum between 192 and 220 MeV/c. Of the remaining events, 3.6 were expected to arise from $K_{\pi2}^+$ decays in which one of the two γ rays was Compton scattered in the beryllium block before converting in one of the shower spark chambers, and about 3.8 events were expected to arise from $K_{\pi2}^+$ decay in which one of the γ rays showered in the copper absorber producing a secondary

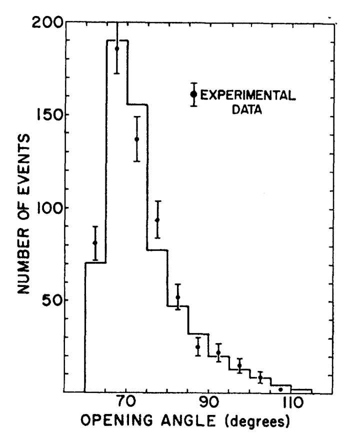

FIG. 5. Distribution of the opening angle of the π^0 for the 623 events in the $K_{\pi2}^+$ sample which satisfied $K_{\pi2}^+$ selection criteria compared with the predicted distribution shown by the histogram. The χ^2 is 9.9 for 7 degrees of freedom.

γ ray which in turn converted in a shower spark chamber.

In the remaining 94 events in the $K_{\pi 2}^+$ sample, which fail at least one of the coplanarity, K^+ stopping position, or opening angle criteria, one of the two observed γ rays was not associated with a K^+ decay, but arose from some interaction of a beam particle such as π^+ charge exchange in the copper absorber. (The ratio π^+/K^+ in the incident beam was about 300/1.) This spurious γ ray was observed if it showered within the 500-nsec sensitive time of the spark chamber. Generally, the shower did not occur at the time of the K^+ decay and hence there was no pulse at the appropriate time in counter $C6$ or $C7$ at the back of the shower spark chambers. This led to rejection of the event since, by selection criterion 3, a pulse was required if the chamber exhibited sparks in either of its last two gaps. However, occasionally a pulse was present due to an accidental coincidence, or alternatively a pulse was absent legitimately because some showers did not penetrate to counters $C6$ or $C7$.

Hence, it was anticipated that about 94 background events in the $K_{\pi 2}^+$ sample arose from $K_{\pi 2}^+$ decays in accidental coincidence with a γ ray not associated with the K^+ decay. To confirm this hypothesis, a sample of 835 $K_{\mu 2}^+$ events was analyzed. Any γ ray associated with such events must be spurious. The $K_{\mu 2}^+$ sample was obtained with the Čerenkov counter and counters $C6$ and $C7$ removed from the triggering logic. The selection criterion 1 pertaining to the charged decay particle was applied and in addition this particle was required to have a measured momentum between 231 and 252 MeV/c. Events were then scanned for γ rays. Of the 835 $K_{\mu 2}^+$ events, 110 had an associated, spurious γ ray in one shower spark chamber and 16 had associated γ rays in both chambers. Thus there was a probability of 0.066 ± 0.006 for a spurious γ ray in a given chamber, and a probability of 0.019 ± 0.005 for spurious γ rays in both chambers. 92 of the 142 spurious γ rays exhibited sparks in at least one of the last two gaps, but only 10 of these 92 γ rays had an appropriate pulse from counter $C6$ or $C7$ and hence satisfied selection criterion 3. Four of the remaining 50 γ rays, all of which satisfied selection criterion 3 by the absence of a spark in one of the last two gaps of the shower spark chamber, also had an associated pulse in $C6$ or $C7$.

The same incident beam associated sources of spurious γ rays that existed for $K_{\mu 2}^+$ decay also existed for $K_{\pi 2}^+$ and K_{e3}^+ decay. Hence it was possible to understand, from the preceding analysis of the $K_{\mu 2}^+$ sample, the total number of background events in the $K_{\pi 2}^+$ and K_{e3}^+ samples and also the number of background events in these samples rejected by selection criterion 3 alone. Using the rate measured in the $K_{\mu 2}^+$ sample for spurious γ rays which satisfied selection criterion 3, rough calculations were made of the backgrounds expected in the $K_{\pi 2}^+$ and K_{e3}^+ sample. For $K_{\pi 2}^+$ decay the calculated background was $(11.6\pm2.5)\%$ compared to the actual background of $(15.2\pm1.5)\%$ in the $K_{\pi 2}^+$ sample. For K_{e3}^+ decay the calculated background for the sample of events which reconstructed properly as nominal K_{e3}^+ was $(20\pm3)\%$.

A more accurate estimate of the background in the K_{e3}^+ sample was obtained using the observed distribution of spurious γ-ray vertices in the $K_{\mu 2}^+$ sample. A Monte Carlo calculation was made to determine the effect of spurious γ rays on the K_{e3}^+ sample. For those Monte Carlo events for which a γ ray converted in only one shower chamber, a spurious γ ray was positioned randomly in the other shower chamber according to the measured (essentially uniform) distribution of spurious γ-ray positions in the $K_{\mu 2}^+$ sample. It was found that $(35.0\pm0.7)\%$ of the events in this Monte Carlo-generated sample of spurious K_{e3}^+ events failed to reconstruct. In the experimental K_{e3}^+ sample there were 286 events that failed to reconstruct and 1867 events that reconstructed properly. A small fraction, 3.0%, of the experimentally observed good K_{e3}^+ events, were expected to fail to reconstruct because of the experimental resolution. Hence the number N of spurious events in the experimental K_{e3}^+ sample which accidentally satisfied the reconstruction criterion is given by

$$N=\left[\frac{\text{fraction of spurious Monte Carlo events which reconstruct}}{\text{fraction of spurious Monte Carlo events which fail to reconstruct}}\right]\times[\text{number of events in the experimental sample which fail to reconstruct }-R],$$

where R is the number of good K_{e3}^+ events which should fail to reconstruct because of the experimental resolution. Inserting numbers from above yields

$$N=\left[\frac{0.65}{0.35}\right]\times[286-0.03(1867-N)]=452.$$

Hence the fraction B of background events in the sample of 1867 events which reconstructed and therefore appeared as K_{e3}^+ events is given by

$$B=452/1867=0.242\pm0.02.$$

Another measure of this background was obtained from the fraction of events with an opening angle between the two γ rays of less than 61.3 degrees, the minimum kinematically allowed opening angle. Here

$(22.0\pm0.7)\%$ of the Monte Carlo-generated spurious K_{e3}^+ events and 135 of the events in the experimental K_{e3}^+ sample had an opening angle less than 61.3 degrees. Hence the expected fraction B is given by

$$B=\left(\frac{0.65}{0.22}\right)\times\left(\frac{135}{1867}\right)=0.214\pm0.02.$$

Combining the two values of B, we obtained $(23\pm2)\%$ as the best measure of the fraction of background events in the K_{e3}^+ sample. This agrees with the earlier rough calculation of $(20\pm3)\%$. In addition to the 23% background previously calculated, $(2.4\pm0.7)\%$ of the events in the K_{e3}^+ sample of reconstructed events arose from $K_{\pi2}^+$ decays. This number of $K_{\pi2}^+$ events in the K_{e3}^+ sample had a negligible effect on the determination of λ.

Finally, background subtractions were calculated for the sample of 1867 nominal K_{e3}^+ events which satisfied all selection criteria. The total background subtraction, exclusive of $K_{\pi2}^+$ events, was normalized to 23% of the 1867 events. Including $K_{\pi2}^+$ subtraction, a total of 1393 events remain. It was also necessary to know the energy distributions relating to background events. These distributions were obtained from the Monte Carlo-generated sample of spurious K_{e3}^+ events. Indications of the validity of this calculation were obtained from the distributions of positron momentum, γ-ray energy, and π^0 energy for those *events which were rejected by selection criterion 3.* This criterion required that the counter behind each shower spark chamber give a pulse if the shower in that chamber contained sparks in either of its last two gaps. These rejected events should differ from the background events in the final sample of 1867 nominal K_{e3}^+ events only with regard to selection criterion 3. Thus, when reconstructed as K_{e3}^+ decays, the background events that passed and failed criterion 3 should have exactly the same distributions. These are shown in Figs. 6–8, along

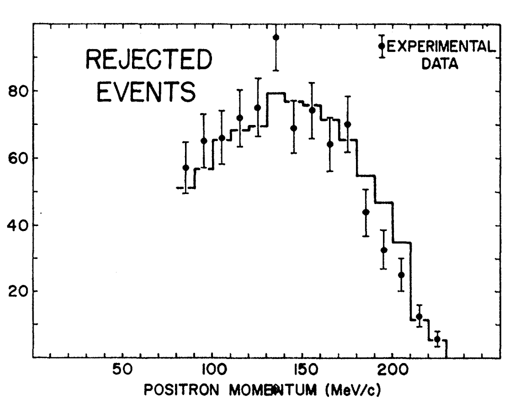

FIG. 6. Positron momentum distribution for the events in the K_{e3}^+ sample which were rejected by selection criterion 3, compared to the predicted distribution for spurious events shown by the histogram. The χ^2 is 14.3 for 12 degrees of freedom.

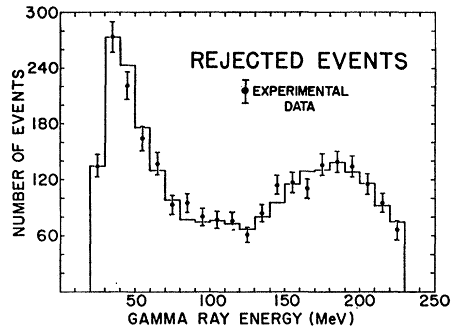

FIG. 7. γ-ray distribution for the events in the K_{e3}^+ sample which were rejected by selection criterion 3, compared to the predicted distribution for spurious events shown by the histogram. Because of a kinematic ambiguity there were two solutions for the γ-ray energies. Both solutions were given equal weight in both the experimental and the predicted distributions. The χ^2 is 15.4 for 20 degrees of freedom.

with the calculated distributions for Monte Carlo-generated spurious events. The good agreement provides strong support for the background calculation, particularly since these distributions differ substantially from the corresponding distributions for good events.

Figure 9 shows the corrected (for background) and uncorrected γ-ray energy distributions for the 1867 nominal K_{e3}^+ events and the expected Monte Carlo distribution for $\lambda=0.016$; the γ-ray energy distribution is insensitive to the value of λ. The good agreement of the corrected and calculated distribution supports the estimates of both the magnitude and the energy distribution of the background. Further, the agreement for low γ-ray energies indicates that there was no significant loss of events with low-energy γ rays; such a loss would lead to an erroneous value of λ.

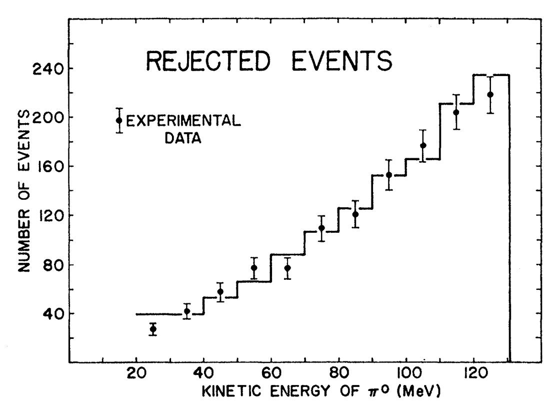

FIG. 8. Distribution of π^0 kinetic energy for the 631 events in the K_{e3}^+ sample of reconstructed events which were rejected by selection criterion 3, compared to the predicted distribution for spurious events shown by a histogram. Because of a kinematic ambiguity, there were two solutions for the π^0 kinetic energy, and both solutions are shown. All solutions below 30 MeV are shown in the same bin. The χ^2 is 11.9 for 10 degrees of freedom.

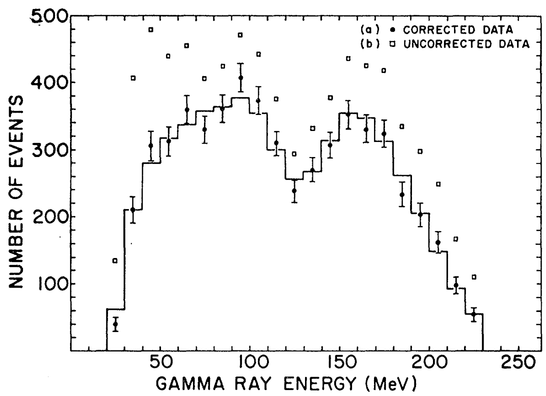

FIG. 9. The corrected (for background) (a) and uncorrected (b) γ-ray energy distributions for the 1867 events in the final K_{e3}^+ sample compared with the predicted distribution for $\lambda=0.016$ shown by the histogram. Because of a kinematic ambiguity there are two solutions for the γ-ray energy and both solutions are shown. The χ^2 is 16.1 for 20 degrees of freedom.

For approximately 25% of the data no pulse was required in the triggering logic from counters $C6$ and $C7$. It was found that of 547 events which satisfied all other selection criteria, 21 events had no pulse in either counter, compared to 17.7 events expected, due either to the 2% inefficiency of $C6$ and $C7$ or to events with spurious γ rays. Hence, the requirement of at least one pulse from counters $C6$ and $C7$ introduced no significant γ-ray energy-dependent bias.

Figure 10 gives the Dalitz plot of the 1393 K_{e3}^+ events. In each bin the percentage of events subtracted as background is shown. Figure 11 shows the final positron momentum distribution, which is not sensitive to the value of λ. The good fit supports the estimates of the background and indicates that there was no serious positron momentum-dependent bias. Figure 12 shows the final π^0 energy distribution. This distribution was used to obtain λ because $f_+(q^2)=f_+(0)(1+\lambda q^2/M_\pi^2)$ is a function only of the π^0 energy.

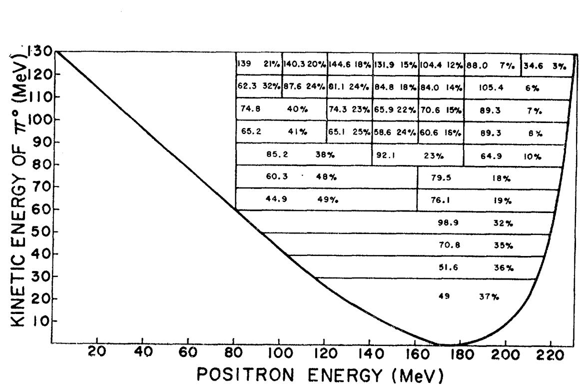

FIG. 10. Dalitz plot of the 1393 K_{e3}^+ events. Both solutions appear. The number of events after background subtraction and the calculated percentage of background are shown in each bin. A χ^2 of 36.2 for 33 degrees of freedom was obtained with $\lambda=0.016$.

Determination of λ

To find the energy dependence of $f_+(q^2)$ we have compared the experimental π^0 energy distribution for the 1393 K_{e3}^+ events with distributions for various values of λ obtained from an extensive Monte Carlo simulation of the experiment. This simulation took into consideration the following: (1) Uncertainty in the calculated positron momentum of 3.1%; (2) bremsstrahlung of the positron in the beryllium block, which caused 10% of the positrons to lose more than 3.1% of their momentum; (3) solid angle for the decay positron as determined horizontally by counter $C4$ and vertically by the Čerenkov counter; (4) multiple scattering of the decay positron in the beryllium block; the mean multiple-scattering angle was 0.037 rad. for an 80-MeV/c positron and was inversely proportional to the positron momentum; (5) solid angle of each of the shower spark chambers; (6) γ-ray conversion efficiency of the shower spark chambers; (7) distribution of the stopped K^+ mesons in the stopping region; (8) uncertainty, when reconstructing an event, in the position of the stopped K^+ to one half the thickness of the beryllium block; the Monte Carlo events were reconstructed with a stopped K^+ position determined in exactly the same manner as described for real events; (9) conversion of a γ ray in the stopping region, which caused about 5% of the events to be lost; (10) conversion of a γ ray in the copper absorber, which caused about 1.5% of the events to be lost.

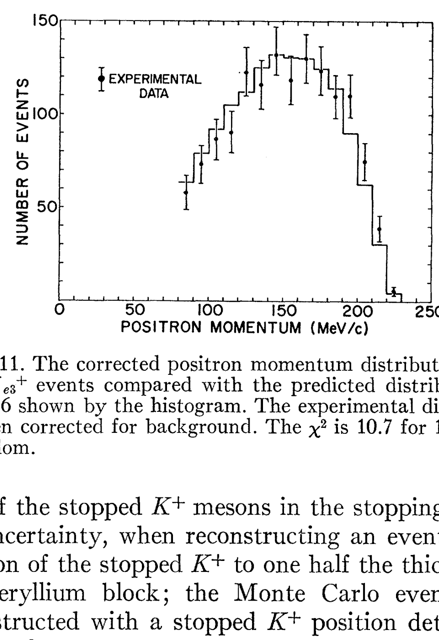

FIG. 11. The corrected positron momentum distribution of the 1393 K_{e3}^+ events compared with the predicted distribution for $\lambda=0.016$ shown by the histogram. The experimental distribution has been corrected for background. The χ^2 is 10.7 for 12 degrees of freedom.

A χ^2-versus-λ curve, shown in Fig. 13, was obtained by comparing the final π^0 energy distribution for K_{e3}^+ events, shown in Fig. 12, with Monte Carlo distributions calculated for various values of λ. For each value of λ approximately 6×10^4 Monte Carlo events were used. A minimum χ^2 of 12.2 for 10 degrees of freedom occurred for $\lambda=0.016$. Without any background substraction a minimum χ^2 was obtained for $\lambda=0.065$. Errors in λ arose from uncertainties in the background; the 2% uncertainty in the magnitude of

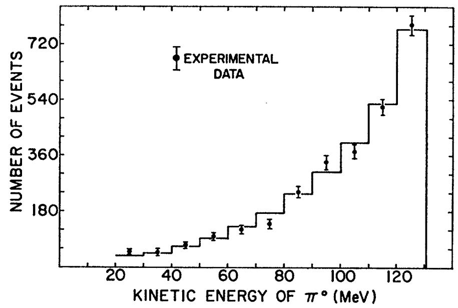

FIG. 12. The corrected π^0 kinetic energy distribution of the 1393 K_{e3}^+ events compared to the predicted distribution for $\lambda=0.016$ shown by the histogram. The experimental distribution has been corrected for background. Due to a kinematic ambiguity, there are two solutions for the π^0 energy and both are shown. All solutions below 30 MeV are shown in the same bin.

the background led to an error in λ of 0.005, while the uncertainty in the background energy distribution led to an error in λ of 0.006. A total error of 0.01 was assigned to λ because of these uncertainties in the background.

The two solutions for the pion energy were not statistically independent and therefore the statistical error could not be obtained directly from the χ^2 curve. We chose to obtain the statistical error from an analysis of a series of Monte Carlo samples of 1500 events. For each 1500 event sample a χ^2-versus-λ curve was obtained by comparing the π^0 energy distribution for that sample with the much larger (6×10^4 events) Monte Carlo energy distributions of the π^0 for various values of λ. The χ^2 curve was used to obtain the value of λ for which χ^2 was a minimum for that sample. The rms deviation of the best fit λ from the λ value used to generate the 40 Monte Carlo samples of 1500 events was then the statistical error associated with a 1500 event sample. Small samples of 500 K_{e3}^+ events were similarly analyzed. On this basis a statistical error of ±0.013 was assigned to the experimental value of λ.

CONCLUSIONS

Combining the statistical error of ±0.013 with the error in the background subtraction ±0.010, we obtain $\lambda=0.016\pm0.016$ in $f_+(q^2)=f_+(0)(1+\lambda q^2/M_\pi^2)$.

The Dalitz plot, shown in Fig. 10, was compared with a Monte Carlo Dalitz plot for $\lambda=0.016$ and a χ^2 of 36.2 for 33 degrees of freedom was obtained. The good agreement supports the assumption of vector coupling for K_{e3}^+ and agrees with previous experiments[2–6] on the nature of the coupling in K_{e3} and $K_{\mu3}$ decays.

The value of λ obtained in this experiment, $\lambda=+0.016\pm0.016$, is in agreement with other measurements of

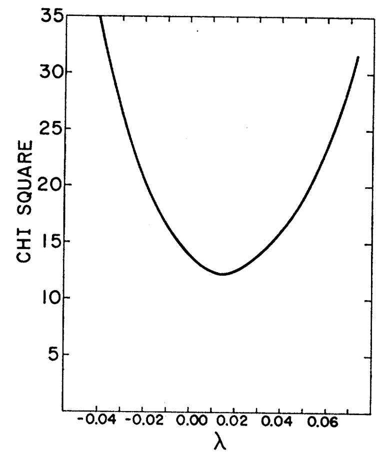

FIG. 13. Goodness of fit for vector interaction with a form factor $f_+(q^2)=f_+(0)(1+\lambda q^2/M_\pi^2)$. A minimum χ^2 12.2 for 10 degrees of freedom occurred at $\lambda=0.016$.

λ for K_{e3}^+, as shown in Table I. The weighted mean of the values for K^+ in Table I is $\langle\lambda^+\rangle=+0.019\pm0.009$.

If the energy dependence of f_+ is due to a dominant $J=1^-$, $I=\frac{1}{2}$ $K\pi$ intermediate state of mass M, such as the K^* resonance at 890 MeV, then f_+ has the form $f_+(q^2)=f_+(0)M^2/(M^2-q^2)$, and $\lambda\cong M_\pi^2/M^2$. We find $M=(1180_{-387}^{+\infty})$ MeV for our value of λ, which is consistent with K^* dominance, but is also consistent with no energy dependence for f_+. At the 95% confidence level, $M>630$ MeV.

The experimental situation for K_{e3}^0 decay is not as clear as that for K_{e3}^+ because of the wider spread in the measured values of λ. The weighted mean of the values for K_{e3}^0 in Table I is, however, $\langle\lambda^0\rangle=+0.010\pm0.018$ which is equal within the errors to $\langle\lambda^+\rangle$ and thus offers no evidence for violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.

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