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THE TONES FROM BELLS.

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SYNOPSIS.

Normal Modes of Vibration.—The positions of the *nodal lines* are examined for partial tones up to the ninth. Certain partials are heard only when the bell is tapped on the sound bow, others only when it is tapped above the sound bow.

Elasticity.—A value is obtained for the *Young's modulus* of bell metal.

Striking Notes.—The *principal striking note* is the note which is best heard when tunes are played on bells, and which gives its name to a bell. It cannot be picked up from the bell by a resonator, it cannot be elicited from the bell by resonance, and it does not beat with a tuning fork of nearly its own pitch. No partial tone of the bell has the same pitch as this striking note, but the fifth partial is an octave above it. A striking note is not a difference tone, and it does not arise from compressional waves running through the material of the bell. The pitch of the striking note seems to be determined by the fifth partial, the octave in which it lies being, however, generally misjudged. A possible reason for this general failure in correct estimation of the octave is found in the rates at which the different partials reach their maximum intensities.

A *secondary striking note*, an octave below the fourth partial of the bell, can be heard when the bell is tapped above the sound bow.

1. INTRODUCTION.

BELLS have been a subject of interest to many investigators,[1] but from the physical point of view very little work has been done. Papers by Lord Rayleigh,[2] Canon A. B. Simpson,[3] and Mr. P. J. Blessing[4] contain practically all that is known.[5]

[1] For bibliography of more than 250 titles see H. B. Walters, Church Bells of England, London, Frowde, 1912.

[2] Phil. Mag., (5), 29, p. 1, 1890, or Theory of Sound, § 235*a*.

[3] Simpson's two popularly written articles on "Why Bells Sound Out of Tune" and "How to Cure Them" were published in the Pall Mall Magazine, Oct., 1895, and Sept., 1896. They have also been published in the form of a booklet, which I believe is out of print. London, Skeffington, 1897.

[4] Physikal. Zeitschr., 12, p. 597, 1911.

[5] From the last number of Science Abstracts (Jan., 1920) I learn of two publications on bells by J. Biehle. The copy of the Physikalische Zeitschrift in which these are reviewed (Vol. 20, pp. 429–431, Sept. 15, 1919) has not yet reached Smith College. I have, however, succeeded in seeing the review, and from it I judge that the material in my paper is sufficiently different from Biehle's to make it worth publishing. From the review of Biehle's work, from my study of the bells of the Dorothea Carlile chime, and from Simpson's statements about English and continental bells I think it likely that the bells of the Dorothea Carlile chime are more like English bells than like the 450 bells which Biehle has examined.

Rayleigh used a set of Helmholtz resonators and examined the partial tones given out by each of a number of bells. Each resonator was tuned by covering its mouth to a greater or less extent with a finger, and then the various partial tones of the bells were picked out by ear. In seven out of the eight church bells which Rayleigh examined the fifth partial was the only tone, irrespective of octave, which was close to the note that the bell was said to give, and in the case of at least one of the bells this fifth partial was an octave higher than the note which the founders gave as the pitch of the bell.

Simpson's description of his method of study is given in a single sentence: "Any note of a bell can be elicited separately by touching the bell with the stem of a vibrating fork which is of the same pitch as the note in question." Simpson examined a considerable number of bells and came to regard as the most important partials three tones which are usually in the neighborhood of three successive octaves. The lowest of these three is the *hum note*, which is the lowest tone given by the bell. The next is the so-called *fundamental*, and for the highest of the three Simpson introduced the term *nominal*. Simpson adduced reasons for believing that when English tuners tuned a peal of bells they tuned the nominals and paid little attention to the other tones, whereas continental tuners paid most attention to the fundamentals. He also remarks, "There is further this curious fact: That while a tuner [English] always gave the nominal as the note of the bell, he invariably gave the pitch an octave lower than it really was." Simpson pointed out that for the finest musical quality all the partial tones of a bell should harmonize. This they seldom do. The interval from the hum note for instance to the fundamental he found was usually less than an octave, and the interval from the fundamental to the nominal somewhat over an octave. Simpson suggested a method of tuning bells which appears to depend on the fact that different partials have nodal circles at different distances up the bell. Thus by thinning the bell at certain distances from the bottom certain partials may be more affected than others. The results thus obtained are said to be very fine.[1]

Blessing distinguished the *principal note* or *striking note* of a bell from its *secondary notes*. The striking note is the note which is usually most noticed when tunes are played on bells. The secondary notes are pro-

[1] Simpson's method of tuning was at once taken up and developed by John Taylor and Co. of Loughborough, Eng. As to the results see T. L. Papillon, Encycl. Brit., ed. 11, article Bell; Lord Grimthorpe, Clocks, Watches, and Bells, ed. 8, p. 393, London, Lockwood, 1903; W. W. Starmer, Carillons, p. 12, London, Novello, 1915; W. W. Starmer, Musical Times, 60, p. 522, Oct. 1, 1919. There are in North America three chimes by Messrs. Taylor—one of 10 bells at the Iowa State College, one of 13 bells at St. John's Church, Peterborough, Ontario, and one of 12 bells at the University of California—cast respectively in 1899, 1911, and 1915.

duced by division of the bell into segments separated by nodal lines, and are therefore the various partial tones of the bell. These partial tones can be picked out by properly tuned resonators and can be elicited from the bell by resonance. The striking note, on the contrary, sound no louder with a resonator than without it and cannot be elicited from the bell by resonance. The pitch of the striking note is usually not far from that of the second partial, but the manner in which the striking note is produced is a mystery.

These remarkable statements as to the striking note of a bell, and the curious fact observed by Rayleigh and Simpson that the pitch of the bell appears to be an octave below the fifth partial of the bell certainly suggest some interesting problems. The installing of the Dorothea Carlile chime of twelve bells[1] at Smith College provided opportunity for a study of the striking note and also for comparison of the partials of a number of bells, all cast in the same year and by the same founder.

2. Method of Study.

Pitch Determinations.—The partial tones were found by Rayleigh's method with the use of Helmholtz resonators. The striking notes were picked up without resonators. The pitches, both of partial tones and of striking notes, were obtained by ear by comparison with a sonometer which carried a wire one meter long. Two positions of the sonometer bridge were found such that a note from the sonometer was in one case just perceptibly sharper and in the other just perceptibly flatter than the tone in question, and the mean of these two settings of the bridge was used. In order to avoid errors due to large amplitude the sonometer wire was plucked gently and the tuning fork was tapped lightly. As standard a c_4 [= 512 vd] tuning fork by König was used, and the sonometer wire was frequently compared with it.

The accuracy of the pitch determinations seemed at the time of making an observation to be often as good as 5 cents,[2] but observations made at different times usually differed by much more than this. The average deviation of half a dozen to a dozen observations made at different times was usually not far from 15 cents to 20 cents.[3]

Calibration of Sonometer.—For the shorter lengths of the sonometer wire the frequency of the note from the wire was not accurately proportional to the reciprocal of the length. This appeared to be due

[1] Cast in 1919 by the Meneely Bell Company, Troy, N. Y. Total weight 11,838 lbs. [= 5370 kg.] Weights of largest and smallest bells respectively 3006 lbs. [= 1364 kg.] and 268 lbs. [= 122 kg.].

[2] In designating intervals I use Mr. Ellis's very convenient interval the *cent*, 100 of which make an equally tempered half step.

[3] 1 per cent. change in frequency corresponds to a change in pitch of 17.2 cents.

partly to the natural stiffness of the wire and partly to the increased tension when the bridge approached the end of the wire. In order to calibrate the sonometer two methods were used. For the range from c_3 to c_5 settings of the bridge were made at pitches which corresponded to a dozen selected notes on a reed organ which had a stop tuned to the untempered scale. Care was used to blow the organ at a pressure at which the tuning was good. For the range above c_5 it was at first thought that the first, second, third, etc., overtones of any part of the wire might be taken as the octave, twelfth, double octave, etc., of the note given by that part of the wire, and therefore that corresponding to any chosen position A of the bridge the positions for the octave, twelfth, double octave, etc., would be those which gave the same notes as the corresponding overtones obtained with the bridge at A. This, however, assumed that the natural stiffness of the wire was negligible and did not prove satisfactory. The method was therefore modified. No attempt was made to get the overtones of the wire, but the octave was estimated by ear, and only the octave was used. Thus corresponding to any chosen position A of the bridge two other positions were found, one at which the pitch sounded a trifle flatter and one at which it sounded a trifle sharper than the octave of the note given at A. The mean of these two was taken as the setting for the octave of the note given at A. In the range where this method overlapped the calibration from the reed organ the agreement of the two methods was very satisfactory. The corrections obtained have been applied to all readings, although for pitches below f_4 they are not more than 10 cents. In the neighborhood of c_5 they amount to about 18 cents, and in the neighborhood of g_5 to nearly 25 cents.

Temperature Correction.—The pitches if the bells appear to be very little affected by changes in temperature. Throughout the later part of the work the temperature was read at frequent intervals on a thermometer hung near the bells. The temperatures ranged from 10° C. to 30° C. The pitch of the hour bell—low f of the chime—in the range from − 15° C. to 0° C., and the pitches of both the transverse and the longitudinal vibrations of a straight bar of bell metal in the range from − 10° C. to + 20° C. were also determined. The temperature coefficients obtained in the different cases were small and were not consistent.

It is not difficult to show that for the transverse and longitudinal vibrations of a straight bar, and for the extensional and flexural vibrations of a thin ring,

$$n = n_0 \left(1 + \frac{\alpha + \gamma}{2} \cdot t \right), \qquad (1)$$

where n and n_0 stand for the frequencies when the bar is respectively at temperatures t and 0°, and α and γ stand for the temperature coefficients respectively of linear expansion and of Young's modulus. For many metals $\alpha + \gamma$ is negative, and the pitches of such bars and rings therefore fall with rise of temperature. Since equation (1) holds for all four of these cases it seems likely that it will also hold for bells. Taking α for bell metal as 0.000018 per degree C.,[1] and γ as $-$ 0.0003 per degree C.,[2] equation (1) leads to a temperature coefficient of about $-$ 0.24 cent per degree C.

In view of the lack of consistency in the experimental values for the temperature coefficient of pitch of the bells, and of the small value which such a correction would have, it was finally decided to make no attempt to reduce readings to a common temperature.

Pitch as a Function of Amplitude.—Throughout most of the work it was thought that the pitches of many of the partial tones—especially of the lower partials of the larger bells—were somewhat lower when the bells were first struck than when the sound had nearly died out. This effect was not confirmed by the frequency with which properly tuned forks would beat with the tones in question, and was probably a case of the subjective lowering of pitch which has been discussed by Dr. C. V. Burton.[3] The pitches given below are averages from values which were obtained when tapping the bells gently.

3. The Partial Tones of the Bells.

Nodal Lines.—Rayleigh determined the number of nodal meridians for the first five partials of two bells. His method was to find a number of successive meridians at which the beats of the tone in question vanished. The number of nodal meridians is then half the total number of the meridians at which the beats vanish. In the case of the sixth partial of the largest bell of the Dorothea Carlile chime this method failed. Instead of two normal modes of vibration which gave a single set of beats for this partial there seemed to be several normal modes of nearly the same frequency, so that there were beats of several different frequencies. The frequencies of the most prominent beats varied in an

[1] From the Landolt and Börnstein tables. This is about the value given for brass and for a bronze.

[2] From values found by Kiewiet [Winkelmann, Hbduch. d. Phys., ed. 2, Vol. 1, p. 567] for the temperature coefficients of various copper-tin alloys. Kiewiet's values have evidently been multiplied by some factor. On comparing his values with those for various substances as given in the Landolt and Börnstein tables I think it likely that this factor is 10^4, and have so assumed. The coefficient given above is for the proportion of Cu 78 per cent. to Sn 22 per cent, which is the proportion used by the Meneely Bell Company.

[3] E. H. Barton, A Text-book on Sound, p. 579.

erratic manner with the position at which the bell was tapped and the position at which the resonator was held, both along a circle of latitude and along a meridian. These variations may be partly due to the long inscription on this bell.

After some time spent in trying to unravel these different beats another method of determining the number of nodal meridians was hit upon which proved entirely satisfactory and was in most cases much more expeditious. The resonator was connected to the observer's head by a piece of rubber tubing and the binaurals of a stethoscope. A single position was found where the beats vanished, the bell was tapped at this one position, and the resonator was moved quickly some distance around the bell. At the nodal meridians the sound grew faint and between them swelled out. This method proved to be also of service in cases where no beats could be detected and Rayleigh's method would have involved fastening to the bell a local load.

As regards nodal circles, Rayleigh observed that certain partials were

TABLE I.

Nodal Lines.

In this table the positions of nodal circles are indicated by fractions. The unit chosen is the distance measured along the outer surface from the bottom of the bell to the point where the vertical "shoulder" joins the more or less horizontal "crown." Numbers in parentheses give the average deviations of the measurements on the different bells. The middle of the "sound bow" is at 0.164 (0.007).

Partial.	Bells on which Observed.	No. of Nodal Meridians.	Positions of Nodal Circles.	Remarks.
1 = hum note	All	4	None	
2 = fundamental	All	4	0.33 (0.015)	
3	All	6	0.47 (0.012)	
4	All	6	None found	Not certainly detected when bell was tapped below 0.21 (0.014).
5 = nominal	All	8	0.48 (0.017)	Very clear when bell is tapped on sound bow. Faint for other positions of tapping. On five smallest bells not detected when tapping above sound bow.
6	6 largest	8	0.20 (0.009) 0.53 (0.033)	
7	6 largest	10	None found	Detected only when tapping below 0.37 (0.035).
8	2 largest	10	None found	Detected only when tapping above 0.46 (0.025).
9	1 largest	12	None found	Detected only when tapping below 0.37.

very faint, if heard at all, when the bell was tapped at certain latitudes. My results check his very well and extend the observations beyond the first five partials. They are given in Table I. It will be seen that vibration of the material in the sound bow—on which the clapper

TABLE II.

Pitches of the Tones.

In the upper half of this table pitches are given on the basis of $c_3 = 256$ vd. Calculations on the basis of $a_3 = 435$ vd. would indicate that all but one of the bells are more or less flat. A note with no sign following it means that the value obtained did not differ from that equally tempered note by more than 10 cents; a note followed by + or − means that the value obtained did not differ from that note by more than 35 cents; and two notes mean that the value obtained lay between them, but did not lie as close to either as 35 cents.

In the lower half of the table the pitches are in cents above the principal striking note. The three bottom lines give the average, the average deviation, and the range of values for the lower half of the table.

Bell.	Principal Striking Note.	Partial Tones.								
		1.	2.	3.	4.	5.	6.	7.	8.	9.
$e\flat$	$e\flat_3$	f_2+	$e\flat_3$	$g\flat_3 g_3$	$c_4 d\flat_4$	$e\flat_4+$	a_4	$b\flat_4+$	$e\flat_5+$	$e\flat_5 e_5$
f	f_3	$a\flat_2$	$e\flat_3$	$a\flat_3$	d_4+	f_4	$b\flat_4$	c_5+	$e\flat_5+$	
g	g_3	$a_2 b\flat_2$	$g\flat_3-$	$b\flat_3 b_3$	$e_4 f_4$	g_4+	$d\flat_5+$	d_5+		
$a\flat$	$a\flat_3$	b_2-	$g\flat_3$	b_3	$f_4 g\flat_4$	$a\flat_4+$	$d\flat_5$	$e\flat_5 e_5$		
a	a_3+	c_3	g_3	c_4	g_4-	$a_4 b\flat_4$	$e\flat_5-$	e_5+		
$b\flat$	$b\flat_3-$	$c_3 d\flat_3$	a_3	$d\flat_4+$	g_4	$b\flat_4$	e_5-	f_5		
c	c_4	$d_3 e\flat_3$	c_4+	$e\flat_4+$	$a_4 b\flat_4$	c_5+				
$d\flat$	$d\flat_4$	e_3-	b_3+	e_4-	$b\flat_4-$	$d\flat_5$				
d	d_4-	f_3	$d\flat_4+$	f_4	c_5-	d_5-				
$e\flat$	$e\flat_4+$	$g\flat_3$	$d\flat_4$	$g\flat_4$	$d\flat_5-$	$e\flat_5+$				
f	f_4+	$a\flat_3$	$e\flat_4+$	$a\flat_4$	$e\flat_5-$	f_5+				
g	g_4-	$a_3 b\flat_3$	$f_4 g\flat_4$	$b\flat_4+$	e_5	$g\flat_5 g_5$				
$e\flat$		−969	+ 2	+349	+ 948	+1235	+1813	+1934	+2414	+2466
f		−891	−203	+298	+ 930	+1200	+1693	+1923	+2213	
g		−936	−118	+341	+ 950	+1220	+1830	+1906		
$a\flat$		−939	−206	+292	+ 929	+1224	+1692	+1946		
a		−922	−220	+289	+ 964	+1239	+1768	+1910		
$b\flat$		−909	− 71	+358	+ 930	+1239	+1808	+1933		
c		−956	+ 9	+310	+ 943	+1210				
$d\flat$		−930	−182	+265	+ 872	+1197				
d		−875	− 65	+334	+1000	+1210				
$e\flat$		−915	−224	+283	+ 955	+1202				
f		−904	−188	+293	+ 961	+1204				
g		−926	−131	+333	+ 922	+1172				
Average		−923	−133	+312	+ 942	+1213	+1768	+1925	+2314	+2466
Av. Dev.		20	71	26	21	16	50	12	100	
Range		94	233	93	128	67	138	40	201	

strikes—has little to do with the production of partials 4, 6, and 8, so that these partials are relatively faint when the bell is struck in the usual manner by a clapper, whereas in the production of partials 5, 7, and 9 the vibration of the material above the sound bow appears to be unimportant, and these partials—especially 5, which is the lowest of them —will be more strongly brought out by a blow of the clapper.

The Pitches of the Patrial Tones.—The pitches of the partial tones are given in Table II. It will be seen that it is only in a rather rough way that the successive partials form the same intervals for the different bells of the Dorothea Carlile chime. It will also be seen that in every case a partial of even order lies closer to the partial next above it than to the one next below, and an odd partial lies closer to the partial next below it than to the one next above. The averages for all bells are given in Table III.

TABLE III.

Average Interval from One Partial to the Next.

Partials	1, 2	2, 3	3, 4	4, 5	5, 6	6, 7	7, 8	8, 9
Intervals in cents	790	445	630	271	541	158	385	52

This difference in the intervals according as we pass up from an odd or an even partial may to some extent be understood by referring to Table I., where it is seen that the transition from an even partial to the next above it involves an increase in the number of nodal meridians, whereas the transition from an odd partial to the next above does not. For instance, on passing from the fourth partial to the fifth the width of each vibrating segment is reduced by about one fourth, whereas on passing from the fifth partial to the sixth the height of a vibrating segment is reduced by something like a half. If nothing else changed we should therefore expect a larger interval between the fifth and sixth partials than between the fourth and fifth. Table III. shows that this expectation is justified.

4. THE STRIKING NOTES.

Principal and Secondary Striking Notes.—From Table II. it will be seen that there is in general no partial which approximates at all closely to the pitch of the striking note. Thus Blessing's statement that a resonator does not respond to a striking note appears to be correct. Now when a bell is tapped on the waist or shoulder, *i.e.*, above the sound bow, it is known[1] that the bell sounds flatter than when tapped on the sound bow. And when tapping on the waist or shoulder I seemed to hear a note, flatter than the striking note, to which a resonator would not

[1] Helmholtz, Sensations of Tone, 4th Eng. ed., p. 72.

respond. I am therefore using the name *principal striking note* for the note obtained from the sound bow and not reinforced by a resonator, and the name *secondary striking note* for the note obtained from the waist or shoulder and not reinforced by a resonator. The secondary striking note is not as readily heard as the principal striking note. It is usually some 200 cents to 300 cents flatter than the principal striking note and somewhat flatter than the second partial of the bell.

Resonance.—On two of the bells of the Dorothea Carlile chime the principal striking note is very close to the second partial. At some half dozen different times I thought that I picked up with a resonator the principal striking note of some one of the other bells, but later attempts with the same bells seem to show that I was mistaken. A resonator does not, in general, respond to a striking note.

The results were similar when I attempted to get the bells to respond to a tuning fork which could be adjusted by moveable loads to give pitches throughout a range of about an octave. The bell was tapped and the fork adjusted by ear until its pitch was about that of the partial in question. The fork was then struck and its stem pressed against the bell. Various partial tones within the range of the fork responded clearly, but the striking notes did not thus respond.

Beats.—There was no difficulty in tuning a fork until its pitch was about that of a given partial and then hearing distinct beats when both fork and bell were tapped. But no beats could be detected when the fork had approximately the pitch of a striking note.

Difference Tones.—The first explanation of the striking notes which suggests itself is perhaps that they may be combination tones. Blessing says that Rudolph König suggested this possibility, but that no combination of the partial tones of a bell would give the proper frequency. Blessing, however, gives no data in support of his statement, and it seemed worth while to examine the question.

If a striking note is a combination tone of any sort it is most likely that it is a first order difference tone. From the observed frequencies of the various partials the difference tones given in Table IV. have been calculated. This table has reference to the principal striking notes. A calculation for the secondary striking notes shows similar results.

On comparing with Table II. it will be seen that Table IV. includes the difference tones arising from the fifth and seventh partials for all the bells on which the seventh partial was observed. But it will also be seen that in only half of these cases does the difference tone which arises from the fifth and seventh partials lie within a quarter of a step from the principal striking note. Moreover if the striking note were a combina-

TABLE IV.

Difference Tones.

This table includes all the first order difference tones which lie within half a step from the principal striking note. Those which lie within quarter of a step are starred.

Partials.	Bell.	Cents from Prin. Strik. Note Up to Dif. Tone.
2,5	Low *e♭*	+58
	c	+13*
	d	+85
	High *g*	+75
4,6	Low *f*	−91
	a♭	−79
	a	+59
5,7	Low *e♭*	+28*
	Low *f*	+71
	Low *g*	−22*
	a♭	+79
	a	−55
	b♭	+15*
7,8	Low *e♭*	−40*

tion tone we should expect it to be very faint, if heard at all, when the bell is tapped gently. As a matter of fact, the principal striking note comes out clearly when the bell is tapped gently. It seems then to be clear that a striking note is not a difference tone.

Compressional Waves.—It seemed possible that the striking notes might be due to compressional waves which spread through the material of a bell and returned periodically to the point where the bell had been struck. If this were the case the frequency of the principal striking note would be roughly the same as that of the longitudinal vibration of a straight bar which had a length equal to half the circumference of the sound bow.

To enable me to examine this matter the Meneely Bell Company kindly cast for me a rod of bell metal. After the ends had been trued off this rod had a length of 93.0 cm. The frequency of longitudinal vibration of the lowest mode was about 1810 vd.[1] To have the same frequency as that of the low *e♭* bell (304 vd) a rod of bell metal would therefore have to be 553 cm. long. The diameter of the mouth of the

[1] The density of this rod is about 8.86 g./c.c. Its Young's modulus is therefore about $10.0 \cdot 10^{11}$ dynes/cm². This value for the Young's modulus [bell metal = 78 per cent. Cu and 22 per cent. Sn] fits excellently on a curve coördinating the values obtained by Voigt [Wied. An., 48, p. 674, 1893] for copper, tin, and an alloy of 88 per cent. Cu and 12 per cent. Sn. Voigt's values were obtained by forcing a short bar of the given material to vibrate with a frequency of from one to two vibrations per second.

low *e♭* bell is 134 cm., so that the rod required would have a length much greater than half the circumference of the sound bow—in fact considerably in excess of the entire circumference of the mouth. Similar statements hold for the other bells.

Moreover, if the principal striking note were due to a compressional wave running, say, around through the sound bow, the diameters of the sound bows of the various bells should be inversely proportional to the frequencies of the principal striking notes. A calculation of the relative frequencies of the bells on this basis leads to values which are considerably in error—in the cases of three of the bells by something like 100 cents.

From this failure of the diameters of the bells to be proportional to the periods of the principal striking notes, and from the fact that a compressional wave would run through the material of a bell too quickly to give the observed pitch, it is clear that the principal striking note is not produced by a compressional wave.

Misjudged Octave.—From Table II. it will be seen that the pitch of the principal striking note is not far from an octave below the fifth partial of the bell. This checks with Rayleigh's and Simpson's observations that in most cases the only partial which is close to the pitch of a bell is the fifth, and that this fifth partial is an octave higher than the pitch of the bell is supposed to be.

It is well known that an error of an octave in judging the pitch of a note is easily made, and Simpson evidently believed that there is in general no note which has the pitch of the principal striking note, but that we hear most clearly the fifth partial of the bell and think it is an octave lower than it is. If that is the case it would explain why a striking note cannot be picked up from a bell by a resonator nor elicited from the bell by resonance, and why it will not beat with a tuning fork. It would probably also explain a statement made by Blessing that if the sound bow of a bell is gradually turned thinner and thinner the striking note grows fainter and fainter and finally disappears. Blessing says nothing as to how this process affects the partial tones of the bell. But since the fifth partial appears to be produced almost entirely by vibration of material in the sound bow it seems likely that reducing the thickness of the sound bow would weaken the fifth partial.

What evidence is there as to the octave in which the principal striking note lies? There is the judgment of the founders and tuners referred to by Rayleigh and Simpson. As to myself, I have at times felt very sure that the note I heard was really of the pitch which I have called that of the principal striking note and was not an octave higher. At other times

I did not feel so sure. When the chime was being played I have compared the notes which I heard with those on a reed organ. When doing this it seemed to me there could be no question but that the pitches really are those which they are ordinarily supposed to be, *i.e.*, each one an octave below the fifth partial. A statement from the Meneely Bell Company also corroborates this. With reference to the low *f* bell of the Dorothea Carlile chime they write me, "This is supposed to be about F natural above middle C, although bells as a rule, from their nature, seem to sound lower." The evidence seems to show that if the pitch of the principal striking note is determined by the fifth partial the octave is very generally misjudged—even by bell founders.

Is there any probable reason for such an error in judgment? In attempting to answer this question we may exclude from consideration the fourth, sixth, and eighth partials, all of which are faint, if heard at all, when the bell is struck on the sound bow. From Table II. it will be seen that the seventh partial is almost exactly a musical fifth (700 cents) above the fifth partial. Now in the harmonic series of tones to which the notes from strings, pipes, etc., approximate, we are accustomed to hear a fundamental accompanied by its octave and twelfth, so that even if the fifth and seventh partials do not produce a combination tone an octave below the fifth partial, they may nevertheless help to suggest a fundamental note of that pitch.

Another, and probably much more important, reason for misjudging the octave lies in the rates at which the different partials reach their full intensities. The fifth partial seems to reach its maximum intensity almost as soon as the bell is struck, the second and third, especially on the larger bells, not quite so soon. Thus it is possible that when a bell is struck the fifth partial at once attracts attention, and the second and third add a considerable volume of sound so soon afterward as to make the pitch seem an octave lower than that of the fifth partial. On the smaller bells the second and third partials seem to be more prompt in their response, and this may have something to do with the difficulty which bell founders are said to have usually experienced in casting small church bells of good musical quality.

My present hypothesis as to the principal striking note of a bell is, then, that it is a note of which the pitch, except for octave, is determined by the fifth partial, and that the octave in which we think we hear it is determined by the more sluggishly responding second and third partials. It is desirable that a considerable number of bells should be investigated, and that photographic records should be obtained showing the rates at which the various partials, especially the second, third, and fifth, grow to their maximum intensity when the bells are struck in the usual manner by a clapper.

In conclusion I wish to express my thanks to the Meneely Bell Company for their kindness in giving me various data and in casting for me the rod of bell metal.

Smith College,
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