Pair Production by Photons on Electrons*

K. J. Mork†
National Bureau of Standards, Washington, D. C.
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The cross section for production of electron pairs by unpolarized photons in the field of an electron is calculated. The total cross section is obtained by numerical integration. The contributions to the total cross section from different diagrams (exchange and γ-e terms) are calculated separately, and it is shown that Borsellino's result for the cross section is valid for photon energies above about 8 MeV, the deviation from the correct cross section being of order 1% at this energy. Votruba's results for the total cross section are confirmed for energies very close to threshold (within about 10 keV of threshold). The results of Kopylov *et al.* appear to be in error by a factor of 2. Dividing their results by this factor, we find agreement with ours within limits of error. The recoil momentum distribution is computed numerically for some energies and it is found to agree with the results of Borsellino. At high energies it is also in agreement with the results of Suh and Bethe. The momentum distribution for pair production in the field of a very heavy particle is also calculated for some energies from the formulas of Borsellino and Jost *et al.* From a comparison of the recoil distributions for pair production on electrons with that on very heavy particles, an estimate of the error in the Wheeler-Lamb results can be made. Comparison with some experiments is given.

I. INTRODUCTION

THE differential cross section for triplet production was first obtained by Votruba[1] in lowest-order perturbation theory. Votruba also made an approximate calculation of the total cross section at high energies. Borsellino[2] neglected exchange and γ-e interactions (see Sec. II) and obtained a cross section which, compared with Votruba's results, is very much simpler. Borsellino and Ghizzetti[3] also obtained analytic expressions for the momentum distribution of the recoiling electron and for the total cross section. Earlier Wheeler and Lamb[4] had used the Weizsäcker-Williams method to calculate the total cross section at very high photon energies. They also included effects of atomic binding and screening. Essentially they integrated the incoherent scattering function multiplied by the momentum distribution derived from the Bethe-Heitler formula for pair production in the field of a very heavy particle. Lately Kopylov *et al.*[5] calculated the total cross section for energies below 30 MeV using numerical computation of matrix elements, and numerical integration over final states. References to other works on triplet production can be found in the review paper by Joseph and Rohrlich.[6]

The cross sections derived by Votruba and by Borsellino differed both at high and low energies, and it was assumed that this is due to an effect of the terms neglected by Borsellino. While this is correct for photon energies close to threshold ($4\,mc^2$ in the lab system), it is incorrect at high energies as was pointed out by Suh and Bethe.[7] These authors showed that if terms of order $1/\omega$ are neglected, where ω is the energy of the incoming photon in the lab system, integration of the formulas of Votruba and of Borsellino gives the same results for the total cross section, and further that these results are identical with those for pair production in the field of a very heavy particle. They also showed that the recoil momentum distribution derived by Borsellino becomes identical to the one derived from the Bethe-Heithler formula[8] at high energies, and this justifies the Wheeler-Lamb calculations.

Kopylov *et al.*[5] calculated the total cross section by the Monte Carlo method. They chose samples of random variables for the spins and the final-state momenta and angles. They have, however, lost a factor $\frac{1}{2}$. When their result is corrected by this factor, it agrees with Votrubas formula at threshold, but above 5 MeV the accuracy becomes poor (uncertainty larger than 10%).

In Sec. II of this paper we calculate the triplet cross section summed over spin and polarization, and we integrate it by the Monte Carlo method. The contributions from the different diagrams and their interferences are computed separately.

Section III contains calculations of recoil momentum distributions for some photon energies, and in Sec. IV we compare with some experiments.

We use units in which $\hbar=1$, $c=1$, $m=1$.

II. TOTAL CROSS SECTION FOR TRIPLET PRODUCTION

A. Differential Cross Section

The 8 diagrams for pair production in the field of an electron are shown in Fig. 1. The incoming photon and electron have four-momenta k and p, and the outgoing

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† Present address: Institute of Theoretical Physics, Norwegian Institute of Technology, Trondheim, Norway.

[1] V. Votruba, Bull. Intern. Acad. Tcheque Sci. 49, 19 (1948).
[2] A. Borsellino, Nuovo Cimento, 4, 112 (1947); Rev. Univ. Nacl. Tucuman A6, 7 (1947).
[3] A. G. Ghizzetti, Rev. Univ. Nacl. Tucuman A6, 37 (1947).
[4] J. A. Wheeler and W. E. Lamb, Phys. Rev. 55, 858 (1939); *ibid.* 101, 1836 (1956).
[5] G. I. Kopylov, L. A. Kulyukina and I. V. Polubarinov, Zh. Experim. i Teor. Fiz. 46, 1715 (1964) [English transl.: Soviet Phys—JETP 19, 1158 (1964)].
[6] J. Joseph and F. Rohrlich, Rev. Mod. Phys. 30, 354 (1958); see also J. Reyntjens, Ann. Soc. Sci. Bruxelles 71, 189 (1957).
[7] K. S. Suh and H. A. Bethe, Phys. Rev. 115, 672 (1959).
[8] H. A. Bethe, Proc. Cambridge Phil. Soc. 30, 524 (1934).

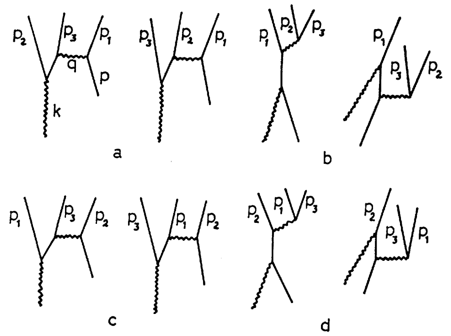

FIG. 1. Diagrams for triplet production.

electrons and positron have momenta p_1, p_2, and p_3, respectively. The quantity $q=p-p_1$ is called the four-momentum transfer. Energies and momenta of the electrons are ϵ, ϵ_1, ϵ_2 and $\mathbf{p}$, $\mathbf{p}_1$, $\mathbf{p}_2$, respectively, and energies and momenta of the photon and positron are ω, ϵ_3 and $\mathbf{k}$, $\mathbf{p}_3$, respectively. We shall call the diagrams a Borsellino diagrams since they correspond to the Borsellino cross section. The diagrams *b* will be referred to as γ-e diagrams, and diagrams *c* and *d* as Borsellino exchange and γ-e exchange diagrams, respectively. Evaluation of the covariant differential cross section for unpolarized particles using the technique of covariant polarization sums has been done in part in the book of Jauch and Rohrlich.[9] We found that the simplest way to obtain the differential cross section in a convenient form is to follow Jauch and Rohrlich and to complete the evaluation of the traces left there, making full use of the symmetry properties of the cross section. The differential cross section can then be written in the form

$$d\sigma=\frac{\alpha r_0^2}{|p\cdot k|}\frac{d^3p_1d^3p_2d^3p_3}{4\pi^2\epsilon_1\epsilon_2\epsilon_3}\delta^4(p+k-p_1-p_2-p_3)X\,, \quad (1.1)$$

where $\alpha=1/137$, r_0 is the classical radius of the electron, and X can be split in 6 parts,

$$X=X_B+X_\gamma+X_{B\gamma}+X_{BE}+X_{\gamma E}+X_{B\gamma E}. \quad (1.2)$$

Here X_B comes from the squares of the diagrams *a* and *c*, and $X\gamma$ from the squares of diagrams *b* and *d*. The interference terms $ab+cd$, ac, bd, and $ad+bc$ are represented by $X_{B\gamma}$, X_{BE}, $X_{\gamma E}$ and $X_{B\gamma E}$, respectively. All these quantities can be expressed as a function of invariant products between four-vectors of the particles. The energy-momentum conservation laws $p+k=p_1+p_2+p_3$ can be used to eliminate all products except four independent ones (five if $p\cdot k$ is included). The expression for X is very long, and is not given here. It can be found in Ref. 10.

[9] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Cambridge, 1955) p. 248.

[10] K. Mork, in Proceedings of The Physics Seminar in Trondheim nr.7, 1965 (unpublished).

B. Integrations

The integration of the cross section in Eq. (1.1) over final states is performed in the center-of-mass system since the kinematics are simplest there. The integration over d^3p is trivial using $\delta^3(\mathbf{k}+\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)$. Choosing $\mathbf{p}_1$ as polar axis we use the energy conserving δ function $\delta(\epsilon+\omega-\epsilon_1-\epsilon_2-[(\mathbf{p}_1+\mathbf{p}_2)^2+1]^{1/2})$ to integrate over the angle θ_{12} between $\mathbf{p}_1$ and $\mathbf{p}_2$, and we keep the azimuthal angle φ_2 (cf. Fig. 2a) as a variable. Since φ_2 does not enter into the energy-momentum conservation, it can be integrated from 0 to 2π. Next we integrate over the angle θ_1 between $\mathbf{p}_1$ and $\mathbf{k}$, which also can be integrated over its full physical region 0 to π. The integration over φ_1 (cf. Fig. 2b) gives just a factor 2π, and as final variables we choose ϵ_1 and ϵ_2. The range of integration of ϵ_2 is determined by the condition

$$|\cos\theta_{12}|=\left|\frac{(\epsilon+\omega)(\epsilon-\epsilon_1-\epsilon_2)+\epsilon_1\epsilon_2}{|\mathbf{p}_1||\mathbf{p}_2|}\right|\leq 1\,, \quad (1.3)$$

and ϵ_1 is limited by the condition that ϵ_2 should be real. Since we integrate over two identical particles in the final state we obtain identical final states twice, and the cross section should therefore be multiplied by $\frac{1}{2}$. Thus we obtain the total cross section as

$$\sigma_T=\frac{1}{2}\frac{\alpha r_0^2}{\omega(\epsilon+\omega)}\frac{1}{2\pi}\int_1^{\epsilon_1+}d\epsilon_1\int_{\epsilon_2-}^{\epsilon_2+}d\epsilon_2 \times\int_{-1}^{+1}d(\cos\theta_1)\int_0^{2\pi}d\varphi_2\,X\,, \quad (1.4)$$

where

$$\epsilon_2{}^{\pm}=\frac{ab\pm|\mathbf{p}_1|(a^2+b^2-|\mathbf{p}_1|^2)^{1/2}}{a^2-|\mathbf{p}_1|^2}\,, \quad (1.5)$$

$$a=\epsilon+\omega-\epsilon_1\,,$$
$$b=(\epsilon+\omega)(\epsilon-\epsilon_1)\,,$$
$$\epsilon_1{}^{+}=2\omega-\epsilon\,,$$

and $|\mathbf{k}|=\omega=|\mathbf{p}|=(\epsilon^2-1)^{1/2}$. The integrations are now performed by the Monte Carlo method. That is, we choose samples of random values of the variables and take the average over a large number of samples. Also the rms deviation is computed in order to get an estimate of the accuracy of the method. With the IBM 7094 at the National Bureau of Standards a run with 10 000 samples took about 8 min, and the computed relative

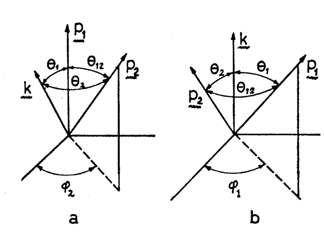

FIG. 2. Relative directions in triplet production.

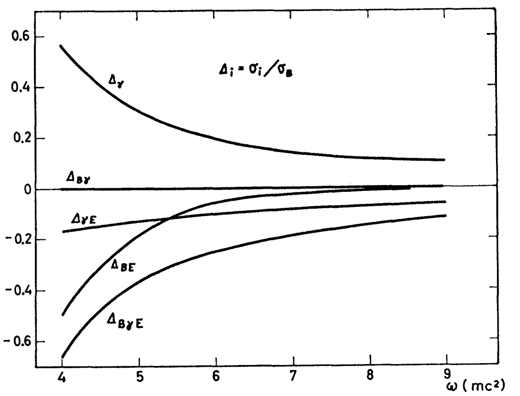

FIG. 3. The contributions Δi to the triplet total cross section from exchange and γ-e terms relative to the Borsellino cross section.

error was below 1% for energies close to threshold ($\omega = 4/3$ in the c.m. system, and $\omega = 4$ in the lab system).

C. Results

The contributions σ_B, $\sigma_\gamma \cdots$ to the total cross section from the different parts X_B, $X_\gamma \cdots$ (cf. Eq. (1.2)) of X were computed separately. Since the Borsellino terms are known exactly[2,3], we have plotted the quantities $\Delta_\gamma = \sigma_\gamma/\sigma_B$, $\Delta_{B\gamma} = \sigma_{B\gamma}/\sigma_B$, $\cdots$ in Fig. 3 as a function of the photon lab energy. Summing the contributions from exchange and γ-e terms, we obtain the triplet cross section as the Borsellino cross section times a correction factor

$$\sigma_T = \sigma_B(1-\Delta), \tag{1.6}$$

where Δ is given in Fig. 4 as a function of the photon lab energy. The quantities σ_T and σ_B are tabulated in Table I. The relative error in the integrations increases rapidly with increasing photon energy, but fortunately $\Delta\sigma_B$ decreases with surprising rapidity, and is zero within an accuracy of 2% relative to the Borsellino cross section at $\omega = 15$. The reason for this rapid decrease is seen to be (cf. Fig. 3) the cancellations between exchange and γ-e terms, as was expected by Suh and Bethe.[7] Since there is no reason to believe that the importance of exchange and γ-e terms will increase with increasing energy, we conclude that Borsellino's results for the triplet cross section can be used for photon energies above 7 MeV. Our results agree with the ones obtained by Kopylov *et al.*[5] within limits of error, if their results are corrected by a factor $\frac{1}{2}$.

At photon energies close to threshold we get agreement with results obtained previously. The lowest lab energy we used was $\omega = 4.00225$ for which Votruba's formula[11]

$$\sigma^{(V)}/\alpha r_0^2 = \frac{\pi\sqrt{3}}{4\times 3^5}(\omega-4)^2 \tag{1.7}$$

gives $\sigma^{(V)}/\alpha r_0^2 = 2.834\times 10^{-8}$, and Borsellino's formula

$$\sigma^{(B)}/\alpha r_0^2 = \frac{\pi\sqrt{3}}{2^3\times 3^3}(\omega-4)^2 \tag{1.8}$$

gives $\sigma^{(B)}/\alpha r_0^2 = 12.76\times 10^{-8}$. We find $\sigma_T/\alpha r_0^2 = \sigma^{(V)}/\alpha r_0^2 = 2.83\times 10^{-8}$, and $\sigma_T^{(B)}/\alpha r_0^2 = 12.85\times 10^{-8}$ with an uncertainty of 1%. Votrubas threshold approximation holds only very close to threshold. For $\omega = 4.211$ Votruba's formula gives $\sigma^{(V)}/\alpha r_0^2 = 2.49\times 10^{-4}$, but we find $\sigma_T/\alpha r_0^2 = 5.04\times 10^{-4}$. The same conclusion was also reached by Kopylov *et al.*[5]

Close to threshold the quantity X in Eq. (1.4) becomes a constant and all energy and angular dependence of the cross section is contained in the final-state factors. As a check of our differential cross section we computed σ_T analytically in the threshold limit, finding that in order to obtain the correct limit of Votruba X must be $\frac{2}{3}$. Numerical calculation of X gave this number to four figures which was the limit put on the computer.

Our calculation of the Borsellino terms confirmed the results[12] of Borsellino and Ghizzetti.[2,3] For photon energies above $\omega = 15$ the method described in Sec. B gave rapidly increasing uncertainties. This is due mainly to the increasing importance of triplet production at small angles and with small momentum transfer. In order that the square of the momentum transfer

$$q^2 = \epsilon\epsilon_1 + |\mathbf{p}||\mathbf{p}_1|\cos\theta_1 - 1 \tag{1.9}$$

shall be small, $\cos\theta_1$ has to be close to -1 and ϵ_1 has to be close to its maximum value $2\omega - \epsilon$. Changing from $\cos\theta_1$ and ϵ_1 to the variables $u = (\epsilon\epsilon_1 - |\mathbf{p}||\mathbf{p}_1| - 1)/(\epsilon\epsilon_1$

TABLE I. The Borsellino cross section σ_B, and the triplet total cross section σ_T in mb (no screening or binding) as functions of the photon lab energy ω in mc^2. The relative uncertainty in σ_T increases from about 1% at $\omega = 5$ to about 2% at $\omega = 14$. For $\omega \gtrsim 16$ σ_T is equal to σ_B. For high energies ($\omega \gtrsim 100$) σ_B is given by[a] $\sigma_B = 3.111 \ln 2\omega - 8.074 - (1.333 \ln^3 2\omega - 3 \ln^2 2\omega + 6.84 \ln 2\omega - 21.51)\omega^{-1}$ if terms of order ω^{-2} are neglected.

ω	σ_B	σ_T	ω	σ_B
4.0	0.00	0.00	40	2.35
4.4	0.0023	0.0011	45	2.57
5.2	0.022	0.0148	50	2.77
6.0	0.0592	0.0462	60	3.12
7.2	0.136	0.118	70	3.43
8.0	0.196	0.176	80	3.69
10	0.364	0.340	90	3.93
12	0.539	0.518	100	4.14
14	0.711	0.699	150	4.97
16	0.878	0.878	200	5.56
18	1.035		300	6.38
20	1.187		500	7.40
25	1.536		1000	8.76
30	1.840		5000	11.84
35	2.11		10000	13.12

[a] References 2 and 3.

[11] The paper of Joseph and Rohrlich [Ref. 6] has a misprint in Eq. (2).

[12] It seems that Borsellino has an error in his high-energy formula (56) of Ref. 2. The term 21.51 should have a minus sign as can be seen from the expansions given by Ghizzetti [Ref. 3]. This error is repeated several places in the literature.

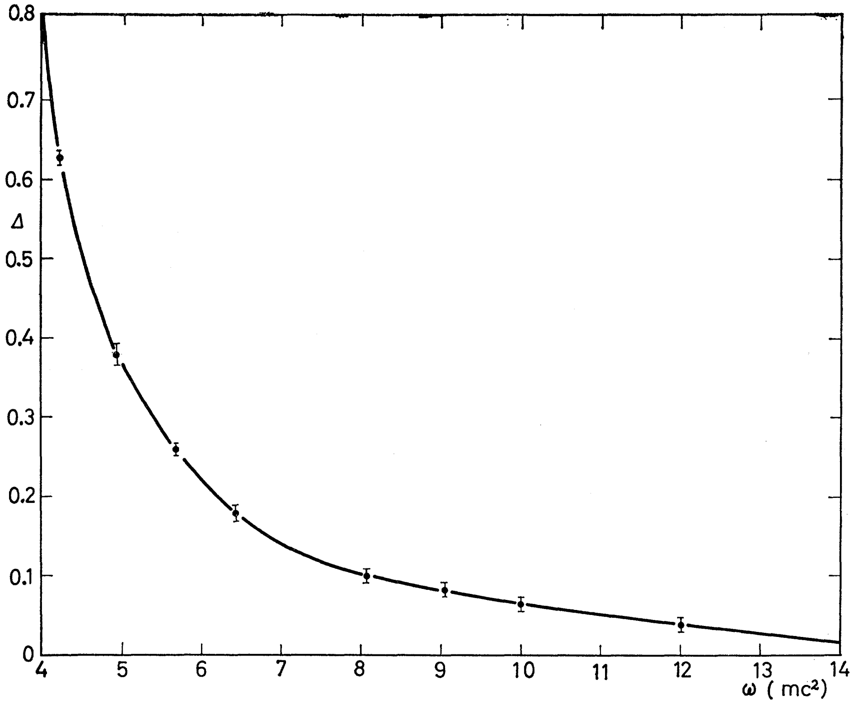

FIG. 4. The quantity Δ by which the triplet cross section σ_T can be obtained from the Borsellino cross section σ_B. $\sigma_T=\sigma_B(1-\Delta)$.

$+|\mathbf{p}||\mathbf{p}_1|\cos\theta_1-1)$ and $v=2(\epsilon-\omega)/(\epsilon-\epsilon_1)$, and taking random values for u and v within their range of variation, we find that the accuracy of the numerical integration is greatly improved. We are thus able to confirm Borsellino's results for $\omega=30$, 50, 100, 200, and 500 (in the lab system) with an accuracy of 3, 3.5, 6, 6.5, and 8%, respectively.

III. RECOIL MOMENTUM DISTRIBUTIONS

The momentum distribution of the recoiling electron has been derived by Borsellino.[2] This distribution is valid for the triplet process if exchange and γ-e terms can be neglected. These terms are of importance only for low photon energies, or for large momentum transfers such that the recoiling electron can not be distinguished from the one that is produced. For these cases the concept of momentum transfer is not very useful. Since it appears from our results that exchange and γ-e terms do not contribute to the total cross section for photon energies above 8 MeV, we conclude that Borsellino's distribution is valid for energies above this value if the energy of the recoiling electron is well below the energy of the electron produced. Suh and Bethe[7] have shown that Borsellino's distribution is identical to the one derived from the Bethe-Heitler formula if terms of order $1/\omega$ are neglected.

We have computed the momentum distribution as a function of the momentum $|\mathbf{q}|$ of the recoiling electron in the lab system, for photon energies $\omega=20$, 50, 100, and 1000, calculating numerically the integrals left in Borsellino's[2] formula. As a check we have also computed the same distributions by integration of the cross section derived by us in Eq. (1.1), keeping only the Borsellino term X_B of X. Since the momentum $|\mathbf{q}|$ is determined by the square of the four-momentum transfer in the c.m. system, $q^2=2[(1+|\mathbf{q}_{\text{lab}}|^2)^{1/2}-1]$, this integration may be performed conveniently in the c.m. system. We obtain an integral which is similar to the one in Eq. (1.4). We have only to express $\cos\theta_1$ by q^2 using Eq. (1.9), integrate φ_2 and ϵ_2 as in Eq. (1.4), and replace the lower limit of integration of ϵ_1 in this formula by $\epsilon(1+q^2/4)-\omega[q^2(1+q^2/4)]^{1/2}$. The results[12a] are shown in Fig. 5, where the recoil momentum distribution

$$\Gamma(\omega,|\mathbf{q}|)=(1/\alpha r_0^2)d\sigma_B/d|\mathbf{q}| \tag{2.1}$$

has been plotted as a function of $|\mathbf{q}|$ for energies $\omega=1000$, 100, 50 and 20 (lab system). The uncertainty in the numerical integrations was never more than a few percent. In order to show how $\Gamma(\omega,|\mathbf{q}|)$ depends on ω we have also plotted $\Gamma(\omega,|\mathbf{q}|)$ as a function of ω for the values $|\mathbf{q}|=0.2$, 1.0, 2.0, 5.0 and 10.0 in Fig. 6.[12a] These results agree with the ones derived by Suh and Bethe[7] for $\omega=1000$ and 100.

We have also computed the recoil momentum distribution for production of pairs on an infinitely heavy particle $\Gamma_\infty(\omega,|\mathbf{q}|)$ using the formula of Borsellino.[2] As a check we have also calculated $\Gamma_\infty(\omega,|\mathbf{q}|)$ from the

[12a] These results in tabular form have been deposited as Document number 9467 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D. C. 20540. A copy may be secured by citing the Document number and by remitting \$1.25 for photoprints, or \$1.25 for 35-mm microfilm. Advance payment is required. Make checks or money orders payable to: Chief, Photoduplication Service, Library of Congress.

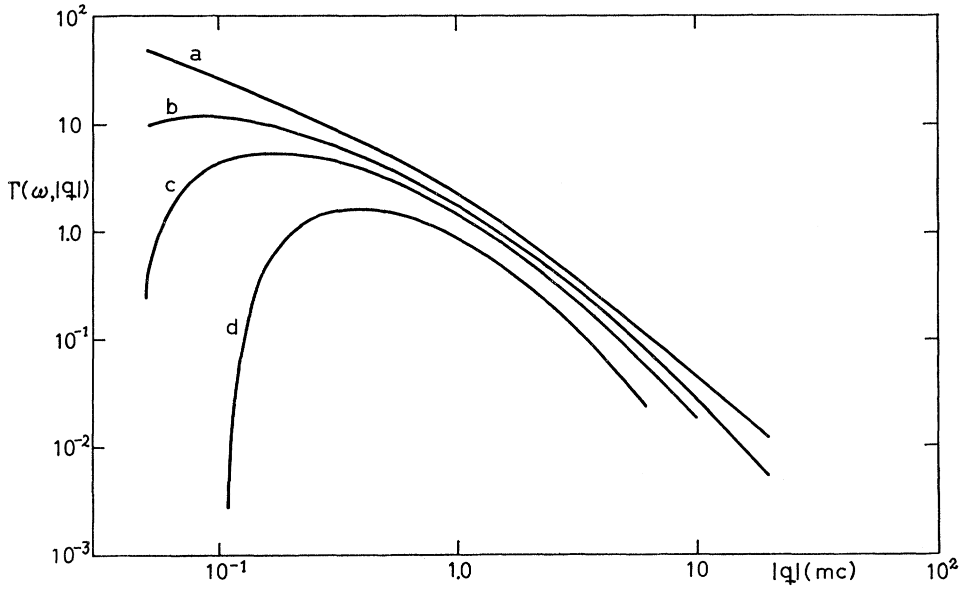

FIG. 5. Triplet recoil momentum distributions derived from the Borsellino diagrams as a function of the momentum $|\mathbf{q}|$. The curves a, b, c, and d correspond to lab photon energies $\omega=1000$, 100, 50, and 20 mc^2, respectively.

formula of Jost *et al.*[13] Both formulas contain integrals which we performed numerically. In order to compare the triplet distribution $\Gamma(\omega,|\mathbf{q}|)$ with $\Gamma_\infty(\omega,|\mathbf{q}|)$ we have plotted the ratio $R=\Gamma(\omega,|\mathbf{q}|)/\Gamma_\infty(\omega,|\mathbf{q}|)$ as a function of $|\mathbf{q}|$ for $\omega=1000$, 100, 50 and 20 in Fig. 7. The ratio R is close to 1 for $\omega=1000$ and small $|\mathbf{q}|$, but it is considerably smaller than 1 for lower energies if $|\mathbf{q}|$ is smaller than 1. The ratio approaches 0 when $|\mathbf{q}|$ approaches its lower limit since the minimum momentum transfer for the triplet process is higher than the minimum for pair production on a very heavy particle.

IV. RELATIONS TO EXPERIMENTS

The triplet process has been observed in emulsions, cloud chambers and bubble chambers. In these experiments it is possible to identify all three triplet particles and to measure their momenta, angular distributions, etc. However, if the momentum of the recoiling electron is very small, it is difficult to observe this particle, and triplet production can not be separated from pair production in the field of the nuclei.

A more accurate type of experiment[14–29] is the photon absorption measurement in which the attenuation of a beam of photons is measured. The triplet total cross section can be obtained from the attenuation coefficient since the cross section for all other processes which contribute to the attenuation are known. The accuracy to which these cross sections are known depends, however, on the photon energy and on the atomic number of the target.

The formula, Eq. (1.6), for the triplet total cross section is valid if effects of screening and binding can be neglected. Exception must also be taken for energies very close to threshold for which the triplet particles are nonrelativistic and for which therefore the Born approximation is not valid. The effects of exchange and γ-e terms decrease rapidly with the photon energy, and may only be observed for energies between threshold and about 8 MeV. Because of uncertainties of order $(\alpha Z)^2 \ln\omega/\omega$ in the Coulomb-corrected pair cross section an experiment would have to be performed with low-Z materials. According to Wheeler and Lamb[4] the relative correction to the triplet cross section from screening and binding is smaller than the screening correction to pair production. It should therefore be correct to use the Borsellino formula[2] at least for energies and Z values for which screening can be neglected in the Bethe-Heitler formula. However, it may be incorrect for some cases since small values of the momentum transfer are relatively more important

[13] R. Jost, J. M. Luttinger and M. Slotnick, Phys. Rev. **80**, 189 (1950).
[14] J. A. Phillips and P. G. Kruger, Phys. Rev. **76**, 1471 (1949).
[15] J. M. Wyckoff and H. W. Koch, Phys. Rev. **117**, 1261 (1960); J. M. Wyckoff, B. Ziegler, H. W. Koch and R. Uhlig, *ibid.* **137**, B576 (1965).
[16] E. R. Gaerttner and M. L. Yeater, Phys. Rev. **78**, 621 (1950).
[17] R. I. Walker, Phys. Rev. **76**, 527 (1949).
[18] A. I. Berman, Phys. Rev. **90**, 210 (1953).
[19] E. L. Hart, G. Cocconi, V. T. Cocconi, and J. M. Sellen, Phys. Rev. **115**, 678 (1959).
[20] J. Moffat, J. J. Thresher, G. C. Weeks, and R. Wilson, Proc. Roy. Soc. (London) **244A**, 245 (1958); J. Moffat and G. C. Weeks, *ibid.* **73**, 114 (1959).
[21] S. D. Warshaw and S. Courtenay Wright, Nuovo Cimento **31**, 756 (1964).
[22] J. L. Lawson, Phys. Rev. **75**, 433 (1949).
[23] J. W. DeWire, A. Ashkin, and L. A. Beach, Phys. Rev. **82**, 447 (1951); **83**, 505 (1951).
[24] C. R. Emigh, Phys. Rev. **86**, 1028 (1952).
[25] J. D. Anderson, R. W. Kenney, and C. A. McDonald, Phys. Rev. **102**, 1626 (1956); J. D. Anderson, R. W. Kenney, C. A. McDonald, and R. F. Post, *ibid.* **102**, 1632 (1956).
[26] D. H. Cooper, thesis, California Institute of Technology, 1955 (unpublished).
[27] E. Malamud, Phys. Rev. **115**, 687 (1959).
[28] J. K. Walker, M. Wong, R. Fessel, R. Little, and H. Winick, Phys. Rev. **144**, 1126 (1966).
[29] M. Fidecaro, G. Finocchiaro, and G. Giacomelli, Nuovo Cimento **23**, 800 (1962).

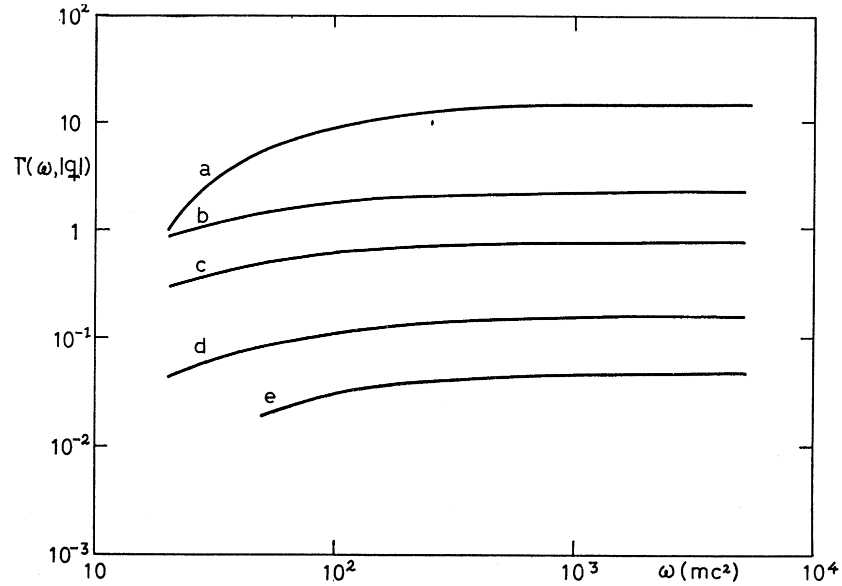

FIG. 6. Triplet recoil momentum distributions derived from the Borsellino diagrams as a function of the lab photon energy ω. The curves a, b, c, d, and e refer to momenta $|\mathbf{q}|=0.2$, 1.0, 2.0, 5.0, and 10 mc, respectively.

for the Borsellino cross section than for the Bethe-Heitler cross section as can be seen from Fig. 7.

For very high energies (ω of order BeV) our calculation confirms the conclusion of Suh and Bethe[7] that the results of Wheeler and Lamb[4] are valid. It should be noted that Wheeler and Lamb used an incoherent scattering function which (except for hydrogen) was based on the Fermi-Thomas model of the atom. This model works well for high-Z elements. However, triplet production is largest relative to pair production for low-Z elements. For absorption experiments with accuracy of order 1% or better, more accurate incoherent scattering functions are needed. For a review of calculations of such functions see Grodstein.[30]

For photon energies in the region from a few hundred MeV to a few BeV one can (as was suggested by Bethe[31]) obtain the triplet cross section by subtracting from the Wheeler-Lamb cross section Z times the difference between the unscreened Bethe-Heitler cross section (for $Z=1$) and the Borsellino cross section. As is shown in Fig. 7, curve a, the momentum distributions derived from the Bethe-Heitler and the Borsellino cross sections are approximately equal for small $|\mathbf{q}|$, and it is only in this region that binding and screening are important. Therefore the difference between the Bethe-Heitler and the Borsellino cross sections comes only from regions of $|\mathbf{q}|$ of order one. Correct cross sections can of course be obtained directly by integration of the Borsellino momentum distribution multiplied by the correct incoherent scattering function. For energies below a few hundred MeV and down to energies for which screening and binding are not important this procedure is necessary in order to get a correct triplet cross section. As is shown in Fig. 7 the difference between the Borsellino and the Bethe-Heitler momentum distributions is considerable for all values of $|\mathbf{q}|$ if ω is 50 MeV or smaller.

It should be noted that Coulomb corrections are not of importance for triplet production since the electron has only unit charge. The calculated cross section is therefore correct to order $\alpha=1/137$. For very high energies we can also include the radiative corrections (to order α). For these energies the triplet process is similar to the pair-production process, and we therefore assume that the radiative corrections to the triplet total cross section are equal to the radiative corrections to pair production. These corrections have been

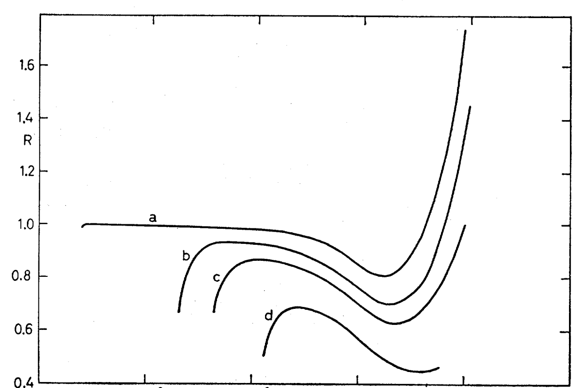

FIG. 7. The ratio R between the recoil distributions for pair production by electrons and by a very heavy nucleus. The curves a, b, c, and d refer to lab photon energies $\omega=1000$, 100, 50, and 20 mc^2, respectively.

[30] G. White Grodstein, Natl. Bur. Std. (U. S.) Circ. 583 (United States Government Printing Office, Washington, D. C., 1957).

[31] H. A. Bethe and J. Ashkin, in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, 1953). Vol. I, Part II, p. 263.

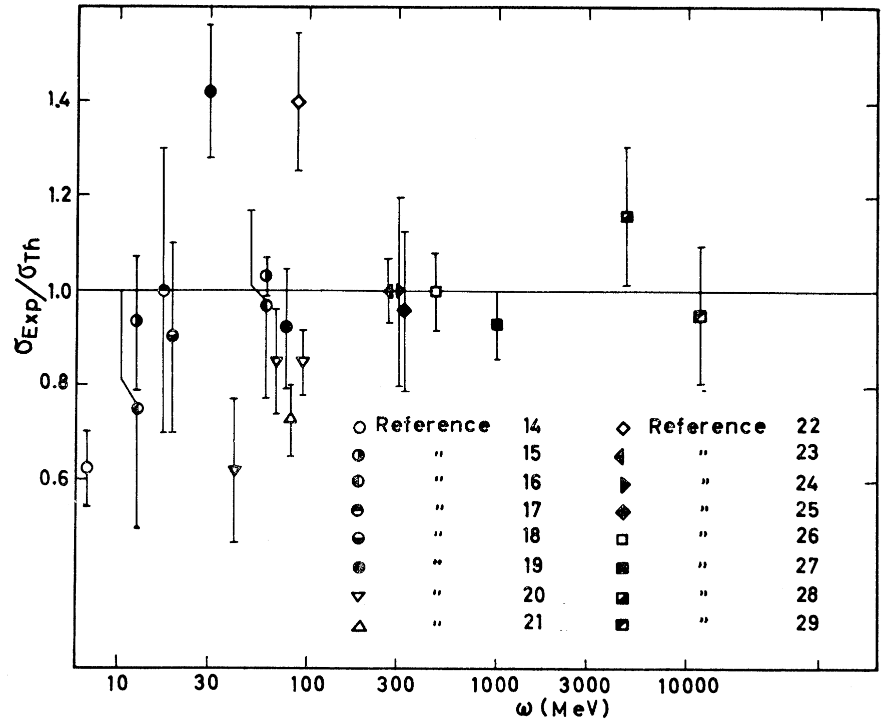

FIG. 8. The ratio of the experimental to the theoretical total cross section for triplet production as a function of the photon energy ω.

calculated by Mork and Olsen.[32] In the limit of complete screening (very high energies) the corrections are only 0.93%, and do not become more than a couple of tenths of a percent larger at lower energies. Although the cross section for triplet differs from that of pair production at medium energies it should still be a good approximation to use the same radiative corrections for these processes.

A comparison between experiments and theory is shown in Fig. 8. The ratios between the experimental and the theoretical total cross sections have been plotted as a function of the photon lab energy. Most of these experiments are photon absorption measurements. The experimental cross section has been obtained by subtracting from the observed absorption cross section the cross section for pair production including radiative corrections[32] and Coulomb corrections and screening,[33] the Compton cross section including radiative corrections and double Compton effect,[34] and the cross section for photonuclear effects[15] and for meson production.[27] The results of one bubble chamber experiment[21] and three cloud chamber experiments[14,16,19] are also shown. Mohanty, Webb, Sandhu and Roy[35] have made experiments on triplet production in the 5–90 MeV energy range using emulsions. They have, however, compared their results with a wrong theory (Wheeler-Lamb including "exchange correction").

The momentum distribution of the recoil electron has been measured by Hart *et al.*[19] for recoil momenta above 0.15 *mc*. The results agree well with theory. Mohanty *et al.*[35] measured the recoil momentum distribution between 0.1 and 0.8 *mc*. The shape of the distribution agrees fairly well with theory, but the distribution seems to fall off more rapidly than the theoretical one for momenta above 0.5 *mc*. The accuracy, however, is too poor to allow a definite conclusion.

ACKNOWLEDGMENT

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[32] K. Mork and H. Olsen, Phys. Rev. **140**, B1661 (1965).
[33] A. Sørenssen, Nuovo Cimento **38**, 745 (1965).
[34] K. Mork (to be published).
[35] R. C. Mohanty, E. H. Webb, H. S. Sandhu, and R. R. Roy, Phys. Rev. **124**, 202 (1961); H. S. Sandhu, E. H. Webb, R. C. Mohanty and R. R. Roy, Phys. Rev. **125**, 1017 (1962).