Phenomenological Λ-Nucleon Potentials from s-Shell Hypernuclei. II. Dependence on Intrinsic Range*

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By examining the binding-energy data of the s-shell hypernuclei and the Λ-proton scattering data with a number of effective central Λ-nucleon potentials of various hard-core radii (0–0.6 F) and intrinsic ranges (1.5–2.5 F), a Λ-nucleon potential has been found which yields a very good fit to these experimental data. This potential has an intrinsic range of 2.1 F, a hard-core radius of 0.6 F, and an odd-parity-state strength which is equal to 60% of the even-parity-state strength. Also, it has a rather small degree of spin dependence in the Λ-proton case, with the consequence that the predicted $J=1$ excited state in $_{\Lambda}H^4$ has a small excitation energy of only about 0.1 MeV. The Λ-proton effective-range parameters have the following values: $a_t^p=-2.08$ F, $r_{0t}^p=3.40$ F, $a_s^p=-2.25$ F, and $r_{0s}^p=3.29$ F. The case of Λ-neutron scattering has also been considered. Here it is found that the total cross sections differ appreciably from those of the corresponding Λ-proton case only when the c.m. energy is less than about 4 MeV.

I. INTRODUCTION

In a previous publication[1] (hereafter referred to as HTI), the binding-energy data of the s-shell hypernuclei and the total cross sections of the Λ-proton scattering were examined with two-body, spin-dependent, central Λ-nucleon potentials which have an intrinsic range of 1.5 F and a hard core with a radius of 0, 0.3, 0.45, or 0.6 F. From that study, it was found that the Λ-nucleon interaction very likely has a hard core of radius greater than 0.3 F and an intrinsic range greater than 1.5 F. In this investigation, we continue the study by examining Λ-nucleon potentials with intrinsic ranges equal to 2.0 and 2.5 F. It is hoped that from these studies we can gain a better understanding about the characteristics of the Λ-nucleon interaction.

Recently, some improved experimental data on the binding energies of the s-shell hypernuclei[2] and the total cross sections of Λ-proton scattering[3–5] have appeared. These newer data differ from the older data which we analyzed in HTI in essentially two aspects: (i) the binding energy of $_{\Lambda}H^3$ has a smaller value, and (ii) the Λ-proton scattering cross sections are increased. As far as the scattering cross sections are concerned, both of these features tend to aggravate the discrepancy between the experimental values and the values calculated with Λ-nucleon potentials of intrinsic range equal to 1.5 F. For this latter reason, we feel that an analysis with intrinsic ranges other than 1.5 F becomes even more desirable.

Except for two changes, the procedure which we use here to analyze the experimental data will be similar to that used in HTI. These two changes are as follows. First, we shall take into account quantitatively the charge-symmetry-breaking (CSB) effect when the Λ-proton scattering problem is considered. Second, we shall use the binding-energy data on the three- and four-body hypernuclei to obtain the charge-symmetric (CS) part of the Λ-nucleon interaction, in contrast with what was done in HTI where we used the data on the three- and five-body hypernuclei. This seems to be a desirable modification, since the inclusion of the hypernucleus $_{\Lambda}He^5$ in the anslysis may lead to an underestimate of the triplet interaction appropriate to an isolated Λ-nucleon system. Recently, Dalitz[6] has mentioned a number of situations which could give rise to this underestimate. The most important of these is probably the existence of an isospin suppression effect, originally pointed out by Bodmer.[7] This suppression effect might be quite important in $_{\Lambda}He^5$, but is not expected to be important in $_{\Lambda}H^4$ or $_{\Lambda}He^4$. Also, it should be mentioned that we are analyzing the experimental data with effective central Λ-nucleon potentials which are considered to contain the effect of a possible tensor component. It is well known that, because He^4 is a strongly bound, spin-saturated system, the effectiveness of the tensor interaction is suppressed to a significant degree, depending upon the range of the interaction.

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[5] L. Piekenbrock (private communication). Earlier results have been reported by L. Piekenbrock and F. Oppenheimer, Phys. Rev. Letters 12, 625 (1964).
[6] R. H. Dalitz, invited paper presented at the Topical Conference on the Use of Elementary Particles in Nuclear Structure Studies held at Brussels, September 1965 (unpublished).
[7] A. R. Bodmer, Phys. Rev. 141, 1387 (1966).

TABLE I. Parameters of the CS Λ-nucleon potentials.

Potential type	b (F)	$r_{\Lambda N}$ (F)	λ (F^{-1})
A	1.5	0	2.361
B	1.5	0.30	3.935
C	1.5	0.45	5.902
D	1.5	0.60	11.804
E	2.0	0.45	3.219
F	2.0	0.60	4.427
G	2.5	0.60	2.724

Thus, if ${}_\Lambda He^5$ is used in the analysis, the strength of the triplet Λ-nucleon interaction will also be somewhat underestimated. On the other hand, the tensor suppression effect is expected to be less severe in ${}_\Lambda H^4$, since the structure of this latter hypernucleus is more diffused. In fact, here, we believe that this suppression effect should be relatively unimportant, since, as was mentioned by Dalitz,[6] the tensor component is very likely short-ranged ($\hbar/mc \approx 0.4$ F).

Because of the change in experimental data and the change in the procedure of analysis, we shall include here also the results obtained with potential A, B, C, D of HTI, which have an intrinsic range of 1.5 F.

The effect of S'-state mixing in ${}_\Lambda H^3$, as discussed by Bodmer,[7] will not be included in our calculation. As will be shown below, all the Λ-nucleon potentials which give a reasonable fit to the experimental data have only a small degree of spin dependence, indicating that such a mixing effect cannot be important at all.

In Sec. II, the results of our analysis on the hypernuclear systems with $A=3$–5 will be presented. From these results, we determine the strengths of the Λ-nucleon interactions in the triplet and singlet states. Section III is devoted to a study of the Λ-proton scattering problem. Here, we shall show that even with the relatively crude experimental data presently available, we can still obtain useful information not only about the intrinsic range and the hard-core size of the Λ-nucleon potential but also about the strength of the interaction in odd orbital-angular-momentum states. In Sec. IV, we assess the importance of the isospin and tensor suppression effects by computing the Λ-nucleon potentials from the binding-energy data of ${}_\Lambda H^3$ and ${}_\Lambda He^5$. By comparing the properties of these potentials with those given in Secs. II and III, based on the binding-energy data of the three- and four-body hypernuclei, a quantitative estimate of the importance of these effects in ${}_\Lambda He^5$ can be obtained. Finally, in Sec. V, we discuss and summarize the results of this investigation.

II. ANALYSIS OF s-SHELL HYPERNUCLEI AND Λ-NUCLEON POTENTIALS

A. Analysis of s-Shell Hypernuclei with CS Λ-Nucleon Potentials

Except as otherwise noted, the notations used here have the same meaning as those appearing in HTI. The only point we wish to emphasize is that the depths U_{0t} and U_{0s} refer only to the CS part of the Λ-nucleon interaction.

The nucleon-nucleon potential is the same as that used in HTI. For the CS Λ-nucleon potentials, we consider these with intrinsic ranges equal to 2.0 and 2.5 F. The parameters of these potentials are listed in Table I, where they are referred to as potential E, F, and G, respectively. For convenience, we have also listed in this table the parameters for potential A, B, C, and D, which have been given in HTI.

All the Λ-nucleon potentials listed in Table I have the feature that the intrinsic range of the attractive part when it is centered at the origin has a value less than or equal to 1.5 F, which corresponds to a range of $\hbar/2m_\pi c$ for a Yukawa potential without a hard core. Potentials with a longer range for the attractive part need not be considered, since the mechanism of 2π exchange is the one which gives rise to the longest range potential consistent with charge symmetry.

With trial wave functions which have the same form as those discussed in HTI, the results obtained for E_A or B_Λ as a function of U_{0A} are given in Table II. Using these results and Eq. (12) of HTI, the constants a_A and b_A can then be determined; these are listed also in Table II.

For the hypernucleus ${}_\Lambda H^3$, the CSB contribution cancels out. Therefore, the experimental fact that its spin is equal to $\frac{1}{2}$ [8,9] indicates that the singlet part of the CS Λ-nucleon interaction is stronger than the triplet part. Thus, in the following, we shall only consider the case where U_{0s} is greater than U_{0t}.

We wish to mention that, because of CSB effects, the experimental determination of $J=0$ for ${}_\Lambda H^4$ [8] does not guarantee that U_{0s} is greater than U_{0t}. To reach this latter conclusion about the relative strength of the CS triplet and singlet potentials from the experimental data for the four-body hypernuclei alone, without recourse to any theoretical argument about the nature of the CSB potential, it is necessary to establish experimentally that the spin of ${}_\Lambda He^4$ is also zero.

On the other hand, if one uses the facts that U_{0s} is greater than U_{0t} and the binding energy B_Λ of ${}_\Lambda He^4$ is larger than that of ${}_\Lambda H^4$, then by adopting the conclusion reached by Downs[10] that the CSB potential is predominantly spin dependent (proportional to $\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N$), one can easily see that the spin of ${}_\Lambda He^4$ must be equal to zero.[11]

[8] M. M. Block, R. Gessaroli, J. Kopelman, S. Ratti, M. Schneeberger, L. Grimellini, T. Kikuchi, L. Lendinara, L. Monari, W. Becker, and E. Harth, in *Proceedings of the International Conference on Hyperfragments, St. Cergue, Switzerland, 1963* (CERN, Geneva, 1964), p. 63.

[9] R. H. Dalitz and L. Liu, Phys. Rev. **116**, 1312 (1959).

[10] B. W. Downs, Nuovo Cimento **43**, 459 (1966). See also, B. W. Downs and R. J. N. Phillips, *ibid.* **41**, 374 (1966); R. H. Dalitz and F. von Hippel, Phys. Letters **10**, 153 (1964).

[11] In the following analysis, we shall always consider ${}_\Lambda He^4$ to have a spin equal to zero.

TABLE II. Results of the variational calculation for the s-shell hypernuclei.[a]

Hypernucleus ${}_\Lambda Z^A$	Potential type	U_{0A} (MeV)	E_A (MeV)	B_Λ (MeV)	$\langle r_{NN}^2\rangle^{1/2}$ (F)	a_A (MeV)	b_A (MeV)$^{1/2}$
${}_\Lambda H^3$	E	440.0	-2.41 ± 0.09	0.18 ± 0.09	3.38 ± 0.07	418.9	49.5
		450.0	-2.62 ± 0.08	0.40 ± 0.08	3.24 ± 0.07		
	F	910.0	-2.53 ± 0.08	0.30 ± 0.08	3.28 ± 0.07	863.7	83.9
		925.0	-2.76 ± 0.08	0.53 ± 0.08	3.19 ± 0.07		
	G	325.0	-2.51 ± 0.10	0.29 ± 0.10	3.33 ± 0.07	302.9	41.2
		332.0	-2.72 ± 0.09	0.50 ± 0.09	3.24 ± 0.07		
${}_\Lambda H^4$	E	430.0	-9.76 ± 0.16	2.34 ± 0.17	2.57 ± 0.05	341.0	58.2
		450.0	-10.93 ± 0.19	3.51 ± 0.20	2.50 ± 0.05		
	F	890.0	-9.74 ± 0.16	2.32 ± 0.17	2.57 ± 0.05	760.6	85.0
		920.0	-10.94 ± 0.19	3.52 ± 0.20	2.51 ± 0.05		
	G	320.0	-9.62 ± 0.20	2.20 ± 0.21	2.56 ± 0.05	252.5	45.5
		330.0	-10.32 ± 0.20	2.90 ± 0.21	2.54 ± 0.05		
${}_\Lambda He^5$	E	395.0	-30.99 ± 0.53	2.68 ± 0.56	2.22 ± 0.04	342.1	32.3
		405.0	-32.10 ± 0.48	3.79 ± 0.52	2.21 ± 0.04		
	F	834.0	-30.28 ± 0.56	1.97 ± 0.59	2.22 ± 0.04	763.7	50.1
		865.0	-32.40 ± 0.50	4.09 ± 0.53	2.21 ± 0.04		
	G	300.0	-30.31 ± 0.84	2.00 ± 0.86	2.21 ± 0.04	263.2	26.1
		315.0	-32.27 ± 0.71	3.96 ± 0.73	2.18 ± 0.04		

[a] The statistical accuracy in the value of E_A is obtained with 50 000 estimates for ${}_\Lambda H^3$ and 200 000 estimates for ${}_\Lambda H^4$ and ${}_\Lambda He^5$.

The experimental values of the binding energies of the s-shell hypernuclei are as follows[2]:

$$\begin{aligned} B_\Lambda({}_\Lambda H^3) &= 0.17\pm0.13 \text{ MeV},\\ B_\Lambda({}_\Lambda H^4) &= 1.91\pm0.10 \text{ MeV},\\ B_\Lambda({}_\Lambda He^4) &= 2.20\pm0.06 \text{ MeV},\\ B_\Lambda({}_\Lambda He^5) &= 3.09\pm0.03 \text{ MeV}. \end{aligned} \tag{1}$$

The major point to note is that $B_\Lambda({}_\Lambda H^3)$ has now a smaller value of 0.17 ± 0.13 MeV, instead of the older value of 0.32 ± 0.17 MeV which was used in HTI.

From Eq. (1), we find that

$$\Delta B_\Lambda = B_\Lambda({}_\Lambda He^4) - B_\Lambda({}_\Lambda H^4) = 0.29\pm0.12 \text{ MeV}. \tag{2}$$

As indicated by a number of authors,[1,10] ΔB_Λ would be a negative quantity due to Coulomb effects if the Λ-nucleon interaction were totally charge symmetric. To find the contribution from the Coulomb effects, we shall use the following procedure. We choose a suitable value for U_{04} which yields a value for $B_\Lambda({}_\Lambda H^4)$ approximately equal to 2 MeV. With this same value for U_{04}, the value of $B_\Lambda({}_\Lambda He^4)$ is then computed. Four types of Λ-nucleon potentials have been examined in this way; these are potential C, D, E, and F given in Table I. For all these potentials, $(\Delta B_\Lambda)_{\text{Coulomb}}$ is found to be about -0.25 MeV.[12] Together with Eq. (2), this means that the CSB component of the Λ-nucleon interaction would be required to account for a value of ΔB_Λ equal to 0.54 ± 0.12 MeV.

[12] As mentioned in HTI, the value of $(\Delta B_\Lambda)_{\text{Coulomb}}$ given here is smaller in magnitude than that given by Downs (Ref. 10). The main reason for this is that the difference in the matter radii of H^3 and He^3 ($\approx$0.03 F) obtained by us with a variational calculation is only about half as much as that estimated by Downs with the help of a simple model devised by Dalitz and Thacker, Phys. Rev. Letters **15**, 204 (1965).

With the above estimate for the contribution of the CSB potential, we obtain

$$B_\Lambda^{\text{CS}}({}_\Lambda H^4) = 2.18\pm0.06 \text{ MeV}, \tag{3}$$

and

$$B_\Lambda^{\text{CS}}({}_\Lambda He^4) = 1.93\pm0.06 \text{ MeV}, \tag{4}$$

where $B_\Lambda^{\text{CS}}({}_\Lambda H^4)$ and $B_\Lambda^{\text{CS}}({}_\Lambda He^4)$ denote the binding energies of the Λ particle in ${}_\Lambda H^4$ and ${}_\Lambda He^4$, respectively, if the CS part of the Λ-nucleon interaction alone is considered.[13]

With the values of $B_\Lambda({}_\Lambda H^3)$ given in Eq. (1) and $B_\Lambda^{\text{CS}}({}_\Lambda H^4)$ given in Eq. (3), we obtain from Eq. (12) of HTI the values of U_{03} and U_{04} which correspond to these binding energies. These are listed in Table III, together with the values of U_{0t} and U_{0s} obtained by using Eq. (5) of HTI. From this table, it is seen that for potential A, B, C, and D, the values of U_{0t} and U_{0s} are substantially different from those given in Table VII of HTI, which is a consequence not only of the change in the procedure of analysis but also of the change in the experimental values of B_Λ.

TABLE III. Values of potential depths U_{03}, U_{04}, U_{0t}, and U_{0s}.[a]

Potential type	U_{03} (MeV)	U_{04} (MeV)	U_{0t} (MeV)	U_{0s} (MeV)
A	181.7 ± 6.8	159.4 ± 1.9	114.8 ± 6.6	204.0 ± 9.4
B	646.9 ± 13.1	607.9 ± 3.8	529.8 ± 12.7	685.9 ± 18.4
C	1611.1 ± 22.4	1557.7 ± 6.4	1450.8 ± 21.3	1664.5 ± 30.8
D	7078.4 ± 37.1	6969.5 ± 15.2	6751.9 ± 50.2	7187.2 ± 57.9
E	439.3 ± 9.3	426.9 ± 3.5	402.1 ± 8.8	451.7 ± 14.2
F	898.3 ± 15.8	886.0 ± 5.2	861.4 ± 14.4	910.6 ± 22.3
G	319.9 ± 8.1	319.7 ± 3.4	319.3 ± 9.7	320.1 ± 12.5

[a] The values of ΔU_{0t} and ΔU_{0s} are optimized by considering the uncertainties in the binding-energy data of all the s-shell hypernuclei.

[13] The procedure of obtaining the values of $B_\Lambda^{\text{CS}}({}_\Lambda H^4)$ and $B_\Lambda^{\text{CS}}({}_\Lambda He^4)$ is described in Ref. 10.

TABLE IV. Depths of CSB, Λ-proton, and Λ-neutron potentials.

Potential type	W_0 (MeV)	Λ-proton U_{0t}^p (MeV)	U_{0s}^p (MeV)	s_t^p	s_s^p	Λ-neutron U_{0t}^n (MeV)	U_{0s}^n (MeV)	s_t^n	s_s^n
A	3.5	118.2±6.6	193.6±9.4	0.384±0.022	0.629±0.031	111.3±6.6	214.4±9.4	0.362±0.022	0.697±0.031
B	6.6	536.4±12.7	666.2±18.4	0.628±0.015	0.779±0.022	523.2±12.7	705.6±18.4	0.612±0.015	0.825±0.022
C	10.4	1461.2±21.3	1633.2±30.8	0.760±0.011	0.849±0.016	1440.4±21.3	1695.8±30.8	0.749±0.011	0.882±0.016
D	21.1	6773.0±50.2	7123.8±57.9	0.880±0.007	0.926±0.008	6730.7±50.2	7250.6±57.9	0.875±0.007	0.943±0.008
E	5.5	407.6±8.8	435.3±14.2	0.712±0.015	0.761±0.025	396.6±8.8	468.1±14.2	0.693±0.015	0.818±0.025
F	8.0	869.4±14.4	886.6±22.3	0.804±0.013	0.820±0.021	853.4±14.4	934.7±22.3	0.789±0.013	0.864±0.021
G	4.3	323.6±9.7	307.2±12.5	0.790±0.024	0.750±0.031	315.0±9.7	333.0±12.5	0.769±0.024	0.813±0.031

From Eq. (5) of HTI, it is seen that the maximum values of U_{04} and U_{05} are both equal to U_{03}. Thus, using the central values of U_{03} given in Table III, we find that the maximum values of $B_\Lambda{}^{\rm CS}({}_\Lambda{\rm H}^4)$ consistent with the experimental value of $B_\Lambda({}_\Lambda{\rm H}^3)$ are 4.36, 4.15, 3.85, 3.84, 2.86, 2.64, and 2.20 MeV for potential A, B, C, D, E, F, and G, respectively. Comparing these values with the value of $B_\Lambda{}^{\rm CS}({}_\Lambda{\rm H}^4)$ given in Eq. (3), we note that for potential G, the experimental value is almost equal to the maximum possible value. This indicates that for a hard-core Λ-nucleon potential with a core radius of 0.6 F, it is not necessary to consider cases with an intrinsic range greater than 2.5 F. In fact, since the maximum possible value of $B_\Lambda{}^{\rm CS}({}_\Lambda{\rm H}^4)$ is rather insensitive to the hard-core size, we can liberalize the above statement to make it applicable to Λ-nucleon interactions of smaller core size. Similar calculations can also be made to find the maximum possible values of $B_\Lambda({}_\Lambda{\rm He}^5)$; these turn out to be equal to 16.30, 13.28, 11.48, 10.84, 9.07, 7.24, and 4.75 MeV for potential A, B, C, D, E, F, and G, respectively. Here, one sees that these values are much larger than the experimental value, indicating that no useful information about the intrinsic range can be obtained.

Except for potential A and B, it is seen from Table III that the spin dependence of the CS Λ-nucleon potential is rather small. In particular, for potential G, U_{0s} is almost equal to U_{0t}. This has the consequence that with this potential, the hypernucleus ${}_\Lambda{\rm H}^3$ will have a particle-stable excited state of $J=\frac{3}{2}$ and small excitation energy. If the latter should indeed be the case, then an accurate, experimental determination of $B_\Lambda({}_\Lambda{\rm H}^3)$ might become quite difficult. Fortunately, however, there exists considerable evidence, to be discussed below, which could be used to rule out potential G as a possible candidate to represent the correct Λ-nucleon interaction.

B. CSB, Λ-Proton, and Λ-Neutron Potentials

The CSB part of the Λ-nucleon potential is assumed to have the form

$$U_{\rm CSB}(r)=-\tau_3{}^N\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N W_0\exp[-\lambda(r-r_{\Lambda N})],\quad(5)$$

where $\tau_3{}^N$ denotes the isospin operator which is equal to $+1$ and -1 for Λ-proton and Λ-neutron systems, respectively. This particular form is chosen mainly to simplify numerical computation, but does contain a feature found by Downs,[10] using a theory based on particle mixing in SU_3, that the dominant term in the CSB potential has the factor $\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N$. Also, we have, for simplicity, chosen the range parameter λ to be the same as that in the CS part of the Λ-nucleon potential. This is not a serious limitation, since it has been found by Downs and Phillips[14] that the low-energy properties of the hypernuclear systems seem to be quite independent of the choice of the range parameter in the CSB potential.

Because of the particular choice of $U_{\rm CSB}(r)$, the values of the depth parameter W_0 for the various types of CSB potential[15] can be found, to a very good approximation, by simply using Eq. (12) of HTI. With the CSB potential, the depth of the spin-averaged Λ-nucleon potential in the hypernucleus ${}_\Lambda{\rm H}^4$ is

$$U_{04}({}_\Lambda{\rm H}^4)=U_{04}-W_0,\quad(6)$$

which is equal to $a_4+(1.91)^{1/2}b_4$ by using the experimental value of $B_\Lambda({}_\Lambda{\rm H}^4)$ given in Eq. (1). Thus, with a_4, b_4, and U_{04} already determined, the values of W_0 can be easily calculated; these are listed in Table IV.

The Λ-proton potential has the form

$$\begin{aligned} U_t{}^p(r)&=\infty & (r<r_{\Lambda N})\\ &=-U_{0t}{}^p\exp[-\lambda(r-r_{\Lambda N})], & (r>r_{\Lambda N})\\ U_s{}^p(r)&=\infty & (r<r_{\Lambda N})\quad(7)\\ &=-U_{0s}{}^p\exp[-\lambda(r-r_{\Lambda N})], & (r>r_{\Lambda N})\end{aligned}$$

where the depth parameters $U_{0t}{}^p$ and $U_{0s}{}^p$ are given by

$$U_{0t}{}^p=U_{0t}+W_0,\quad(8)$$

and

$$U_{0s}{}^p=U_{0s}-3W_0.\quad(9)$$

The Λ-neutron potential takes on a similar form, with the depth parameters given by

$$U_{0t}{}^n=U_{0t}-W_0,\quad(10)$$

and

$$U_{0s}{}^n=U_{0s}+3W_0.\quad(11)$$

[14] B. W. Downs and R. J. N. Phillips, Nuovo Cimento **41**, 374 (1966).

[15] There will also be seven types of CSB potential considered. Each of these types has the same values of $r_{\Lambda N}$ and λ as the corresponding CS potential.

TABLE V. Λ-proton and Λ-neutron effective-range parameters.

Potential type	Λ-proton a_t^p (F)	r_{0t}^p (F)	a_s^p (F)	r_{0s}^p (F)	Λ-neutron a_t^n (F)	a_s^n (F)
A	-0.74 ± 0.06	3.48 ± 0.18	-1.94 ± 0.25	2.25 ± 0.10	-0.68	-2.60
B	-0.85 ± 0.07	3.32 ± 0.18	-2.07 ± 0.29	2.16 ± 0.10	-0.78	-2.85
C	-0.97 ± 0.08	3.02 ± 0.16	-2.07 ± 0.30	2.14 ± 0.10	-0.89	-2.87
D	-1.04 ± 0.10	2.87 ± 0.17	-2.16 ± 0.31	2.08 ± 0.10	-0.95	-2.99
E	-1.60 ± 0.15	3.61 ± 0.17	-2.16 ± 0.36	3.15 ± 0.21	-1.44	-3.22
F	-1.84 ± 0.20	3.34 ± 0.17	-2.09 ± 0.37	3.15 ± 0.23	-1.63	-3.15
G	-3.04 ± 0.52	3.75 ± 0.23	-2.33 ± 0.47	4.19 ± 0.39	-2.64	-3.60

For the various types of Λ-nucleon potential, the depths U_{0t}^p, U_{0s}^p, U_{0t}^n, and U_{0s}^n are given, together with the values of W_0, in Table IV, where the values of the well-depth parameters s_t^p, s_s^p, s_t^n, and s_s^n are also listed.

From Table IV, we see that with the types of Λ-nucleon potential considered here, the values of the well-depth parameters are all less than 1, indicating that a bound Λ-proton or Λ-neutron system does not exist. Since it is quite likely that the values of 0.6 F for the hard-core radius and 2.5 F for the intrinsic range represent the upper limits of these quantities, our calculation serves to rule out quite definitely the possibility for the existence of a particle-stable, two-body, Λ-hypernuclear system.

Another interesting feature in Table IV is that for potential G, the value of s_t^p is greater than that of s_s^p. This has the consequence that in this particular case, the ground-state spin of $_\Lambda H^4$ is equal to 1, which is in disagreement with experiment.[16] Thus, we have here a convincing piece of evidence that potential G cannot be a candidate to represent the actual Λ-nucleon interaction. In fact, by a simple interpolation, we can conclude that if the Λ-nucleon interaction has a hard core of radius 0.6 F, the fact that $_\Lambda H^4$ has a ground-state spin of 0 limits the intrinsic range to values less than about 2.3 F.

The values of the effective-range parameters of the Λ-proton potentials are listed in Table V. From this table, we see that the singlet scattering lengths are almost constant at about -2.2 F, independent of the choice of the type of potential. On the other hand, the triplet scattering lengths are sensitively dependent upon the values of the intrinsic range, with the value at $b=2.5$ F about three times as large as that at $b=1.5$ F. Also, it is noted that the values of the triplet and singlet parameters are quite similar when the intrinsic range is equal to 2.0 F, which is not the case when the intrinsic range is equal to 1.5 F.

Also, we have listed the values of the Λ-neutron scattering lengths in Table V.[17] Comparing with the corresponding values for the Λ-proton case, we note that, in magnitude, the value of the singlet scattering length is about 40% larger, while the value of the triplet scattering length is about 10% smaller.[18]

With U_{0t}, U_{0s}, and W_0 determined, we can compute the binding energy $B_\Lambda^*(_\Lambda H^4)$ or $B_\Lambda^*(_\Lambda He^4)$ of a $J=1$ excited state of the hypernucleus $_\Lambda H^4$ or $_\Lambda He^4$.[19] For these states, the spin-averaged well depths are

$$U_{04}^*(_\Lambda H^4)=\tfrac{5}{6}U_{0t}+\tfrac{1}{6}U_{0s}+\tfrac{1}{3}W_0\,, \qquad (12)$$

and

$$U_{04}^*(_\Lambda He^4)=\tfrac{5}{6}U_{0t}+\tfrac{1}{6}U_{0s}-\tfrac{1}{3}W_0\,. \qquad (13)$$

Using these values and Eq. (12) of HTI, the values of B_Λ^* can be easily calculated; these are listed in Table VI.[20] From this table, we see that for the cases with $b=2.0$ F, the values of the excitation energies in $_\Lambda H^4$ are quite small, in distinct contrast with the rather large values found in the cases with $b=1.5$ F.

The values of $B_\Lambda(_\Lambda He^5)$ can be computed by using the values of U_{0t} and U_{0s} given in Table III. For potentials A–G, these turn out to be equal to 5.58, 4.86, 4.73, 4.45, 5.02, 4.82, and 4.68 MeV, respectively. Comparing with the experimental value of 3.09 ± 0.03 MeV, these calculated values of B_Λ are clearly too large.[21] This shows that the isospin and tensor suppression effect in $_\Lambda He^5$ is not unimportant. It is, of course, not possible to determine from this calculation which effect is mainly responsible for this difference, although

TABLE VI. B_Λ^* of $_\Lambda H^4$ or $_\Lambda He^4$.

Potential type	$_\Lambda H^4$: $B_\Lambda^*(_\Lambda H^4)$ (MeV)	Excitation energy (MeV)	$_\Lambda He^4$: $B_\Lambda^*(_\Lambda He^4)$ (MeV)	Excitation energy (MeV)
A	0.49	1.42	0.15	2.05
B	0.58	1.33	0.24	1.96
C	0.75	1.16	0.39	1.81
D	0.75	1.16	0.39	1.81
E	1.50	0.41	1.10	1.10
F	1.73	0.18	1.32	0.88

[16] We should point out also that with potential G, there is no internal consistency in our procedure; this is so, since Eq. (3) is obtained by taking the ground-state spin of $_\Lambda H^4$ as zero.

[17] Since there is not much interest in the Λ-neutron interaction at the present time, we have not computed the uncertainties of the Λ-neutron scattering lengths.

[18] Similar finding has also been reported by Downs (Ref. 10).

[19] Because of the peculiar property of potential G mentioned in a previous paragraph, this particular potential will not be considered in this discussion.

[20] In the calculation of $B_\Lambda^*(_\Lambda He^4)$, we have made a very small error by using the value of $(\Delta B_\Lambda)_{\text{Coulomb}}$ appropriate to the ground state.

[21] Similar finding has also been reported by R. K. Bhaduri, Y. Nogami, and W. van Dijk, Phys. Rev. **155**, 1671 (1967).

TABLE VII. Weizmann-Heidelberg-Maryland results.

E (MeV)	c.m. energy range (MeV)	σ (mb)
3.7	2.7–4.6	223±38
5.5	4.6–6.6	173±24
7.7	6.6–9.0	154±17
10.3	9.0–11.7	109±11
12.9	11.7–14.7	84±8
17.1	14.7–21.6	48±8

we tend to believe that the isospin suppression effect might be the more important one.

III. Λ-PROTON SCATTERING

A. Analysis of Λ-Proton Scattering Data

When the Λ-nucleon interaction is represented by an effective central potential, the binding energies of the s-shell hypernuclei are determined primarily by the s-wave interaction.[22] Therefore, the depths given in Sec. II are those which are appropriate to a Λ-nucleon system in a relative s state.

To analyze the experimental data on Λ-proton scattering in the c.m. energy range of 0–40 MeV, it is, however, necessary to know also the interaction in states with relative orbital angular momentum $l>0$. For this reason, we shall assume the following form for the Λ-proton potentials[23]:

$$U^p(r)=[(1-x)+x\cdot P_{\Lambda N}{}^r]\times\left[\frac{1+P_{\Lambda N}{}^\sigma}{2}U_t{}^p(r)+\frac{1-P_{\Lambda N}{}^\sigma}{2}U_s{}^p(r)\right], \quad (14)$$

where $P_{\Lambda N}{}^r$ is a space-exchange operator, and $U_t{}^p$ and $U_s{}^p$ are given by Eq. (7). The quantity x is a reduction factor; it determines the relative strength of the interactions in even- and odd-parity states. For instance, if x is equal to 0.1, the Λ-proton interaction in odd-parity states is only 80% as strong as that in even-parity states.

The experimental data to be analyzed consist of: (i) six data points of the Weizmann-Heidelberg group[3] in the c.m. energy range of about 2 to 20 MeV; (ii) six data points of the Maryland group[4] in the same energy range; (iii) five data points of Piekenbrock[5] in the range 8–50 MeV; and (iv) one data point of Groves[24] at an average c.m. energy of 37 MeV. Of all these data, those of the Weizmann-Heidelberg and Maryland groups will be emphasized, since they are generally of higher accuracy and contain information on both the total cross sections σ and the forward-to-backward ratios F/B in the angular distributions. Also, because they cover the same energy range, we have further combined them into one set of six data points; this set will be referred to as the Weizmann-Heidelberg-Maryland (WHM) results and its values are listed in Table VII.

The quantity x, which is the only variable parameter in Eq. (14) will be determined by using the information on the total cross sections in the c.m. energy region 20–40 MeV and the F/B ratios. It is necessary to employ these comparatively less accurate data, since the total cross sections in the region with E less than 10 MeV are insensitive to the value of x. Also, we should mention that with a particular type of Λ-proton potential, it is certainly not always possible to obtain a good fit to all the available experimental data with a single variable parameter x; hence, the values of x quoted below are merely those values which give a best possible fit to experiment. After the values of x are determined for all seven types of potential considered in this investigation, we can then use the more accurate WHM data to determine the type of Λ-proton potential which fits the experimental data.

To illustrate the above procedure, let us consider potential F. In Figs. 1 and 2, we have shown the total cross section and the F/B ratio as a function of c.m. energy. From these figures, it is seen that although the value of x cannot be determined precisely due to the crude nature of the experimental data, we can still say quite definitely that it lies in the range from 0.15 to 0.25.

Similarly, we have considered all the other types of Λ-proton potential. The results are as follows. For Λ-proton potentials with an intrinsic range equal to 1.5 F, the value of x is close to 0, indicating that there is very little reduction in strength in odd-parity states.

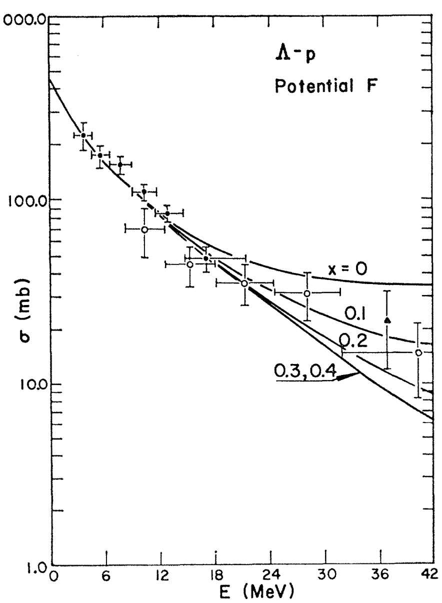

FIG. 1. Λ-proton total cross section as a function of c.m. energy for potential F and various values of x. The experimental data are from Table VII (closed circle), Ref. 5 (open circle), and Ref. 24 (closed triangle).

[22] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958).

[23] For $x>0.5$, the potential represented by Eq. (14) will have a deep hole instead of a hard core, but this need not worry us, since it will not be necessary to examine values of x greater than 0.5 in the analysis of the Λ-proton scattering data.

[24] T. H. Groves, Phys. Rev. **129**, 1372 (1963).

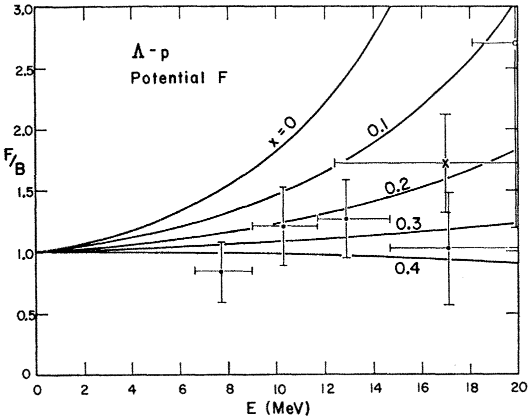

FIG. 2. Λ-proton F/B ratio as a function of c.m. energy for potential F and various values of x. The experimental data are from Ref. 3 (closed circle), Ref. 4 (cross), and Ref. 5 (open circle).

On the other hand, for Λ-proton potentials with intrinsic ranges equal to 2.0 and 2.5 F, the best values of x lie within the range of 0.15–0.25 and 0.25–0.35, respectively, which indicates that with longer-ranged potentials, the reduction in odd-state strength is quite considerable.[25]

A comparison between the calculated values of the total cross sections and the experimental values is shown in Fig. 3. In this figure, the curves are obtained with x equal to 0, 0.2, and 0.3 for the cases with intrinsic ranges equal to 1.5, 2.0, and 2.5 F, respectively. Also, in order to have a quantitative measure of the goodness of fit, we have adopted the usual χ^2 criterion; that is, we define

$$\chi^2=\frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_{\text{calc}}(i)-\sigma_{\text{exp}}(i)}{\Delta\sigma_{\text{exp}}(i)}\right]^2, \tag{15}$$

where $N=6$ is the number of data points in the WHM set, σ_{calc} and σ_{exp} are the calculated and experimental values of the total cross section, respectively, and $\Delta\sigma_{\text{exp}}$ is the associated experimental error. For the various types of Λ-proton potential (A–G) considered here, the values of χ^2 turn out to be equal to 14.0, 10.3, 7.4, 5.4, 3.0, 1.2, and 2.1, respectively. For the cases with $b=1.5$ F, the values of χ^2 are quite large, indicating that Λ-proton potentials with this intrinsic range yield bad fits to the experimental data and can be ruled out for future consideration. The case with the best fit to experiment is that with potential F, which has a core radius of 0.6 F and an intrinsic range of 2.0 F.

From the behavior of the various curves in Fig. 3, one can in fact make the statement that for an effective, central Λ-nucleon potential to yield agreement with the binding-energy data of the three- and four-body hypernuclei, and, at the same time, fit the experimental Λ-proton scattering data, it should have an intrinsic range in the range of 1.9–2.3 F and a hard-core radius rather close to 0.6 F. Also, there must be a reduction in strength in the odd-parity states, with x having a value about equal to 0.2.

To illustrate the influence of the CSB effect, we plot in Fig. 4 the Λ-proton and Λ-neutron total cross sections using potential F with $x=0.2$. Here, we see that the difference between the two cross sections is relatively large only in the very-low-energy region. When the energy becomes larger than about 10 MeV, the difference is almost entirely negligible.

B. An Interpolated Λ-Nucleon Potential

Based on the results in the previous sections, we propose here a Λ-nucleon potential, to be called potential H, which yields a good fit to the binding-energy data of the three- and four-body hypernuclei and the Λ-proton scattering data. This potential has an intrinsic range of 2.1 F and a hard-core radius of 0.6 F. The values of its parameters can be obtained fairly accurately by an interpolation procedure using Tables III and IV and the values of x discussed in a previous paragraph; these values are[26]

$$\begin{aligned}\lambda&=3.935\ \text{F}^{-1},\\ U_{0t}&=676.9\ \text{MeV},\\ U_{0s}&=713.1\ \text{MeV},\\ W_0&=6.9\ \text{MeV},\\ x&=0.2.\end{aligned} \tag{16}$$

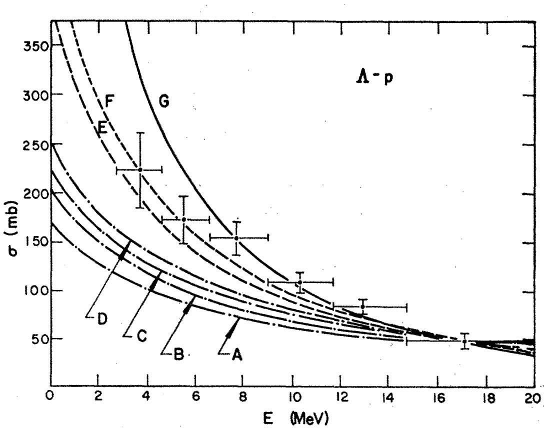

FIG. 3. Λ-proton total cross section as a function of c.m. energy for potential A–G. The experimental data are from Table VII.

[25] Based mostly on calculation of the binding energy of a Λ particle in nuclear matter, a number of authors have commented about the possibility of a reduction in strength in odd-parity states [J. D. Walecka, Nuovo Cimento **16**, 342 (1960); A. R. Bodmer and S. Sampanthar, Nucl. Phys. **31**, 251 (1962); B. W. Downs and W. E. Ware, Phys. Rev. **133**, B133 (1964); B. Ram and B. W. Downs, *ibid.* **133**, B420 (1964); B. W. Downs and R. J. N. Phillips, Nuovo Cimento **33**, 137 (1964); D. M. Brink and M. E. Grypeos, Nucl. Phys. **80**, 681 (1966); B. W. Downs and M. E. Grypeos (to be published); P. Westhaus and J. W. Clark, Phys. Letters **23**, 109 (1966)]. We wish to point out, however, that the Λ-nucleon potentials used by these authors were not the result of a detailed analysis of the hypernuclear binding-energy data.

[26] As was mentioned above, due to the crude nature of the experimental data on F/B, the value of x is not critical.

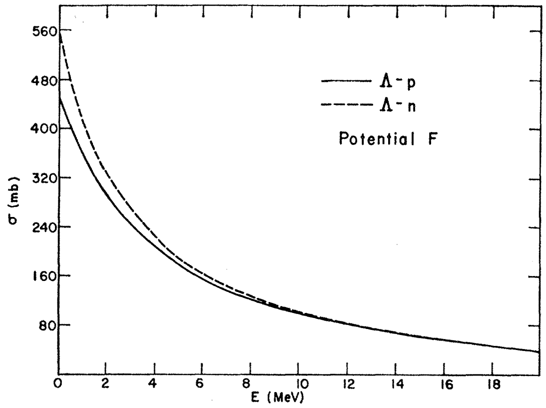

FIG. 4. Λ-proton and Λ-neutron total cross sections as a function of c.m. energy for potential F and x equal to 0.2.

With this potential, σ and F/B are plotted as a function of E in Fig. 5, where we see that the agreement with experiment is indeed quite good. The value of χ^2 is now only about 0.6, which is much smaller than those obtained with the seven types of Λ-nucleon potential considered above.

The values of the well-depth parameters s_t^p and s_s^p are equal to 0.80 and 0.81, respectively. The closeness of these two values indicates that there is only a small degree of spin dependence in the Λ-proton potential, which is also reflected in the fact that the calculated excitation energy of a $J=1$ excited state in ${}_\Lambda H^4$ is only about 0.1 MeV.

The low-energy Λ-proton effective-range parameters are

$$\begin{aligned} a_t^p &= -2.08\text{ F}, \\ r_{0t}^p &= 3.40\text{ F}, \\ a_s^p &= -2.25\text{ F}, \\ r_{0s}^p &= 3.29\text{ F}. \end{aligned} \tag{17}$$

These values are very close to the best values obtained by Karshon[3] from fitting the experimental cross sections using the effective-range theory.

In Table VIII, we list the Λ-proton triplet and singlet phase shifts up to $l=3$. Here, it is seen that all the phase shifts vary slowly with energy, indicating that there is no low-energy narrow resonant level in the Λ-proton system. With these phase shifts, the differential cross sections are illustrated at two energies in Fig. 6. In this figure, the outstanding feature is that the angular distribution is peaked in the forward direction, which is a consequence of the fact that the space-exchange part is not the dominant one in the Λ-proton potential.

TABLE VIII. Λ-proton scattering phase shifts with potential H.

E (MeV)	Triplet phase shifts (deg) δ_0	δ_1	δ_2	δ_3	Singlet phase shifts (deg) δ_0	δ_1	δ_2	δ_3
1.0	17.12	0.04	0	0	18.37	0.04	0	0
2.0	21.81	0.12	0	0	23.28	0.13	0	0
3.7	25.46	0.30	0.01	0	27.00	0.31	0.01	0
5.5	26.99	0.53	0.02	0	28.50	0.54	0.02	0
7.7	27.48	0.85	0.04	0	28.92	0.87	0.04	0
10.3	27.11	1.26	0.08	0	28.47	1.29	0.08	0
12.9	26.24	1.70	0.14	0	27.53	1.74	0.14	0
17.1	24.34	2.41	0.27	0	25.53	2.47	0.27	0
20.0	22.87	2.90	0.39	0.01	24.00	2.98	0.39	0.01
30.0	17.53	4.46	0.95	0.03	18.52	4.58	0.97	0.03
40.0	12.32	5.67	1.76	0.07	13.21	5.83	1.80	0.07

TABLE IX. Properties of Λ-nucleon potentials from analysis of ${}_\Lambda H^3$ and ${}_\Lambda He^5$.

Potential type	U_{03} (MeV)	U_{05} (MeV)	U_{0t} (MeV)	U_{0s} (MeV)	Λ-proton a_t^p (F)	r_{0t}^p (F)	a_s^p (F)	r_{0s}^p (F)
A	181.7	120.9	90.5	212.1	−0.52	4.28	−2.18	2.17
B	646.9	544.6	493.4	698.0	−0.66	3.95	−2.28	2.10
C	1611.1	1467.3	1395.3	1683.0	−0.77	3.52	−2.25	2.08
D	7078.4	6795.8	6654.5	7219.6	−0.85	3.25	−2.31	2.05
E	439.3	398.9	378.7	459.5	−1.26	4.14	−2.37	3.03
F	898.3	851.7	828.5	921.6	−1.44	3.79	−2.29	3.05
G	319.9	309.0	303.5	325.3	−2.35	4.17	−2.52	4.04

IV. STUDY OF Λ-NUCLEON POTENTIALS FROM ${}_\Lambda H^3$ AND ${}_\Lambda He^5$

In this section, we present the results of a brief analysis where we find the values of U_{0t} and U_{0s} from the binding-energy data of ${}_\Lambda H^3$ and ${}_\Lambda He^5$. As is mentioned in the Introduction, the purpose of this brief analysis is to obtain a quantitative estimate about the importance of the isospin and tensor suppression effects in ${}_\Lambda He^5$.

The procedure of analysis is similar to that in Sec. II and the results for the various quantities of interest are given in Table IX. To obtain the values of the Λ-proton effective-range parameters listed in this table, we have also used the values of W_0 in Table IV.

By comparing the values of the effective-range parameters listed in Tables V and IX, we note that there is a relatively large difference between the corresponding triplet parameters. For instance, with the potentials which are of particular interest, namely,

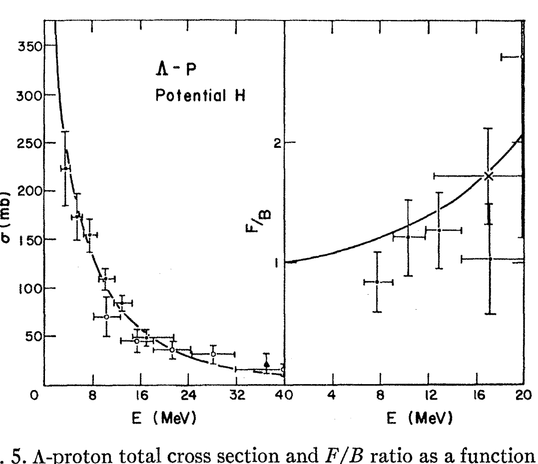

FIG. 5. Λ-proton total cross section and F/B ratio as a function of c.m. energy for potential H.

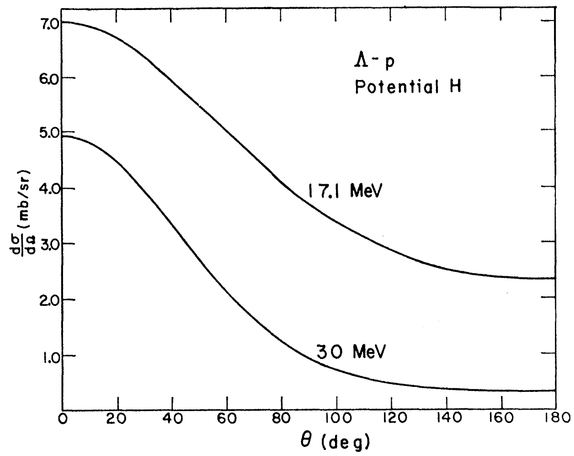

FIG. 6. Λ-proton angular distributions at c.m. energies of 17.1 and 30 MeV for potential H.

potential E and F, the values of a_t are different by about 0.4 F. This shows that for a Λ-nucleon potential with an intrinsic range of about 2.0 F, the isospin and tensor suppression effects may indeed be quite important in $_\Lambda He^5$ and this latter hypernucleus is not suitable in an analysis to obtain the interaction strength appropriate to an isolated Λ-nucleon system.

Also, we have considered the Λ-proton scattering problem in the same way as that in Sec. III. Here again, we have found that with the Λ-proton potentials of this section, the total cross sections computed with potential E and F are smaller than those reported in Sec. III by a rather large amount of about 20%, thus further supporting our statement made in the preceding paragraph about the importance of the isospin and tensor suppression effects in $_\Lambda He^5$.[27]

V. CONCLUSION

In this investigation, the binding-energy data of the s-shell hypernuclei and the Λ-proton scattering data have been examined with seven types of effective two-body central Λ-nucleon potentials, which cover a range of hard-core radius from 0 to 0.6 F and a range of intrinsic range from 1.5 to 2.5 F. The results show that to obtain a good agreement with these experimental data, the Λ-nucleon potential must have an intrinsic range between 1.9 and 2.3 F and a hard-core radius close to 0.6 F. Also, there are strong indications of a reduction in strength in odd-parity states. By examining the experimental data on the forward-to-backward ratios in the angular distributions, we conclude that the interaction strength in odd-parity states is only about half as big as that in even-parity states.

Based on the results obtained with these types of Λ-nucleon potential, we have found, by an interpolation procedure, a Λ-nucleon potential which yields a very good fit to the experimental data. This potential has an intrinsic range of 2.1 F, a hard-core radius of 0.6 F, and an odd-parity-state strength equal to 60% of the even-parity-state strength. Also, it has a rather small degree of spin dependence in the Λ-proton case. This has the consequence that the predicted $J=1$ excited state in $_\Lambda H^4$ has a small excitation energy of only about 0.1 MeV. Further, we have concluded that with this potential, there is no bound or low-lying narrow resonant state in the Λ-nucleon system, which is, of course, consistent with the fact that no hyperdeuteron has ever been found.

Studies of Λ-nucleon interaction with the one-boson-exchange model by Downs and Phillips[28] indicated that the attractive part is given in large part by the exchange of a spin-isospin scalar boson with a mass about equal to that of the K meson.[29] In this respect, it is interesting to note that the attractive part of our proposed Λ-nucleon potential, when it is centered at the origin, has an intrinsic range of 0.9 F which corresponds to the mechanism of such a particle exchange.

Our finding of a reduction in strength in odd-parity states is also an interesting one. In a previous calculation on $_\Lambda C^{13}$,[30] we have found that with an ordinary Λ-nucleon interaction, the calculated value of B_Λ is about 2 MeV larger than the experimental value. With the introduction of a space-exchange component, it is very likely that this discrepancy in the value of B_Λ can be removed.[31]

At present, we are using this proposed Λ-nucleon potential to calculate the binding energy of a Λ particle in nuclear matter and the B_Λ values of a number of p-shell hypernuclei. It is hoped that with these calculations, we cannot only gain an even better understanding about the characteristics of the Λ-nucleon interaction but also learn the structure of the p-shell hypernuclei.

ACKNOWLEDGMENTS

We wish to express our sincere gratitude to Dr. D. H. Davis, Dr. U. Karshon, Professor B. Kehoe, and L. Piekenbrock for sending us their results prior to publication.

[27] For potentials A, B, C, and D which have an intrinsic range of 1.5 F, the total cross sections obtained here are, however, less than 10% smaller than those reported in Sec. III.

[28] B. W. Downs and R. J. N. Phillips, Nuovo Cimento **36**, 120 (1965).

[29] R. A. Bryan and B. L. Scott, Phys. Rev. **135**, B434 (1964).

[30] R. C. Herndon and Y. C. Tang, Phys. Rev. **149**, 735 (1966).

[31] See also the discussion by D. M. Brink and M. E. Grypeos, Nucl. Phys. **80**, 681 (1966).