THE DETECTING EFFICIENCY OF THE SINGLE ELECTRON TUBE.

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SYNOPSIS.

Definition of Detecting Efficiency.—As a definition of the detecting efficiency $\lim \frac{A^2}{b_0}$ is taken. Here b_0 is the average change in the rectified component of the output plate current and A is the amplitude of the input grid potential.

Derivation of Theoretical Formula.—A theoretical formula is derived for the detecting efficiency. (See formula 12).

Measurement of Detecting Efficiency.—Measurements of detecting efficiency were carried out by means of a *condenser potential-divider* and a sensitive *quadrant electrometer.*

Verification of Theory by Experiment.—By the measurements performed the theory has been verified qualitatively. An attempt to verify the theory quantitatively showed the necessity of taking into account the capacities between the tube elements.

Application to Armstrong's Tuned Plate Circuit.—The sudden drop in signal strength observed with Armstrong's circuit has been explained by the above theory.

1. INTRODUCTORY.

IN a previous paper by the authors,[1] it has been pointed out that the detecting efficiency means, in general words, the efficiency of an amplifier to render a weak signal intelligible. It was shown that the detecting efficiency depended upon the relation between the input grid potential and the resulting output plate current. In the case of the high frequency transformer-coupled amplifier the detecting efficiency was defined by the relation $\lim_{A \doteq 0} \frac{b_0}{A^2}$ where A and b_0 are the amplitudes, respectively, of the input grid potential, and of the rectified component of the output plate current. Experimental methods of measuring b_0 and A were devised, and the detecting efficiency was thus determined by direct measurement. In order to arrive at a complete understanding of the detecting efficiency, it is necessary to know how the detecting efficiency depends upon the characteristics of the tubes and upon the constants of the circuits of the amplifier. The present paper takes up this phase of the problem for a simple case and describes an investigation, both theoretical and experimental, of the detecting efficiency of a single electron tube. A theoretical analysis, in which certain simplifying

[1] "The Detecting Efficiency of the Electron Tube Amplifier." (As yet unpublished.)

assumptions were made, was carried out which yielded an expression for the detecting efficiency of the tube in terms of the constants of the tube and the circuits. Experiments were then performed to test the theory.

2. Theoretical.

In the current literature on electron tubes Van der Bijl's[1] relation is usually taken as the starting point in the derivation of the concept of the internal resistance of the tube. This procedure is unnecessarily special, for the behavior of a tube may be represented by an amplification factor and an internal resistance even though it does not conform to Van der Bijl's relation.

The following treatment considers only the case of negative grid voltage *i.e.*, $E_g < 0$. In this case the grid current is zero. A more general treatment of the problem will be taken up in a future paper. It is assumed that the plate current I_p is a function only of the plate voltage E_p and grid voltage E_g. We write

$$I_p = f(E_p, E_g). \qquad (1)$$

It is assumed that this function within the range considered has first, second and third derivatives, and that the function and its derivatives are finite, single-valued and continuous. It is necessary to specify accurately the conditions just stipulated because there are cases for which these conditions do not hold. For example, the recent experiments of R. Whittington[2] show clearly that there are resonance effects of the positive ions within the tube. For frequencies close to these resonant frequencies it is not legitimate to say that I_p is a single-valued function of E_p and E_g only, because it depends on the time rate of change of these quantities. But these frequencies are so high that at the ordinary frequencies used in radio one is not troubled by the complex effects observed by Whittington.

Denoting by Δx the change in any quantity x, the expansion of (1) is written

$$\begin{aligned}\Delta I_p &= I_p(E_{p_0} + \Delta E_p, E_{g_0} + \Delta E_g) - I_p(E_{p_0}, E_{g_0}) \\ &= \frac{\partial I_p}{\partial E_g}\Delta E_g + \frac{\partial I_p}{\partial E_p}\Delta E_p + \frac{1}{2}\frac{\partial^2 I_p}{\partial E_g^2}(\Delta E_g)^2 + \frac{\partial^2 I_p}{\partial E_g \partial E_p}\Delta E_g \Delta E_p \\ &\qquad + \frac{1}{2}\frac{\partial^2 I_p}{\partial E_p^2}(\Delta E_p)^2 + \cdots. \qquad (2)\end{aligned}$$

E_{p0} and E_{g0} are the values of the plate and grid voltages, respectively,

[1] Proc. Inst. Radio Eng., 7, 97, 1919.

[2] Radio Review, 1, 53, 1919.

when there is no high frequency voltage impressed on the grid. The values of the derivatives in (2) are to be evaluated for $E_p = E_{p_0}$ and $E_g = E_{g_0}$.

It is assumed that we deal with small values of ΔE_g and ΔE_p, so that the differentials in (2) of higher order than the second can be neglected. It is also assumed that the internal capacities of the tube are negligible.

Let us assume that the grid voltage varies with the time according to a sine law. So that

$$\Delta E_g = A \cos \omega t. \tag{3}$$

Then after initial conditions have been obliterated, the plate current can be expanded as a Fourier series in ωt.

$$\Delta I_p = b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \cdots + a_1 \sin \omega t + a_2 \sin 2\omega t + \cdots. \tag{4}$$

It is seen that the constant term b_0 means physically the rectified component of the change in the plate current.

We denote the absolute value of the plate circuit impedance for the frequency $m(\omega/2\pi)$ by Z_m, the resistance of that impedance by R_m, and the reactance by X_m. Then from (4)

$$\begin{aligned}-\Delta E_p = {} & Z_0 b_0 + Z_1 b_1 \cos\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + Z_2 b_2 \cos\left(2\omega t + \tan^{-1}\frac{X_2}{R_2}\right) \\ & + Z_1 a_1 \sin\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + Z_2 a_2 \sin\left(2\omega t + \tan^{-1}\frac{X_2}{R_2}\right).\end{aligned} \tag{5}$$

Setting (3) and (5) in (2) gives

$$\begin{aligned}\Delta I_p = {} & -\frac{\partial I_p}{\partial E_p}\left\{ Z_0 b_0 + Z_1 b_1 \cos\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right. \\ & \left. + Z_1 a_1 \sin\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right\} + \frac{\partial I_p}{\partial E_p} A \cos \omega t \\ & + \frac{1}{2}\frac{\partial I_p^2}{\partial E_p^2}\left\{ Z_0 b_0 + Z_1 b_1 \cos\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right. \\ & \left. + Z_1 a_1 \sin\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right\}^2 + \frac{1}{2}\frac{\partial^2 I_p}{\partial E_g^2} A^2 \cos^2 \omega t \\ & - \frac{\partial^2 I_p}{\partial E_p \partial E_g} A \cos \omega t \left\{ Z_0 b_0 + Z_1 b_1 \cos\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right. \\ & \left. + Z_1 a_1 \sin\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + \cdots \right\} \\ & + \text{etc.}\end{aligned} \tag{6}$$

The values of a_0, a_1, $a_2 \cdots$, b_0, b_1, $b_2 \cdots$, are obtained by identifying the terms of (4) and (6). In so doing we make use of the method of successive approximations and neglect higher orders than the second.

Equating the constant terms we find

$$b_0 = \frac{\frac{Z_1^2}{4}\frac{\partial^2 I_p}{\partial E_p^2}(a_1^2 + b_1^2) + \frac{A^2}{4}\frac{\partial^2 I_p}{\partial E_g^2} - \frac{A}{2}\frac{\partial^2 I_p}{\partial E_p \partial E_g}(b_1 R_1 + a_1 X_1)}{1 + Z_0 \frac{\partial I_p}{\partial E_p}}. \tag{7}$$

To find a_1 and b_1 we identify the coefficients of $\cos \omega t$ and $\sin \omega t$ in (4) and (6) neglecting quantities of higher order than the first. This can be done conveniently by observing that the equations involved are all linear, the terms of (4) being $b_1 \cos \omega t + a_1 \sin \omega t$, and the terms of (6) being

$$-\frac{\partial I_p}{\partial E_p}\left[Z_1 b_1 \cos\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right) + Z_1 a_1 \sin\left(\omega t + \tan^{-1}\frac{X_1}{R_1}\right)\right] + A \cos \omega t \cdot \frac{\partial I_p}{\partial E_g}.$$

We now introduce the symbol $\bar{e} = A\epsilon^{i\omega t}$, where $i = \sqrt{-1}$, and ϵ is the base of natural logarithms. Let $(b_1 \cos \omega t + a_1 \sin \omega t)$ be the real part of $I_p'\epsilon^{i\omega t}$. It is seen that I_p' means the fundamental radio frequency component of the plate current.

Then

$$I_p' = -\frac{\partial I_p}{\partial E_p}\bar{Z}_1 I_p' + \frac{\partial I_p}{\partial E_g}\bar{e},$$

where

$$\bar{Z}_1 = Z_1 \epsilon^{i \tan^{-1}(X_1/R_1)}.$$

Therefore

$$I_p' = \frac{k\bar{e}}{\bar{Z}_1 + r_p},$$

where

$$r_p = \frac{1}{\frac{\partial I_p}{\partial E_g}}$$

and

$$k = \frac{\frac{\partial I_p}{\partial E_g}}{\frac{\partial I_p}{\partial E_p}}. \tag{8}$$

The expression (8) shows the relation between the internal resistance of the tube r_p and $\partial I_p/\partial E_p$. It also gives a general meaning to the amplification factor k. It is to be noted that no use of Van der Bijl's relation has been made in the above derivation.

Further, expanding (8) we have

$$I_p' = \frac{Ak[(r_p + R_1) - iX_1][\cos \omega t + i \sin \omega t]}{(r_p + R_1)^2 + X_1^2}. \tag{9}$$

The real part of this is

$$\frac{Ak}{(r_p + R_1)^2 + X_1^2}[(r_p + R_1) \cos \omega t + X_1 \sin \omega t]. \tag{10}$$

Comparing (4) and (10) we see that

$$b_1 = \frac{Ak(r_p + R_1)}{(r_p + R_1)^2 + X_1^2} \quad \text{and} \quad a_1 = \frac{AkX_1}{(r_p + R_1)^2 + X_1^2}.$$

Substituting these values in (7) gives finally the expression for b_0/A^2, which is the detecting efficiency for an amplitude A at frequency $\omega/2\pi$. It results that

$$\frac{b_0}{A^2} = \frac{(R_1^2 + X_1^2)k^2 \dfrac{\partial^2 I_p}{\partial E_p^2} + [(R_1 + r_p)^2 + X_1^2] \dfrac{\partial^2 I_p}{\partial E_g^2} - 2k[R_1(r_p + R_1) + X_1^2] \dfrac{\partial^2 I_p}{\partial E_p \partial E_g}}{4\left(1 + \dfrac{Z_0}{r_p}\right)[(r_p + R_1)^2 + X_1^2]}. \tag{11}$$

If the tube obeys Van der Bijl's relation

$$k^2 \frac{\partial^2 I_p}{\partial E_p^2} = k \frac{\partial^2 I_p}{\partial E_p \partial E_g} = \frac{\partial^2 I_p}{\partial E_g^2}.$$

In this case (11) becomes

$$\frac{b_0}{A^2} = \frac{r_p^2 \dfrac{\partial^2 I_p}{\partial E_g^2}}{4\left(1 + \dfrac{Z_0}{r_p}\right)[(r_p + R_1)^2 + X_1^2]}. \tag{12}$$

At the risk of repetition let us state again the meaning of the symbols used in (12). Z_0, R_1, X_1 are the direct current resistance, the high frequency resistance, and the reactance of the plate circuit, respectively. r_p is the internal resistance of the tube. r_p and the derivative $\partial^2 I_p/\partial E_g^2$ are obtained from the static characteristics of the tube.

Formula (12) depicts the manner in which the detecting efficiency depends on the constants of the circuits and of the tube. In particular it states that b_0 is proportional to A^2 for a specified frequency. This has been shown to be approximately true by direct experiment. (See the curves of the paper by the authors referred to in Section I.) It remained for further experiment to determine to what extent the assumptions underlying the derivation of formula (12) were permissible. This is taken up in the following sections.

3. Apparatus.

The apparatus was arranged as shown schematically in Fig. 1. The condenser potential divider consisting of the three condensers C_1, C_2, C_3 the inductance coil L' and the thermogalvanometer T, were the same as that described in a previous paper by the authors (loc. cit.). By coupling L' to a suitable generating set high frequency voltage of small known amplitude and frequency was impressed on the grid of the electron tube. The device for measuring the change in the rectified component

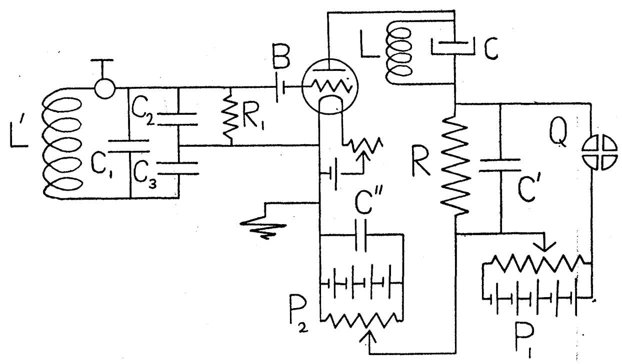

Fig. 1.

of the plate current consisted of a quadrant electrometer Q connected across a resistance R, the potential drop across R due to the plate battery P_2 being compensated by a potential divider P_1. This has also been described in the earlier paper (loc. cit.). The resistance R was a non-inductive resistance made of a dilute solution of copper sulphate in water. This was shunted by a variable condenser C'. C'' was a large fixed paper condenser having a low frequency capacity of about $2\mu F$.

In the plate circuit was inserted an inductance coil L shunted by a variable condenser C. The inductance of L was $1738\mu h$, its distributed capacity about $16\mu\mu F$, and its high frequency resistance at 1150 meters was 6 ohms. The condenser C was a General Radio Company Air Variable Condenser, Type 182 A. This had a shield which was always connected to the electrometer as shown in Fig. I. The electron tube was a Western Electric Company tube, Type V T I. The tube was used always with a filament current of 1.10 amp. and a plate voltage of 22.0 volts. The negative terminal of a standard cell B, Fig. 1, was always connected to the grid. A high resistance leak R_1 (about 2 megohms) kept the grid potential at a definite value.

4. Experimental Tests.

It is seen that when R_1 and X_1 in (12) become sufficiently large b_0/A^2 becomes very small. This was verified in the following manner: Keeping the current through T constant, *i.e.*, A constant, the condenser C was varied and the electrometer deflections were observed. Curves 1 and 2, Fig. 2, show the electrometer deflection in millimeters plotted

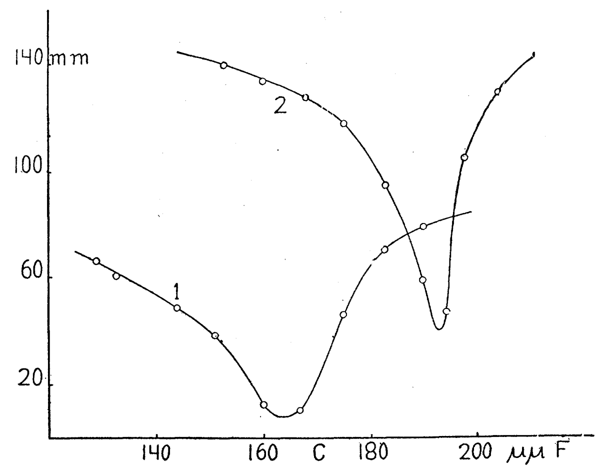

Fig. 2.

against the capacity of C in $\mu\mu F$. Curve I was taken for smaller deflections than Curve 2. Also in curve I the resistance R was about 21,000 ohms, while in Curve 2 R consisted of 421,000 ohms in series with a pair of telephones. The condenser C' was disconnected in the case of Curve 2. It is seen that in both cases the deflection dropped down for a proper value of capacity.

The explanation of these curves lies in the fact that R_1 and X_1 become very large for certain values of C. This can be seen from the expressions for the effective resistance and effective reactance of the parallel combination of an inductance coil having an inductance L and resistance R with a condenser of capacity C. If R_e and X_e are the effective resistance and reactance, respectively, it may be shown that

$$R_e = \frac{R}{R^2C^2\omega^2 + (LC\omega^2 - 1)^2}, \qquad X_e = -\frac{R^2C\omega + L\omega(LC\omega^2 - 1)^2}{R^2C^2\omega^2 + (LC\omega^2 - 1)^2}. \tag{13}$$

The maximum value o R_e (when C is varied) occurs very nearly at

$C = 1/L\omega^2$. The value of R_e for this value of C is $1/RC^2\omega^2 = L^2\omega^2/R$. Using the values obtained by measurement in the case of the present experiment, namely, $R = 6$ ohms, $L = 1.738 \times 10^3\ \mu h$, $\omega = 1.63 \times 10^6$,

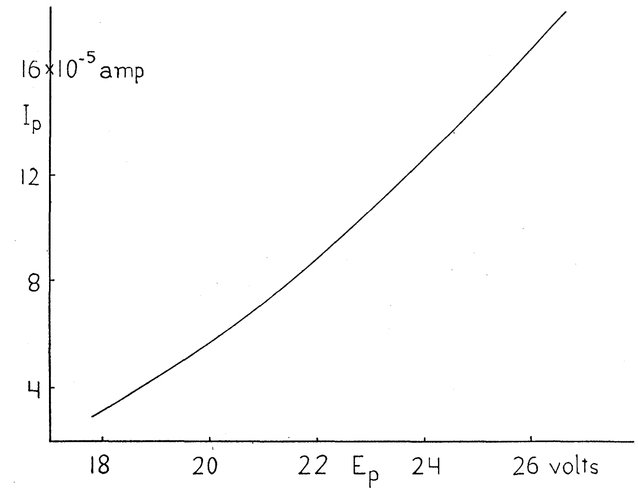

Fig. 3.

the effective resistance is found to be 1.34×10^6 ohms. This is R_1 in the notation of formula (12).

In order to determine r_p the static characteristic curves of the electron tube were drawn. This was done by measuring the plate current I_p

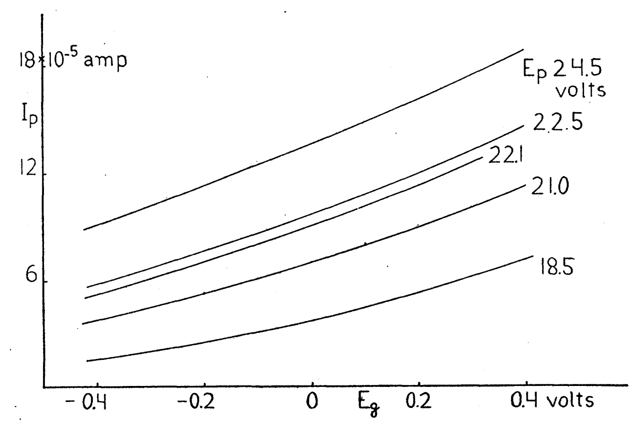

Fig. 4.

for various values of the plate voltage and grid voltage. The curves are shown in Fig. 3 and 4.

From the slope of the curve of Fig. 3 at the operating point $\partial I_p/\partial E_p$ was obtained. In obtaining the operating voltage E_{p_0} account was taken of the potential drop in the plate circuit resistance. The reciprocal of $\partial I_p/\partial E_p$ gives r_p, which was found to be 6.5×10^4 ohms. The high value was due to the use of a small filament current (see section 3).

Returning to the experiment under discussion, R was 2.1×10^4 ohms and when C was set so that the combination of L and C were at resonance for the impressed frequency, we have

$$R_1 = 1.34 \times 10^6 + 2 \times 10^4 = 1.36 \times 10^6 \text{ ohms}$$

and

$$(R_1 + r_p)^2 = 2.01 \times 10^{12}.$$

If now C were to differ from its value at resonance by $20\mu\mu F$ the values of R_e and X_e (as computed from (13)) are 714 ohms and 3.12×10^4 ohms, respectively. Then

$$R_1 = 2.1 \times 10^4 + 0.7 \times 10^4 = 2.2 \times 10^4 \text{ ohms}$$

$$R_1 + r_p = 8.7 \times 10^4 \text{ ohms}; \; X_1 = X_e = 3.12 \times 10^4 \text{ ohms};$$

and

$$(R_1 + r_p)^2 + X_1^2 = 8.55 \times 10^9.$$

Hence the electrometer deflection must have decreased in the ratio of at least

$$\frac{2.01 \times 10^{12}}{8.55 \times 10^9} = 2.35.$$

This calculation applies to Curve I, Fig. 2. An inspection of the curve shows at once that quantitatively the simple theory presented above is not borne out by experiment, for the electrometer deflection at the minimum point is about one-eighth of the deflection at $20\mu\mu F$ from the minimum. The discrepancy is to be attributed to the neglect of the effect of the internal capacities of the tube. If these capacities are taken into account there results a formula which gives better agreement with the observations. This will be taken up in a future paper. It was possible to demonstrate experimentally that the larger the capacity between the grid and plate of the tube became the less marked was the effect of making R_1 very large. This was done as follows: A variable air condenser (General Radio Company, Type 101 L) was connected from the grid to the plate of the tube. For a low condenser setting the electrometer deflection decreased perceptibly when R_1 became sufficiently high. When the condenser reached a value of about $500\mu\mu F$ the effect could no longer be observed.

Another cause of error to be considered arises from the assumption of Van der Bijl's relation. In the above case, however, the tube obeyed

Van der Bijl's law closely, as seen from the curves of Fig. 4, and so the discrepancies cannot be attributed to a deviation from Van der Bijl's relation.

There are various other factors which cause discrepancies between formula (12) and the measurements. An effect which is difficult to

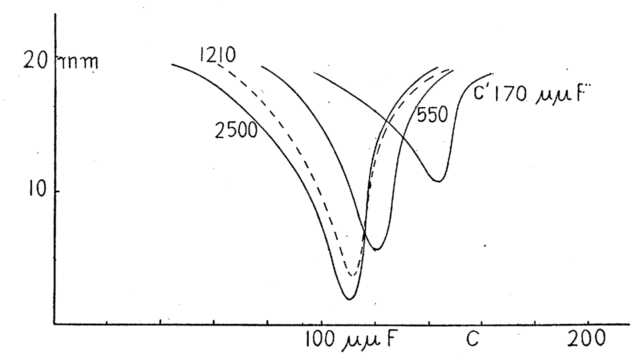

Fig. 5.

take rigorously into account is that introduced by small capacities between the various parts of the circuits. As an illustration of this effect the curves shown in Fig. 5 were drawn. In order to obtain the data for this family of curves the connections shown schematically in Fig. 6 were used in the plate circuit. It is seen that these connections differed from those used previously in that the electrometer was connected across all of the resistance in the plate circuit instead of across only a portion.

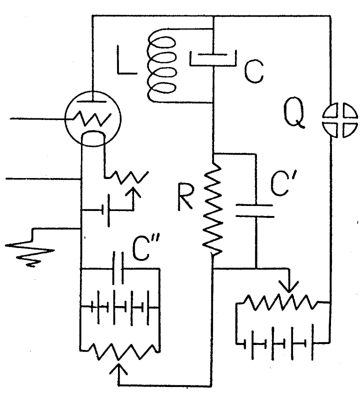

Fig. 6.

By giving C' smaller and smaller values the effect of a variation of C becomes less and less, as is seen from the curves of Fig. 5. These curves are plotted with electrometer deflections in mm. (which are proportional to the detecting efficiency) as ordinates against capacities of C in $\mu\mu F$ as abscissas for several values of C'. The grid voltage was $-$ 1.018 and the impressed wave-length 1,150 meters. It is noticed that the position of the minimum shifts. The explanation of the family of curves lies in the fact that the quadrant electrometer acted as a condenser. The position of the minimum of a curve occurs when R_1 is a maximum.

R_1 is a maximum approximately when the sum of the resistances of the parallel combination of L and C, and the parallel combination of R and C', becomes zero. The total resistance of these parts in series depends on the setting of C' because the effective resistance of the parallel combination of R and C' is

$$\frac{R}{1 + R^2C'^2\omega^2}.$$

If C' is large the above quantity is small and consequently the total resistance of the plate circuit at its maximum is lessened because that resistance varies inversely with the resistance of all the parts in series.

It was of interest to study the effect with a positive voltage on the grid. With $E_g = +1.51$ volts curves similar to those of Fig. 5 were obtained and are shown in Fig. 7. It is seen that the electrometer

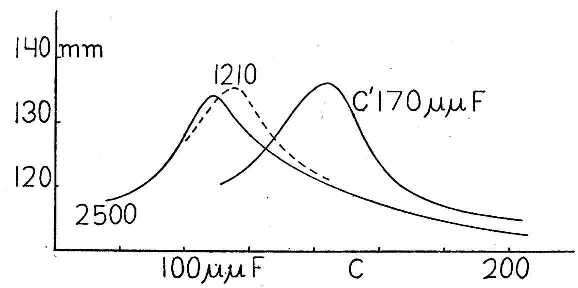

Fig. 7.

deflection, and hence the detecting efficiency, increased in absolute value, when R_1 was increased, instead of decreasing as it did in the case of negative grid voltage. It was observed, however, that the deflection in the case of a positive grid voltage and grid leak was opposite to that in the case of negative grid voltage. Thus, roughly speaking, the phenomenon may be explained by saying that there are two actions going on in the tube. One of these is due to the curvature of the grid voltage grid current curve, the other (the smaller of the two) is the same effect which has been studied with negative grid voltages. The effects on the average plate current due to these two actions are opposite. By making R_1 very large we decrease the second effect and thus increase the absolute value of the deflection. Strictly speaking this way of describing the phenomenon is not entirely correct. It gives, however, an easily memorized picture of the experimental facts.

We wish to point out the practical significance of the effects observed. We refer here to an effect frequently observed in the Armstrong regenerative circuit which to our knowledge has not been explained. In Armstrong's circuit the parallel combination of L and C is used just as we used it in our experiments. It is found that as C is increased, starting with small values, the intensity of the sound increases up to a certain point where it suddenly drops. Using the same circuit which we studied above and connecting the tube to an antenna it was found that this point is precisely the point where, with the same plate circuit connections, the detecting efficiency was found to be least. Further, by taking out a sufficient amount of loading inductance in the antenna and then retuning with a larger series capacity the behavior of the circuit was changed altogether. The signal no longer increased with an increase of C but remained fairly constant except in a narrow region where it became very faint. This narrow region coincided with the position of sudden decrease observed before.

Johns Hopkins University,
March, 1920.