Nuclear Levels of Dv^{162} [†]

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The nuclear level scheme of Dy^{162} has been studied utilizing a variety of experimental methods. From the reaction Dy¹⁶¹ (n, γ) Dy¹⁶², low-energy γ -ray transitions were measured between 40 and \approx 1350 keV with a bent-crystal spectrometer, and the (n, e^-) conversion-electron spectrum was investigated in the energy range between 160 and 1300 keV with a double-focusing β spectrometer. Altogether, some 160 transitions were found, and multipolarities were assigned for about 50 of these. High-energy γ rays from neutron capture in Dy¹⁶¹ were measured in the energy region between 4.9 and 8.2 MeV with a Ge(Li) detector operated in the pair mode; 49 transitions were found. Furthermore, the reactions $Dy^{162}(p, p')$, $Dy^{162}(d, d')$, $Dy^{161}(d, p)$, and $Dv^{163}(d,t)$ have been studied at 12-MeV beam energy, using a broad-range magnetic spectrograph. Combining the results of all these experiments, a level scheme containing 90% of the intensity of the low-energy (n, γ) spectrum of Dy¹⁶³ was constructed consisting of the following levels: the ground-state rotational band up to the 8⁺ level; the $K=2^+$ (γ -vibrational) band at 888.22 keV including all members up to the possible $7^{\frac{1}{4}}$ state; the $K=2^-$ octupole-vibrational or $[411\uparrow -523\uparrow]$ band with the band head at 1148.29 keV and including members up to the tentative 5 and 6 levels; the $K = 4+$ [5211+523], band at 1535.89 keV containing members up to the 6⁺ and possibly the 7⁺ level; a $K = 0^-$ band (probably octupole) with 1⁻ and 3⁻ members at 1275.4 and 1357.0 keV, and possibly the 5⁻ member at 1520 keV; a tentative $K=0^+$ band with 2⁺ and 4⁺ members at 1206.1 and 1390.3 keV, suggesting the 0⁺ level at 1127 keV; the $K = 1 + \frac{521}{1} - 523$] band at 1745 keV containing states up to the 6⁺ level; the $K=3$ ^{-[642}]+521] band at 1770 keV containing levels up to the 6⁻ member; and finally the $K=2$ ⁻ $[642\uparrow-521\downarrow]$ band at 1866 keV containing levels up to the 5⁻ state. Other unassigned levels were found above 1.4 MeV in the (d, p) , (d, t) , and high-energy (n, γ) measurements. The agreement between experimental and calculated cross sections for the (d,t) and (d,p) reactions was found to be good for the $[521]\pm523\downarrow$ and the $[642]\pm521\downarrow$ states, but not so good for the $\lceil 642 \uparrow \pm 523 \rfloor$ states. The neutron binding energy in Dy¹⁶² was determined as 8192.8 \pm 3 keV.

I. INTRODUCTION

N comparison with the neighboring even dysprosium \blacktriangle isotopes,¹ relatively little information has been available for Dy^{162} . The ground-state rotational band

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 i *Nuclear Data Sheets*, compiled by K. Way et al. (Printing and ¹ Nuclear Data Sheets, compiled by K. Way et al. (Printing and Publishing Office, National Academy of Sciences—National Res earch Council, Washington 25, D. C., 1964), No. NRC 20418.

of Dy¹⁶² has been studied in Coulomb-excitation experiments,²⁻⁸ and several of its members have also been observed by Morinaga and Gugelot utilizing the $(\alpha, 2n\gamma)$

² T. Huus, J. H. Bjerregaard, and B. Elbek, Kgl. Danske
Videnskab. Selskab, Mat. Fys. Medd. 30, No. 17 (1956).

³ N. P. Heydenburg and G. F. Pieper, Phys. Rev. 107, 1297 (1957).

⁴ E. L. Chupp, J. W. M. DuMond, F. J. Gordon, R. C. Jopson, and H. Mark, Phys. Rev. 112, 518 (1958).

⁵ B. Elbek, M. C. Olesen, and O. Skilbreid, Nucl. Phys. 19, 523 (1960).
6 G. Goldring and Z. Vager, Nucl. Phys. 26, 250 (1961).

⁷ J. deBoer, G. Goldring, and H. Winkler, in Proceedings of the Third Conference on Reactions Between Complex Nuclei, edited by A. Ghorsov, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, California, 1963), p. 317.

^s R. Graetzer and W. M. Bernstein, Phys. Rev. 129, 1772 (1963).

160 ioii reaction.⁹ The collective properties of the lowest excited states have been confirmed through lifetime excited states have been confirmed through lifetine
measurements.^{10,11} The $I^{\pi}=2^+$ state of the γ vibration: measurements.^{10,11} The *I* ^π = 2⁺ state of the γ vibrationa
band has been identified by Yoshizawa *et al*.¹² Mihelici band has been identified by Yoshizawa *et al*.12 Mihelich
et al.,¹³ Jørgensen *et al*.,¹⁴ and Harmatz *et al*.¹⁵ have Iil
15 investigated the decay of Ho^{162} and established the I^{π} =5 $\lceil 642 \rceil$ +523 \lceil level at about 1489 keV. Martin and Harlan¹⁶ have seen a few γ transitions in Dy¹⁶² during the investigation of the Tb¹⁶² decay. This decay has also been studied by Schneider and Münzel.¹⁷ Recently, the decay of Tb^{162} has been carefully examined by Funke *et al.*,¹⁸ who found a strong β branch $(\log ft < 5.0)$ populating the 1148-keV level. Accordingly, this state was assigned as the $K, I^* = 2, 2$ [411 \uparrow - 523 \uparrow] two-quasiproton level. In addition, Schima¹⁹ and two-quasiproton level. In addition, Schima
Gujrathi et al.²⁰ have studied the decay of Tb¹⁶². σ (barn)

In spite of the high quality of the previous work on Dy¹⁶², few excited levels have been observed except for the ground-state band. For this reason, we have decided to study Dy¹⁶² through reaction spectroscopy. In the reaction $Dy^{161}(n, \gamma)Dy^{162}$ using thermal neutrons, the compound system resulting from s-wave neutron capture has spin and parity 2^+ or 3^+ . Therefore, all low-lying levels with spins up to about 6 are expected to be excited²¹ during the decay of this compound state. From a detailed examination of the low-energy γ -ray and conversion-electron spectrum following neutron capture in Dv^{161} it should therefore be possible to assign further levels in Dy¹⁶². In addition, the measurement of the high-energy (n,γ) spectrum will reveal primary transitions, which directly yield levels in Dy^{162} . Furthermore, excited states in Dy^{162} can be investigated through $(\rho, \rho'), (d,d'), (d,\rho)$, and (d,t) reactions.

The examination of low-energy (n, γ) radiation with the curved-crystal spectrometer at Risg permitted the construction of a level scheme $2²$ including the groundstate rotational band up to the $I^* = 8^+$ state, the γ -vibrational band, up to the 6+ level, states at 1148, 1210,

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¹⁵ B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev.
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¹⁸ F. J. Schima, Phys. Rev. **151**, 950 (1966).
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- Letters 17, A6 (1966).

²¹ O. W. B. Schult, B. P. Maier, U. Gruber, and R. Koch, Z.

Physik 185, 295 (1965).

²² O. W. B. Schult, U. Gruber, and B. P. Maier, in *Proceedings of*
- the International Conference on the Study of Nuclear Structure with Neutrons, Antwerp, 1065 (North-Holland Publishing Company, Amsterdam, 1966), p. 518.

TABLE I. Properties of the dysprosium isotopes in the targets. The abundance, the neutron-capture cross section, the percent of the total neutron absorption in the target, and the binding energy of the last neutron for the daughter isotope Dy^{A+1} are listed. The binding energies are from Ref. 39. For comments on the cross sections, see Ref. 31.

Isotope	Dv^{160}	Dv^{161}	Dv^{162}	Dv^{163}	$\mathrm{Dv^{164}}$
$\%$ abundance σ (barn) % absorption Neutron binding energy (keV) of Dv^{A+1}	0.59 $55 + 9*$ 0.06 6410	90.0 $617+40b$ 94.2 $8192.8 + 3$	7.75 $215 + 16b$ 2.83 $6270.2 + 3$	0.25 $7653.9 + 3$	0.59 $135 + 13b$ 2600 + 410 ^a 2.61 $5716.3 + 3$

^a J. J. Scoville, J. W. Rogers, and E. Fast, Nucl. Sci. Eng. (to be pub-
lished).
b See Ref. 34.

and 1297 keV which probably are K, $I^* = 2$, 2^- , 2 , 3^- , and 2, ⁴—levels and ^a state at ¹⁵³⁵ keV.

In the present work we combine all information which has been obtained during the investigation of Dy^{162} utilizing the following experimental methods: (1) conversion-electron spectroscopy at Studsvik using the $\mathrm{Dy^{161}}(n,e^-)\mathrm{Dy^{162}}$ process; (2) low-energy γ -ray spectroscopy at Risø from the neutron capture in Dy^{161} ; (3) the determination of the high-energy γ -ray transition spectrum from the same process at Los Alamos; and (4) charged-particle reaction spectroscopy at the Florida. State University, yielding excited states in Dy^{162} .

II. METHODS AND RESULTS

A. The Internal-Conversion Electron Spectrum

The internal-conversion electron spectrum was measured with the 50-cm radius double-focusing β spectromsured with the 50-cm radius double focusing β spectrometer at Studsvik.²³ A beam of thermal neutrons was extracted from the $R2$ reactor into the spectrometer, giving a neutron flux density at the source of $10⁸$ neutrons/cm' sec. A voltage gradient source arrangement^{23,24} of 20 strips, 1.5 mm wide, having a total area of approximately 12 cm', was employed. Four sets of source strips with different thicknesses ranging from 0.2 to 2 mg/cm' were prepared for the present measurements. The thinnest sources were made by electroplatments. The thinnest sources were made by electroplating dysprosium nitrate in acetone.²⁵ The thicker sources were prepared by mixing dysprosium oxide with diluted Zapon lacquer and spraying the suspension on the backing, which for both kinds of sources was 3-mg/cm' Al foil. The sources were made of enriched material²⁶ (see Table I). Owing to the comparatively high capture cross sections of other Dy isotopes, even small amounts of these may give rise to spurious lines in the spectra.

The spectrometer was adjusted to a resolution of 0.2% (full width at half-maximum, FWHM) and the magnetic field was measured and controlled with an accuracy of

²⁶ Obtained from Oak Ridge National Laboratory.

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¹³ J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. 108, 989 (1957).

²⁸ G. Bäckström, A. Bäcklin, N. E. Holmberg, and K. E. Bergkvist, Nucl. Instr. Methods 16, 199 (1962); A. Bäcklin, Arkiv Fysik (to be published). ~ K. E. Bergkvist, Arkiv Fysik 27, ⁴³⁹ (1964). "We are grateful to Ing. L. Wahlman for preparing these

sources.

FIG. 1. Part of the internal conversion spectrum from the reaction
Dy¹⁶¹ (n,γ) Dy¹⁶² recorded with the Studsvik β spectrometer. The ordinate gives the total number of counts recorded at each point.

better than 1 part in 104. The detector was a GM counter equipped with a 0.6 mg/cm² aluminized Mylar window. The conversion spectrum was scanned from 160—1300-keV electron energy, with the exception of the region between 300 and 400 keV, where no or few conversion lines strong enough to be detected in the present measurements were expected. The source thickness was chosen according to the energy region so that the FWHM of the lines never exceeded 0.3% .

A part of the conversion spectrum is shown in Fig. 1. About 60 conversion lines belonging to 46 transitions were assigned to Dy¹⁶². Furthermore, upper limits of the X-conversion intensities could be determined for 8 transitions. The energies of the conversion lines were measured relative to the K line of the strong 282-keV transition, the energy of which was accurately meas-

ured with the Risø crystal diffraction spectrometer (cf. Sec. II 8). The result of the energy and intensity determinations is given in Table II, columns 6 to 11. The electron intensities are given in absolute units, i.e., electrons per 100 captured neutrons. The conversion factor from the measured relative intensity to absolute units was obtained with the aid of the γ intensity measured in absolute units for a transition with known multipolarity in combination with the theoretical value multipolarity in combination with the theoretical value
of the K-conversion coefficient of the transition.^{27,28} Two lines were employed for this normalization procedure: Transitions below 600 keV were normalized on the 282 keV E2 transition and transitions above that energy were normalized on the 888-keV E2 transition. The conversion coefficients calculated from the conversion electron intensities and the γ intensities obtained with

FIG. 2. Theoretical (Ref. 27) and experimental values of the E-conversion coefficients of some of the strongest transitions in Dy^{162} .

27 L. A. Sliv and I. M. Band, Coefficients of Internal Conversion of Gamma Radiation (Academy of Sciences of the USSR, Moscow-Leningrad, 1956–1958).
²⁸ M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

TABLE II (continued)

160

NUCLEAR LEVELS OF Dy¹⁶²

TABLE II (continued)

160

NUCLEAR LEVELS OF Dy¹⁶²

FIG. 3. Part of the low-energy γ -ray
spectrum from $Dy^{161}(n,\gamma)Dy^{162}$ recorded with the Risø crystal diffraction spectrometer in the third order of reflection from the 110 plane in the bent quartz crystal.

the Risø diffraction spectrometer are given in Table II together with theoretical values for various multipolarities.^{27,28} In the last column of Table II are given the multipolarity assignments and mixing ratios obtained from the conversion coefficients. The general agreement between the experimentally obtained conversion coefficients and the theoretical values is illustrated by Fig. 2.

B. The Low-Energy (n, γ) Spectrum

The low-energy (n, γ) spectrum was measured with the Ris ϕ curved-crystal spectrometer.²⁹ The source consisted of 70-mg Dy₂O₃ enriched in Dy¹⁶¹ as given in Table I. A resolution $\delta E/E \approx 1.5 \times 10^{-5} \times E/n$ was obtained, where δE is the FWHM, E is the energy in keV, and n is the order of reflection. The procedures for measuring the energies and intensities have been described previously.³⁰ A part of the spectrum is shown in Fig. 3. About 230 transitions with energies ranging from 30-1.3 MeV were detected. A considerable number of these lines have been assigned earlier to transitions in Dy¹⁶³ (Ref. 31), Dy¹⁶⁴ (Ref. 30), Dy¹⁶⁵ and Ho¹⁶⁵ (Ref. 32). Energies and intensities of the transitions assigned to Dy¹⁶² are given in Table II. Absolute γ energies were obtained from a calibration of the spectrometer using Bergvall's energies for the $K\alpha_1$ and $K\alpha_2$ x-ray lines of dysprosium.³³ The absolute γ intensities (number of quanta per 100 captured neutrons) were obtained from

32 O. W. B. Schult, B. Maier, and U. Gruber, Z. Physik 182, 171 (1964)

a comparison with the strongest lines of Dv^{165} , the absolute intensities of which are given in Ref. 32 (see in particular p. 190 of Ref. 32). In this comparison, the relative abundances of the isotopes Dy¹⁶¹ and Dy¹⁶⁴ in the source material were considered together with the neutron absorption cross sections.³⁴ The intensity errors (column 5 of Table II) include only the uncertainty due to our measurement and not the errors of the capture cross sections and abundances. Column 3 in Table II shows how the transitions fit into the level scheme (Fig. 9). The level energies were computed using the energies of all transitions involved. ΔE is the difference between the expected energy and the measured γ -ray energy.

C. The High-Energy (n, γ) Spectrum

The high-energy (n, γ) spectrum was measured with a spectrometer consisting of a Ge(Li) detector with a 3-mm-thick depletion depth placed inside a large NaI annulus. The spectrometer views a sample placed in the thermal column of the Los Alamos Omega West Reactor. Further details of this arrangement are given in Ref. 31. The spectrometer was operated in the pair mode requiring both annihilation quanta to be absorbed in the annulus for an event in the Ge detector to be accepted. With a field-effect transistor preamplifier, the resolution was 8.0 keV (FWHM) at 8 MeV. The spectrometer calibration was performed by using as standards the energies³⁵ and cross sections³⁶ of the

²⁹ U. Gruber, P. P. Maier, and O. W. B. Schult, Kerntechnik
5, 17 (1963); B. Maier, U. Gruber, and O. W. B. Schult, ibid. 5,

^{31 (1963);} *J*. *Maler*, *O. Gruber, and O. W. B. Schult, <i>total.* 3,

³⁰ O. W. B. Schult, U. Gruber, B. Maier, and F. W. Stanek,

³⁰ O. W. B. Schult, U. Gruber, B. Maier, and F. W. Stanek,

²¹ O. W. B. Schult *et al*

³³ P. Bergyall, Arkiv Fysik 16, 57 (1959).

³⁴ L. S. Danelyan, Yu. V. Adamchuk, S. S. Moskalev, M. I, Pevzner, and S. S. Yastrebov, At. Energ. (USSR) 16, 58 (1964).
³⁵ R. E. Carter and H. T. Motz, in *Proceedings of the Inter-*

national Conference on Nuclear Physics with Reactor Neutrons. edited by F. E. Throw (Argonne National Laboratory, Argonne,

Illinois, 1963), Report No. ANL-6769, p. 181.

³⁶ H. T. Motz, R. E. Carter, and W. D. Barfield, *Pile Neutron* Research in Physics (International Atomic Energy Agency, Vienna, 1962), p. 234

FIG. 4. The high-energy γ -ray spectrum from the reaction Dy¹⁶¹(n, γ)Dy¹⁶² measured with a Ge(Li) spectrometer at Los Alamos.

nitrogen lines from neutron capture in melamine. The Dy^{161} target²⁶ consisted of 82 mg of Dy_2O_3 enriched to 90% in Dy¹⁶¹. In Table I the percent abundance, the capture cross section, the percent of total neutron capture, and the binding energy are summarized for each of the dysprosium isotopes in the target. The necessity to use enriched material is evident. A typical spectrum is shown in Fig. 4. In order to make the isotopic assignments for the recorded lines, the capture γ -ray spectrum of each of the remaining stable dysprosium isotopes must be investigated. Because the binding energies of the other stable dysprosium isotopes are all lower than the binding energy of Dy^{162} , the radiations from the Dy¹⁶², Dy¹⁶³, and Dy¹⁶⁴ isotopes in the target do not appear in the upper 1.9, 0.5, and 2.5 MeV of the $Dy^{161}(n, \gamma)Dy^{162}$ spectrum, respectively. The two most energetic (and most intense) γ rays³⁷ from the reaction $\mathrm{Dy^{164}}(n, \gamma) \mathrm{Dy^{165}}$ contribute to peaks 33 and 34 (Fig. 4). Studies^{31,38} of the capture γ -ray spectra from Dy¹⁶², Dy¹⁶³, and Dy¹⁶⁴ targets in the 3.5 to 8.3 MeV energy region indicate that other radiations from these isotopes do not significantly contribute to the present spectra. Thus all of the remaining lines are assigned to the reaction $\mathrm{Dy^{161}}(n,\gamma)\mathrm{Dy^{162}}$.

The measured γ -ray energies and intensities from the reaction $\mathrm{Dy}^{161}(n,\gamma) \mathrm{Dy}^{162}$ are listed in Table III. As-
suming, as is overwhelmingly the case,³⁹ that only E1 suming, as is overwhelmingly the case,³⁹ that only $E1$ and $\overline{M}1$ transitions occur from the 2⁺ or 3⁺ compound state to the lower levels, transitions are permitted only to states with spin 1, 2, 3, or 4. The transitions, whose energies differ from the binding energy by less than about 2 MeV, are assumed to correspond to primary transitions from the capture state directly to a lowlying level of energy E_{ex} . It follows that

$$
E_{\rm ex} = E_B - E_\gamma, \tag{1}
$$

where E_B is the binding energy of the captured neutron and E_{γ} is the primary transition energy. The less

FIG. 5. Spectrum of protons scattered from Dy¹⁶² and contaminating impurities.

³⁷ R. K. Sheline, W. N. Shelton, H. T. Motz, and R. E. Carter, Phys. Rev. 136, B351 (1964).
³⁸ D. W. Hafemeister and E. B. Shera, Phys. Rev. 1**52**, 1084 (1966).
³⁹ B. B. Kinsev and G. A. Bartholomew, Phys. Rev. **93**, 1

TABLE III. The high-energy γ -ray energies and intensities $(\gamma$ rays emitted per 1000 neutrons captured by the parent isotope). The excitation energies have been derived from Eq. (1) using the binding energy of 8192.8 keV, as determined in this experiment.

Line no.	γ-ray energy (keV)	Excitation energy (keV)	Intensity $\gamma/(1000)$ neutrons) captured by parent	Comments
1	8112.9 ± 5.0	79.9±4.0	0.29 ± 0.10	
2	7926.5±5.0	266.3 ± 4.0	$_{\rm 0.24\pm0.10}$	
3	7304.7±5.0	888.1 ± 4.0	$0.34{\pm}0.12$	
$\overline{\mathbf{4}}$	7233.1±5.0	959.7 ± 4.0	$0.23{\pm}0.10$	
$\frac{5}{6}$	7131.5 ± 3.0	1061.3 ± 1.0	1.0 ± 0.3	
	$7044.7 \!\pm\! 3.0$	$1148.1 \!\pm\! 1.0$	2.2 ± 0.4	
7	6982.3 ± 3.0	1210.5 ± 1.0	$0.77 + 0.19$	
8 9	6896.1 \pm 3.0	$1296.7 \!\pm\! 1.0$	$0.95{\pm}0.24$	
10	6654.2 ± 4.0 6621.8 ± 3.0	$1538.6{\pm}3.0$ 1571.0 ± 1.5	$^{0.27\pm 0.8}$ $2.7 \ \pm 0.5$	
11	6523.5 \pm 3.0	1669.3 ± 1.5	$0.98{\pm}0.25$	
12	6361.3 \pm 5.0	$1831.5 \!\pm\! 4.0$	$0.88{\pm}0.3$	Complex
13	6328.3 ± 3.0	$1864.5 \!\pm\! 2.0$	$0.90{\pm}0.23$	
14	6306.9 ± 4.0	1885.9 ± 3.0	0.37 ± 0.09	
15	6282.3 ± 3.0	1910.5 ± 2.0	$0.86{\pm}0.22$	
16	6241.0 \pm 5.0	$1951.8 + 4.0$	$0.45\!\pm\!0.15$	Complex
17	6220.6 \pm 4.0	1972.1 ± 3.0	$0.63\!\pm\!0.16$	
18	6074.7 ± 4.0	2118.1 ± 3.0	$^{0.29\pm 0.09}$	
19	6006.5 \pm 5.0	2186.3 ± 4.0	$0.48{\pm}0.16$	Complex
20	5954.9 \pm 3.0	2237.9 \pm 2.0	3.6 ± 0.8	
21	5917.1±4.0	$2275.7 + 3.0$	$0.45{\pm0.14}$	
22 23	5895.9 ± 4.0	2296.9 ± 3.0	$0.81{\pm}0.24$	
24	5878.3 ± 3.0 5853.2 ± 5.0	2314.5 ± 2.0	$2.9\ \pm0.7$ $0.38{\pm}0.10$	Complex
25	5842.6 ± 4.0	2339.6±4.0 2350.2 ± 3.0	1.7 ± 0.4	
26	5822.4 ± 4.0	2370.9 \pm 3.0	$_{0.59\pm0.15}$	
27	5782.9 \pm 4.0	2409.9 \pm 3.0	$0.33 \!\pm\! 0.08$	
28	$5756.1 \!\pm\! 4.0$	2436.7 ± 3.0	1.3 ± 0.3	
29	5735.7 \pm 4.0	$2457.1 \!\pm\! 3.0$	1.2 ± 0.3	
30	5703.4 ± 4.0	2489.4 ± 3.0	1.9 ± 0.5	Complex
31	5676.9 ± 5.0	2515.9±4.0	$0.51{\pm}0.13$	Complex
32	5629.9±5.0	2562.9 ± 4.0	2.3 ± 0.6	Complex
33	$5610.5 \!\pm\! 3.0$	2582.3 ± 2.0	3.1	$\mathrm{Dy^{16\tilde{1}}(}n,\!\gamma\mathrm{)}\mathrm{Dy^{162}}$
			45	(70%), complex $\mathrm{Dy^{164}}(n,\gamma)\mathrm{Dy^{165}}$
34	5554.3 ± 5.0	2638.6 ± 4.0	0.99	(30%) Complex, partly $\mathrm{Dy^{164}}(n, \gamma) \mathrm{Dy^{165}}$
35	$5544.7 + 4.0$	2648.1 ± 3.0	1.6 ± 0.4	
36	5529.0 ± 4.0	2663.8 ± 3.0	1.1 ± 0.3	
37	5507.8 ± 4.0	2685.0 ± 3.0	$^{0.34\pm 0.09}$	
38	5468.6 ± 5.0	$2724.2 \!\pm\! 4.0$	$_{0.68\pm0.17}$	Complex
39	5450.4 ± 4.0	$^{2742.4\pm3.0}$	1.2 ± 0.3	
40	$5427.3 \!\pm\! 4.0$	2765.5 \pm 3.0	1.2 ± 0.3	
41	5420.2 ± 5.0	2772.6 ± 4.0	$0.75{\pm}0.19$	$\mathop{\rm Complex}\nolimits$
42	5396.6±4.0	2796.2 \pm 3.0	$0.38{\pm}0.10$	
43	5371.0 \pm 3.0	2821.8 ± 2.0	2.6 ± 0.6	
44	5347.3 ± 4.0	2845.5 ± 3.0	$0.48 + 0.12$	
45 46	5243.0 ± 4.0 5196.5 \pm 4.0	2949.8±3.0	2.5 ± 0.8 2.1 ± 0.6	
47	5174.5 ± 4.0	2996.3 \pm 3.0 3018.3 ± 3.0	2.5 ± 0.8	
48	5163.4 ± 4.0	3029.4±3.0	2.5 ± 0.8	
49	4951.7 \pm 4.0	3241.1 ± 3.0	1.3 ± 0.4	

energetic γ rays may have been preceded by a primary transition, in which case Eq. (1) is inapplicable.

The binding energy of the last neutron in Dv^{162} was determined to be 8192.8 ± 3 keV. Since there is no direct ground-state transition, this number was calculated by combining the γ -ray energy of lines 5, 6, 7, and 8 with the excitation energy of the corresponding levels determined by the low-energy-capture γ -ray studies. This value of the binding energy may be compared with the results obtained from the $Dy^{161}(d, p)Dy^{162}$ reaction. Combining the (d,p) Q value 5969 \pm 5 keV (see Sec. D) with the binding energy of the deuteron $(2224.61 \pm 0.07 \text{ keV})^{40}$ yields $8193.6 \pm 5 \text{ keV}$, which agrees very well with the (n, γ) value. The excitation energies of the remaining levels of Dv^{162} calculated using the (n,γ) value for the binding energy are listed in Table III. A minimum error of ± 3 keV has been assigned to all of the primary γ -ray energies, since these values include any possible uncertainty in the $N¹⁵$ neutron binding energy upon which the energy calibration is based. The excitation energies, which involve only energy differences, are assigned smaller errors. Errors arising from the uncertainties in the calibration are considered signilicantly larger than the errors resulting from the statistical uncertainty in locating line centroids.

The first nine lines from the reaction $Dy^{161}(n,\gamma)Dy^{162}$ (lines ¹—9) correspond to levels deduced from the lowenergy (n, γ) data. Comparing the level energies which have been derived from the high-energy (n,γ) data with those obtained from the low-energy data (Sec. III and Fig. 9), we note that the agreement is good; the mean difference is 0.95 keV and the largest deviation is 3.3 keV for line 4, which is very weak. The spins assigned to the levels populated by the nine most energetic primary transitions are either 1, 2, 3, or 4, which is consistent with the range of spin values that can be populated by primary dipole transitions.

Three of the primary γ rays (lines 6, 7, and 8) can be interpreted as $E1$ transitions since they populate states of negative parity, while six of the primary γ rays (lines ¹—5, and 9) populate states of positive parity and

TABLE IV. Excitation energies (in keV) of levels in Dv^{162} observed during the inelastic scattering of protons and deuterons at various laboratory angles. Only strong impurity groups are included in this table.

	$\mathrm{Dy^{162}}(p,p')$	${\rm D}{\rm v}^{162}(d,d')$		
90°	130°	130°		Remarks
~ 0	~ 0	~ 0	0.0^{+}	
$\sim \!\!80$	$\sim \!\!80$	\sim 78	0.2^{+}	
	162.3 ± 5	$163 + 5$		$9/2^-$ Dy ¹⁶³
264.6 ± 3	263.6 ± 3	$262.6 + 3$	0.4^{+}	
\sim 544	546.9 ± 3	$547.7 + 3$	0.6^{+}	90° : +Cl ³⁵
$861.5a \pm 4$	$858.7^{\circ}+4$			130° : Cl^{35} , 90° : Na ²³
$884.7a+4$	$889.74 + 4$	$890.2 + 3$	2.2^{+}	
1063.5 ± 3	1064.9 ± 4	$1063.5 + 3$	2.4^{+}	90° : + F^{19}
1130.4 ± 4				ဂၤး
\sim 1212ª	$1211.7 + 4$	$1212.9 + 3$	$2.3-$	
		\sim 1280b		
$1286.4 + 3$				O ¹⁶
	\sim 1358	$1359.3 + 5$		
.	\sim 1395	\sim 1393		$(p, p') 130^\circ$: Na ²³
1476.8 ± 3				N^{14}
		$1630.3 + 5$		C^{135}
		$1739.6 + 5$		

^a Comple**x** group.
^b Questionable peak

R. C. Greenwood and W. W. Black, Phys. Letters 21, 702 $(1966).$

FIG. 6. Deuteron spectrum predominantly from the reaction $\mathrm{Dy^{162}}(d,d').$

should be interpreted as $M1$ transitions. On the average, the branching ratios of these γ rays are in agreement with this assignment of multipolarity, since the E1 transitions $(I_x=2.2, 0.77, \text{ and } 0.95 \gamma/1000n)$ are on the average three times stronger than the $M1$ transitions $(I_7=0.29, 0.24, 0.34, 0.23, 1.04,$ and $0.27 \gamma/1000n$. However, because the most intense $M1$ transition (line 5) has a strength practically equal to the weakest E1 transitions (lines 7 and 8), it is obvious that the strengths of the high-energy-capture γ rays cannot alone be used to assign parity to the low-lying levels. Both this and previous (n, γ) experiments³¹ show that some of the primary transitions that should be easily observable from spin considerations are indeed not seen, which statistically may be explained in terms of Porter-Thomas fluctuations⁴¹ of the partial radiation widths.

D. Charged-Particle Reaction Spectroscoyy

Many levels in Dy¹⁶² have been excited during charged-particle reaction experiments performed, with Florida State University Tandem Van de Graaff ac-Florida State University Tandem Van de Graaff accelerator.⁴² The projectiles, 12-MeV protons or deuterons with a beam current between 0.3 and 0.1 μ A, impinged on dysprosium targets $(100-250 \mu g/cm^2)$, which were evaporated on thin $(20-60 \mu g/cm^2)$ carbon backings. The charged particles emerging from the target after the reaction were analyzed in a broad-range magnetic spectrograph⁴³ and stopped in Kodak nuclear emulsion plates. ⁴⁴ The over-all energy resolution of this system was approximately 0.1% for these experiments. The resultant data were fitted both by computer⁴⁵ and by hand.

I. Inelastic Scattering Experiments

The reaction $Dy^{162}(p,p')$ was studied at laboratory angles of 90° and 130° . The 90° spectrum is shown in Fig. 5. The $Dy^{162}(d,d')$ reaction was investigated at 130', and the corresponding spectrum of inelastically scattered deuterons is shown in Fig. 6. The target for these experiments was enriched $\rm Dy^{162}$: 0.2% $\rm Dy^{160}$, 7.8% Dy¹⁶¹, 87.2% Dy¹⁶², 3.4% Dy¹⁶³, and 1.7% Dy¹⁶⁴. The levels observed in these experiments are listed in Table IV.

The (p, p') and (d,d') spectra are quite similar. Above the lowest members of the ground-state rotational band follow groups, corresponding to the 2^+ and 4^+ states of the γ -vibrational band (cf. Sec. III). The 3⁺ and 5⁺ members of the γ -vibrational band are not observed at all. These two states have "unnatural parity" and should thus be excited only weakly, if at all. A strong group is observed at 1212 keV. This state is identified with the state at 1210.1 keV populated in the (n,γ) experiments. The $I^* = 2^-$ (band head) and 4^- members of the octupole band which were observed in the (n,γ) experiments, cf. Sec. III, are not excited sufficiently to be observed due to their "unnatural parity. "

2. The (d,p) and (d,t) Reactions

The (d,p) stripping and (d,t) pickup cross sections of deformed nuclei are calculated according to Satchler.⁴⁶ The inclusion of configuration mixing gives the following

 $\overline{^{41}$ C. E. Porter and R. G. Thomas, Phys. Rev. 104, 483 (1956). ⁴² Supported by the U. S. Atomic Energy Commission, the Nuclear Program of the State of Florida, and the U. S. Air Force Office of Scientific Research.

^{~%.} N. Shelton and R. N. Kenehck, Ph.D. thesis, Florida

State University, 1962 (unpublished). ⁴⁴ We would like to thank Mrs. Mary Jones, Mrs. Sue Hipps, Mrs. Hazel Benton, and Miss Veronica Bryant for their careful plate scanning.

⁴⁵ We are much indebted to Mr. H. Kaufman for making his program STRIP available to us.
⁴⁶ G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

formula for an odd- N , even- Z target nucleus:

$$
\frac{d\sigma}{d\omega} = g^2 \langle \Phi_2 | \Phi_1 \rangle^2 \sum_l \sum_{j=l \pm \frac{1}{2}} \langle I_2 K_2 | I_1 j \mp K_1 \Omega_2 \pm \Omega_1 \rangle^2
$$

$$
\times C_{jl}^2 (\Omega_2 \pm \Omega_1) \sigma_l(\theta) S(\Omega_2 \pm \Omega_1), \quad (2)
$$

where

$$
g=\sqrt{2}
$$
, if $K_1=\Omega_1=0$ or $K_2=\Omega_2=0$
 $g=1$, otherwise.

The target nucleus has spin I_1 , and $\Omega_1 = K_1$. The residual nucleus has spin I_2 , and $\Omega_2=K_2$. The \pm signs correspond to the two possible cases $\Omega = \Omega_2 + \Omega_1$ or $\Omega = |\Omega_2 - \Omega_1|$. The symbols C_{jl} are the expansion coefficients for the Nilsson orbitals in terms of the eigenfunctions of a spherical potential. The factor $\sigma_l(\bar{\theta})$ is the intrinsic single-particle differential cross section for capture or pickup of a nucleon with angular momentum *l*. This factor $\sigma_l(\theta)$ is obtained from distorted-wave Bornapproximation⁴⁷ calculations, using interpolated opticalapproximation calculations, using interpolated optical
model parameters.^{48,49} The factor $\langle\Phi_2|\Phi_1\rangle^2$ is the overlap integral between the initial and final vibrational wave functions. The intrinsic spectroscopic factor $S(\Omega)$ depends on the configurations of the single-particle states in the initial and final nuclear levels and may be calculated approximately by taking into account the residual nuclear interactions.

In the case of a (d,p) stripping reaction leading to an even-even residual nucleus the intrinsic spectroscopic factor for the ground state is given by $S(\Omega) = V_f^2(\Omega)$, whereas the corresponding factor for the two-quasiparticle states is given by $S(\Omega) = U_i^2(\Omega)$. The quantities $V_f(\Omega)$ and $U_i(\Omega)$ refer to the residual and target nuclei, respectively, and are obtained from pairing theory. In the case of a (d,t) reaction leading to an even-even residual nucleus the intrinsic spectroscopic factor to the ground state is $S(\Omega) = U_f^2(\Omega)$, whereas the corresponding factor to the two-quasiparticle states is $S(\Omega) = V_i^2(\Omega)$.

Only very few of the two-quasiparticle states of an even-even nucleus may be appreciably excited by the (d,p) or (d,t) reactions. For example, if the even-even residual nucleus is formed by capture of a neutron in the (d,p) process by an odd-A target nucleus, then one expects to form only two-quasiparticle neutron states in which the captured neutron is coupled to the original unpaired neutron in the odd-A target. An analogous argument can be made for the (d,t) reaction, except that the possible two-quasineutron states which can be formed are different, since the odd-A target is different.
The ground state is a special case.⁵⁰ The passage fron The ground state is a special case.⁵⁰ The passage from this extreme single-particle picture to the case where there is a residual pairing force between the nucleons simply introduces the intrinsic spectroscopic factor given above. We neglect the small admixture of threequasiparticle states into the ground state of the odd- A target nucleus.

The reaction $Dy^{161}(d, p)Dy^{162}$ was studied at laboratory angles 45° , 57° , and 65° . A typical spectrum is shown in the lower half of Fig. 7. The targets consisted of enriched (77%–99%) Dy¹⁶¹. The (d, t) data were taken at 60' with respect to the incident beam. In this case, the Scandinavian-type mass separator of the Florida State University has been used to make a pure target⁵¹ of Dy¹⁶³. The triton spectrum is seen in the upper part of Fig. 7. The energies of the excited states in Dy¹⁶² populated during the (d,p) and (d,t) reactions are listed in Table V. A Q value of 5969 ± 5 keV is found for the ground state for the (d,p) reaction. The ground state Q value for the (d,t) reaction is -13.5 ± 5 keV.

III. DISCUSSION

A. The Ground-State Rotational Band

The total intensity of the 80.6-keV E2 transition is approximately 0.87 per captured neutron. This transition, the strongest in the spectrum, must lead to the ground state, thereby defining a 2^+ level at 80.660 keV. The level is populated through a direct high-energy γ ray from the $I=2^+$ or 3^+ compound state (cf. Table III). Furthermore, the level was observed both in the (d,p) and (d,t) reactions (Table V). The second excited level was observed in all reactions studied in the present work. Adding the energy of the strong (total intensity 0.51 per captured neutron) 185.0-keV E2 line to the first excited level determines the energy of the 4+ level as 265.665 keV. Another state is clearly seen in the charged-particle spectra 283 keV above the 4+ state. This level spacing agrees well with the energy of the 282.8-keV line, which is fairly strong, as is to be expected for a transition between low-lying states. Ke thus obtain an energy of 548.529 keV for this level. The absence of a high-energy γ ray from the 2⁺ or 3⁺ compound state together with the missing transitions to the ground state and first excited state indicate a 5+ or 6+ state. On the basis of the rotational characteristics expected for this nucleus, we assign the level as a 6+ state.

Thus the 2^+ , 4^+ , and 6^+ states of the ground-state rotational band are strongly excited in the (n, γ) reaction. Applying the energy formula for rotational levels⁵²

$$
E = AI(I+1) + BI^2(I+1)^2, \tag{3}
$$

we obtain from the precise transition energies 80.660 and 185.005 keV,

 $A_0 = 13.5119 \text{ keV}, B_0 = -11.44 \text{ eV}.$

⁴⁷ R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240, 1962 (unpublished).
⁴⁸ F. G. Perey, Phys. Rev. 131, 745 (1963).
⁴⁹ C. M. Perey and F. G. Perey, Phys. Rev. 132,

⁵¹ We are very grateful to Ken Chellis and Vagn Toft for preparation of the Dyt⁸⁸ target.
¹³, A. Bohr and B. R. Mottelson, At. Energ. (USSR) 14, 41 (1963) .

FIG. 7. The upper part shows the triton spectrum from the reaction $Dy^{163}(d,t)Dy^{162}$ as observed at a laboratory angle of 60°.
The lower part shows the proton spectrum from $Dy^{161}(d,t)Dy^{162}$ observed at a laboratory a

	Excitation energies (keV) and energy errors of levels ^a in $\mathrm{Dy^{162}}$				Relative (d, p) and (d, t) cross sections for states in Dy ¹⁶²		
(d,p) 45°	(d,p) 57°	(d,p) 65°	(d,t) 60°	(d,p) 45°	(d,p) 57°	(d,p) 65°	(d,t) 60°
\ddotsc $80 + 2$ 265b $548 + 4$ $891 + 4$ $931 + 5$ \cdots	\cdots $78 + 3$ $263^\circ \pm 3$ \sim 553 \sim 895 \sim 938 ^{e,d} \sim 9660,d	80 ± 6 265 ^b 541 ~1.885 919	~ 0 $79 + 3$ $268 + 3$ \sim 547 \cdots	$10 - \pm 1.5$ 11 ± 1.5 4.3 ± 1.1 $5.7 + 1.1$ $\sim1.5 \pm 0.7$	3.9 ± 1.2 8 ± 3 5.5 ± 1.5 4.7 ± 1.5 ~1.0 ~1.5	4.5 ± 1.3 $7.6 + 1.5$ $7.2 + 1.4$ 4 ± 1.1 $3.8 + 1.0$	1.2 ± 0.5 $5.6 + 0.8$ $5.1 + 1.0$ $1.4 + 0.5$
\cdots $1149 + 2$ $1185 + 6$ $1207 + 4$ $1276 + 3$ $1359 + 2$	~1063 ^d \sim 1150 $^{\circ}$ $1282 + 4$ $1362 + 3$	1145 1178 1208 1278 1358	~1355 \sim 1388	$11.5 + 1.6$ \sim 1.2 \pm 0.6 $9.6 + 1.3$ 13 ± 1.8 $28.9 + 2.6$	\sim 2.0 \sim 11 13 ± 2 19 \pm 3	4.5 ± 1.1 $3.2 + 1.0$ $8.4 + 1.7$ $6.7 + 1.4$ $25.5 + 3$	$_{\rm 0.8\pm 0.4}$ $0.8 + 0.4$
~1452	\sim 1459 \circ	1450	\sim 1453 \sim 1481	3 ± 0.9	\sim 5		$2.1 + 0.6$ 1.4 ± 0.5
1485 $1520 + 4$	$1496°+4$ ~1529	1487 1524		25.2 ± 2.5 17 ± 2.1	49 ± 8 \sim 11	18 ± 2.4 9.4 ± 1.8	
$1576 + 2$	$1579 + 4$	1575	$1534 + 3$ $1574 + 3$ $1633 + 3$	42 ± 3	52 ± 7	68 \pm 5	28 ± 3 5 ± 1 $12.5 + 1.5$
$1644 + 5$	$1641 + 5$	1640	\sim 1667 \circ	$3.8 + 0.9$	8 \pm 2	3.2 ± 1.0	
1689 ^c	$1687 \text{°} \pm 5$	∫1672° 1691 ^e	\sim 1681 $^{\circ}$ \sim 1734 ^{c, d}	24.9 ± 2.5	24 ± 5	(15.7 ± 2.2) 15.5 ± 2.1	$1.9 + 1$ $5.2 + 2$ ~1.0
$1743 + 4$	$1743 \times \pm 5$	$1746 + 4$	\sim 1745 $^{\circ}$ \sim 1762°	17.3 ± 2.1	16 ± 6	$12.8 + 2.0$	15 \pm 3 4 ± 2
$1770 + 1$	$1770 + 2$	$1770 + 1$	\sim 1778 $^{\circ}$	144 ± 6	110 ± 12	115 ± 6	$8.2 + 1.5$
$1833 + 3$ 1866 ± 1	$1832 \in \pm 5$ 1866 ± 2	$1831 + 4$ $1866 + 2$	$1835 + 4$ 1906 ± 4	38 ± 3 103 ± 5	28 ± 8 94 ± 11	30 ± 3 83 ± 5	$9.2 + 1.5$ $5.4 + 1$
$1915 + 2$ $1961 + 5$	$1913 + 3$ 1959 [°] ± 4	$1912 + 2$		67 ± 4 \sim 2.3 \pm 0.8	69 ± 8 33 ± 8	60 ± 4	
$1981 + 3$ 2006 ± 3	\sim 1991ª	1978 $1998 + 4$	\sim 1975 $^{\circ}$ \sim 1996 \circ	$18.7 + 2$ $24.5 + 2.5$	\sim 27	23 ± 3 21 ± 3	2.0 ± 0.7 $2.7 + 0.9$
$2057 + 5$	$2055 + 6$	$2061 + 5$	\sim 2031 ^d	6.5 ± 1.3	\sim 8	$10.1\!\pm\!1.6$	~1.0
$2087 + 3$	2087	$2087 + 3$	\sim 2078 $^{\circ}$	13.6 ± 1.7	25 ± 6	26.4 ± 3	$1.3 + 0.6$
			\sim 2100 $^{\circ}$				$1.3 + 0.6$
$2123 + 5$	$2123 + 4$	$2127 + 5$	$2121 + 4$ $2156 + 4$ $2184 + 4$	37 ± 3	33 ± 6	30 ± 3	13.5 ± 2.0 $8.0 + 1.5$ $8.7 + 1.5$
$2195 + 4$	$2190 + 4$	$2193 + 4$	2214 ± 4	37 ± 3	58 ± 8	46 ± 4	5.3 ± 1.5
$2245 + 2$ 2274 ^d	$2243 + 3$	$2244 + 3$	$2244 + 6$ $2273 + 4$	93 ± 5 58 ± 4	143 ± 16	104 ± 6	$6.0 + 1.6$ 20 ± 3
	$2282 + 3$	$2281 + 2$			104 ± 16	110 ± 6	
$2300 + 6$ $2329 + 3$	$2328c + 4$	$2328 + 4$	$2297 + 5$ \sim 2328° \sim 2342°	15 ± 2 33 ± 3	\sim 71	35 ± 4	$9.2 + 2$ $19.5 + 4$ $15.5 + 4$
2356 ± 2	$2355\text{·} \pm 3$	$2353 + 5$	\sim 2372°	90 ± 5	\sim 86	90 ± 5	10 ± 3
$2384 + 5$		$2382 + 4$		$5.7 + 1.1$		26 ± 3	
$2405 + 5$			\sim 2412 \circ \sim 2425 \circ	19 ± 2			2.6 ± 1.3 8 ± 3
$2448 + 3$		2444±4	\sim 2440° \sim 2470 \circ	57 ± 4		96 \pm 5	$2.6 + 1.3$ 4.5 ± 2
$2496 + 3$		2492°±5	2489±4	40 ± 3		75 ± 5	21 ± 3

TABLE V. Excitation energies and relative cross sections for the population of levels in Dy^{162} , observed during the charged-particle reactions (d,t) and (d,p) at various laboratory angles.

A Most of the charged-particle groups which are due to impurities or other reactions have been eliminated from this table. However, the first four columns agree within the experimental energy error may indicate a single

Using the 282.864-keV line and including a term $CI^{3}(I+1)^{3}$ in the rotational formula, the following parameters are obtained:

$$
A=13.5188
$$
 keV, $B=-12.85$ eV, $C=+0.045$ eV.

From these numbers an energy of 375.1 keV is computed for the $8^+ \rightarrow 6^+$ transition, while 366.2 keV is found using A_0 and B_0 . The true value should lie between these numbers. Only one transition, the weak 372.40-keV γ line, was found between these energies. Its intensity agrees well with what is expected from Its intensity agrees well with what is expected from
population considerations.²¹ We may therefore identify this weak line with the $8^+ \rightarrow 6^+$ transition. Accordingly, the energy of the 8+ member of the ground-state rotational band is determined as 920.93 keV, in agreement with the result of Ref. 9.

Contributions to the B term in the series-expansion formula are expected from mixing between the γ - and β -vibrational bands and the ground-state rotational band. However, it has been shown that neither for the γ -vibrational band⁵³ nor for the β -vibrational band¹² is this effect strong enough to explain the magnitude of the coefficient. Very recently calculations have been performed to study the influence of $K=0^+$ bands, including the pairing vibrational band on the groundstate rotational band.⁵⁴ The coefficient B calculated with this theory reproduces rather well the experimental values, especially in Dy¹⁶², where a value of $B \cong -12$ eV was obtained after adding a contribution of approximately -1 eV from the γ ground-state band mixing.

B. The y-Vibrational Band

The energy spacing between the 0^+ and 2^+ levels of the ground-state rotational band is reproduced by the energy difference between the very intense 888- and 807 keV E2 transitions. This energy combination requires a level at 888.22 keV, which is also seen directly in the high-energy (n,γ) spectrum as well as in the (d,p) and inelastic-scattering experiments. In addition, the weak 622-keV line 6ts between this level and the 4+ level of the ground-state band. The decay mode of this 888-keV level implies that it must be a 2+ state. Further energy combinations yield states at 963 and 1061 keV, which are confirmed through the observation of high-energy (n,γ) lines showing the population of these states through primary γ transitions. The 1061-keV level was also observed in the inelastic-scattering experiments.

The E2 character of the transitions from the 963 keV level to the 2+ and 4+ levels of the ground-state band together with the fact that no transitions are seen to other members of this band requires a 3+ character for the 963-keV level. Similarly, the decay mode of the 1061-keV level implies spin and parity 4+ for this state. Another combination involving strong high-energy E2 transitions and precisely measured low-energy transitions establishes a level at 1182 keV. Its mode of depopulation implies that it is a 5+ state.

The energy sequence of these four states and, in particular, the fact that the low-energy transitions between the levels are intense enough to compete with high-energy transitions to the levels of the ground-state band, strongly suggest that these four levels form a rotational band. The absence of members of spin lower than 2 and the E2 character of the transitions to levels in the ground-state rotational band implies $K=2$ for the band. One may thus apply the energy formula of rotational levels⁵²:

 $E = \text{const.} + A_{\gamma}I(I+1)+B_{\gamma}I^{2}(I+1)^{2}+D_{\gamma}(-1)^{I}(I-1)$ $\chi I(I+1)(I+2)$. (4)

From the observed level spacings one obtains

$$
A_{\gamma}
$$
=12.6273 keV, B_{γ} =-10.31 eV, D_{γ} =-1.375 eV.

Using these parameters, the energy between the 6+ and 5+ levels is calculated as 141.46 keV. This is to be compared with the experimentally obtained value of 141.73 ± 0.10 keV, which is the energy between the 1182keV 5+ state and a fairly well established level at 1324 keV. The energy of the 1324-keV state is defined by an energy combination of transitions to the K, $I^* = 2$, 4^+ , and 0, 6+ states, which are the only transitions expected to be strong enough to be seen from the 2, 6+ level. Although the probability of finding an accidental energy combination in general is high, the probability of finding an accidental energy combination involving the given levels within 2 keV from a given energy is only about 5%. Furthermore, both the branching ratio of the transitions and their absolute intensities agree with the expectations²¹ for a 2, $6+$ state.

The calculated energy between the 7+ and 5+ states is 305.94 keV. This agrees well with the energy of the 305.90 ± 0.10 -keV transition. The intensity of this line also agrees with what can be expected for the $7^+ \!\rightarrow 5^+$ transition. We therefore tentatively assign the 305-keV line as the $7^+ \rightarrow 5^+$ transition, thus defining the 7^+ level at 1488.72 keV. The agreement between the experimental level energies and the simple rotation formula is illustrated for the γ -vibrational band and ground-state band in Fig. 8, where the ratios A_{I+1}/A_I are shown for the $K=2$ band together with the ratios A_{I+2}/A_I for the ground-state rotational band $(K=0)$. A_I $E_{\gamma}(I \rightarrow I - 1)/2I$ for $K \neq 0$ and $A_{I} = E_{\gamma}(I \rightarrow I - 2)$ $\frac{(-1)^{n+1}}{(4I-2)}$ for $K=0$. The importance of the term with A_I

2)/
 A_{i+1} the alternating sign in Eq. (4) is clearly seen in this figure.

The observation of intraband and interband transitions permits further investigation of the nuclear structure of the $K=2^+$ band (we follow here the notation of Bohr and Mottelson and call a band " $K=2^+$ band" if

⁶³ O. B. Nielsen, in Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961, edited by T. B. Birks

⁽Heywood & Company, Ltd., London, 1961), p. 317.
⁵⁴ O. Mikoshiba, T. Udagawa, R. K. Sheline, and S. Yoshida
in Gatlinburg Conference on Nuclear Physics, 1966 (unpublished)

.97-

FIG. 8. The quantities A_{I+2}/A_I with $A_I = E_{\gamma}(I \rightarrow I - 2)/(4I - 2)$
and A_{I+1}/A_I with $A_I = E_{\gamma}(I \rightarrow I - 1)/2I$ for the ground-state rotational band and the γ -vibrational band, respectively.

the lowest spin of its members is 2^{+}). If we adopt the extreme adiabatic picture and neglect pairing and mixing effects, one would expect the quadrupole moment Q_{22} within the $K=2^+$ band to be equal to that of the ground-state rotational band: $Q_{00} = 7.19 \times 10^{-24}$ cm² the ground-state rotational band: $Q_{00} = 7.19 \times 10^{-24}$ cm³
(Ref. 5). The collective model^{55,56} then permits the calculation of the transition probability of the 172.8-keV transition and, via the experimentally observed branching ratio $I_{\gamma}(172.8)/I_{\gamma}(980.2)$, the calculation of the E2 transition amplitude Q_{20} from the $K=2$ band to the $K=0$ ground-state rotational band.

From Q_{20} we find $B(E2)\uparrow=B(E2, K=0, I=0^+ \rightarrow$ $K=2$, $I=2^+\approx 4$ single-particle units.⁵⁷ This value is characteristic and agrees well with the result of Yoshizawa et al.,¹² of $B(E2)$ \uparrow = (0.094 \pm 0.018)e² \times 10⁻⁴⁸ cm⁴; $E(2,2^+) = 0.89$ MeV, obtained by means of Coulomb excitation. The agreement with Soloviev's⁵⁸ prediction for this state is very good. The quadrupole moment Q_{22} of the γ -vibrational band may be calculated from the value of the reduced transition probability $B(E2)$ found by Yoshizawa et al. and the experimental branching ratios $I_{\gamma}(I_4 2^+ \rightarrow I_1 2^+)/I_{\gamma}(I_4 2^+ \rightarrow I_1 0^+)$. As an average we obtain $Q_{22} = (5.7 \pm 1.5) \times 10^{-24}$ cm². Applying the s correction (see below) yields the value $Q_{22} = (5.8 \pm 1.5) \times 10^{-24}$ cm².

No conversion coefficients were obtained for the $\Delta I = 1$ transitions within the γ -vibrational band, as the intensities of these transitions are very weak. However, assuming Alaga's rule⁵⁶ to be valid, some information on the $\overline{M1}$ transition strengths in the band may be gained from the branching ratios of the intraband transitions. Thus, for both the 98.0- and the 121.7-keV transitions a lower limit of 2 is found for the mixing ratio δ^2 , i.e., $\delta^2 = I_\gamma(E2, I \rightarrow I-1)/I_\gamma(M1, I \rightarrow I-1) > 2$, which implies that the $M1$ transition strengths within the γ -vibrational band are fairly small.

bution in deformed nuclei is less deformed than the neutron distribution, Greiner⁵⁹ has recently calculated magnetic transition probabilities in even deformed nuclei. A result of $\delta^2 = 2.4$ is obtained for the 2, $4^+ \rightarrow 2, 3^+$ transition in Dv^{162} . The M1 admixture in transitions from the γ -vibrational band to the ground-state band was calculated to be ~ 0.3 to 0.5%, in agreement with the experimental observations that these transitions are

predominantly E2 (Table II). In Table VI we have listed the calculated ratios of the reduced transition probabilities for transitions from members of the γ -vibrational band to the ground-state band, assuming all transitions to be pure E2. A comparison of these values with those theoretically predicted by Alaga et $al.^{56}$ shows that the transitions to the lower spin members of the ground-state rotational band are retarded in comparison with those to the higher spin members. A better fit to the experimental data may be obtained if the mixing of the γ -vibrational band and the ground-state rotational band is taken into account. This may be done by applying the so-called z correction.⁶⁰ The average value of z_2 obtained from the various intensity ratios is 0.028 ± 0.020 , which is considerably smaller than the value 0.08 found by siderably smaller than the value 0.08 found by
Yoshizawa et al.¹² In column 5 of Table VI are given the ratios of the reduced transition probabilities calculated with the *z* correction. Column 7 of the same table shows the individual values of z_2 for the various pairs of transitions. A possible systematic variation of z_2 may be inferred from the table: The z_2 values seem to be larger for the $I \rightarrow I-2$ transitions than for the $I \rightarrow I+2$ transitions $(I$ even). However, the errors are too large for any definite conclusions. The same tendency seems to be exhibited by the corresponding transitions in Dy^{164} (Ref. 30) and Er^{168} (Ref. 61).

In addition to the mixing between the γ -vibrational band and the ground-state rotational band, one may also consider the mixing between these two bands and the β -vibrational band as has been done by Faessler the β -vibrational band as has been done by Faessler *et al*.^{62,63} (the RV model). Using Eq. (21) of Ref. 63 we have calculated the ratios of the reduced transition probabilities given in column 6 of Table UI. The mixing coefficients were interpolated from Table VII of Ref. 63 with the aid of a computer.⁶⁴ The β -vibrational energy E_{β} was taken as 1.2 MeV (cf. Sec. III F), although its value is not very critical; repeating the calculations with $E_8=1.4$ MeV changed the values by only a few percent. In general, the RV model appears to agree with the experimental data slightly better than the z_2

Starting from the assumption that the proton distri-

⁵⁵ Å. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, No. 16 (1953).
⁵⁶ G. Alaga, K. Alder, Å. Bohr, and B. R. Mottelson, Kgl. Danske

Videnskab. Selskab, Mat. Fys. Medd. 29, No. 9 (1955).
⁵⁷ J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physic*
(John Wiley & Sons, Inc., New York, 1962), p. 627. The nuclea
radius constant R_0 was taken as $1.$

⁶⁹ W. Greiner, Nucl. Phys. 80, 417 (1966).
⁶⁹ W. Greiner, Nucl. Phys. 80, 417 (1966).
Phys. 12, 389 (1959).
⁶¹ H. R. Koch, Z. Physik 192, 142 (1966).
⁶² A. Faessler and W. Greiner, Z. Physik 168, 525 (1962); 170,

^{11.} A. Facssler and W. Greener, Z. Physik 168, 525 (1962); 170,
105 (1962); 177, 190 (1964). A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. 70, 33

⁽¹⁹⁶⁵

 64 We owe our thanks to AB Atomenergi for the use of their ${\rm IBM}$ 360/30 computer

				Relative reduced $E2$ transition probability		
Transition $K_iI_i\pi_i \to K_fI_f\pi_f$	Energy keV	Experiment	Alagaa	z - corr. $(z_2=0.028)$	RV (Ref. 63) $(E_8 = 1.2 \text{ MeV})$	100_{22}
$22^+ \rightarrow 00^+$ $22^+ \rightarrow 02^+$	888.2 807.7	0.52 ± 0.14	0.70	0.60	0.57	$5.3 + 4.5$
$22^+ \rightarrow 04^+$	622	~ 0.045	0.05	0.068	0.055	~ 0
$23^+ \rightarrow 02^+$ $23^+ \rightarrow 04^+$	882.3 697.4	2.02 ± 0.7	2.50	1.73	1.82	$1.6 - 1.6 + 3.5$
$24^{+} \rightarrow 02^{+}$ $24^+ \rightarrow 04^+$	980.2 795.5	$0.145 + 0.055$	0.34	0.23	0.20	$5.5 - 1.8 + 2.5$
$24^+ \rightarrow 06^+$	512.3	~ 0.113	0.09	0.15	~ 0.083	\sim 1.1
$25^+ \rightarrow 04^+$ $25^+ \rightarrow 06^+$	917.1 634.2	0.97 ± 0.4	1.75	0.98	1.11	$2.9 + 1.5$
					Average value:	$2.8 + 2.0$

TABLE VI. Relative reduced E2 transition probabilities of transitions from the $K=2^+$ band to the ground-state rotational band in Dy¹⁶².

^s Reference 56.

correction, but the experimental uncertainties are too large for any definite conclusions.

The experimental value of z_2 is in excellent agreement
th the theoretical prediction $z_2=0.028$ by Bès *et al.*,⁶⁵ with the theoretical prediction $z_2=0.028$ by Bès et al., based on the description of the γ -vibrational state as a superposition of many two-quasiparticle states
Using this approach, Kern et al.⁶⁶ have re

Using this approach, Kern et al.⁶⁶ have recently calculated (d,p) cross sections for γ -vibrational levels in various even nuclei, including Dv^{162} . These calculations reproduce fairly well the experimental cross sections obtained for the lowest members of the γ -vibrational band, thus lending further support to the microscopic description of these states.

C. The $K=2^-$ Band

The application of the energy-combination principle to most of the remaining strong transitions below 350 keV yields a set of levels at 1148.29 , 1210.15, and 1297.⁷ keV (see Fig. 9). The E1 multipolarity found for the transitions from these levels to the γ -vibrational band determines the parity of the levels to be negative. As the levels are populated through direct high-energy (n, γ) transitions, their spins must be either 2, 3, or 4. The de-excitation mode of these levels then requires that the spins be $2^-, 3^-,$ and 4^- for the 1148-, 1210-, and. 1297-keV levels, respectively. As shown in Table VII, the branching ratios of the transitions to the γ -vibrational band are in good agreement with those theoretitional band are in good agreement with those theoretically predicted⁵⁶ for a $K=2^-$ band, which strongly supports the assignment of these levels as forming a rotational band. The rotational formula $[Eq. (4)]$ with parameters A_2 -, B_2 -, and D_2 - gives, with B_2 -=0,

$$
A_2 = 10.4688 \text{ keV}, \quad D_2 = 6.59 \text{ eV},
$$

and predicts 96.77 keV for the energy between the 2, 5 and 2, 4 levels. Setting $D_2=0$, we obtain, on the other hand,

 A_2 = 9.5981 keV, B_2 = 39.58 eV,

and expect 115.77 keV as the 2, $5 - \leftrightarrow 2$, 4 energy. The 2, 5 level can thus be expected between 1394 and 1413 keV. In the present investigation, the most easily observed decay process of this level can be expected to take place to the γ -vibrational band. In fact, two transitions at 255 and 347 keV were found to combine with two members of the γ -vibrational band to yield a level at 1408.35 ± 0.05 keV. The probability of obtaining this combination by accident is approximately 25%, but the level is further supported by a possible transition to the 4+ level of the ground-state band. On the other hand, the conversion coefficient of the 347-keV transition is too high for E1 to be the most probable multipolarity of the 347-keV transition. We therefore only tentatively suggest the 2, 5 ⁻ level at 1408.35 keV. The intensity ratio of the 255 and 347 keV transitions agree with Alaga's prediction within a factor of 3.

Fitting the parameters of the rotational energy formula (4) to all 4 states of the $K=2^-$ band gives the

TABLE VII. Relative reduced $E1$ transition probabilities of transitions from the $K=2^-$ band to the $K=2^+$ band in Dy¹⁶².

Transition $K_iI_i\pi i \rightarrow K_fI_f\pi f$	Energy (keV)	Theory ^a	Relative reduced transition probability Experiment
$22^- \rightarrow 22^+$ $22^- \rightarrow 23^+$	260.0 185.3	0.50	$0.46 + 0.08$
$23^- \rightarrow 22^+$ $23 - \rightarrow 23^+$ $23^- \rightarrow 24^+$	321.9 247.1 149.1	1.4 1.8	$1.54 + 0.35$ $1.61 + 0.40$
$24 - \rightarrow 23^{+}$ $24 - \rightarrow 24 +$ $24^- \rightarrow 25$	334.0 236.0 114.2	0.60 1.4	$0.62 + 0.15$ $1.05 + 0.20$

^a Reference \$6.

^{&#}x27;~ D. R. Bhs, P. Federman, E. Maqueda, and A. Zuker, Nucl. Phys. 65, 1 (1965). J. Kern, 0. Mikoshiba, R. K. Sheline, T. Vdagawa, and S. Voshida, Gatlinburg Conference on Nuclear Physics, 1966 (unpublished) .

Dy¹⁶²

following result:

$$
A_2 = 9.8029 \text{ keV}, B_2 = 39.26 \text{ eV}, D_2 = 1.564 \text{ eV}.
$$

These parameters yield an energy of 1548.4 keV for the $2, 6$ ⁻ level. The strongest transitions from this level can be expected to take place to the 5+ and 6+ members of the γ -vibrational band with transition energies of around 365 and 224 keV, respectively. Two transitions are found near to these energies, at 363.47 and 221.762 keV, which add up almost perfectly to the same energy. The probability of finding two transitions within 5 keV from the expected values, which add up by accident to the same energy within the errors is approximately 10%.Further support for the level is obtained from the fact that the total intensity of the two transitions is of the expected order of magnitude. On the other hand, the branching ratio of the transitions differs more than an order of magnitude from Alaga's rule, and therefore we only tentatively suggest the 2, 6 ⁻ level at 1546.30 keV.

Assuming a quadrupole moment of about 8 b to be associated with the $2⁻$ band, the transition probabilities of the E1 transitions from the 1297-keV level to the γ -vibrational band may be estimated. From the branching ratio of the 148.7 and 334.0 keV transitions a hindrance factor H relative to the Weisskopf singleparticle estimate⁵⁷ was derived for the 334.0 keV $E1$ transition: $H = 3500$. Approximately the same value was obtained for the hindrance factors of the competing 114.2 and 236.0 keV transitions.

From the branching ratio of the 148.7 and 86.9 keV intraband transitions one may calculate the mixing ratio δ^2 of the latter assuming the E2 transitions to follow Alaga's rule. A 50% error of the intensity of the 86.9 keV transition gives $\delta^2 = [I_{\gamma}(E2)/I_{\gamma}(M1)] \gtrsim 2$.

The E1 transitions from the $K=2^-$ band to the ground-state rotational band are once K forbidden and are therefore expected to be below the detection limit of the present experiments. However, two strong transitions at 944.6 and 1130.1 keV were found to fit within their errors between the 1210-keV 2, 3 ⁻ level and the 4^+ and 2^+ members, respectively, of the groundstate band. As the probability of obtaining an accidental fit of two $E1$ transitions between the three lowest levels of the $K=2^-$ band and appropriate levels in the ground-state band is less than 1% , we are fairly well convinced that these two transitions really take place. Further support is obtained from population considerations: The total intensity of the transitions found to leave the 2, 2^- and 2, 4^- states is 9 and 7 quanta per 100 neutron captures, respectively. The intensity of the transitions from the 2 , 3 ⁻ state can accordingly be expected to be around 9% per capture. However, the total intensity of the transitions from this state to the γ -vibrational band is only 1.6% per neutron capture. By introducing the transitions to the ground-state rotational band, the de-exciting intensity rises to the much more reasonable value of about 12% per neutron cap-

TABLE VIII. Ratios of reduced $E1$ transition probabilities for transitions from the $K=2^-$ band to the ground-state rotations
band and to the $K=2^+$ band in Dy¹⁶².

	Theory $(RV \text{ model}^{\mathbf{a}})$	Experiment
$B(E1, 22^- \rightarrow 02^+)$	2.8×10^{-4}	2.9×10^{-4}
$B(E1, 22^- \rightarrow 22^+)$		0.6×10^{-4}
$B(E1, 23^{-} \rightarrow 02^{+})$	2.8×10^{-4}	$0.15 + 0.07$ ^d
$B(E1, 23^- \rightarrow 22^+)$		
$B(E1, 23^- \rightarrow 04^+)$		
$B(E1, 23^- \rightarrow 24^+)$	3.8×10^{-3}	$0.11 + 0.04$ ^d
$B(E1, 24^- \rightarrow 04^+)$		
$B(E1, 24^- \rightarrow 24^+)$	3.8×10^{-3}	${<}10^{-2d}$
$B(E1, 25^{-} \rightarrow 04^{+})$		
$B(E1, 25^{-} \rightarrow 24^{+})$	3.8×10^{-3}	0.14d

^a Reference 62.
^b Boneau *et al.* (Ref. 68).
^c Martin and Harlan (Ref. 67).
^d Present work.

ture. Also the tentative $2, 5$ level is probably de-excited mainly to the ground-state band.

The transitions from the even-spin members of the $K=2^-$ band to the ground-state band seem to be considerably weaker than those from the odd-spin members, as no such transitions were detected in the present work. However, the 1068-keV transition from the 2, 2^- level to the 0, 2^+ level has been found by Martin and Harlan⁶⁷ and by Boneau *et al.*⁶⁸ in the decay of Tb^{162} . They find the intensity of the 1068-keV transition to be 0.4% ⁶⁷ and 2% ⁶⁸ of the intensity of the 260-keV transition, which is far below the detection limit of our measurements.

The mixing between the γ -vibrational band and the ground-state band will to some extent cancel the K forbiddenness of the E1 transitions to the ground-state band. Assuming this band mixing to be the only cause of these E1 transitions, the admixed amplitudes may
be calculated,⁶⁹ e.g., using the Faessler-Greiner model.⁶² be calculated, 69 e.g., using the Faessler-Greiner model

In Table VIII the ratios of reduced transition probabilities obtained from these calculations are compared with the experimental values for transitions from the levels in the $K=2^-$ band. The agreement is good for the even-spin levels, but for the odd-spin levels the theoretically predicted values are around two orders of magnitude too small. The $E1$ transitions from these levels must therefore be due to some other mechanism than the γ -band—ground-state-band mixing. Using the approximate hindrance factor of 3500 deduced for the 334-keV transition and the other E1 transitions from

⁶⁷ H. R. Martin and R. A. Harlan, Phys. Rev. (to be published)
⁶⁸ D. Boneau, A. J. Bureau, and J. van Klinken, Bull. Am. Phys

Soc. 11, 459 (1966). "We are very much indebted to Dr. A. Faessler for computing these amplitudes.

TABLE IX. Relative reduced E2 transition probabilities of transitions from the $K=4^+$, $I=4$ level at 1535.9 keV to members of the $K=2^+$ band in Dy¹⁶².

Transition	Energy		Relative reduced E2 transition probability
$K_iI_i\pi_i \rightarrow K_iI_i\pi_i$	(keV)	Theory ^a	Experiment
$44 + \rightarrow 22 +$	647.6		
$44+ \rightarrow 23+$	572.8	0.56	$0.97 + 0.30$
$44 + \rightarrow 24 +$	474.8	0.196	$0.47 + 0.15$
$44 + \rightarrow 25 +$	352.9	0.040	-0.137
$44 + \rightarrow 26 +$	211.3	0.0036	${<}0.3$

a Reference 56.

the $K=2^-$ band to the $K=2^+$ band, one may calculate from the data given in Table VIII a hindrance factor of about 3×10^4 for the three transitions from the oddspin levels to the ground-state band and of about 2×10^7 for the 2, $2^-\rightarrow 0$, 2^+ transition.

The $K = 2^-$ state has been assigned as the $[411 \uparrow -523 \uparrow]$
o-quasiproton state by Funke *et al.*¹⁸ This state then two-quasiproton state by Funke et al.¹⁸ This state then lies 250 keV below the energy predicted by Gallaghe
and Soloviev.⁷⁰ On the other hand, the strong excitatio and Soloviev. On the other hand, the strong excitation of the 2 , 3 ⁻ state in the elastic-scattering experiments of the 2, 3^- state in the elastic-scattering experiments
definitely indicates its collective nature,⁷¹ i.e., an octupole-vibrational. state, and the energy of the 2 state agrees completely with the energy predicted by Soloviev⁵⁷ for the $K=2^-$ octupole state in Dy¹⁶². However, there is no basic difference between the two interpretations of the $K=2^-$ state, as the wave function of the $K=2^-$ -ocutpole state as given by Soloviev for Dy¹⁶⁰ consists of 95% of the $[411\uparrow -523\uparrow]$ twoquasiparticle state (see Ref. 58, p. 185, Table XXIII).

D. The $K = 4 + \lceil 521 \rceil + 523 \rceil$ Band

Energy combinations using the intense 647-keV E2 and 572-keV $M1(+E2)$ transitions suggest a level at 1535.8 keV, which is supported by two further combinations and the existence of a directly feeding high-energy (n,γ) line. The multipolarities of the de-exciting transitions and the decay of this state to the 2+, 3+, 4+, and 5⁺ levels of the γ band require spin and parity 4⁺ for this level. This is supported to some extent by a comparison between the theoretical and experimental reduced transition probabilities (see Table IX). Theoretica1.values are those obtained from Alaga's rule assuming all transitions to be pure E2. The experimental values indicate that this assumption is not wholly justified, which also follows from the fact that the magnitudes of the conversion coefficients definitely imply an $M1$ component in the 572-keV transition and possibly also in the 474-keV transition. Other combinations yield. levels at 1634.6 and 1752.1 keV, which are connected with the 1535.8-keV state through low-energy transitions, indicating that these levels are members of a rotational band based on the 1535.8-keV level. Further

evidence of this is obtained from the (d,t) reaction, in which the 4+ and 5+ levels of this band are strongly excited (see Fig. 7). Also the 6+ level was populated in this experiment.

According to Gallagher and Soloviev,⁷⁰ the two-quasi particle 4+ state of the lowest energy is the [521 \uparrow +523 J] two-neutron state, which is expected around 1600 keV above the ground state. A comparison of the experimental relative cross sections $\sigma_{e,r}$ and the theoretical relative cross sections $\sigma_{t,r}$ calculated for the [521] $+523$] band confirms this assignment (see Table X). However, it is entirely possible that these states partly consist of the $K=4^+$ 2-phonon γ vibration, which is expected in approximately this energy range.

The 1634.6-keV level decays to the 3^+ and 5^+ members of the γ -vibrational band. A possible line to the 4⁺ member is obscured by the strong 572.88-keV line. This experimental information is consistent with the assumption that this state at 1634 keV is $K=4$, $I=5^+$. Possible transitions to higher spin members of the γ band were too weak to be detected in the present experiments.

The application of the rotational-energy formula, Eq. (3), yields the parameters

$$
A_4=10.0704 \text{ keV}, B_4=3.90 \text{ eV},
$$

and 132.83 keV as the difference between the 7+ and 6+ members of this band. Extending the rotational formula, Eq. (4), to a $K=4$ band⁵² yields $E=const.+A_4'I(I+1)$ $+B_4'I^2(I+1)^2+D_4(-1)^{I+4}(I-3)(I-2)\cdots (I+4);$ with $B_4' = 0$ one obtains

$$
A_4' = 9.860 \text{ keV}, D_4 = -0.392 \text{ meV},
$$

and 138.23 keV as the $7^+ \rightarrow 6^+$ transition energy. Of the two transitions found between 132 and 139 keV the 135.73-keV line seems to correspond best to this transition, but the 133.81-keV line cannot be excluded as a candidate. The intensities of both these transitions are in agreement with what is expected from population are in agreement with what is expected from population considerations.²¹ Assuming the intraband $E2$ transi considerations.²¹ Assuming the intraband $\overline{E2}$ transitions to follow Alaga's rule,⁵⁶ the branching ratio of the 216.2- and 117.4-keV lines permits an estimate of the M1 transition strengths in the $K=4^+$ band. Using a quadrupole moment $Q_{44} \approx 7$ b, we find for the parameter C which is analogous to $[(g_K-g_R)\times K]^2$ in odd-A nuclei: $C=0.57$.

From these two parameters the transition probability of the 98.7-keV line can be estimated to be $\sim 5.5 \times 10^8$ sec^{-1} . Using this value in combination with the branching ratio of the 98.7- and 671.4-keV E2 transitions one obtains about 0.21 b for the transition amplitude Q_4 ⁺2⁺. This number explains why no transitions from the 6⁺ and 7⁺ states to the members of the γ -vibrational band were detected and it seems to agree reasonably with what one expects for the $E2$ transition strength from the $[521 \uparrow +523 \downarrow]$ two-quasineutron band to the levels in the γ -vibrational band if the detailed structure

^{&#}x27; C.J. Gallagher, Jr., and V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter 2, No. ² {1962). "W. N. Shelton, Phys. Letters 20, ⁶⁵¹ (1966).

Reaction	Angle	Configuration	$I=0$	$\mathbf{1}$	$\boldsymbol{2}$	3	4	5	6	7
${\rm Dy}^{163}(d,t){\rm Dy}^{162}$	60°	$\lceil 521 \uparrow + 523 \downarrow \rceil K = 4^+$					28	15	10	0.5
		$\lceil 521 \rceil - 523 \rfloor$ $K = 1^+$		9.8 $(4-15)^{a}$	10 (8.2 ± 1.5)	11 (9.2 ± 1.5)	(28 ± 3) 8.8 (5.4 ± 1.0)	(12.5 ± 1.5) 4.2 (2.7 ± 0.9)	$(4-15)a$ (1.3 ± 0.6)	0.7
$\mathrm{Dy^{161}}(d,p)\mathrm{Dy^{162}}$	45°	$[642 \uparrow + 521 \downarrow] K = 3$				151	26.3	18.0	5.6	
	57°					(144 ± 6) 118	(38 ± 3) 25.3	$(0-67)$ ^b 18.4	$(24.5 + 2.5)$ 6.3	
	65°					(110 ± 12) 111 (115 ± 6)	(28 ± 8) 22.5 (30 ± 3)	$(0-69)$ ^c 16.5 $(0-60)$ ^e	$(0-27)$ ^d 5.9 (21 ± 3)	
	45°	$\lceil 642 \rceil - 521 \rfloor$ K = 2-			123	43.8	20.4	10.9	2.8	
	57°				$(103+5)$ 96.2	$(0-67)$ ^b 37.4	(18.7 ± 2) 20.4	(6.5 ± 1.3) 11.5	3.3	
	65°				(94 ± 11) 90 $(83 + 5)$	$(0-69)$ ^e 34.2 $(0-60)$ ^e	$(0-27)$ ^d 18.2 $(23 + 3)$	(~8) 10.5 (10.1 ± 1.6)	3.2	
	45°	$[642 \uparrow +523 \downarrow] K = 5$						10.8 [°]	7.0	2.5
	57°							(25.2 ± 2.5) 11.2 (< 50)	8.8	4.0
	65°							10.1 Ä. (18 ± 3)	8.8	4.6
	45° 57°	$\left[642\right]\n-523\downarrow$ $K=0$	2.3 2.3	6.6 6.5	9.5 10.0	9.9 11.9	7.5 10.3	3.8 5.7		
	65°		2.0	5.7	9.1	11.6	10.5	6.2		

TABLE X. Comparison of calculated cross sections (in μ b/sr) for the Dy¹⁶³(d,t) and Dy¹⁶¹(d,p) reactions and the experimentally observed relative cross sections (in parentheses).

 $^{\bf a}$ The two-particle groups are unresolved. The sum of the experimental cross sections is 15.
 $^{\bf b}$ The two-particle groups are unresolved. The sum of the experimental cross sections is \sim 67. The two-particle gr

of the γ -band wave function as given by Bès et al.,⁶⁵ is considered. Transitions from the 4^+ band to the $2^$ band have not been observed. In addition to K forbiddenness, this fact can well be ascribed to intrinsic forbiddenness, because such a transition would require a two-particle transition.

E. The $K=0^-$ Band

Of the transitions not yet included in the level scheme, one may notice a group of strong $E1$ transitions in the energy region around 1 MeV. Three of these transitions were found to combine with the lowest members of the ground-state band, suggesting levels at 1275.4 and 1357.0 keV (see Fig. 9). The probability to form both these combinations by accident is approximately 1% . Further support for these levels is obtained from the (d,d') experiment, in which a weak and medium-strong peaks were found at these energies (see Fig. 6). Both these states were also strongly excited in the (d,p) reaction (see Table V). The E1 character of the transitions to the ground-state band implies negative parity for both these levels. Assuming the 1275-keV transition, which energetically fits to both levels, to be a close doublet determines the spin as 1 and 3 for the 1275and the 1356-keV levels, respectively. It may also be noted that energy coincidence between $3 - \rightarrow 0$, 2^+ and

 $1^- \rightarrow 0$, 0⁺ transitions has been found in other nuclei.^{12,72} If the 1275-keV transition is removed from one of the levels there are in principle other alternatives for the spin of that level. However, none of these seem to be very plausible.

One should expect both these levels to be directly fed by high-energy γ rays. From an inspection of Fig. 4 the intensity of these γ rays can be determined to be at most 0.15 quanta per 1000 captured neutrons.

The similarities between these two levels suggest that they are members of the same rotational band. Using the rotational formula

$$
E = \text{const.} + AI(I+1),\tag{5}
$$

one obtains $A = 8.09$ keV, which gives an energy of 1501.9 keV for the 5⁻ level. Two levels close to this energy were found in the (d,p) measurements, at 1485 and 1520 keV (see Table V). Although the cross section of the first level seems to be somewhat high (cf. Table X), it may possibly be identified with the $K=5^-$ [642↑+523↓] level, which has been found at 1485 keV in the β decay of Ho¹⁶² (Ref. 14) and is expected to be populated in the (d, p) reaction. We therefore tentatively assign the 1520-keV level as the 5⁻ level belonging to the 1^- and 3^- levels. No transitions to the ground-state band were found from that level; population considera-

⁷² B. Elbek, thesis, Copenhagen, 1963 (unpublished).

TABLE XI. Total intensity of the 1276 keV transition in Dy¹⁶² calculated from the theoretical branching ratios (Ref. 56) for $K_i=0$ and 1. Columns 2 and 3 give the theoretical branching ratios, column 4 the total γ intensity of the 1275-keV transitions deduced from columns 2 and 3 and the experimental intensity of the competing transitions. Column 5 gives the deviation of the values of column 4 from the experimental value.

	$B(E1, K_11^- \rightarrow 00^+)$ $B(E1, K_11^- \rightarrow 02^+)$	$B(E1, Ki3^- \rightarrow 02^+)$ $B(E1, Ki3^- \rightarrow 04^+)$	Expected total I_{γ} for the 1275 keV transitions	Deviation from experimental value
$K_i = 0^-$	0.50	0.75	7.2	$-20\% + 100\%$
$K_i = 1^-$	2.0	1.33	18.3	

tions indicate that these transitions probably are too weak to be detected in the present experiment.

Using Eq. (5), the energies of the possible even-spin members of this band can be calculated as 1259, 1308, and 1502 keV for the 0^- , 2^- , and 4^- states, respectively. None of these states was noticeably populated in any of the reactions employed in this work. While the 1^- and 3^- states are populated to about 5 and 10% per capture, respectively, the corresponding figure for the 2^- and 4^- states is less than 1% . The (d,p) cross sections of the even-spin levels are less than a few μ b/sr (cf. Fig. 7). The absence of even-spin states suggests a $K=0^-$ assignment for this band. This is supported by the intensities of the de-exciting $E1$ transitions (cf. Table XI).Unfortunately the branching ratios could not be compared directly with Alaga's predictions as the intensity of the 1275-keV transition is believed to be shared between the levels, but the total intensity of the 1275-keV transitions is best predicted by assuming $K=0^-$ (see column 5 of Table XI).

Two $K=0^-$ states are predicted at low energy in Dy¹⁶². According to Gallagher and Soloviev⁷⁰ the $[642\uparrow -523\downarrow]$ two-quasineutron state should have an energy around 1.3 MeV. Soloviev⁵⁸ predicts the $K=0^$ octupole state at 1.36 MeV (which, although its composition is not given in Ref. 58, one might conjecture consists to a considerable part of the two-particle state mentioned). The second interpretation is somewhat supported by the fact that the $1⁻$ and $3⁻$ states were (although weakly) excited in the (d,d') reaction, which may indicate collective properties of these levels. Furthermore, the agreement is rather bad between the experimental and the theoretical (d,p) cross sections for the $\lceil 642 \rceil - 523 \rfloor$ state (Table X), indicating that also other $K=0^-$ two-particle states take considerable part in forming the state.

F. A Possible $K=0^+$ Band

Excited $K=0^+$ rotational bands of possible collective origin $(\beta$ vibrations) have been found in the nearby nuclei Dy¹⁵⁸ and Er¹⁶⁴ at 991 and 1238 keV, respectively.⁷³ One may therefore expect to find an excited $K=0^+$ band also in Dy¹⁶² in this energy region. Soloviev⁵⁸ predicts the first $K=0^+$ collective excited state (mainly

consisting of the two low-lying two-quasineutron states $\lceil 642 \uparrow -642 \uparrow \rceil$ and $\lceil 523 \downarrow -523 \downarrow \rceil$ at 1.3-1.4 MeV. As Soloviev's predictions for these states in general seem to be 0.1-0.² MeV too high in this mass region, we may expect the $K=0^+$ band around 1.2 MeV above the ground state. The decay of this band should take place almost exclusively to the ground-state rotational band through E2 and E0 transitions. The (n, e^-) and (n, γ) spectra above 1 MeV yielded six transitions of possible E2, M1, or EO character. Three of these were found to combine with the 0^+ , 2^+ , and 4^+ members of the groundstate band in a way that suggested levels at 1206.1 and 1390.3 keV (see level scheme, Fig. 9). The probability of obtaining by accident these combinations within 2 keV was estimated as a few percent. From the highenergy γ -ray spectrum (Fig. 4), the direct feeding of the 1390-keV level can be estimated to be less than 0.1 quanta per 1000 captures. The possible peak corresponding to the 1206-keV transition could not be resolved from the transition to the 1210-keV level. The resolution of the charged-particle reaction experiments was not high enough to separate a possible peak at 1206 keV from the strong 1210-keV peak. The 1390-keV level is supported by the fact that levels at 1393 and 1388 keV were found in the (d,d') and (d,t) reactions, respectively (see Tables IV and V).

Under the somewhat arbitrary assumption that the 1206- and 1390-keV levels are members of the same rotational band, the mode of decay of these transitions to the ground-state band and the absence of transitions to the γ -vibrational band indicate that these are either the spin 2 and 4 members of a $K=0^+$ band or members of a $K=1^+$ band. In the latter case, the distance between the levels is too large for the levels to have consecutive spin values. With the aid of the rotational formula Eq. (5) the energy of the intermediate level was calculated for the various spin alternatives. The fact that no evidence for a level at these energies was found speaks against the $K=1^+$ alternative. Furthermore, no $K=1^+$ state except the $\lceil 521\uparrow -523\downarrow \rceil$ state, which we found at 1745 keV (see below) is expected below 2 MeV from theory.⁷⁰

Therefore, we tentatively assign the 1206.1- and 1390.3-keV levels as the spin 2 and 4 members of a $K=0^+$ band. Only limits could be set for the branching ratios, but they do not contradict this assignment. Applying the simple rotational energy formula $\lceil \text{Eq. (5)} \rceil$

⁷³ R. Graetzer, G. B. Hagemann, K. A. Hagemann, and B. Elbek, Nucl. Phys. **76**, 1 (1966).

gives

$A = 13.16 \text{ keV}$,

which is not unreasonable. The formula gives the 0⁺ level at an energy of 1127 keV. No evidence for a level at the energy could be found, but it is not expected to be populated strongly enough to be seen in the (n,γ) reaction, and the proximity to the 1148-keV level may make the detection difficult in the (d, p) reaction.

G. The
$$
K=1^+[521\uparrow -523\downarrow]
$$
 Band

States at 1745, 1778, 1835, and 1906 keV are to a considerable extent populated during the (d,t) process. In addition, weak triton groups reveal a level at 1996 keV and indicate another state at 2100 keV. The energy spacings of these levels characterize them as belonging to a $K=1$ rotational band (see the pattern of the triton groups in Fig. 7). The good agreement (see Table X) between the relative experimental cross sections and those calculated for the $K=1^+$ [521 \uparrow - 523] band justify this assignment. The experimentally observed excitation energy of this band (1745 keV) is in fair agreement with the value (1600 keV) predicted by Gallagher and Soloviev. '0

H. The $K=2^{-}$ [642 \uparrow -521 \downarrow] and the $K=3^-$ [642 | +521] Bands

Levels with energies between 1770 and 2000 keV are strongly excited in the (d,p) process. The decomposition of this group of states yields two bands: one with $K=3$ and the members at 1770, 1832, \sim 1913, and 2006 keV and another $K=2$ band with levels at 1866, \sim 1913, 1981, and 2057 keV. The moment of inertia parameters $h^2/2g$ are about 7.9 keV in both bands. The strong excitation of these bands in the $Dy^{161}(d, p)$ reaction and the absence of any of these levels in the (d,t) spectrum indicate that these bands are the twoquasineutron states $\lceil 642 \uparrow \pm 521 \downarrow \rceil$. The $\lceil 521 \downarrow \rceil$ orbit is the lowest particle state, the excitation cross section of which can explain the observed (d,p) cross section. Actually, the experimentally observed and theoretically calculated cross-section ratios for the excitation of the members of the two bands show good agreement. For this reason, we have normalized the experimental relative cross sections against the calculated ones so that the sum of the calculated cross sections (in μ b/sr) for the two configurations agrees with the sum of the experimentally observed relative cross sections. In Table X, the experimental relative cross sections are compared then with the theoretical cross sections, computed in the usual way (DWBA, using proper optical-model parameters). The agreement is very good, except for the $I=6^-$ state of the $K=3^-$ bands, where the larger experimental cross section is probably due to mixing or the presence of an additional degenerate level.

I. Other Levels

Of the 160 low-energy transitions found in the (n,γ) reaction, 55 transitions representing 90% of the total intensity of the low-energy transitions have been placed in the level diagram (Fig. 9). Attempts were made to locate the remaining transitions on basis of energy combinations using the computer program described in Ref. 74. The fact that this program, which has been found to give good results for complicated has been found to give good results for complicate
spectra of spherical even nuclei,^{74,75} gave no statisticall significant suggestions of additional levels in Dv^{162} is probably due to the strongly selective decay of the levels in this nucleus. The remaining transitions are in general rather weak and may therefore take place at higher excitation energies. However, there are also some rather strong transitions not included in the level scheme. In addition to a few transitions in the 1 MeV region, there are two groups of fairly strong $M1$ transitions in the energy regions around 250 and 550 keV.

With two exceptions, the high-energy (n, γ) lines below 1.8 MeV lead to levels shown in the level scheme. The two exceptions are levels at 1571.0 and 1669.3 keV. No spin and parity assignments can be made for these levels, as no statistically significant low-energy combinations could be found for these energies.

Both the (d,t) and (d,p) spectra show that levels at higher-excitation energies are populated with considerable strength. A reliable decomposition of these groups of complex levels would require much higher resolution in the charged-particle spectra. However, one might try to explain the gross structure of these groups of levels. The strongly excited states seen in the (d,t) spectrum are probably two-quasiparticle states, where one neutron is in the orbit $\left[523 \downarrow\right]$ (the ground state of Dy¹⁶³) and the other neutron in the [400] or [4021] orbits. The latter have very high (d,t) cross sections and contribute to the nuclear structure of higher-lying levels in Dy¹⁶³. The states above 2100 keV strongly excited in the (d,p) reaction process are probably the configurations $\lceil 642 \uparrow \pm 512 \uparrow \rceil$, and at still higher energies the two-quasineutron states $\lceil 642 \rceil \pm 510 \rceil$, because the $\lceil 512 \uparrow \rceil$ and the $\lceil 510 \rceil$ orbits contain in the odd-A nuclei rotational members, which are very strongly excited. The levels that have been observed in the (d,p) reaction at energies where one would expect the $I=5$ and $I=6$ members of the [642 \uparrow +523 \downarrow] structure are more strongly populated than one would expect.

IV. CONCLUDING REMARKS

The experiments described in this work have revealed additional information on the low-energy structure of Dy¹⁶². Knowledge of probably 8 intrinsic excitations with superimposed rotational bands in Dy¹⁶²

⁷⁴ A. Bäcklin, N. E. Holmberg, and G. Bäckström, Nucl. Phys.
80, 154 (1966); A. Bäcklin, Nucl. Instr. Methods (to be published).
⁷⁵ O. Bergman and G. Bäckström, Nucl. Phys. **55**, 529 (1964).

make it one of the best studied even nuclei in the deformed region.

Further information on the β -vibrational band should be obtainable from charged-particle reactions. From such experiments it should also be possible to obtain more information on the octupole vibrations. Additional theoretical work is needed to explain some of the features of the decay scheme. In particular, the unexpectedly strong ground-state decay of the odd spin levels of the $K=2^-$ band demands an adequate theoretical explanation. It would be interesting to investigate the effect of a $\Delta K=2$ $(K=0^-, K=2^-)$ mixing on the partial radiation widths of these transitions.

A comparison of the level schemes of the isotopes Dy¹⁵⁸ (Ref. 76), Dy¹⁶⁰ (Ref. 77), and Dy¹⁶⁴ (Ref. 30) shows several systematic trends: The moments of inertia of the ground state and the γ -vibrational bands increase with increasing mass number and the energy of the γ -vibrational band head decreases. $K=4^+$ states have been seen in Dy¹⁵⁸ at 1894 keV and in Dy¹⁶⁰ at 1693 keV. Our result of a $K=4^+$ band at 1536 keV fits nicely into this trend. A $K=2^-$ band has been observed in Dy^{164} . However, the nature of this band seems to be somewhat different from the $K=2^-$ band in Dy¹⁶², since no strong ground-state transitions are found from the odd-spin members of the band in Dy¹⁶⁴.

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⁷⁶ K.Y. Gromov and F.N. Mohtasimov, in 16th Annual Conference on Nuclear Spectroscopy and Structure of Atomic Nuclei, Moscow-Leningrad, 1966 (unpublished).

⁷⁷ F. N. Mohtasimov, in 16th Annual Conference on Nuclear Spectroscopy and Structure of Atomic Nuclei, Moscow-Leningrad, 1966 (unpublished) .