Two-Pion-Exchange Three-Body Λ-Nucleon Interaction in Nuclear Matter*

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The two-pion-exchange three-body Λ-nucleon force is considered in nuclear matter. The π-Λ interaction is consistent with the known hyperon resonances, and contributions from the Σ channel are included without neglecting the Σ-Λ mass difference. Nuclear correlations produced by tensor forces and hard cores are included, as well as the exchange terms, which arise from the antisymmetrization of the nuclear wave functions. The Λ and Σ hyperons are also assumed to have hard cores. The contribution to the potential energy of a Λ hyperon in nuclear matter is found to be 3.6 MeV (repulsion). This number is an overestimate since part of the Λ-N short-range correlations were neglected. Because of its strong density dependence, the contribution in light hypernuclei is probably fairly unimportant.

1. INTRODUCTION

THERE have been several successful attempts to understand the nucleon-nucleon interaction as a one-boson-exchange mechanism.[1] In particular, the one-pion-exchange potential (OPEP) gives the long-range part of the two-nucleon interaction.[2] In S-wave scattering a short-range repulsion dominates at high energies and this is believed to be the result of the exchange of heavy vector mesons, such as the ω. The relatively strong medium-range attraction which is responsible for the nuclear binding is usually assumed to be the result of the exchange of two pions in a resonant or almost resonant S state. An enhancement of the two-pion exchange also makes it possible to explain the π-N phase shifts including the nucleon isobars.[3] Further indications for a π-π attraction are found by analyzing production processes and decays leading to final states with two or more pions, although the evidence may not be quite conclusive.[4]

The qualitative success of this model in the two-nucleon problem makes it natural to use similar ideas for the Λ-N interaction.[5] Since the Λ hyperon is an isospin singlet, $T=1$ particles, such as pions, cannot be exchanged in the usual sense, and the force does therefore not contain the OPEP. Instead, exchange of K mesons becomes possible. Since we shall deal mostly with the long-range part of the interaction, the most important qualitative difference between the N-N and the Λ-N forces is the absence of the OPEP in the latter. We assume that there is a strong short-range repulsion (hard core) in both cases.

The number of Λ-N scattering events measured so far is not large enough to determine the interaction in any detail. Most of the information has therefore been obtained from analyses of hypernuclei. S-shell hypernuclei (i.e., with baryon number $A\leq 5$) have been extensively analyzed using variational methods. These systems give reliable information only about the S-state interaction. In order to obtain knowledge of other angular momentum states it is necessary to investigate heavier hypernuclei. It seems to be possible to understand the binding energies of both S- and P-shell hypernuclei on the basis of a local two-body Λ-N potential with a hard core.[6]

The other extreme, heavy hypernuclei, serves as a test also of higher-partial waves in the Λ-N interaction. As a simplified model, the case of a Λ-hyperon in nuclear matter offer calculational advantages over finite hypernuclei. The "experimental" value of the binding energy B of the hyperon must then be found by extrapolating from actual experiments. Several estimates of this kind have been made, a recent value of the binding is[7] $B=27\pm1$ MeV.

Theoretical estimates of the binding of a Λ hyperon in nuclear matter have also been made.[8–12] Since these have a tendency to predict too much binding, it has been suggested that there might be some suppression of the higher-partial waves of the potential.[8] Another possibility to explain the discrepancy is to introduce many-body forces.[9] A recent calculation by Clark and Westhaus[11] indicates that the local two-body forces obtained from light hypernuclei overbind the Λ particle in nuclear matter by several MeV. Other authors have found similar discrepancies.[12]

The longest possible range in a Λ-N force corresponds to the exchange of two pions. Because of the enhancement via the π-π attraction, this process is likely to give a significant amount of attraction. In a many-body system these two pions might go to different nucleons,

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[1] See, e.g. J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966); R. A. Bryan and B. L. Scott, *ibid.* **135**, B434 (1964).
[2] G. Breit and M. H. Hull, Nucl. Phys. **15**, 216 (1962).
[3] J. Hamilton and W. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).
[4] A. N. Kamel, Nuovo Cimento **44**, 1055 (1966).
[5] B. W. Downs and R. J. Phillips, Nuovo Cimento **36**, 120 (1965).
[6] A. R. Bodmer and J. W. Murphy, Nucl. Phys. **64**, 593 (1965); **73**, 664 (1965).
[7] G. Lemonne *et al.*, Phys. Letters **18**, 354 (1965).
[8] J. D. Walecka, Nuovo Cimento **16**, 342 (1960).
[9] B. Ram and B. W. Downs, Phys. Rev. **133**, B420 (1964).
[10] J. Dabrowski and H. S. Kohler, Phys. Rev. **136**, B1612 (1964).
[11] J. W. Clark and P. Westhaus, Phys. Letters **23**, 109 (1966).
[12] G. Ranft, Nuovo Cimento **43**, 259 (1966); B. W. Downs and M. E. Grypeos, *ibid.* **44**, 306 (1966).

giving rise to a three-body force. That is the process we shall consider, assuming that it corresponds to the main part of the many-body forces.

2. TWO-BODY GROUND-STATE CORRELATIONS

We shall assume that most of the potential energy of hypernuclei comes from two-body forces. It is then sufficient to evaluate the contribution from the many-body forces in first order of their strengths. The procedure we have in mind is the following: We first solve the many-body problem with two-body forces (at least in some approximation). This corresponds to obtaining a correlated many-body wave function which approximates the ground state of the system. We then evaluate the contribution from the many-body forces by taking their expectation value with respect to this wave function. If the resulting energy is small, this procedure is justified.

In order to obtain the approximate ground-state wave function of the Λ+nuclear matter system, we shall have to use the Λ-N as well as the N-N two-body forces. The contribution from the many-body forces depends on these. As a criterion on the ground state, we may demand that it corresponds to approximately the correct binding energy of the system. We shall use the technique of many-body theory; the complete many-body wave function will therefore never explicitly appear in the calculations.

The contribution from the many-body forces should be evaluated at least to the same accuracy as the calculations of the binding from two-body forces. At present this would allow an uncertainty of 2 or 3 MeV. Assuming that all of the difference between the theoretical estimates and the experimental value is due to the three-body force, the permissible uncertainty would still be of the order of 30%. It does therefore not seem necessary to treat the nuclear matter part of the problem in its full complexity; we shall use a simplified version of the Brueckner theory due to Moszkowski and Scott.[13] This method separates the two-body potential into short- and long-range parts. The separation distance d is defined so that the inner part of the force gives no contribution to the phase shift in the scattering of free particles. Since the short-range part of the force tends to scatter particles into intermediate states of high momentum, it is not seriously affected by the exclusion principle in nuclear matter. In a first approximation one can ignore this part of the interaction completely, and the long-range part is weak enough to be treated as a perturbation. It is possible to construct correction terms to this procedure; for our purposes this is not necessary. Strictly speaking, the separation method can only be applied in cases where the force outside the hard core is sufficiently attractive; the separation distance will also depend on the relative energy and angular momentum of the two-body system. Further, the existence of noncentral forces, such as the OPEP, will make the definition of the separation distance less obvious. We shall ignore these difficulties and neglect the correction terms as well as the state dependence of the separation distance. With these approximations nuclear matter will still be bound with a reasonable binding energy. The neglected effects are mainly important for the nuclear saturation, and the density dependence of our results will therefore not be very meaningful. The separation method was first used to explain nuclear spectra in a shell-model calculation by Kallio and Kolltveit.[14]

If we represent the N-N force by the Hamada-Johnston[15] potential, the separation distance turns out to be around $d_{NN}=1$ fm, depending on the energy and angular momentum of the two-body system. We use that value throughout this paper.

The N-N and Λ-N interactions differ by more than the OPEP. The spin-averaged Λ-N interaction appears to be somewhat less attractive than that of the short and medium-range parts of the N-N interaction. We assume the same hard core in Λ-N and N-N interactions, and assign the difference to the medium-range interaction.

The Λ hyperon can emit a pion in nuclear matter and become a virtual Σ particle. It would therefore be desirable to have data about the Σ-N interaction. Unfortunately, there is little knowledge of this. It is not possible to draw any conclusions about the short-range part of the interaction from the available data on Σ absorption in hydrogen.[16] We assume it to be roughly similar to the N-N interaction.

3. TWO-BODY AND THREE-BODY FORCES

Several authors have considered two pion exchange three-body Λ-nucleon forces.[17] A three-body force of this kind arises when the Λ emits two pions which go to different nucleons. Alternatively, we can look upon the process as π-Λ scattering, the pions being virtual (off their energy shell) and created and annihilated by the nucleons. A general graph for this process is given in Fig. 1, where the two nucleons are in the states $\mathbf{k}_1$ and

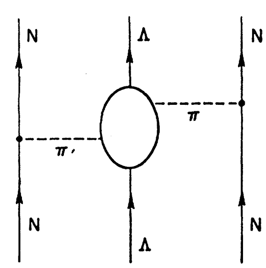

FIG. 1. General graph for the two-pion-exchange three-body Λ-nucleon interaction. The π-Λ interaction is not specified here.

[13] S. Moszkowski and B. Scott, Ann. Phys. (N. Y.) **11**, 65 (1960).

[14] A. Kallio and K. Kolltveit, Nucl. Phys. **53**, 87 (1964).

[15] T. Hamada and I. Johnston, Nucl. Phys. **34**, 382 (1962).

[16] W. M. Dante and E. M. Henley, Phys. Rev. **144**, 1224 (1966).

[17] J. D. Chalk, III, and B. W. Downs, Phys. Rev. **132**, 2727 (1963) and references cited therein.

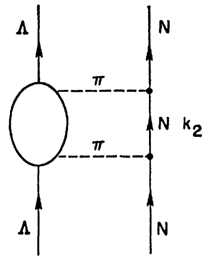

FIG. 2. A contribution to the two-body Λ-N force produced by the exchange of two pions. Since the two-body force is defined to reproduce the interaction between free particles, there is no restriction on the summation over the intermediate states $\mathbf{k}_2$ of the nucleon.

$\mathbf{k}_2$ before and $\mathbf{k}_1'$ and $\mathbf{k}_2'$ after the interaction. The dotted line represents the pion. We assume the Λ to be in a zero-momentum single-particle state since this state has the strongest binding in the Hartree-Fock approximation. The model of the π-Λ interaction is not specified at this stage, primarily we shall need the forward-scattering amplitude.

When evaluating the contribution to the binding from the three-body force in Fig. 1 in a many-body system, it is necessary to make sure that the two nucleons really are not the same one. Considering Λ-N scattering in free space, which we *define* to proceed via two-body forces only, we note that there is a related graph, given in Fig. 2. By comparing the contributions from these two diagrams in nuclear matter, we will make sure that there is no two-body component in the three-body force.

Because the two-body force is defined to reproduce the interaction between two particles in free space, it contains the diagram in Fig. 2 with an implicit summation over the states $\mathbf{k}_2$ of the intermediate nucleon. In a field-theoretical treatment this summation would reduce to a propagator and a projection operator for positive energy states. The contribution to the binding of the Λ particle in nuclear matter from this process is in lowest order obtained by equating $\mathbf{k}_1$ and $\mathbf{k}_1'$ and summing over all states in the Fermi sea. In the Goldstone diagrammatic language this is denoted by joining the two external nucleon lines. We thus get the nuclear matter diagram in Fig. 3, where $\mathbf{k}_1$ is restricted to within the Fermi sea, and $\mathbf{k}_2$ should be summed over all values without respect to the Pauli principle.

When evaluating the expectation value of the three-body force, we also require the state after the interaction to be the same as before. This means that we must put $\mathbf{k}_1=\mathbf{k}_2'$ and $\mathbf{k}_2=\mathbf{k}_1'$, because a nucleon cannot emit even a virtual pion and remain in the same state. Joining the lines, the structure of the graph will again be that of Fig. 3. This time, however, both the nucleon lines correspond to states inside the Fermi sea. The contribution from this graph will also differ by an over-all minus sign. The three-body force therefore cancels part of the two-body force, in fact it cancels exactly the part which is forbidden by the exclusion principle. It is essential to treat the three-body force as such rather than just as a quenching of the two-body force, since it is then possible to discuss the influence of the nuclear correlations in configuration space.

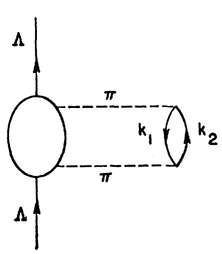

FIG. 3. This graph illustrates the combined expectation value of the two-pion exchange two- and three-body Λ-nucleon forces in nucleon matter.

In the introduction it was noted that the enhanced two-pion exchange can be assumed to be an important source of binding for the Λ hyperon. Since the three-body force can be thought of as a quenching of this two-body force, we expect the three-body force to be repulsive.

4. LOW-ENERGY π-Λ INTERACTION

As mentioned above, the important part of the three-body force is the π-Λ scattering amplitude. It is essential to have a reasonably good model of this process in order to be able to evaluate the strength of the three-body force with any accuracy. There are not enough data on total cross sections to make it possible to evaluate the forward-scattering amplitude from a dispersion relation, in contrast to the situation for π-N scattering.

An ambitious semiphenomenological study of low-energy π-Λ scattering has been made by Martin.[18] He uses the fact that there is a resonance, the $Y_1^*(1385)$, in the $P_{3/2}$ channel. Contributions from Σ and Y_1^* exchange and the exchange of a low-energy S-wave π-π pair were evaluated to dispersion relations for the $P_{3/2}$ π-Λ scattering amplitude. It was possible to find an amplitude which was self-consistent on the physical and crossed physical cut simultaneously, provided the coupling constant to the Σ has the value

$$G_{\Sigma\Lambda\pi}^2/4\pi \cong 10.9. \qquad (1)$$

(This was not possible without the π-π pair.) A constant related to the $P_{3/2}$ scattering length was also evaluated. One may therefore hope that continuing this scattering amplitude below the π-Λ threshold will indeed estimate the actual contribution from this state. We represent the $P_{1/2}$ amplitude by the Σ pole, using the coupling constant given above. This may be a good approximation, since the point at which we shall use the amplitude is close to this pole.

There is little knowledge about the S-wave π-Λ interaction. No resonance has been found in this state. The polarization due to final-state interactions in the weak decay $\Xi^- \to \Lambda^0 + \pi^-$ gives information about the S-wave scattering length, Ref. 17 obtains the value $a_0 = -0.08 \pm 0.20$ in units of the pion Compton wavelength. Since the sign is not determined and the magnitude is small, we neglect the S-wave contribution. For

[18] B. R. Martin, Phys. Rev. **138**, B1136 (1965). We neglect the fact that later experiments have indicated a smaller width of the Y_1^* than assumed by Martin.

lack of data we shall also neglect partial waves higher than P waves.

The pions involved here are virtual, having a space-like four-momentum. Different models for estimating possible changes in the amplitude for this reason are considered for the π-N case.[19] It seems that corrections of this kind are not large, the distance to the mass shell is only one pion mass.

We have split the π-Λ scattering amplitude into two parts. One is the Σ pole, the other could in principle be evaluated from a dispersion integral, and most of the contribution here is due to the first hyperon resonance, the $Y_1^*(1385)$. The situation is somewhat similar to π-N scattering, where the corresponding particles are the nucleon itself and the 3,3 resonance. These two states do indeed dominate the low-energy π-N interaction.

The pions in question here are produced by the long-range part of the OPEP, and in the complex plane of the total pion lab energy ω we are there close to the origin. The nearest singularity of the π-Λ scattering amplitude is the Σ pole, and the distance to it is 75 MeV, the Σ-Λ mass difference. The kinetic energy of a nucleon on the Fermi surface in nuclear matter is 40 MeV. We shall therefore not neglect the single-particle energies of the nucleons compared to the Σ-Λ mass difference nor vice versa when evaluating the contribution from the coupling to the Σ hyperon. On the other hand, the distance to the Y_1^* is 270 MeV, and in this case it seems safe to neglect the kinetic energies. This we denote in the diagrams by drawing the pion lines at equal times. The fact that the pion lines are horizontal indicates that we make the usual static approximation; i.e., neglect the kinetic energies of the nucleons compared to the pion mass. This approximation also allows us to use the standard local form of the long-range part of the OPEP. The separation of the three-body force into two parts is illustrated in Fig. 4; we treat the two parts separately.

5. THE CONTRIBUTION FROM THE $P_{3/2}$ SCATTERING AMPLITUDE

We shall for a moment ignore the Λ-N correlations completely, and make sure that the N-N correlations are properly included. The general graph for the contribution from the three-body force is then given by Fig. 5, where the box symbolizes the sum of all linked graphs with two external pion lines. Because of the way we continue the π-Λ scattering amplitude, it has the structure of a contact interaction; we ignore the kinetic energies compared to the Λ-Y_1^* mass difference. This approximation makes it possible to reduce the problem to that of finding the total-binding energy of nuclear matter.

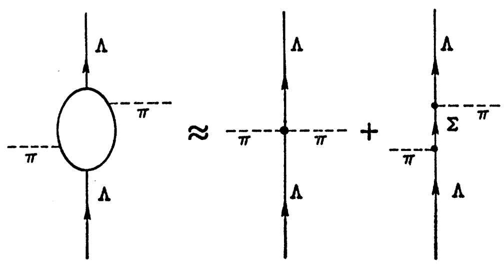

FIG. 4. The Σ pole is extracted from the π-Λ scattering amplitude and will be treated separately.

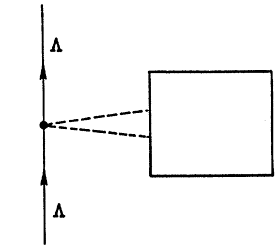

FIG. 5. General graph for the contribution to the binding of the Λ from the exchange of two pions in the absence of Λ-N correlations. The box symbolizes the sum of all linked nucleon clusters with two external pion lines.

The trick we shall use amounts to noting that a diagram part corresponding to a pion line with a contact interaction is simply the (negative) derivative of the pion propagator without the interaction. This equality is very similar to Ward's identity,[20] except that the derivative in our case is to be taken with respect to (the square of) the pion mass. This is an operation which gives the number of pions. The contribution from any diagram of the form in Fig. 5 is then proportional to the derivative of a diagram contributing to the potential energy in nuclear matter. Taking all graphs and including the complete nuclear force gives the total potential energy in nuclear matter. The factor of proportionality is the scattering part of the S matrix for π-Λ scattering excluding E/M factors associated with the external ines (i.e., the T matrix).

It is straightforward to check this prescription to any order of perturbation theory. If we use perturbation theory in the strict sense and represent the pion lines by the complete OPEP, the contribution to the energy density from the simplest diagram, Fig. 6(a), is given by

$$E_1^{(2)}=6\left(\frac{G_{NN\pi}}{2M_N}\right)^2\int_{k_1<k_F,k_2<k_F}\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{(\mathbf{k}_1-\mathbf{k}_2)^2}{(\mathbf{k}_1-\mathbf{k}_2)^2+\mu^2}\,. \tag{2}$$

We take M_N and μ to be the masses of the charged

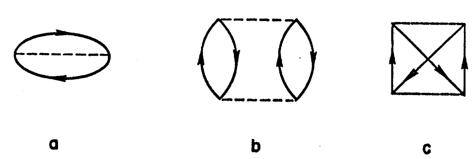

FIG. 6. Part of the linked cluster expansion for the density of energy in nuclear matter. The dotted lines represent the long-range part of the OPEP; the direct term in first order is not shown since it gives no contribution.

[19] L. Resnick, Nuovo Cimento **39**, 641 (1965); M. L. Thiebaux, Jr., Phys. Rev. **144**, 1224 (1966).

[20] J. Ward, Phys. Rev. **84**, 897 (1951).

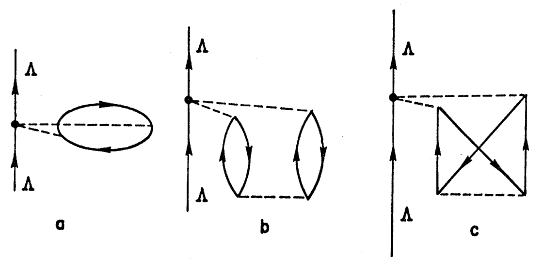

FIG. 7. These are some of the diagrams which are obtained when the general graph in Fig. 5 is expended into a series of linked clusters. The contributions to the potential energy of the Λ particle from these diagrams can be written as derivatives of the corresponding terms in Fig. 6.

states of the nucleon and pion, respectively. The contribution from the corresponding diagram with the three-body force [Fig. 7(a)] is found to be:

$$E_1^{(3)}=6T_{\pi\Lambda}\left(\frac{G_{NN\pi}}{2M_N}\right)^2 \times\int_{k_1<k_E,k_2<k_F}\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{(\mathbf{k}_1-\mathbf{k}_2)^2}{[(k_1-k_2)^2+\mu^2]^2}, \quad (3)$$

where $T_{\pi\Lambda}$ is the part of the π-Λ T matrix which is approximated by a contact interaction. As expected, the formula

$$E^{(3)}=-T_{\pi\Lambda}\frac{\partial}{\partial\mu^2}E^{(2)}(\mu^2) \quad (4)$$

relates Eqs. (2) and (3), and it takes the Pauli principle into account; both nucleons in Fig. 7(a) correspond to states inside the Fermi sea.

The relation $G_{NN\pi}/2M_N=f/\mu$ introduces some ambiguity via the μ dependence of the π-N coupling. These possible additional powers of μ should be regarded as constants when the derivative is evaluated; this is equivalent to using the pseudoscalar coupling constant, as we have done.

The δ function in the central part of the OPEP may be excluded or included at will, it gives no contribution in (4), since it does not depend on the pion mass.

The formula (3) reduces the problem to that of estimating how the binding energy of nuclear matter depends on the pion mass. As stated in Sec. 2, this will be done using the Moszkowski-Scott separation method, which amounts to perturbation theory on the long-range part of the potential. We need only keep those diagrams where at least one interaction is due to the OPEP. The first few diagrams in the linked cluster expansion for the binding energy are given in Fig. 6. The first-order direct term is not included since it gives no contribution from the OPEP. In case of the exchange term in this order only the central part contributes.

In the second-order direct term, we may assume one interaction to be the OPEP, whereas the other in principle is the long-range part of the total two-nucleon interaction. In this case, also the tensor part of the force contributes. Since it is an order of magnitude stronger than the central part of the OPEP, we need only keep the tensor force. Once we have decided, however, that one interaction takes place via the tensor force, the other interaction must be a quadratic spin-orbit or another tensor interaction. Looking at the values of the parameters of the Hamada-Johnston potential,[15] we find that the quadratic spin-orbit term is numerically weaker than the tensor force at large distances, we shall therefore neglect it.

The fact that we include more than the lowest diagram corresponds to evaluating the expectation value of the three-body force in the presence of ground-state correlations. The most important correlations are those produced by the tensor force, these correlations are so strong that the second-order term [Fig. 6(b)] is larger than the first-order one for the OPEP. We emphasize that this does not mean that the perturbation expansion diverges; it is the result of the fact that terms linear in the tensor force average out. We shall include only the first- and second-order terms, which might be thought of as working to lowest nonvanishing orders of the central and tensor forces separately. The second-order graph, Fig. 6(b), will be evaluated with the tensor part of the static OPEP, ignoring the fact that the tensor force of the Hamada-Johnston potential is weaker than that of the OPEP. This could be corrected by increasing the separation distance somewhat.

Using the results of Dahlblom *et al.*,[21] it is straightforward to write down the contributions to the binding energy per volume from the diagrams in Fig. 6. The graph in Fig. 6(a) gives

$$E_a^{(2)}=-6\left(\frac{G_{NN\pi}}{2M_N}\right)^2\mu^2e^{-\mu d} \times\int_{V_1}\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{\cos(qd)+(\mu/q)\sin(qd)}{q^2+\mu^2}, \quad (5)$$

where the volume V_1 of integration is $k_1<k_F$, $k_2<k_F$ and $\mathbf{q}=\mathbf{k}_1-\mathbf{k}_2$. Eq. (5) does not reduce to Eq. (2) in the limit $d\rightarrow 0$. The reason for this is that the separation excludes the zero-range δ-function singularity of the central part of the OPEP. The δ function gives repulsion, whereas the rest of the central part gives attraction in this order.

The contribution from the second-order direct graph can be written in the form

$$E_b^{(2)}=-\frac{1}{8\pi^8}\left(\frac{G_{NN\pi}}{2M_N}\right)^4\mu^4M^*e^{-2\mu d}\int_0^\infty dq[T(q)]^2I(q), \quad (6)$$

[21] T. Dahlblom *et al.*, Nucl. Phys. **56**, 177 (1964).

where

$$T(q)=\frac{q\cos(qd)+\mu\sin(qd)}{q^2+\mu^2}-3\frac{1+\mu d}{\mu^2 d}j_1(qd) \tag{7}$$

and

$$I(q)=\int_{V_2}\frac{d^3k_1d^3k_2}{\mathbf{q}\cdot(\mathbf{k}_1+\mathbf{k}_2+\mathbf{q})}. \tag{8}$$

The nucleons are taken to have an effective mass, $M^*=0.75M_N$ in the intermediate states. The integral in (8) is over the volume V_2, given by $|k_1|$, $|k_2|<k_F$; $|\mathbf{k}_1+\mathbf{q}|$, $|\mathbf{k}_2+\mathbf{q}|>k_F$, and can be evaluated analytically.[22]

The exchange term in second order, Fig. 6(c), is more complicated. Using the results of Dahlblom *et al.*,[21] the contribution to the density of binding energy can be obtained from the following expressions:

$$E_c^{(2)}=-\frac{1}{32\pi^8}\left(\frac{G_{NN\pi}}{2M_N}\right)^4\mu^4M^*e^{-2\mu d}\int_0^\infty q^2dq\,I_{\mathrm{Ex}}(q), \tag{9}$$

$$I_{\mathrm{Ex}}=\frac{3}{4}\int_{V_2}\frac{d^3k_1d^3k_2}{\mathbf{q}\cdot(\mathbf{k}_1+\mathbf{k}_2+\mathbf{q})}\frac{1}{qq'}$$
$$\times[(2A-B)(2A'-B')\cos^2(\mathbf{q},\mathbf{q}')-2AA'-BB'], \tag{10}$$

where $\mathbf{q}'=\mathbf{q}+\mathbf{k}_1-\mathbf{k}_2$ and

$$A=A(q)=\frac{q\cos(qd)+\mu\sin(qd)}{q^2+\mu^2}-B(q),\quad A'=A(q'),$$

$$B=B(q)=\frac{1+\mu d}{\mu^2 d}j_1(qd),\qquad B'=B(q').$$

The formulas (5) and (10) give the contribution from the complete OPEP. We evaluate the integral in this exchange term by the same Monte Carlo method as the above authors. The contribution to the binding of nuclear matter is attractive and roughly one quarter of that of the direct term in second order.

The formula (4) gives the contribution from the three-body force in the absence of Λ-N correlations. It corresponds to the case when the Λ is in a true plane-wave state in a correlated system of nucleons. The two-body Λ-N interactions produce correlations which are not accounted for by this formula. The dominant feature is that the hard cores make "holes" in the wave function, and part of the three-body force is therefore lost. Because of the outer attraction the two-body wave function is enhanced outside the core of the Λ particle. The latter effect is not so strong as in the nuclear case since the force is weaker.

An ambitious treatment of this part of the problem would necessarily have to include three-body correlations. It seems that any simple calculation would overestimate the repulsion. This is the case for the nuclear three-body correlations.[23] All our contributions are finite even in the absence of the Λ-N short-range repulsion, ignoring this means that the contribution is somewhat overestimated (possibly as much as 40–50%).

We use the values

$$k_F=1.36\ \mathrm{fm}^{-1},$$
$$d=1.00\ \mathrm{fm},\quad T_{\pi\Lambda}=-0.79\,\mu^{-1}, \tag{11}$$
$$G_{NN\pi}^2/4\pi=14.6,$$

$T_{\pi\Lambda}$ being obtained from Ref. 18. We find

$$E_a^{(3)}=-0.5\ \mathrm{MeV},$$
$$E_b^{(3)}=+1.3\ \mathrm{MeV}, \tag{12}$$
$$E_c^{(3)}=+0.3\ \mathrm{MeV},$$

which gives a net contribution of 1.1 MeV from the three-body force. (We have defined repulsive contributions to be positive.)

In obtaining these numbers we have ignored the μ dependence of the separation distance (which is quite weak) as well as the fact that the Λ-N correlations introduce nonforward parts of the π-Λ scattering amplitude.

6. THE COUPLING TO THE Σ CHANNEL

It now remains to evaluate the contribution from processes where the Σ hyperon appears as an intermediate state. This is essentially a two-body process, treating it as such will allow us to keep both the Σ-Λ mass difference and the single-particle energies in the intermediate states.

The coupling to the Σ channel may be introduced by the following nonrelativistic interaction Hamiltonian density:

$$H_{\Sigma\Lambda\pi}=\frac{G_{\Sigma\Lambda\pi}}{M_\Sigma+M_\Lambda}[\boldsymbol{\psi}_\Sigma^\dagger\cdot(\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\Phi})\psi_\Lambda+\psi_\Lambda^\dagger(\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\Phi})\cdot\boldsymbol{\psi}_\Sigma], \tag{13}$$

in an obvious notation, the same as in the first paper in Ref. 17. This may be compared to the corresponding operator for the π-N interaction

$$H_{NN\pi}=\frac{G_{NN\pi}}{2M_N}[\psi_N^\dagger\boldsymbol{\tau}\cdot(\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\Phi})\psi_N], \tag{14}$$

which gives the OPEP. Because of the similarity of these interactions, the pion-exchange processes to be considered here are essentially the same as the exchange of pions between nucleons. This means that the structure of the expressions for the graphs is going to be the same as for diagrams with the OPEP. We shall introduce an additional 75 MeV in the energy denominators because of the Σ-Λ mass difference.

In this case the tensor-like property of the pion-

[22] H. Euler, Z. Physik **105**, 553 (1937).

[23] H. Bethe, Phys. Rev. **138**, B804 (1965).

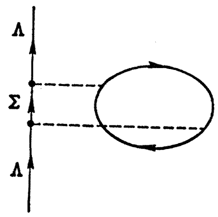

FIG. 8. Lowest-order contribution from the coupling between the Λ and Σ channels. This graph is related to the Σ pole in the π-Λ scattering amplitude. For the three-body part of this process, both nucleon lines correspond to states inside the Fermi sea (hole lines). The dashed lines are taken to be the long-range tensor part of the one-pion-exchange interaction.

exchange mechanism appears already in lowest-non-vanishing order, which corresponds to the exchange of two pions. We shall therefore only evaluate the graph in Fig. 8. The net combinatorial factor of the corresponding diagram for the binding of a nucleon (Fig. 9) is twice that of this graph because the Λ is different from a nucleon. For the same reason there will be no exchange term.

In this case we can include the short-range correlations by inserting Jastrow-type correlation functions. For simplicity, we take the correlation function to be exactly zero inside a correlation radius r_0, and unity outside. We use the value $r_0=0.9$ fm, which is the average between the hard-core radius and an approximate Λ-N separation distance. We assume that the Σ has an effective mass $M_\Sigma{}^*$ in nuclear matter,

$$\frac{M_\Sigma{}^*}{M_\Sigma}=\frac{M^*}{M_N}=0.75. \qquad (15)$$

The contribution from the graph in Fig. 8 now takes the form

$$E_\Sigma=\frac{1}{8\pi^8}\left(\frac{G_{NN\pi}}{2M_N}\right)^2\left(\frac{G_{\Sigma\Lambda\pi}}{M_\Sigma+M_\Lambda}\right)^2\mu^4M^*e^{-2\mu r_0}\times\int_0^\infty dq[T(q)]^2I_\Sigma(q)\,, \qquad (16)$$

where

$$I_\Sigma(q)=\int_{V_\Sigma}\frac{d^3k}{\mathbf{q}\cdot(\mathbf{k}+\mathbf{q})+(M_\Sigma-M_\Lambda)(M^*-q^2/2M_\Sigma)}. \qquad (17)$$

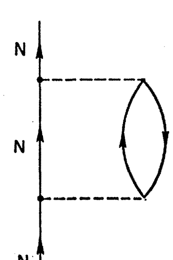

FIG. 9. Second-order contribution to the single-particle energy of a nucleon in nuclear matter. The dashed lines represent the long-range part of the OPEP.

The volume V_Σ of the integration is $k<k_F$, $|\mathbf{k}+\mathbf{q}|<k_F$ for the three-body part of this interaction. The total two-body contribution is obtained by restricting the integration only by the condition $k<k_F$ and inserting a negative sign (cf. Sec. 3). The function $T(q)$ is the same as in Eq. (6) (with d replaced by r_0), and corresponds to the contribution from a tensor interaction; the central part is negligible.

Using the value $r_0=0.9$ fm, we find that the three-body part of the graph in Fig. 8 gives 2.6 MeV (repulsion), the two-body part of the contribution is -9 MeV. The net contribution from the Σ channel is the algebraic sum of these. The contributions from the graphs in Fig. 10 were also evaluated as a check on the convergence of the perturbation expansion; when the long-range part of the N-N potential was approximated by the OPEP, the contribution from each of these was found to be less than 0.5 MeV. (They are attractive.)

Finally, we remark that the Σ admixture in the wave function of the Λ is roughly 2% for the values of the

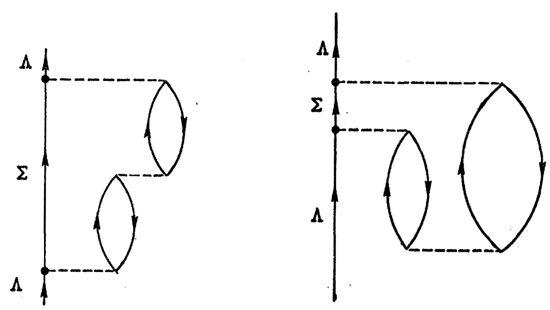

FIG. 10. Higher-order contributions from the coupling between the Λ and Σ channels. These graphs are neglected.

parameters we have chosen; treating it as a perturbation is therefore quite appropriate.

7. CONCLUDING REMARKS

Summing up the results of the preceding sections, we find that the contribution to the potential energy of the Λ from the three-body forces is 3.7 MeV (repulsion). This is the contribution in infinite nuclear matter of a density corresponding to the Fermi momentum $k_F=1.36\ \mathrm{fm}^{-1}$. It is likely that this number overestimates the contribution since we have not extracted the rearrangement energy, and because the hard core of the Λ was neglected in Sec. 5. An idea of the contribution of actual hypernuclei can be found by using a lower value of k_F, corresponding to some average nuclear density. This reduces the contribution significantly, roughly by a factor 2 for, say, ${}_\Lambda He^5$.

The best knowledge of the Λ-N two-body force is obtained from light hypernuclei, and it is therefore an effective force, corrected for a small amount of repulsion from three-body forces. When this effective two-body force is used in nuclear matter, the three-body force will cause an error, since it is more sensitive to the density than the two-body force. From simple estimates it seems that calculations of this kind should be corrected for the three-body force by reducing the binding by 1 or 2 MeV. At the moment this number is of the same order as both the experimental and theorteical uncertainties in the binding of a Λ hyperon in nuclear matter, and does not account for the apparent disagreement between theory and experiment. Reasons for introducing suppression of the odd-state Λ-N force therefore still remain.

The main part of the three-body Λ-nucleon force was due to the coupling to the Σ hyperon, and would therefore not be present in the nuclear case.

ACKNOWLEDGMENTS

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Comparison of Moderate-Energy Proton-Proton Models. III*

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The predictions of 12 proton-proton models and phase-shift representations are compared to a selected but comprehensive set of 9–330-MeV scattering data. The best fit was found to be produced by a quadratic interpolation of Arndt and MacGregor's phase-shift table, with a ratio of χ^2 to its expected value of 1.4. The best potential is that of Hamada and Johnston, with a ratio of 3.1. The ratio for the Tabakin potential is 28.

I. INTRODUCTION

In this paper we bring up to date the comparison[1–3] of published proton-proton models and energy-dependent phase-shift analyses with data that, in our opinion, represent the most accurate experimental information currently available on proton-proton scattering between 9 and 330 MeV. The models we consider were constructed for a variety of purposes and were fitted to various other selections of the data, so that a simple χ^2 listing does not necessarily serve as a figure of merit as to how well the original authors' purposes were served. However, these models are often used for other purposes, accompanied by some statement to the effect that "this model agrees with existing scattering data." To the best of our knowledge, this is not true for existing models according to the usually accepted statistical criteria[4]; on the other hand the discrepancies may be irrelevant for some applications. This point obviously should be investigated in each case. For example, a small adjustment of the parameters might improve the fit to the data without affecting the calculation at hand; but it also might be extremely significant. In other cases, the model does provide a good fit over some energy ranges, but might be applied in an energy range where it is in violent disagreement with the data. Clearly this point should also always be investigated, and one of our purposes is to make this more obvious. Another is to give some indication of where the best existing models should be corrected. A third is to point up the fact that many popular models which are often

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[1] P. Signell and N. R. Yoder, Phys. Rev. **132**, 1707 (1963).

[2] P. Signell and N. R. Yoder, Phys. Rev. **134**, B100 (1964).

[3] A preliminary version of this paper was presented at the New York meeting of the American Physical Society in January, 1967 [N. R. Yoder and P. Signell, Bull. Am. Phys. Soc. **12**, 50 (1967)]; numerical results given here supercede that report.

[4] Since this work was completed, work by both the Yale and the Livermore groups as reported at the Special Topics Conference on the Nucleon-Nucleon Interaction, Gainsville, Florida, March, 1967 (unpublished) is in much closer agreement with the data and with each other than any phase representation discussed in this paper. Also a new potential model was reported from La Jolla, and a new revision of the boundary condition model from M.I.T. For this recent work the reader should consult the abstracts in the Bull. Am. Phys. Soc. (to be published), and the appropriate papers in the July, 1967 issue of the Rev. Mod. Phys. (to be published).