one finds

$$\delta W=\epsilon_\mu[P^\mu(\sigma_1)-P^\mu(\sigma_2)]+\tfrac{1}{2}\omega_{\mu\nu}[J^{\mu\nu}(\sigma_1)-J^{\mu\nu}(\sigma_2)]$$
$$=0,$$

when the coordinate variation is specialized to a Lorentz transformation

$$\delta x^\mu=\epsilon^\mu+\omega^{\mu\nu}x_\nu,\quad \omega^{\mu\nu}=-\omega^{\nu\mu}.$$

Thus

$$P_\mu(\sigma)=\text{const},\quad J^{\mu\nu}(\sigma)=\text{const},\tag{35}$$

where

$$P_\mu(\sigma)=\int_\sigma d\sigma_\nu T^{\mu\nu}(x),$$

$$J^{\mu\nu}(\sigma)=\int_\sigma d\sigma_\lambda[x^\mu T^{\lambda\nu}(x)-x^\nu T^{\lambda\mu}(x)],$$

and

$$T^{\mu\nu}=-F^{\mu\lambda}F_\lambda{}^\nu-\tfrac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa}$$
$$+\sum_{a=1}^{N}\int ds_a m_a\frac{dx_a{}^\mu}{ds_a}\frac{dx_a{}^\nu}{ds_a}\delta(x-x_a(s_a))$$
$$+\sum_{b=1}^{*N}\int ds_b m_b\frac{dx_b{}^\mu}{ds_b}\frac{dx_b{}^\nu}{ds_b}\delta(x-x_b(s_b)).$$

These are recognized to be the conservation laws of total energy momentum and generalized angular momentum. The ten conservation laws (35) also follow from the equation

$$\partial_\nu T^{\mu\nu}(x)=0,$$

which can be verified directly by using Eqs. (33), (34), and (14).

ACKNOWLEDGMENTS

I wish to express my sincere gratitude to Professor J. Schwinger for suggesting this investigation and for his valuable advice and encouragement. It is my pleasure to thank S. J. Chang for an interesting discussion.

PHYSICAL REVIEW VOLUME 160, NUMBER 5 25 AUGUST 1967

Longitudinal Distribution of Čerenkov Light from Extensive Air Showers

Carlo Castagnoli, Mauro Dardo, and Piero Penengo
Gruppo Italiano di Fisica Cosmica G.I.F.C.O. del C.N.R., Istituto di Fisica Generale, Università di Torino, Torino, Italia
(Received 12 April 1967)

The longitudinal development of extensive air showers of $\sim 10^{13}$ eV is studied both experimentally and theoretically. The risetime and duration of Čerenkov pulses in uv light at 3500 m above sea level are studied experimentally with high-speed techniques. The results are in quite good agreement with an accurate Monte Carlo estimate in the "*B*" approximation of electromagnetic shower theory.

INTRODUCTION

The development of extensive air showers (EAS) has been extensively studied from a statistical point of view. The information on the different stages of growth of an individual EAS is instead scanty and not very precise.

The methods to obtain such information are based on (a) measurements of the distribution of EAS particles, and (b) detection of the Čerenkov light emitted by the electronic component of the EAS in the atmosphere.

In case (a), the information is obtained through the longitudinal distribution of the particles in the shower by measuring the relative delays between the particles with the method used by Bassi *et al.*[1] This method is today extensively used in an experiment at Haverah Park.[2] A second method is based on the observation of the directions of motion of the muons, these data being related to the distribution of particle production along the shower axis.[3]

In case (b), it is possible to use two methods: the first one based on the comparison between the amount of Čerenkov light and the number of particles reaching the ground[4]; the second one based on the temporal analysis of the Čerenkov light pulse[5] produced by the

[1] P. Bassi, G. Clark, and B. Rossi, Phys. Rev. **92**, 441 (1953).

[2] Bi-annual Newsletter of British Universities HPNL-1, 1966 (unpublished).

[3] K. Greisen, Ann. Rev. Nucl. Sci. **10**, 92 (1960).

[4] A. E. Chudakov, N. M. Nesterova, V. I. Zatsepin, and E. I. Tukish, in *Proceedings of the Moscow Cosmic Ray Conference, 1959* (International Union of Pure and Applied Physics, Moscow, 1960).

[5] F. I. Boley, J. H. Baum, J. A. Palsedge, and J. H. Pereue, Phys. Rev. **124**, 1205 (1961); F. I. Boley, J. A. Palsedge, and J. H. Baum *ibid.* **734**, 126 (1962).

shower electrons. This method offers the following advantages:

(1) Detection devices for Čerenkov light are faster than particle counters.

(2) It is less sensitive to fluctuations than particle-detection methods.

(3) The Čerenkov radiation is proportional not to the local number of particles, but to the integral over the previous shower history.

Moreover, we recall that Čerenkov photons have a greater lateral spread than particles, and this makes the shower detection easier over a more extended area with simple devices.

We took up this problem with the aim of finding (by means of a very-high-speed photomultiplier and a gigascope) the time dependence of the Čerenkov radiation produced by the electronic component of the shower in the atmosphere. Our experimental results were then compared with the results of a Monte Carlo calculation for EAS produced by primary photons of energy 10^{12}, 10^{13}, and 10^{14} eV.

EXPERIMENTAL APPARATUS

The experiment was carried out in the period June to November, 1966 at the Testa Grigia Laboratory at an altitude of 3500 m above sea level (s.l.). The Čerenkov light of the shower was collected by a parabolic mirror in conjunction with an RCA C70045A photomultiplier positioned with the photocathode at the focal point of the mirror. The photomultiplier pulses were sent through a 50-Ω coaxial cable to a Tektronix type-519 gigascope and recorded photographically. The measurements were made with a horizontal sweep speed of 5 nsec/cm. To ensure that the pulses recorded by the fast channel would be due to EAS, our detection device included a second apparatus consisting of three spherical mirrors in conjunction with three Philips 56 AVP photomultipliers. The output pulses of these three photomultipliers were sent to a Tektronix type-555 dual-beam oscilloscope. A block diagram of the apparatus is shown in Fig. 1, and the optical and geometric characteristics of the detectors are reported in Table I.

The lack of a particle-detection device did not bring about appreciable uncertainty in the identification of

TABLE I. Geometric characteristics of the Čerenkov detectors.

Optical system	Mirror diameter (cm)	Focal length (cm)	Angular acceptance (°)	Soild angle (sr)	Geometric factor (cm² sr)
Spherical mirror + P.M. 56 AVP	50	25	2°30′	5.9×10^{-3}	8.9
Parabolic mirror + P.M. RCA C70045A	60	29	1°30′	2.1×10^{-3}	4.2

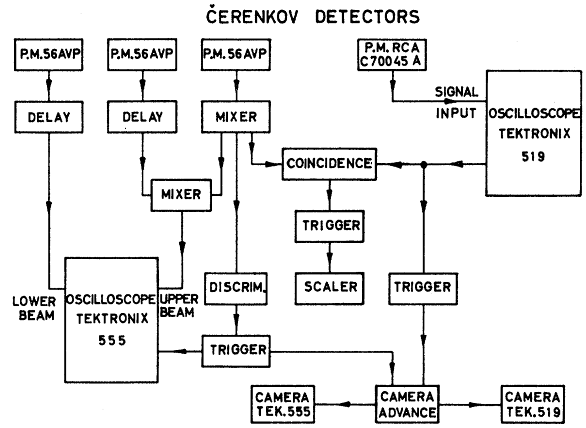

FIG. 1. Block diagram of the electronic circuits.

the shower. In fact, in previous experiments[6] made at the same elevation the Čerenkov light pulses were accompanied by particle-counter coincidences in 96% of the cases. The RCA C70045A is a high-speed 14-stage photomultiplier. The response of our detecting apparatus (photomultiplier, coaxial cable, oscilloscope, impedance matchers) to a light pulse of duration $\lesssim 0.1$ nsec had a risetime $\tau = 0.5$ nsec in agreement with the results of Birk *et al.*[7]

We screened the photocathode with a uv filter (aqueous solution of 50% Co $SO_4 \cdot 7H_2O$ in a quartz container), whose transmittance curve $T(\lambda)$ is shown in Fig. 2 together with the quantum efficiency $\epsilon(\lambda)$ of the photocathode and the resultant sensitivity $T(\lambda)\ \epsilon(\lambda)$.

This filter arrangement is suggested by the fact that on clear moonless nights the requirement of a minimum signal of 1 V from a shower of $\simeq 10^{12}$ eV (the vertical sensitivity of the oscilloscope being 9.1 V/cm) leads to a too intense night-sky current. Consequently it is impossible to observe showers of primary energy $E_0 \simeq 10^{12}$–10^{13} eV when using the complete spectral region of the photocathode. However, for average atmospheric conditions, the ozone concentration has a pronounced maximum at an altitude of 20–25 km and thus constitutes a screen for the star light in the ultraviolet region.[8]

On the other hand, the EAS Čerenkov light is produced at lower altitudes and does not suffer much attenuation.

The relative number of Čerenkov photons per unit path and per unit wavelength interval, under the assumption $\beta \simeq 1$, is proportional to η/λ^2, where $\eta = n - 1$ (n is the air refractive index). The value of η varies by $\sim 2\%$ over the spectral range used. The yield per unit

[6] A. E. Chudakov and N. M. Nesterova Nuovo Cimento **607**, 8 (1958).

[7] M. Birk, Q. A. Kerns, and R. F. Tusting, IEEE Trans. Nucl. Sci. **NS11**, 129 (1964).

[8] W. N. Charman in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society, London, 1966), Vol. 2, p. 1066.

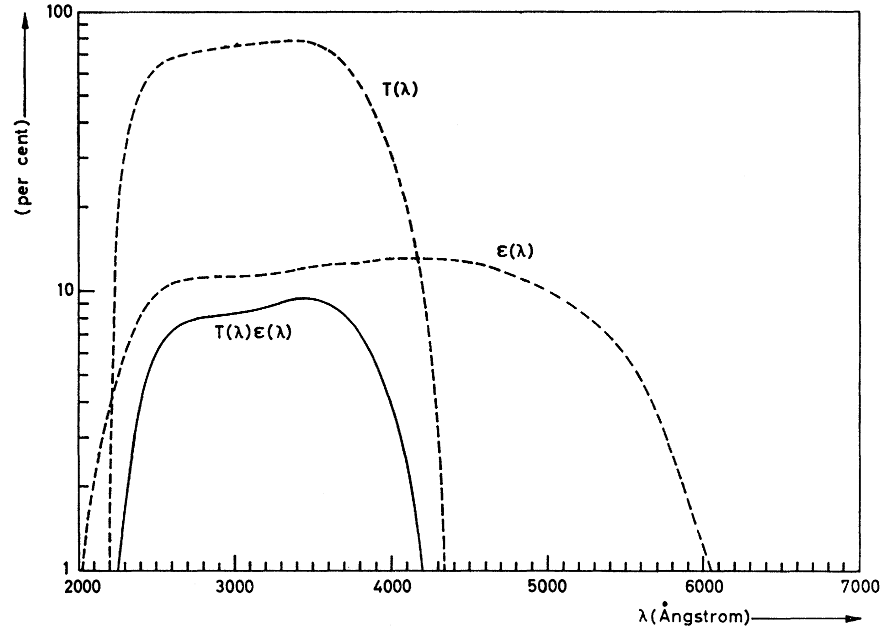

Fig. 2. Transmittance $T(\lambda)$ of the uv filter, quantum efficiency $\epsilon(\lambda)$ of RCA C70045A photomultiplier, and product curve.

wavelength interval is therefore greater in the ultraviolet than in the visual region.

We have also estimated the threshold photon flux density recorded by our apparatus by calibration with Čerenkov radiation of known intensity. The Čerenkov flashes were produced by ultrarelativistic muons of cosmic radiation at 70 m water equivalent under rock in 3 cm of distilled water in direct optical contact with the photocathode and contained in a cylindrical mirror. From considerations of the geometry of the device and of the flux of Čerenkov photons in EAS, we

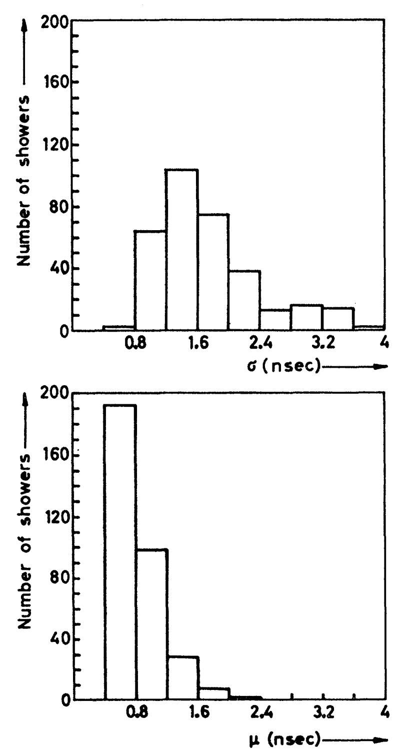

Fig. 3. Frequency distributions of μ and σ of the pulses recorded by the fast channel.

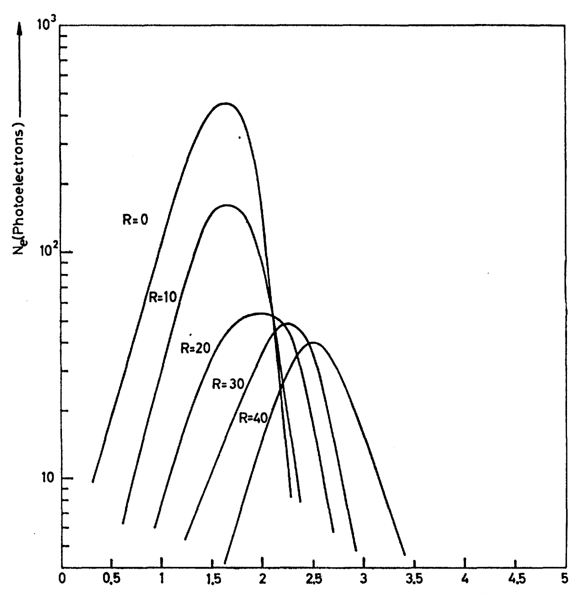

Fig. 4. Number of photoelectrons N_e from the cathode of RCA C70045A against the time t elapsed from the arrival of the EAS front, at different distances R(m) from the axis, for showers of primary energy $E_0 = 10^{13}$ eV.

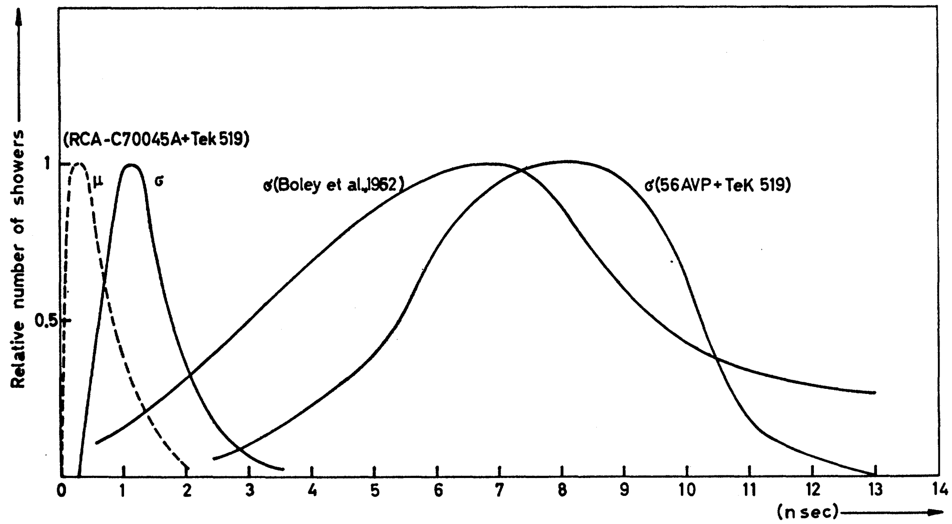

FIG. 5. σ and μ distributions obtained with different experimental apparatus. (The curves have been normalized to give equal maxima.)

were able to estimate that the energy of detected showers is $E_0 \gtrsim 10^{13}$ eV.[9]

RESULTS AND DISCUSSION

We measured the duration time (σ) of the pulse at half-maximum ($\frac{1}{2}V_{\max}$) and the risetime (μ) between $\frac{1}{2}V_{\max}$ and $V_{\max}$.

The results of these measurements without any correction for instrumental effects are reported in Fig. 3.

These instrumental effects are due to (1) risetime of photomultiplier and oscilloscope, (2) dispersion caused by connecting signal cables, and (3) nonlinearity of the oscilloscope sweep.

These effects were estimated using the measurements on cosmic-ray muons discussed in the previous section. These pulses have a calculated duration time of $\simeq 5\times10^{-2}$ nsec. Out of 1300 pulses due to cosmic-ray muons, we obtained the mean values $\langle\mu\rangle = 0.3$ nsec and $\langle\sigma\rangle = 0.6$ nsec which display our instrumental effects. The μ and σ mean values of the EAS corrected for these last values are, respectively, $\langle\mu\rangle_{\text{expt}} = 0.6$ nsec, and $\langle\sigma\rangle_{\text{expt}} = 1.4$ nsec.

To discuss the experimental information so obtained, we used the results of a calculation of Castagnoli *et al.*[10] on the development in Čerenkov light of an electrophotonic cascade generated by a primary photon of energy 10^{12}, 10^{13}, and 10^{14} eV. In fact the Čerenkov production in the atmosphere is known to be mainly due to the electronic component of the shower. The calculations were carried out with a Monte Carlo method in the B approximation, that is, taking into account multiple scattering, pair production, and energy loss by ionization and radiation besides Čerenkov emission. For this last process the Tamm-Frank theory was used with the air refraction index given by the relation $\eta = \eta_0 \exp(-h/h_0)$ ($\eta_0 = 2.9\times10^{-4}$, and $h_0 = 7.1$ km, standard atmosphere height). Thus the radial and temporal development of the Čerenkov light disk were calculated at several altitudes above sea level.

Figure 4 reports the results computed for an EAS of primary energy $E_0 = 10^{13}$ eV produced at $h = 15$ km and observed at 3500 m above sea level. The abscissa reports the time t(nsec) elapsed from the arrival of the EAS front and the ordinate the number of photoelectrons observed by our phototube; the curves are for different values (in meters) of the distance R from the shower axis. Averaging over R up to 30 m, we obtained

$$\langle\mu\rangle_{\text{theor}} = 0.5 \text{ nsec},$$

in good agreement with our experimental results.

We have to point out some simplifications, namely, (1) We assume a purely electrophotonic cascade, and do not consider the nuclear cascade; (2) We did not average over h and E_0. Simple evaluations show, however, that the results are not sensibly dependent on these points.

From Fig. 5 we see that previous measurements made at the same height (but perhaps with EAS of energy somewhat higher) by Boley *et al.*[5] with an EMI photomultiplier give a remarkably higher value of σ.

We tested how the risetime of the experimental arrangement screens the physical event by carrying out some measurements at sea level with a Philips 56 AVP photomultiplier whose characteristics are similar to the ones of the EMI tube used by Boley. Figure 5 compares the various experimental results.

---

[9] L. Bonaudo, G. Bosia, and M. Dardo, Cosmic Rays Group of Turin University, Internal Report No. 39, 1967 (unpublished).

[10] C. Castagnoli, M. Locci, P. Picchi, and G. Verri, Nucl. Phys. (to be published).