Resonating-Group Calculation of He^3-He^3 Scattering*

D. R. THOMPSON AND Y. C. TANG
School of Physics, University of Minnesota, Minneapolis, Minnesota
(Received 8 March 1967)

The He^3-He^3 elastic scattering is considered in the c.m. energy range of 0–20 MeV using the resonating-group method in the one-channel approximation. A two-body central potential of Gaussian form which fits the low-energy nucleon-nucleon scattering data as well as possible is used. The saturation character which is not contained in this potential is approximately taken into account by fixing the radii of the clusters according to experimental data. Phase shifts up to $l=6$ are calculated, and a rather broad $l=3$ resonant level in Be^6 with an excitation energy of about 25 MeV is predicted. Angular distributions are calculated at ten energies. The agreement with experimental data in the range 1.5 to 6.0 MeV is excellent, while that with the data from 6.0 to 12.0 MeV is somewhat worse, but still satisfactory. An optical-model analysis is also performed at an energy of 12 MeV and a reaction cross section of about 450 mb is predicted. This indicates that, at higher energies, channels other than the He^3-He^3 channel should be included in the resonating-group calculation to achieve a better agreement with experiment.

I. INTRODUCTION

In this work the method of resonating-group structure[1] is used to study the elastic scattering of incident He^3 particles from a He^3 target. The main objective is to determine if the resonating-group method in the one-channel approximation can give the proper scattering cross sections for the He^3-He^3 system. So far, this method has been used to treat quite successfully the scattering problems of n-d,[2] n-t,[3] n-He^3,[3] n-α,[4] d-d,[5] d-t,[6] He^3-α,[7] and α-α.[8] The He^3-He^3 case has also been studied previously by Bransden and Hamilton,[9] but the results of that calculation showed marked disagreement with the experimental data of Tombrello and Bacher,[10] and Gammel *et al.*[11] This is rather surprising, since, because of the Pauli exclusion principle, the two He^3 clusters are not expected to overlap with each other strongly, which means that, at least in the low-energy region, a calculation with the one-channel approximation should yield results in reasonable agreement with experiment.

Also, we hope to gain from this calculation some information about the structure of the compound nucleus Be^6. Recently, Batty *et al.*[12] have found from experiment two excited levels of Be^6 above the He^3-He^3 threshold. If these levels should have predominantly a He^3 plus He^3 cluster structure, then their presence would be predicted from our results on the scattering phase shifts. Indeed, in a similar calculation on He^3-α scattering,[7] the presence of ^2F excited states has been correctly determined in this way.

The main advantage of the resonating-group method lies in the fact that a two-body potential is used in the calculation. This avoids the difficulty of the optical model, for example, which has a number of variable parameters. Another advantage is that the indistinguishability of the nucleons is taken into account correctly. This latter feature is achieved through the use of a totally antisymmetrized wave function in the calculation.

To simplify the calculation, only the He^3-He^3 channel is considered in computing the elastic-scattering cross sections. In particular, the open p-Li^5 and $2p$-α reaction channels will be ignored. This seems to be a reasonable assumption especially in the low-energy region, since it has been determined experimentally[13] that the total reaction cross sections at incident energies less than 6 MeV [14] are rather small. On the other hand, for energies greater than 6 MeV, the reaction cross sections begin to increase quite rapidly, indicating that our assumption may lead to less accurate results.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] J. A. Wheeler, Phys. Rev. **52**, 1083 (1937); see also K. Wildermuth and T. Kanellopoulos, Nucl. Phys. **7**, 150 (1958); **9**, 449 (1958).

[2] R. A. Buckingham, S. J. Hubbard, and H. S. W. Massey, Proc. Roy. Soc. (London) **A211**, 183 (1952); P. G. Burke and H. H. Robertson, Proc. Phys. Soc. (London) **A70**, 777 (1957).

[3] B. H. Bransden, H. H. Robertson, and P. Swan, Proc. Phys. Soc. (London) **A69**, 877 (1956).

[4] S. Hochberg, H. S. W. Massey, and L. H. Underhill, Proc. Phys. Soc. (London) **A67**, 957 (1954); S. Hochberg, H. S. W. Massey, H. H. Robertson, and L. H. Underhill, Proc. Phys. Soc. (London) **A68**, 746 (1955).

[5] W. Laskar, C. Tate, and P. G. Burke, in *Nuclear Forces and the Few Nucleon Problem*, edited by T. C. Griffith and E. A. Power (Pergamon Press, Inc., New York, 1960), Vol. II, p. 559.

[6] W. Laskar, C. Tate, B. Pardoe, and P. G. Burke, Proc. Phys. Soc. (London) **77**, 1014 (1961).

[7] Y. C. Tang, E. Schmid, and K. Wildermuth, Phys. Rev. **131**, 2631 (1963).

[8] A. C. Butcher and J. M. McNamee, Proc. Phys. Soc. (London) **74**, 529 (1959); E. W. Schmid and K. Wildermuth, Nucl. Phys. **26**, 463 (1961); S. Okai and S. C. Park, Phys. Rev. **145**, 787 (1966).

[9] B. H. Bransden and R. A. H. Hamilton, Proc. Phys. Soc. (London) **76**, 987 (1960); also in *Nuclear Forces and the Few Nucleon Problem*, edited by T. C. Griffith and E. A. Power (Pergamon Press, Inc., New York, 1960).

[10] T. A. Tombrello and A. D. Bacher, Phys. Rev. **130**, 1108 (1963).

[11] J. L. Gammel, J. E. Brolley, L. Rosen, and L. Stewart, in *Proceedings of the International Conference on Nuclear Structure*, edited by D. A. Bromely and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 215.

[12] C. J. Batty, E. Friedman, P. C. Rowe, and J. B. Hunt, Phys. Letters **19**, 35 (1965).

[13] A. D. Bacher, Ph.D. thesis, California Institute of Technology, 1966 (unpublished).

[14] Unless otherwise stated, all the energy values referred to are those in the c.m. system.

The potential used to represent the two-body interaction is assumed to have a Gaussian form and to be purely central. It is chosen to fit the low-energy two-nucleon scattering data as well as possible. The saturating character, which is not contained in this potential, is crudely accounted for by fixing the size of the clusters according to experimental data. As for the space-exchange nature, the nucleon-nucleon scattering data requires that it should be close to a Serber type. However, in this calculation, since a number of open and closed channels has been neglected, we shall vary the exchange mixture to yield the best agreement with experiment. If the resultant mixture should turn out to be very different from a Serber mixture, this would indicate that the approximations used in this calculation are not too accurate.

An optical-model analysis is also performed at an energy of 12 MeV. The purpose of this analysis is to see, in a crude manner, how the inclusion of the reaction channels can affect the He^3-He^3 elastic-scattering cross sections.

A brief discussion of the formulation of the resonating-group method, as well as an account of the computational technique, is given in Sec. II. Sections III and IV contain the results of the resonating-group calculation and the optical-model analysis, respectively. Finally, in Sec. V we summarize the results of this investigation and discuss the various approximations which have been made.

II. FORMULATION

Using only the He^3-He^3 channel in the calculation, one writes the wave function of the six-body scattering system as

$$\Psi=A\{\phi_1\phi_2F(\mathbf{R}_1-\mathbf{R}_2)\xi(\sigma,\tau)\}, \tag{1}$$

where the operator A signifies the complete antisymmetrization of the wave function with respect to the exchange of all pairs of particles and $\xi(\sigma,\tau)$ is a function describing the charge and spin of the system. The functions ϕ_1 and ϕ_2 describe the spatial behavior of the two He^3 clusters; they have the form

$$\phi_1=\exp[-\tfrac{1}{2}\alpha\sum_{i=1}^{3}(\mathbf{r}_i-\mathbf{R}_1)^2], \tag{2}$$

and

$$\phi_2=\exp[-\tfrac{1}{2}\alpha\sum_{i=4}^{6}(\mathbf{r}_i-\mathbf{R}_2)^2], \tag{3}$$

with $\mathbf{R}_1$ and $\mathbf{R}_2$ being the position vectors of the center of mass of the two clusters, respectively. A Gaussian form is chosen for these functions to facilitate the analytical evaluation of the various integrals involved in this problem. The scattering function $F(\mathbf{R}_1-\mathbf{R}_2)$, which describes the relative motion of the two clusters, is determined from the variational principle

$$\delta\int\Psi^*(H-E')\Psi d\tau=0, \tag{4}$$

where E' is the total energy of the system and H is the Hamiltonian given by

$$H=-\frac{\hbar^2}{2m}\sum_{i=1}^{6}\nabla_i^2+\sum_{i>j=1}^{6}V_{ij}. \tag{5}$$

The two-body interaction V_{ij} is assumed to be purely central and of the form

$$V_{ij}=-V_0\exp(-\kappa r_{ij}^2)(w+mP_{ij}^r+bP_{ij}^\sigma-hP_{ij}^\tau)+\frac{e^2\epsilon_{ij}}{r_{ij}}, \tag{6}$$

where P_{ij}^r, P_{ij}^σ, and P_{ij}^τ are the space, spin, and isospin exchange operators, respectively, and ϵ_{ij} is equal to one if particles i and j are protons, and zero otherwise. The constants w, m, b, and h satisfy the equations

$$\begin{aligned}&w+m+b+h=1,\\&w+m-b-h=x,\end{aligned} \tag{7}$$

where x is the ratio of the n-p singlet to triplet interaction.

By integrating over the internal coordinates of the clusters, it is possible to derive an integrodifferential equation satisfied by the scattering function $F(\mathbf{r})$. This equation has the form

$$\left\{\frac{\hbar^2}{3m}\nabla^2+E-V_D(\mathbf{r})-V_C(\mathbf{r})\right\}F(\mathbf{r})=\int K(\mathbf{r},\mathbf{r}')F(\mathbf{r}')d\mathbf{r}', \tag{8}$$

where E is the relative energy of the two clusters in the c.m. system. The potential $V_D(r)$ denotes the direct interaction between the clusters and is given by

$$V_D(r)=-V_0\left(\frac{3\alpha}{3\alpha+4\kappa}\right)^{3/2}[9w-2m+4b-5h]\times\exp\left(\frac{-3\alpha\kappa r^2}{3\alpha+4\kappa}\right). \tag{9}$$

The kernel $K(\mathbf{r},\mathbf{r}')$ represents the nonlocal interaction between the clusters; it can be expanded in terms of Legendre polynomials as

$$K(\mathbf{r},\mathbf{r}')=\frac{1}{4\pi rr'}\sum_{l=0}^{\infty}(2l+1)k_l(r,r')P_l(\mu), \tag{10a}$$

where $\mu=\mathbf{r}\cdot\mathbf{r}'/rr'$. With Eq. (10a), $k_l(r,r')$ can be

obtained from the equation

$$k_l(r,r')=2\pi rr'\int_{-1}^{1}K(\mathbf{r},\mathbf{r}')P_l(\mu)d\mu. \tag{10b}$$

The explicit form of $k_l(r,r')$ is

$$k_l(r,r')=-\frac{\hbar^2}{2m}\left(\frac{27\alpha}{16\pi}\right)^{3/2}C_l\exp\left[-\frac{15\alpha}{16}(r^2+r'^2)\right]\left[\frac{45\alpha}{4}S_l\left(-\frac{9\alpha}{8}\right)-\frac{81}{32}\alpha^2(r^2+r'^2)S_l\left(-\frac{9\alpha}{8}\right)+\frac{63}{16}\alpha^2rr'T_l\left(-\frac{9\alpha}{8}\right)\right]$$
$$-\left(\frac{27\alpha}{16\pi}\right)^{3/2}V_0\left\{\left(\frac{\alpha}{\alpha+2\kappa}\right)^{3/2}[(-2w-2m)C_l-(2b+2h)(3-C_l)]S_l\left(-\frac{9\alpha}{8}\right)\exp\left[-\frac{15\alpha}{16}(r^2+r'^2)\right]+\left(\frac{\alpha}{\alpha+\kappa}\right)^{3/2}\right.$$
$$\times[-4wC_l+4m(2C_l-3)-2b(2C_l-1)+2h(3C_l-4)]S_l\left(-\frac{9}{8}\frac{\alpha^2}{\alpha+\kappa}\right)\exp\left[-\frac{15\alpha^2+24\alpha\kappa}{16(\alpha+\kappa)}(r^2+r'^2)\right]$$
$$+[9m-5b-wC_l+h(C_l+2)]S_l\left(-\frac{9\alpha+36\kappa}{8}\right)\exp\left[-\frac{15\alpha+36\kappa}{16}(r^2+r'^2)\right]-2\left(\frac{4\alpha}{4\alpha+5\kappa}\right)^{3/2}[(2w+2m)C_l$$
$$-(b+h)(3-C_l)]S_l\left[-\frac{9\alpha}{2}\left(\frac{\alpha+2\kappa}{4\alpha+5\kappa}\right)\right]\left[\exp\left(-\frac{15\alpha^2+21\alpha\kappa}{16\alpha+20\kappa}r^2-\frac{15\alpha^2+39\alpha\kappa}{16\alpha+20\kappa}r'^2\right)+\exp\left(-\frac{15\alpha^2+39\alpha\kappa}{16\alpha+20\kappa}r^2\right.\right.$$
$$\left.\left.-\frac{15\alpha+21\alpha\kappa}{16\alpha+20\kappa}r'^2\right)\right]-\left(\frac{16\pi}{27\alpha+36\kappa}\right)^{3/2}(b-m)(2-C_l)\exp\left(-\frac{3\alpha\kappa}{3\alpha+4\kappa}r'^2\right)\delta(r-r')\left.\right\}+\left(\frac{27\alpha}{16\pi}\right)^{3/2}C_lE'S_l\left(-\frac{9\alpha}{8}\right)$$
$$\times\exp\left[-\frac{15\alpha}{16}(r^2+r'^2)\right], \tag{11}$$

where $C_l=2-(-1)^l$.[15] In the above expression, $S_l(\lambda)$ and $T_l(\lambda)$ are given by

$$S_l(\lambda)=\frac{4\pi}{\lambda}\mathcal{J}_{l+\frac{1}{2}}(\lambda rr'), \tag{12}$$

and

$$T_l(\lambda)=\frac{4\pi}{\lambda}\left[\mathcal{J}_{l+\frac{3}{2}}(\lambda rr')-\frac{l}{\lambda rr'}\mathcal{J}_{l+\frac{1}{2}}(\lambda rr')\right], \tag{13}$$

where $\mathcal{J}(x)$ is a hyperbolic spherical Bessel function. For the computation of the Coulomb potential V_c between the clusters, only the unantisymmetrized part of the wave function will be used. This is a reasonable approximation, since the Coulomb interaction between two protons is long-ranged.[16] Thus, with this approximation, we obtain

$$V_c(r)=\frac{4e^2}{r}\Phi\left[\left(\frac{3\alpha}{4}\right)^{1/2}r\right], \tag{14}$$

where $\Phi(u)$ is defined as

$$\Phi(u)=\frac{2}{\sqrt{\pi}}\int_0^u\exp(-t^2)dt. \tag{15}$$

To solve Eq. (8), $F(\mathbf{r})$ is expanded as

$$F(\mathbf{r})=\sum_l\frac{f_l(r)}{r}P_l(\cos\theta). \tag{16}$$

Together with Eq. (10), this yields the equation

$$\left\{\frac{\hbar^2}{3m}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+E-V_D(r)-V_c(r)\right\}f_l(r)=\int_0^\infty k_l(r,r')f_l(r')dr', \tag{17}$$

which will be solved by a numerical procedure.

The scattering phase shifts can be found by solving Eq. (17) with the boundary conditions

$$f_l(0)=0, \tag{18}$$

and

$$f_l(r)\sim\sin(kr-\tfrac{1}{2}l\pi-\eta\ln 2kr+\sigma_l+\delta_l), \tag{19}$$

where $\eta=4e^2/\hbar v$, with v being the relative velocity of the two clusters at infinity. The quantities σ_l and δ_l are the Coulomb and nuclear phase shifts, respectively. The differential cross section is given by

$$\sigma(\theta)=\tfrac{3}{4}\sigma_t(\theta)+\tfrac{1}{4}\sigma_s(\theta), \tag{20}$$

with

$$\sigma_t(\theta)=|f(\theta)-f(\pi-\theta)|^2, \tag{21}$$

and

$$\sigma_s(\theta)=|f(\theta)+f(\pi-\theta)|^2. \tag{22}$$

[15] Factors such as C_l occur, since for the He^3-He^3 system, the total spin S and the orbital angular momentum l are correlated by the equation $S=\frac{1}{2}[1-(-1)^l]$.

[16] In a similar calculation on α-α scattering [S. Okai and S. C. Park, Phys. Rev. **145**, 787 (1966)], it has indeed been shown that the exchange Coulomb potential makes very little contribution.

In the above equations, the subscripts t and s refer to triplet and singlet spin states, respectively, and the scattering amplitude $f(\theta)$ has the form

$$f(\theta)=-\frac{\eta}{2k\sin^2\frac{1}{2}\theta}\exp[-i\eta\ln(\sin^2\tfrac{1}{2}\theta)]$$
$$+\sum_{l=0}^{\infty}\frac{1}{k}(2l+1)\exp[2i(\sigma_l-\sigma_0)+i\delta_l]$$
$$\times\sin\delta_l P_l(\cos\theta). \quad (23)$$

As mentioned in the Introduction, the two-body potential of Eq. (6) is chosen to give the best possible fit to the low-energy scattering data. The parameters for this potential are

$$V_0=72.98\text{ MeV},$$
$$\kappa=0.46\text{ F}^{-2}, \quad (24)$$
$$x=0.63.$$

With this potential, the values of the n-p effective range parameters are

$$a_t=5.36\text{ F},$$
$$r_{0t}=1.70\text{ F},$$
$$a_s=-14.63\text{ F}, \quad (25)$$
$$r_{0s}=2.28\text{ F}.$$

These values are in fairly good agreement with those determined experimentally.[17]

The width parameter α of the He^3 cluster is chosen to yield the experimentally determined value for the rms radius of the nucleon distribution. This latter value[18] of 1.70 ± 0.10 F is obtained from the bare form factor tabulated by Srivastava[19] for the three-nucleon system using the experimental data of Collard *et al.*[20] and an expression derived by Schiff.[21] With this procedure, we obtain

$$\alpha=0.36\text{ F}^{-2}, \quad (26)$$

which corresponds to a rms radius of 1.67 F.

The quantities w, m, b, and h in Eq. (6) can be varied, subject to the conditions expressed in Eq. (7), to yield the best agreement with experiment. In this calculation we have, however, not varied the two free parameters in this way. Rather, we have expressed the two-body potential in the form

$$V_{ij}=yV_{\text{Serber}}+(1-y)V_{\text{symmetric}}, \quad (27)$$

where V_{Serber} is a two-body potential given by Eq. (6)

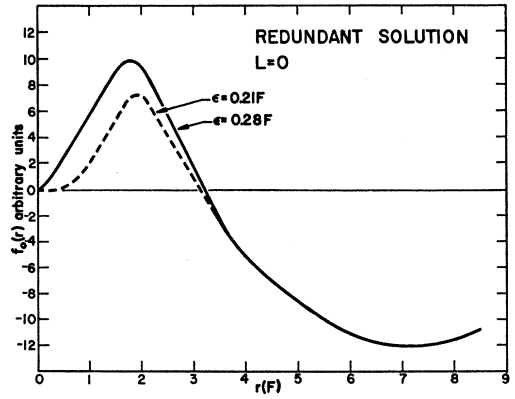

FIG. 1. Demonstration of the $l=0$ redundant solution with $\alpha=0.50$ F^{-2}, $y=1$, and a c.m. energy of 1.7 MeV.

with $w=m$, and $b=h$, and $V_{\text{symmetric}}$ is obtained from Eq. (6) with $m=2b$ and $h=2w$. The parameter y is then varied. If the approximations used in this calculation are reasonably valid, then the resultant value for y should not be too different from 1.

The solution of Eq. (17) is carried out numerically in a manner similar to that used in other calculations of this type.[7] Briefly, what one does is to divide the region of integration into two parts, separated at the point $r=R_M$ such that $k_l(r,r')$ has a vanishingly small value for $r>R_M$. In the region $r\leq R_M$, the kernel is tabulated in the form of a 40×40 matrix at intervals ϵ in r and r'. The integrodifferential equation is then converted into a system of algebraic equations and solved by a method described in a paper by Robertson.[22] In the region $r>R_M$, the kernel is set as zero and Eq. (17) becomes an ordinary differential equation which can be solved using a method of Fox and Goodwin.[23] Since the kernel is fairly long-ranged and has a rapid variation for small values of r and r',[24] care should be taken in choosing a proper value for ϵ. In this calculation it is found that values of ϵ between 0.20 and 0.30 F yield phase shifts which are equal to within 0.1 deg. Therefore, we have set $\epsilon=0.24$ F which corresponds to $R_M=9.6$ F in the calculations that follow.

It is a standard procedure in this type of calculation to check the derivation of the kernel $k_l(r,r')$ by looking for redundant solutions.[4] In our calculation, if the Coulomb term in Eq. (17) is set equal to zero, redundant solutions should appear for $l\leq1$.[25] These redundant solutions may be exhibited by plotting $f_l(r)$ obtained by using different values of ϵ. If the kernel is derived correctly, the functions $f_l(r)$ for $l\leq1$ should differ at small values of r but remain the same in the asymptotic region. On the other hand, for $l>1$, $f_l(r)$

[17] H. Pierre Noyes, Phys. Rev. **130**, 2025 (1963).
[18] Y. C. Tang, E. W. Schmid, and R. C. Herndon, Nucl. Phys. **65**, 203 (1965).
[19] B. K. Srivastava, Phys. Rev. **133**, B545 (1964).
[20] H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. Letters **11**, 132 (1963).
[21] L. I. Schiff, Phys. Rev. **133**, B802 (1964).
[22] H. H. Robertson, Proc. Cambridge Phil. Soc. **52**, 538 (1956).
[23] L. Fox and E. T. Goodwin, Proc. Cambridge Phil. Soc. **45**, 373 (1949).
[24] See Ref. 2 and also J. W. Humberston, Proc. Phys. Soc. (London) **78**, 1157 (1961).
[25] The Coulomb term must be excluded, since it is derived using an unantisymmetrized wave function.

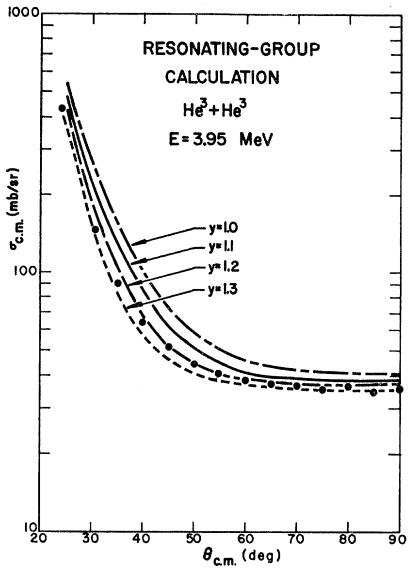

FIG. 2. Differential cross section as a function of y at a c.m. energy of 3.95 MeV with $\alpha=0.36$ F^{-2} and the potential of Eq. (6) parametrized by Eq. (24). The experimental data are those of Tombrello and Bacher (Ref. 10).

should be independent of ϵ for all values of r.[26] Figure 1 demonstrates the $l=0$ redundant solution for $\epsilon=0.21$ and 0.28 F. A similar behavior was also observed for $f_1(r)$. Thus, we conclude that the kernel given by Eq. (11) is indeed correct.

The problem is solved on a CDC 1604 computer. For a given incident energy, phase shifts up to $l=6$ and differential cross sections at 5-deg intervals are computed. A run for one incident energy value takes about 1 min.

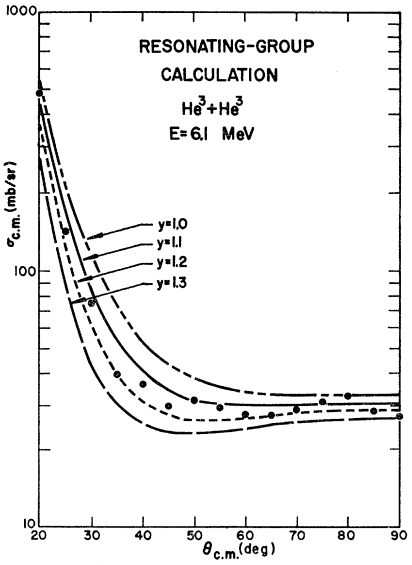

FIG. 3. Differential cross section as a function of y at a c.m. energy of 6.10 MeV with $\alpha=0.36$ F^{-2} and the potential of Eq. (6) parametrized by Eq. (24). The experimental data are those of Leland *et al.* (Ref. 30).

[26] K. Wildermuth and T. Kanellopoulos, CERN Report No. 59-23, 1963 (unpublished).

III. RESULTS

A. Results of the Resonating-Group Calculation

Using the two-body potential described in Sec. II and the value of α given by Eq. (26), we have calculated differential cross sections with $y=1.0$, 1.1, 1.2, and 1.3 at energies of 3.95, 6.10, and 8.40 MeV. The results are shown in Figs. 2, 3, and 4, respectively. From these figures it can clearly be seen that $y=1.2$ yields the best fit to the experimental data at each of these energies.

The finding that the optimum value of y is 1.2 means that the two-body potential in odd orbital-angular-momentum states required in the case of He^3-He^3 scattering is somewhat stronger than that of an isolated nucleon-nucleon system. Qualitatively, this can be explained by remembering that a number of open and closed channels has been omitted in this calculation. Thus, the polarization potential arising from the mutual distortion of the He^3 clusters has not been taken into account. Since it has been shown in the related case of α-α scattering that this latter potential is weakly attractive,[27] its omission can be compensated for by using a value of y slightly greater than 1.[28]

If the above argument is indeed correct, one would expect that in similar calculations on α-α and He^3-α scattering, where the α cluster involved is not as easily distorted, the value of y should be less than 1.2. This,

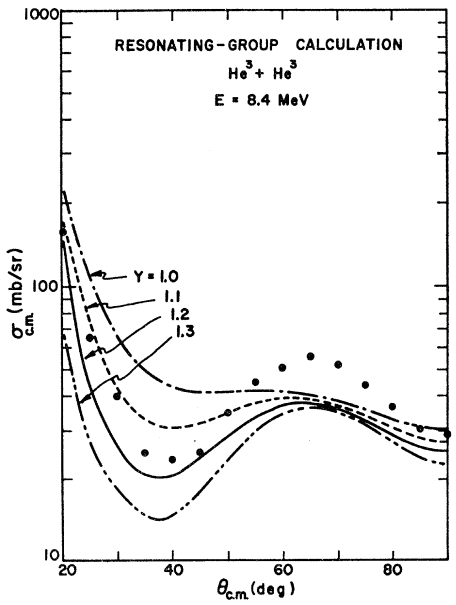

FIG. 4. Differential cross section as a function of y at a c.m. energy of 8.40 MeV with $\alpha=0.36$ F^{-2} and the potential of Eq. (6) parametrized by Eq. (24). The experimental data are those of Leland *et al.* (Ref. 30).

[27] A. Herzenberg and A. S. Roberts, Nucl. Phys. **3**, 314 (1957).

[28] The omission of the Coulomb exchange term in our calculation also slightly overestimates the value of y.

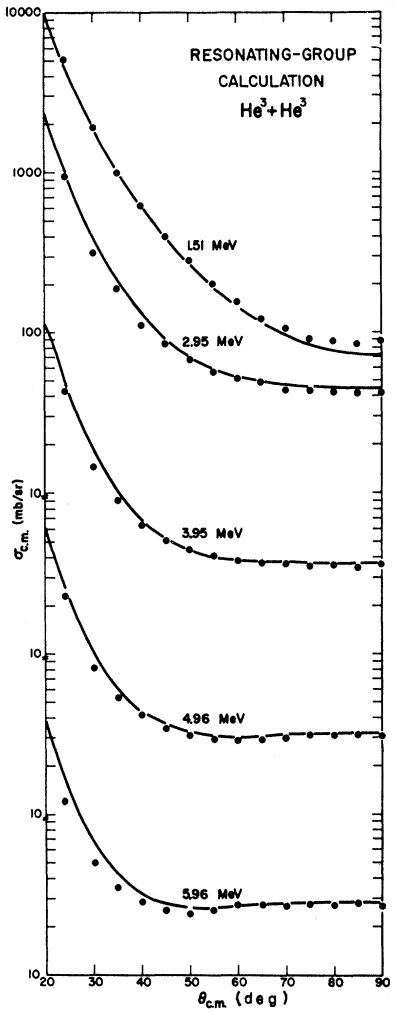

FIG. 5. Differential cross sections in the c.m. energy range 1.5–5.96 MeV with $y=1.2$ and $\alpha=0.36$ F^{-2}. The experimental data are those of Tombrello and Bacher (Ref. 10).

in fact, was found to be the case. In a recent calculation on α-α scattering by Okai and Park,[8] the optimum value of y was close to 0.9, while in the calculation of He^3-α scattering by Tang *et al.*,[29] a value of y equal to 1.02 was required.

In Figs. 5 and 6, the values of the calculated differential cross sections with $y=1.2$ are compared with those determined experimentally[10,30] at 10 energies from 1.5 to 12.0 MeV. From these figures, it can be seen that for $E<6.0$ MeV, the fit is excellent. On the other hand, for $E>6.0$ MeV, the fit becomes progressively worse as the energy increases, although the general features of the experimental data are still reproduced quite correctly. To understand this behavior we note that for $E<6.0$ MeV, the total reaction cross sections determined experimentally by Bacher[13] are rather small ($\cong$100 mb), while for $E>6.0$ MeV, where the He^3+d +p channel is also open, the reaction cross section begins to increase fairly rapidly with energy, reaching a value of about 300 mb at 9 MeV. Thus, the lack of a close agreement between the calculated and experimental values for the He^3+He^3 elastic-scattering cross section at higher energies is very likely due to the one-channel approximation. In Sec. IV, we shall attempt to see the influence of the reaction channels on the elastic-differential cross sections by making an optical-model analysis where these channels can be crudely incorporated into the calculation by the introduction of an imaginary optical potential.

Figure 7 shows the behavior of the calculated phase shifts up to $l=5$ as a function of the c.m. energy.[31]

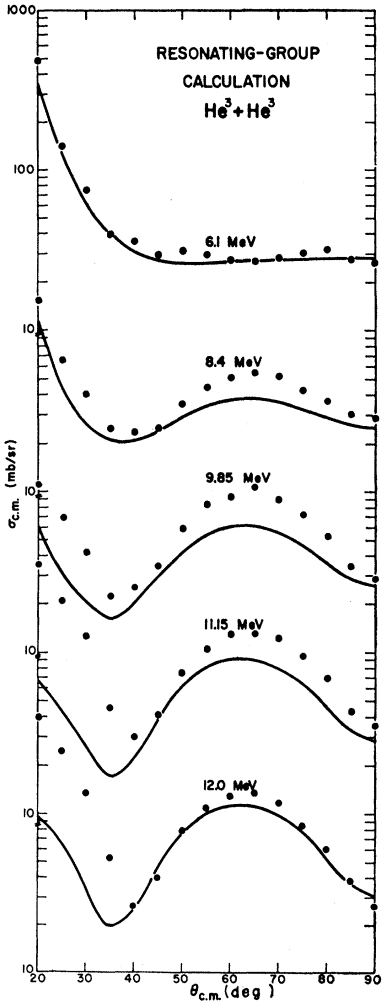

FIG. 6. Differential cross sections in the c.m. energy range 6.1–12.0 MeV with $y=1.2$ and $\alpha=0.36$ F^{-2}. The experimental data are those of Leland *et al.* (Ref. 30).

[29] The value of y given in Ref. 7 was obtained with a value of 0.438 F^{-2} for the width parameter of the He^3 cluster, while that given here is for a width parameter equal to 0.36 F^{-2} [Y. C. Tang (unpublished)].

[30] W. T. Leland, J. E. Brolley, Jr., and L. Rosen, Bull. Am. Phys. Soc. **10**, 51 (1965); also W. T. Leland (private communication).

[31] The phase shifts for $l=6$ are less than 2° for $E<20$ MeV; hence, they are not shown.

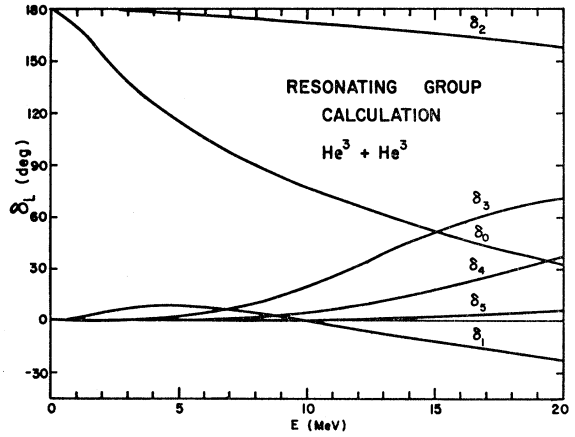

FIG. 7. Calculated phase shifts as a function of c.m. energy using $y=1.2$ and $\alpha=0.36$ F^{-2}.

Here it is seen that the $l=3$ phase shift exhibits a rather interesting feature. It begins to rise quite rapidly at about 5 MeV and reaches a value of 71° at 20 MeV. This indicates that there is a rather broad $l=3$ resonant level with an excitation energy of about 25 MeV in Be^6.[32] In the region where the levels reported by Batty *et al.*[12] are located ($E<6$ MeV), there is, however, only a very broad bump in the curve for the $l=1$ phase shift. If this should correspond to a resonant $l=1$ level, then its width must be very large.

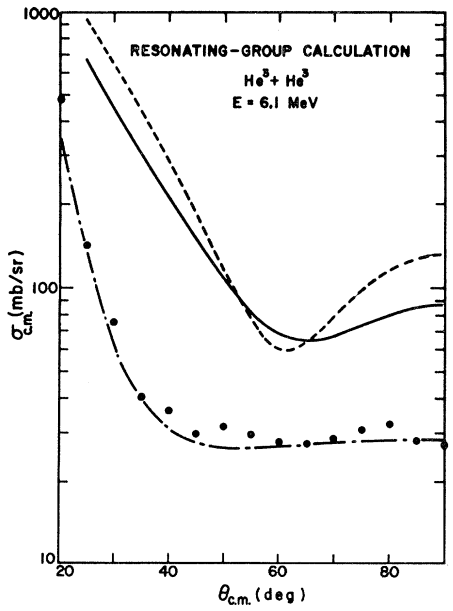

FIG. 8. Comparison of the differential cross sections obtained using different exchange mixtures at a c.m. energy of 6.1 MeV with $\alpha=0.36$ F^{-2}. The solid curve is calculated with $y=0$, the dotted curve is calculated with $m=h=0$ in Eq. (6), and the dot-dash curve is calculated with $y=1.2$. The experimental data are those of Leland *et al.* (Ref. 30).

[32] This level will be split if a noncentral force is included in our calculation.

B. Influence of Exchange Mixture, Cluster Size, and Potential Range

To examine the sensitivity of our results on the exchange character of the nucleon-nucleon potential, we have made calculations with a pure symmetric force ($y=0$) and a force obtained by setting $m=h=0$ in Eq. (6). The results at 6.1 MeV are compared with those obtained with $y=1.2$ in Fig. 8. From this figure it can easily be seen that the curve with $y=1.2$ fits the data much better. This is gratifying, since it means that the exchange nature of the two-body potential must indeed be nearly Serber to yield a good agreement with experiment.

Figures 9 and 10 show the dependence of the differential cross sections on the width parameter α at energies of 3.95 and 8.4 MeV, respectively. In these calculations, the value of y is chosen as 1.2. Here, we

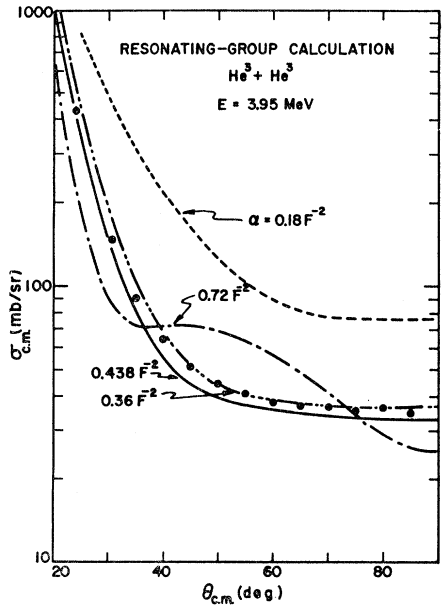

FIG. 9. Differential cross section as a function of α at a c.m. energy of 3.95 MeV with $y=1.2$. The experimental data are those of Tombrello and Bacher (Ref. 10).

note that the curve obtained with $\alpha=0.438$ F^{-2}, corresponding to a value of 1.51 F for the rms radius of He^3, fits the experimental data somewhat worse than the curve obtained with $\alpha=0.36$ F^{-2}, but the fit can be improved slightly by varying the parameter y. For the cases where $\alpha=0.18$ and 0.72 F^{-2}, it is, however, definitely not possible to fit the experimental data even if one allows y to take on values very different from 1. Thus, from this study, we conclude that the results of the resonating-group calculation do depend in a fairly sensitive manner on the size of the clusters, and agreement with experimental data can be obtained only when the rms radii of the clusters are chosen to have values close to those determined experimentally.

A study to see the influence of the range of the nucleon-nucleon potential has also been made. For this purpose we have examined the scattering cross sections with a potential used by Laskar *et al.*[6] (to be referred to as the London potential). This potential has the parameters

$$V_0=46.80 \text{ MeV},$$
$$\kappa=0.2669 \text{ F}^{-2}, \quad (28)$$
$$x=0.60,$$

which yield the following values for the n-p effective range parameters:

$$a_t=5.68 \text{ F},$$
$$r_{0t}=2.08 \text{ F},$$
$$a_s=-39.0 \text{ F}, \quad (29)$$
$$r_{0s}=2.86 \text{ F}.$$

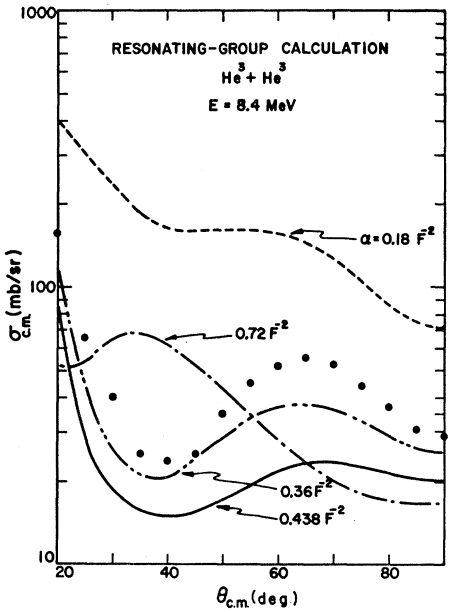

FIG. 10. Differential cross section as a function of α at a c.m. energy of 8.4 MeV with $y=1.2$. The experimental data are those of Leland *et al.* (Ref. 30).

This potential does not fit the two-body scattering data very well; in particular, the effective ranges are about 20% too large. The calculations using this potential are made at energies of 3.95 and 8.4 MeV with $\alpha=0.36$ F^{-2} and $y=0.8$, 0.9, and 1.0. The results are shown in Figs. 11 and 12, where it is seen that the optimum value of y is 0.8. Comparing these results with those obtained using the two-body potential parametrized by Eq. (24) (see Figs. 5 and 6) we note that the fit obtained here is worse, but not to a large extent.[33] This indicates that the results of the resonating-group calculation are not overly sensitive to the range of the nucleon-nucleon potential. This is unfortunate, since, otherwise, one could use the low-energy scattering data where both the incident and target particles are light nuclei to determine accurately the range of the two-body potential.

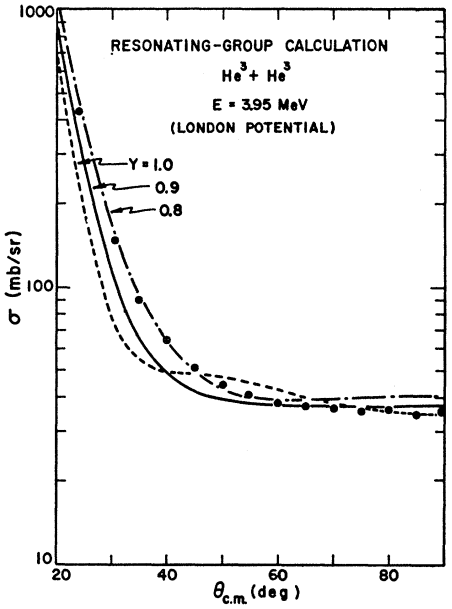

FIG. 11. Differential cross section as a function of y at a c.m. energy of 3.95 MeV using the London potential and $\alpha=0.36$ F^{-2}. The experimental data are those of Tombrello and Bacher (Ref. 10).

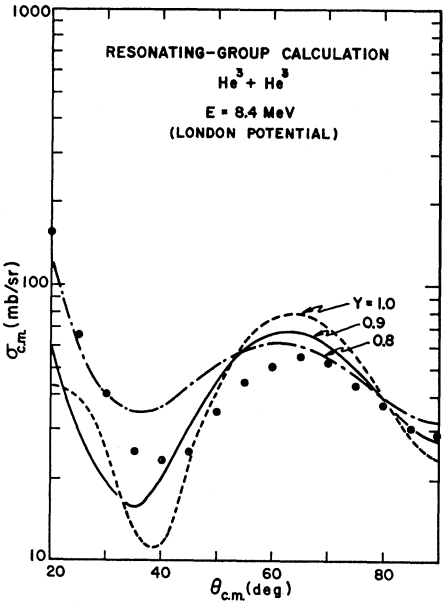

FIG. 12. Differential cross section as a function of y at a c.m. energy of 8.4 MeV using the London potential and $\alpha=0.36$ F^{-2}. The experimental data are those of Leland *et al.* (Ref. 30).

[33] The potential used by Bransden and Hamilton (Ref. 9) has also been considered. With their value of α, we have obtained, by varying y, a fairly good agreement with experiment. Comparing our values of the phase shifts with theirs, we note that there is, in particular, no agreement at all for the odd phase shifts.

TABLE I. A comparison of resonating-group and optical-model phase shifts at 12.0 MeV.

Phase shifts	Resonating-group calculation	Optical-model analysis
δ_0	66.3°	30.6°
δ_1	−5.2°	−5.4°
δ_2	169.6°	148.9°
δ_3	32.1°	46.8°
δ_4	8.3°	2.7°

IV. OPTICAL-MODEL ANALYSIS

In Sec. III, it has been found that at higher energies where a number of reaction channels are open, the results of the resonating-group calculation do not agree with the experimental data too well. In this section we shall perform an optical-model analysis to determine how a crude inclusion of these channels by means of an imaginary optical potential can lead to a better agreement. The analysis will be done only at 12 MeV, since it is here that the fit obtained using the resonating-group calculation is least satisfactory. Also, at this energy, there is a large number of open channels so that the compound elastic cross section need not be considered.

The analysis is done with a volume-absorption potential of the form

$$V_{\mathrm{VA}}=-\frac{V+iW}{1+\exp[(r-R)/a]}+V_{\mathrm{Coul}}, \qquad (30)$$

and surface-absorption potential of the form

$$V_{\mathrm{SA}}=\frac{-V}{1+\exp[(r-R)/a]}-iW\exp\left[-\frac{(r-R)^2}{b^2}\right]+V_{\mathrm{Coul}}, \qquad (31)$$

where V_{Coul} is the Coulomb potential calculated from a uniformly charged sphere of radius R. The parameters which give the best fit to the data are as follows:

Volume absorption: $V=52.53$ MeV, $R=2.49$ F,
$W=1.75$ MeV, $a=0.45$ F,
Surface absorption: $V=53.71$ MeV, $R=2.46$ F,
$W=0.81$ MeV, $a=0.45$ F,
$b=1.15$ F.

The optical-model fit to the experimental data using these parameters is shown in Fig. 13 where it can be seen that both potentials yield a very good fit.

In Table I, a comparison is made between the phase shifts obtained with the resonating-group calculation and the real part of the phase shifts obtained in the optical-model analysis with the volume-absorption potential. From this table, it is seen that there is a general agreement between these two sets. This indicates that the phase shifts from the resonating-group calculation can be used as a starting set for a detailed phase-shift analysis when more data becomes available.

The total reaction cross sections using the volume- and surface-absorption potentials are equal to 471 and 422 mb, respectively. The fact that this cross section is rather large indicates quite clearly that an one-channel assumption is only approximately valid at 12 MeV. If one wishes to obtain a better agreement with experiment, it is necessary to include more channels in the resonating-group calculation. This is, however, not a simple procedure in this particular case of He^3+He^3 scattering. For example, even the inclusion of the Li^5+p channel is a difficult problem, since the Li^5 nucleus is not bound. One could account for the reaction channels in a crude manner by introducing a phenomenological local imaginary potential into Eq. (17). Although this method would introduce a number of variable parameters into our calculation, we could at least estimate the relative importance of the reaction channels in this way.

V. CONCLUSION

The results of this investigation indicate that the resonating-group calculation with the one-channel approximation can be used quite successfully to explain the features of the elastic scattering of He^3 by He^3. The differential cross sections calculated at energies up to about 6 MeV are in excellent agreement with those determined experimentally. The fit to the higher-energy data is somewhat worse, but this is more or less expected since, for energies greater than about 6 MeV, the He^3 clusters can be easily dissociated and many reaction

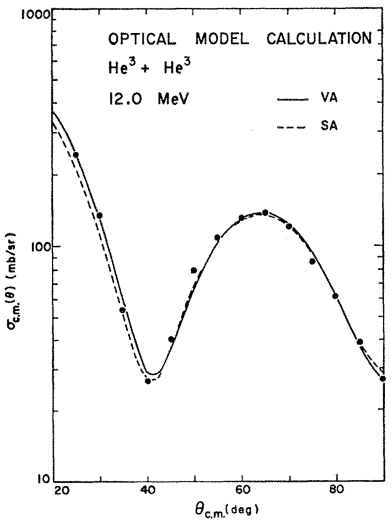

FIG. 13. Optical-model calculation of the differential cross section at a c.m. energy of 12.0 MeV using a volume absorption potential (VA) and a surface absorption potential (SA). The experimental data are those of Leland *et al.* (Ref. 30).

channels are open, which means that, at these energies, the one-channel assumption is only an approximately valid one.

The effect of mutual distortion of the He^3 clusters seems to have some importance. This is exhibited in our calculation by the finding that the two-body interaction in odd angular-momentum states necessary to yield a good agreement with the experimental data on He^3-He^3 scattering is slightly stronger than that determined from the nucleon-nucleon scattering data. The reason for this is again that we have adopted a one-channel approximation, and hence, the polarization potential, being weakly attractive, is not taken into account.

From the behavior of the calculated phase shifts, the presence of a rather broad $l=3$ resonant level in Be^6 with an excitation energy of about 25 MeV is predicted. Aside from this broad resonance, the only other interesting feature we have found is that there is a broad bump in the curve for the $l=1$ phase shift as a function of energy. Whether this broad bump corresponds to the two levels reported by Batty *et al.*[12] is not too clear at this moment. For a further clarification, it is necessary to make a calculation with a noncentral component in our two-body potential. Meanwhile, of course, more detailed experimental studies to determine the nature of the resonant levels in Be^6 would be highly desirable.

The dependence of our results on the cluster size, the exchange mixture, and the range of the two-body potential has also been studied. From this study, the interesting finding is that, to obtain a good agreement with experiment, it is necessary to choose a near-Serber mixture and a rms radius for the He^3 cluster close to the experimental value. As for the range of the two-body potential, we have found that its effect is relatively less important. Indeed, this latter feature seems to be a general one in scattering problems where light nuclei are involved. In a similar calculation on He^3-α scattering[7] the same conclusion has also been reached.

An optical-model analysis is also performed at a c.m. energy of 12 MeV. From this analysis, we find that the reaction cross section is about 450 mb. The fact that this cross reaction is not too small compared with the elastic-scattering cross section supports our assertion that the one-channel assumption is only approximately valid at higher energies.

The fact that the resonating-group calculation in the one-channel approximation yields a good result in the case of He^3-He^3 scattering gives a strong indication that it should also work in the case of He^3-H^3 scattering with a similar exchange mixture for the two-body potential. This latter case is now being examined in detail and the results will be reported in a further publication.

ACKNOWLEDGMENTS

We would like to thank Professor T. A. Tombrello for a valuable discussion and for informing us of the reaction cross sections reported in the thesis of A. D. Bacher. Also, we wish to express our gratitude to Dr. W. T. Leland for sending us results prior to publication.