Small-Signal Current Transients in Insulators with Traps*

R. M. BLAKNEY†
Institute of Optics, University of Rochester, Rochester, New York

AND

H. P. GRUNWALD‡
Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 15 December 1966)

The equations for the time-dependent flow of current in an insulator with traps are derived and solved, subject to the condition that the injected space charge produce a negligible perturbation on the externally applied electric field. The theory discusses the relaxation of a small quantity of charge which is injected into, and isolated within, an insulator and allowed to interact with the trap distribution. Explicit solutions for the current are given for the case of an insulator with one and two distinct trapping levels in which multiple trapping can occur. The parameters in the theory are the mean free time for a mobile carrier between capture events at each level and the mean time-of-dwell for a carrier in the trap. A knowledge of these two parameters as a function of temperature for any level is sufficient to permit a deduction of the capture probability for and density of states in this level. An experiment is described by means of which these parameters may be determined. The analysis is extended to the case of an insulator with a distribution of traps in energy by means of a somewhat arbitrary definition of two classes of traps: shallow and deep. The physical significance of the time constants (parameters) is sacrificed by this device, but the analysis may prove useful in studying trapping processes in materials having several distinct species of traps distributed in the forbidden energy gap, or even a continuous distribution of traps. The practicability of the small-signal technique for the investigation of trapping processes in insulators is discussed briefly.

I. INTRODUCTION

IN many insulators and high-resistivity semiconductors, localized states in the forbidden energy gap (traps and recombination centers) play a dominant role in the mechanism of transport of electrical charge through the material. The identification of these states in the terms of their physical and chemical origins requires first that they be characterized by parameters which can be determined experimentally and also permit interpretation in terms of the physical properties of the state. The two properties of importance are the capture probability of the state and its position in the forbidden energy gap. While the capture probability cannot be measured directly, its value can be inferred from a knowledge of the conduction band density of states, the concentration of trapping centers and two parameters: the mean free time for a mobile carrier between capturing events and the mean time-of-dwell for a carrier in a trap. For a particular insulator, there may be several species of states in competition for the free carriers, each characterized by distinct pairs of values of the trapping parameters and by its position in the energy level structure of the solid. This paper deals with the analysis of charge carrier motion in such a material under conditions in which the localized states act simply as traps. The analysis is subject to well-defined boundary conditions which can be readily met in an experiment, and it will be shown that when the conditions are satisfied, the trapping parameters for each of the competing species of trap may be determined. In the following paper, the theory developed below will be applied to the analysis of some experimental results on vitreous selenium which were obtained by this technique.

The experimental technique is a variation of the minority carrier drift experiment described by Haynes and Shockley.[1] In the modification of interest here, the experiment consists in placing an insulator in a uniform electric field between plane parallel electrodes and observing the transient electrical response when charge carriers of one sign are suddenly injected into the insulator at one electrode. Since only one sign of carrier is involved, the localized states function only as traps for the injected charge and the time history of the observed current pulse reflects the kinetic processes of trapping. This general technique has been used by several investigators[2–12] to study charge-transport processes in insulators.

There are two modes of operation for this experiment which can be described with sufficient precision to allow

* Research supported in part by the National Aeronautics and Space Administration.

† Present address: E. H. Plessett Associates, Incorporated, Santa Monica, California.

‡ This work is based in part on a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Rochester, Rochester, New York. Present Address: Department of Physics, Union College, Schenectady, New York.

[1] J. R. Haynes and W. Schockley, Phys. Rev. **81**, 835 (1951).
[2] J. R. Haynes and W. Schockley, Phys. Rev. **82**, 935 (1951).
[3] W. E. Spear, Proc. Phys. Soc. (London) B **70**, 669 (1957): B **76**, 826 (1960).
[4] W. E. Spear, and J. Mort, Proc. Phys. Soc. (London) **81**, 130 (1963).
[5] R. A. Fotland, J. Appl. Phys. **31**, 1558 (1960).
[6] J. L. Hartke, Phys. Rev. **125**, 1177 (1962).
[7] O. H. LeBlanc. Jr., J. Chem. Phys. **33**, 626 (1960).
[8] R. G. Kepler, Phys. Rev. **119**, 1226 (1960).
[9] R. Van Heyningen, Phys. Rev. **128**, 2112 (1963).
[10] W. Helfrich and P. Mark, Z. Physik **166**, 360 (1960).
[11] A. Many and G. Rakavy, Phys. Rev. **126**, 1980 (1962).
[12] A. Many, S. Z. Weiss, and M. Simhony, Phys. Rev. **126**, 1989 (1962).

a quantitative calculation of the current flow. They are distinguished by the conditions of carrier injection and may be designated as (1) the space-charge-limited (SCL) mode, and (2) the small-signal mode. An analysis of the transient current flow in the SCL mode has been given by Helfrich and Mark[10] for the case of a trap-free insulator, and independently by Many and Rakavy[11] for semiconductors and insulators in the presence of trapping. The theory was applied by Many, Weiss and Simhony[12] in the analysis of SCL current transients in single crystals of iodine, with good agreement between the predicted and observed shape of the current transient in samples exhibiting both weak and strong trapping. In principle, this theory is applicable in the case of an insulator with more than one species of trap, but because of the mathematical complexities arising from the presence of the injected space-charge field, it is practically limited to materials containing a single trapping level. One purpose of this communication is to show how insulators with a more complicated trapping structure can be handled.

The theory for current flow in an insulator with traps in the small-signal mode describes the relaxation of a small quantity of charge which has been isolated within the insulator and allowed to interact with the trap distribution. The small-signal condition requires that the total amount of charge injected into the sample be a small fraction of that stored on the electrodes, and further, that the time for injection be small compared to the free-carrier transit time across the sample. Under these conditions, the current in the external circuit may be regarded as being induced by the drift of a thin sheet of charge in a uniform electric field. By requiring the space-charge field to be negligible by comparison with the externally applied electric field, the equations governing the current flow become linear and can be solved exactly. The trapping process is described in terms of two parameters for each trapping level—the mean free time for a carrier between trapping events and the mean time-of-dwell for a carrier in a trap. The values of these parameters may be determined by fitting the theoretical expression for the current to the experimentally observed small-signal current transient.

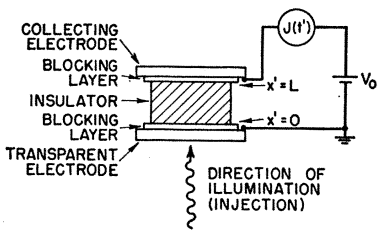
The plan of the paper is as follows: In Sec. II, the physical problem for the time-dependent flow of charge in an insulator with traps is formulated, and the boundary conditions for the small-signal mode are discussed. The equation for the current transient in a material containing a single trapping level is developed in Sec. III, and in Sec. IV, the extension of the analysis to more than one species of trap is given. In Sec. V, the practicability of the small-signal technique is discussed.

II. FORMULATION OF THE PROBLEM

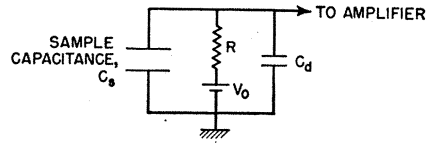
A. General Equations for Current Flow

The time-dependent flow of charge carriers in one dimension will be considered. The sample in the form of a slab is sandwiched between plane parallel electrodes which have the property of preventing the exchange of charge between sample and electrodes, i.e., they are blocking electrodes. As indicated in Fig. 1(a), charge injection is assumed to be accomplished by illuminating the insulator through one transparent electrode with a short flash of light which is strongly absorbed by the insulator. For the polarity given in the figure, a definite number of electrons is thus injected as a thin sheet just inside the electrode at $x'=0$. The sheet of charge subsequently drifts toward the collecting electrode at $x'=L$ under the action of the external electric field. This induces the current $J(t')$ in the external circuit which decreases with time as electrons are lost to traps. The equation for $J(t')$ will be developed first for the idealized circuit of Fig. 1a, and the changes required in the formulation for the more realistic circuit of Fig. 1(b) will be given in the Appendix.

Prior to the injection of the carrier sheet, the sample is electrically neutral, and since we consider only insulators, the equilibrium concentration of free and trapped charge is taken to be zero. After the injection, a space-charge exists in the sample, part of which is



(a)



$R(C_s+C_d) \gg$ TRANSIT TIME

C_d = AMPLIFIER INPUT CAPACITANCE PLUS DISTRIBUTED CAPACITANCE

R = PARALLEL COMBINATION OF AMPLIFIER INPUT RESISTANCE AND LARGE RESISTANCE IN SERIES WITH SAMPLE AND BATTERY

(b)

FIG. 1. (a) Sample configuration and idealized circuit assumed in the calculation of the small-signal current transient. Electrons are injected at $x'=0$ when a short flash light, transmitted by the transparent electrode, is strongly absorbed by the insulator, producing a thin sheet of mobile carriers. As the electrons drift toward the collecting electrode at $x'=L$, a current $J(t')$ is induced in the external circuit and measured by a "zero-impedance" current meter. The potential of the collecting electrode is maintained at V_0 by battery. (b) Schematic drawing of the essential elements of a practical measuring circuit for high-capacitance samples. The current $J(t')$ is integrated by the $R(C_s+C_d)$ network to produce a voltage signal. After amplification, this signal may be differentiated to produce a signal proportional to $J(t')$. The potential of the collecting electrode varies as $V_0-(1/C_d)\int_0^{t'}J(t')dt'$.

mobile with a concentration $n(x',t')$, and the remaining part is in traps with a concentration $n_t(x',t')$. In the absence of sources or sinks for charge, the equation for charge continuity states:

$$\partial J_c(x',t')/\partial x' = -q(\partial/\partial t')[n(x',t')+n_t(x',t')], \quad (1)$$

where $J_c(x',t')$ is the conduction current density, given by

$$J_c(x',t') = q\mu_d n(x',t')E(x',t') - qD\partial n(x',t')/\partial x'. \quad (2)$$

q is the charge on the electron, μ_d and D are the drift mobility and diffusion constant for the electrons, respectively, and $E(x',t')$ is the electric field. If κ is the static dielectric constant for the insulator, the field is related to the space-charge through Poisson's equation:

$$\partial E(x',t')/\partial x' = (4\pi q/\kappa)[n(x't')+n_t(x',t')]. \quad (3)$$

The total current $J(t')$ flowing in the sample is the sum of the conduction and displacement currents and is given by

$$J(t') = J_c(x',t') + (\kappa/4\pi)\partial E(x',t')/\partial t'. \quad (4)$$

The effect of trapping may be taken into account as follows: If there are m independent species of traps distributed in the forbidden energy gap, we characterize each species by a capture probability $C_{ti}(\text{cm}^3\ \text{sec}^{-1})$, a uniform concentration $N_{ti}(\text{cm}^{-3})$, and its position in energy, E_i. We now make the simplifying assumption that $N_{ti} \gg n_{ti}$ throughout the period of the current pulse, so that the number of empty traps may be taken as constant. (For materials with trap densities in excess of 10^{13} cm^{-3}, this condition can always be met in the small-signal mode). The probability per unit time for the capture of a carrier into the ith level is then a constant, denoted by $1/T_i$. T_i is the mean free time for a mobile carrier before capture into the level i, and is defined by the relation

$$C_{ti}T_i = 1/N_{ti}. \quad (5)$$

If $1/T_{ri}$ is the probability per unit time for thermal release of a trapped carrier to the conduction band, we may use the principle of detailed balance to express this quantity as

$$1/T_{ri} = (1/T_i)(n/n_{ti}) = C_{ti}N_c \exp[-(E_c-E_i)/kT]. \quad (6)$$

In this expression N_c is the density of states at the bottom of the conduction band which is located in energy at E_c.

The rate at which the trapped carrier concentration changes with time is given by a set of m equations of the form

$$\partial n_{ti}(x',t')/\partial t' = (1/T_i)n(x',t') - (1/T_{ri})n_{ti}(x',t'), \; i=1, 2, \cdots m. \quad (7)$$

The first term on the right of Eq. (7) gives the rate of capture of mobile carriers into the ith level, and the second term gives the rate at which trapped carriers are released from this level by thermal processes. If N_c is known for a given insulator, a measurement of T_i, T_{ri} and (E_c-E_i) will permit the quantities C_{ti} and N_{ti} to be determined.

It is convenient to express Eqs. (2), (3), (4), and (7) in terms of the following dimensionless quantities:

$$x = x'/L, \quad t = t'/t_0, \quad \tau_i = T_i/t_0,$$
$$\rho = qnL/(C_sV_0), \quad j = Jt_0/(C_sV_0), \quad \epsilon = E/(V_0/L). \quad (8)$$

In these equations, $C_s = \kappa/(4\pi L)$ is the geometrical capacitance per unit area of the sample, and $t_0 = L^2/(\mu_d V_0)$ is the time of transit for a single carrier across the sample. V_0 is the voltage applied to the sample by the battery. With these definitions, the equations governing the motion of the charge carriers are transformed to

$$j(t) = j_c(x,t) + \partial\epsilon(x,t)/\partial t, \quad (9)$$

$$j_c(x,t) = \rho(x,t)\epsilon(x,t) - \gamma\partial\rho(x,t)/\partial x, \quad (10)$$

$$\partial\epsilon(x,t)/\partial x = \rho(x,t) + \rho_t(x,t), \quad (11)$$

$$\partial\rho_{ti}(x,t)/\partial t = (1/\tau_i)[\rho(x,t) - \theta_i\rho_{ti}(x,t)], \quad i=1, 2, \cdots m \quad (12)$$

with

$$\rho_t = \sum^{m} \rho_{ti}\,\theta_i = T_i/T_{ri}, \; \gamma = D/(\mu_d E_0 L). \quad (12')$$

B. Boundary Conditions

We consider the current flow only in the time interval between injection and the time of arrival of the first carrier at the collecting electrode, i.e. $0<t<1$. Since a definite number of carriers, ρ_0, are injected at $t=0$, the total change in the sample remains constant in this time interval, and we have the condition

$$\partial\rho_0/\partial t = 0 = (d/dt)\int_0^1 [\rho(x,t)+\rho_t(x,t)]dx, \quad 0<t<1. \quad (13)$$

At the instant of injection, the free and trapped carrier density is zero everywhere, except for the thin sheet of free carriers at $x=0$. We have, therefore,

$$\epsilon(x,0) = 1, \quad x>0, \quad (14)$$

$$\rho(x,0) = 0, \quad x>0, \quad (15a)$$

$$\rho_t(x,0) = 0, \quad x \geqslant 0. \quad (15b)$$

Furthermore, for the circuit shown in Fig. 1a, the battery maintains a constant potential difference between the sample electrodes, so that

$$\int_0^1 \epsilon(x,t)dx = 1, \quad 0 \leqslant t \leqslant 1. \quad (16)$$

Since the left side of Eq. (9) is independent of x,

we can write this equation equivalently:

$$j(t)=\int_0^1 j_c(x,t)dx+(d/dt)\int_0^1 \epsilon(x,t)dx. \qquad (9')$$

Substituting for j_c from Eq. (10) and applying the boundary conditions (16), the total current becomes

$$j(t)=\int_0^1 \rho(x,t)\epsilon(x,t)dx+\gamma\rho(0,t). \qquad (17)$$

The last term in this equation is the contribution to the total current from diffusion, and was obtained by noting that $\rho(1,t)=0$ for $t<1$. The constant γ, defined in Eq. (12'), is the ratio of a "diffusion velocity" to the drift velocity and will be much smaller than unity for all cases of interest in the small-signal mode, which requires $\rho(0,t)\ll 1$. Therefore the diffusive contribution to the current will always be exceedingly small and can be neglected. Equation (17) can then be rewritten as

$$j(t)=\int_0^1 [\rho(x,t)+\rho_t(x,t)]\epsilon(x,t)dx - \int_0^1 \rho_t(x,t)\epsilon(x,t)dx. \qquad (18)$$

The first term in Eq. (18) is just the initial current j_0, since at $t=0$, the second term vanishes by Eq. (15b). With the help of Eq. (11), the first term in Eq. (18) may be integrated to give the initial current:

$$j_0=(\tfrac{1}{2})[\epsilon^2(1,0)-\epsilon^2(0,0)]. \qquad (19)$$

At the instant of injection, an amount of charge, ρ_0, is removed from the electrode at $x=0$ and made mobile at $x=0+$, so that the field at $x=0$, $t=0$ is

$$\epsilon(0,0)=1-\rho_0. \qquad (20)$$

Substituting Eqs. (14) and (20) into Eq. (19), we obtain

$$j_0=\rho_0(1-\rho_0/2). \qquad (21)$$

With a light flash of sufficient intensity to inject all the charge on the electrode, the injection is space-charge limited so that $\rho_0=1$ and $\epsilon(0,0)=0$. Under these conditions, Eq. (21) gives for the initial current, $j_0=\frac{1}{2}$, and Eq. (19) predicts that the initial current will vary as the square of the applied voltage. This is in agreement with the results for transient SCL currents obtained by Helfrich and Mark[10] and Many and Rakavy.[11]

In the small-signal mode, we require $\rho_0\ll 1$, so that we may set $j_0=\rho_0$ without appreciable error. The equation for the transient current then becomes

$$j(t)=\rho_0-\int_0^1 \rho_t(x,t)\epsilon(x,t)dx. \qquad (22)$$

Equation (22) predicts that the initial current should increase linearly with the voltage, and that for optically generated carriers, its magnitude will be proportional to the intensity of the exciting light. The observation of this behavior in an experiment may be taken as evidence that the conditions for the small-signal mode have been met.

III. TRANSIENT CURRENTS IN THE PRESENCE OF A SINGLE TRAPPING LEVEL

For an insulator with a single level of traps, the general shape of the current transient is easy to predict. At the instant of injection, all the carriers are mobile, and a large current will be observed, which will subsequently decrease with time as carriers are lost to traps. In general, the current is composed of two components: the drift of the primary carrier sheet and the motion of the carriers behind the sheet which have been thermally released from the traps. The relative contributions of the two charge distributions and the rate at which the current decays are determined by the capture and release times τ_1 and τ_r.

The analytical expression for the current is obtained by first taking the time derivative of Eq. (22) and substituting Eq. (12), where the subscript i may be dropped. This yields

$$dj/dt=-j_0/\tau+[(1+\theta)/\tau]\int_0^1 \rho_t\epsilon dx - \int_0^1 (\partial\epsilon/\partial t)\rho_t dx. \qquad (23)$$

The term involving $\int_0^1 \rho_t\epsilon dx$ can now be eliminated between Eqs. (22) and (23), giving

$$dj/dt+[(1+\theta)/\tau]j=(\theta/\tau)j_0-\int_0^1 (\partial\epsilon/\partial t)\rho_t dx. \qquad (24)$$

This equation has a simple solution in the small-signal limit, which requires that

$$\rho_0\ll 1 \quad \text{and} \quad \partial\epsilon/\partial t\ll 1. \qquad (25)$$

Therefore, the last term on the right of Eq. (24) may be neglected, since it is the product of two quantities, each of which is much smaller than unity. This yields a simple, first-order differential equation which has the solution

$$j(t)/j_0=[1/(1+\theta)]\exp[-(1+\theta)/\tau]t+\theta/(1+\theta). \qquad (26)$$

In the case of deep traps, $\theta=0$, and there is no contribution from thermally released carriers. Equation (26) then states that the current decays exponentially with time. When Eq. (26), with $\theta=0$, is expressed in terms of the dimensioned variables according to Eq. (8), and integrated between the limits $t=0$, and $t=L^2/$

$(\mu_d V_0)$, the Hecht formula[13] in one of its conventional forms is obtained:

$$Q=(qN_0\mu_d E_0 T_1/L)\{1-\exp[-L/(\mu_d E_0 T_1)]\}. \quad (27)$$

Here, we have put $J_0=qN_0\mu_d V_0/L^2$. Q is the charge induced onto the sample electrodes by the motion of N_0 charges injected at one electrode and driven through the sample by the electric field $E_0=V_0/L$.

IV. TRANSIENT CURRENTS FOR A DISTRIBUTION OF TRAPS IN ENERGY

The extension of the analysis of the preceding section to an insulator with several species of traps at different energy levels in the energy gap is straightforward. We discuss first the case for two trapping levels characterized by τ_1, θ_1 and τ_2, θ_2. Two equations are now required to describe the trapping kinetics:

$$\partial\rho_{t1}/\partial t=(1/\tau_1)(\rho-\theta_1\rho_{t1}),$$
$$\partial\rho_{t2}/\partial t=(1/\tau_2)(\rho-\theta_2\rho_{t2}). \quad (28)$$

The total density of trapped carriers is $\rho_t=\rho_{t1}+\rho_{t2}$, and the equation for the total current becomes [see Eq. (22)]

$$j(t)=j_0-\int_0^1\rho_{t1}\epsilon dx-\int_0^1\rho_{t2}\epsilon dx. \quad (29)$$

The small-signal condition can be applied at this point by noting that

$$\epsilon(x,t)=1-\epsilon_1(x,t), \quad (30)$$

where $\epsilon_1(x,t)$ is the contribution to the electric field due to the injected space charge, and is a small quantity compared to the applied field. Therefore, terms like $\rho\epsilon_1$, $\rho_t\epsilon_1$, etc., may be dropped without incurring appreciable error. When the first and second time derivatives of $j(t)$ in Eq. (29) are taken, and the appropriate trapping equations from Eqs. (28) are applied whenever $\partial\rho_{t1}/\partial$ and $\partial\rho_{t2}/\partial t$ appear, one obtains

$$dj/dt+(1/\tau_1+1/\tau_2)j=(\theta_1/\tau_1)\int_0^1\rho_{t1}dx+(\theta_2/\tau_2)\int_0^1\rho_{t2}dx, \quad (31)$$

$$d^2j/dt^2+(1/\tau_1+1/\tau_2)(dj/dt)-(\theta_1/\tau_1^2+\theta_2/\tau_2^2)j=-(\theta_1/\tau_1)^2\int_0^1\rho_{t1}dx-(\theta_2/\tau_2)^2\int_0^1\rho_{t2}dx. \quad (32)$$

One can now eliminate the terms involving ρ_{t1} and ρ_{t2} from Eqs. (29), (31), and (32) to obtain a second-order differential equation for the total current:

$$d^2j/dt^2+[(1+\theta_1)/\tau_1+(1+\theta_2)/\tau_2](dj/dt)+(1/\tau_1\tau_2)[\theta_1+\theta_2(1+\theta_1)]j=[\theta_1\theta_2/\tau_1\tau_2]j_0. \quad (33)$$

[13] K. Hecht, Z. Physik **77**, 235 (1932).

The solution of Eq. (33) is

$$j(t)=A\exp(-\alpha t)+B\exp(-\beta t)+j_\infty, \quad (34)$$

with

$$\alpha+\beta=(1+\theta_1)/\tau_1+(1+\theta_2)/\tau_2,$$
$$\alpha\beta=[\theta_1+\theta_2(1+\theta_1)]/(\tau_1\tau_2),$$
$$\alpha A+\beta B=j_0/\tau_1+j_0/\tau_2, A+B+j_\infty=j_0,$$
$$j_\infty=j_0\theta_1\theta_2/[\theta_1+\theta_2(1+\theta_1)]. \quad (35)$$

j_∞ is the steady-state current which flows after the injected carriers have come into equilibrium with the trap distribution. (As soon as the first carrier reaches the collecting electrode, of course, Eq. (34) no longer applies.) In principle, all the constants in Eq. (34) can be determined from a single observation of a current pulse, and the relations (35) can be employed to obtain τ_1, τ_2, θ_1, and θ_2. How well this can be done in practice, however, depends on the relative magnitudes of α and β and the signal-to-noise ratio of the current measurement. Unless the rate processes for the two levels differ significantly, the two-level model might be difficult to distinguish from the single level case discussed previously. However, if one of the levels is deep, in the sense that thermal release of captured carriers is improbable in a time interval equal to the transit time, then $\theta_2=0$ in Eqs. (33), (34), and (35), so that $j_\infty=0$, and the current will decay to zero with all the carriers in the deep traps. This situation is frequently encountered in real insulators. Note that if both $\theta_2=0$ and $\theta_1=0$, Eq. (33) reduces to the equation for a single, deep level.

With the following modifications in the interpretation of the time constants, this analysis may be applied to transient currents obtained with materials having many trapping levels distributed in energy. As suggested above, one can expect to distinguish at least two classes of traps in the distribution: (1) shallow levels (denoted by the subscript $s=1, 2, \cdots$) which release trapped carriers in a time shorter than the free-carrier transit time; (2) deep levels (subscript $d=1, 2, \cdots$) from which thermal excitation of trapped carriers is improbable in a transit time interval. For each class of levels, the trapping kinetics are described by equations of the form

$$\partial\rho_{ts}/\partial t=\rho/\tau_s-\rho_{ts}/\tau_{rs}, \quad \partial\rho_{td}/\partial t=\rho/\tau_d.$$

If $\rho_{t1}(x,t)$ and $\rho_{t2}(x,t)$ are the concentrations of carriers in all of the shallow and deep levels, respectively, then $\rho_{t1}=\sum_s\rho_{ts}$, $\rho_{t2}=\sum_d\rho_{td}$. The corresponding trapping equations become

$$\partial\rho_{t1}/\partial t=\rho/\tau_1-\rho_{t1}/\tau_{r1},$$
$$\partial\rho_{t2}/\partial t=\rho/t_2.$$

The time constants τ_1 and τ_2 may be regarded as the effective capture times for the two classes of traps, and τ_{r1} is the mean time-of-dwell for a carrier in the shallow levels. These three quantities are defined as

follows:

$$1/\tau_1=\sum_s(1/\tau_s)\,,\ 1/\tau_2=\sum_d(1/\tau_d)\,,\ \rho_{t1}/\tau_{r1}=\sum_s(\rho_{ts}/\tau_{rs}).$$

With these definitions, the case of many trapping levels may be discussed in terms of the two-trap model. This device, of course, means that the significance of the observed time constants in terms of the capture probabilities and trap densities is sacrificed. Nevertheless, this approach might be employed with profit to study changes in the relative distribution of shallow and deep levels due, for example, to impurity effects or sample history.

Many insulators display trapping phenomena which can be interpreted in terms of a continuous distribution of traps in the energy gap.[14] Vitreous selenium has been cited as an example of such a material.[6,15] The small-signal transient current analysis may be applied to these materials also, by replacing the summations in the above equations by integrations over the energy range appropriate to each of the two sets of traps. If the distribution extends far enough below the conduction band so that the two classes of traps may be defined, the small-signal current is given by Eq. (33) with $\tau_2/\tau_{r2}=\theta_2=0$.

V. DISCUSSION

The main experimental requirement in the application of the small-signal technique to insulators is that the injected space-charge produce a negligible distortion of the externally applied electric field. This necessarily limits the magnitude of the currents which can be observed, and the signals one expects to encounter will be much smaller than those obtained with space-charge-limited injection. To achieve as large a signal as possible, one wants to use the highest-injection level consistent with the small-signal condition. Therefore, an estimate of the magnitude of the upper limit for injection is of interest.

Using the linear relation between the initial current and the flash intensity as the criterion that the small-signal condition is satisfied, Eq. (21) indicates that as much as one-tenth of the stored charge can be injected without serious error. At this level, the proportionality constant between the initial current and the light intensity is 0.95 instead of unity—a difference which would be hard to detect in experiments of this kind. The experiments of Many, Weiss, and Simphony[12] on flash illumination of iodine crystals provide evidence that this criterion is met when as much as 10% of the stored charge is injected. These authors have plotted the initial current in iodine as a function of applied voltage for a range of flash intensity. When these data are replotted to show explicitly the dependence of the initial current on the flash intensity, the linear relation appears to be well established when the intensity has dropped to 10% of that required for space-charge-limited injection. Thus, it appears that the small-signal condition should still be satisfied for current transients as large as one-tenth the maximum current which can be passed by the insulator.

In practice, the small-signal technique is limited to materials in which the carrier mobility is on the order of 1 cm^2/V sec or less. When such materials are being studied, the interpretation of the trapping parameters obtained from the current transients must be made in the light of the mechanism responsible for the low mobility. Consider, for example, a material in which the carrier drift mobility is limited by a high density of shallow traps, and assume that the injected carriers come into equilibrium with the traps in a time so short as to be unresolvable in the transient current measurement. If deeper traps are also present, the above theory will still predict the form of the current transient, but the trapping parameters so obtained will not all be simply related to the trap densities and capture probabilities. The mobility appearing in the current equations will now be the trap limited mobility, rather than the microscopic mobility. The trapping times deduced from the current transients will be larger than the true trapping times [see Eq. (5)] by a factor of $(1/\theta\mu+1)$, where $\theta\mu$ is the ratio of free carrier density to the density of carriers in the traps which control the mobility. However, the time-of-dwell T_r, for a carrier in the deeper traps should be independent of the presence of other trap species, so that the magnitude and temperature dependence of T_r should appear as given in Eq. (6). Hence, if N_c is known, the capture probability for these levels can be deduced by measuring $1/T_r$ as a function of temperature.

APPENDIX

In this Appendix, we indicate the modifications to the boundary conditions of Sec. IIB required for the more realistic measuring circuit of Fig. 1(b). The capacitance of the sample, C_s, is in parallel with the input capacitance of the amplifier and the distributed capacitance in the circuit. The parallel combination of these latter capacitances is represented as C_d in Fig. 1(b). As soon as charge begins to move in the sample, a current is induced in the external circuit and charges C_d. Consequently, the potential of the collecting electrode is not constant in time but varies with the rate of charging of C_d. The boundary condition Eq. (16) must therefore be replaced by

$$\int_0^1 \epsilon(x,t)dx=1-\omega(t)\,. \tag{A1}$$

Here, $1-\omega(t)=V(t')/V_0$ is a dimensionless quantity corresponding to the potential $V(t')$ of the collecting electrode. Integration of Eq. (9) over the volume of the

[14] A. Rose, R.C.A. Rev. **12**, 362 (1951).
[15] H. P. D. Lanyon, Phys. Rev. **130**, 134 (1963).

sample then gives

$$j(t)=\int_0^1 j_c(x,t)dx-(d/dt)\omega(t). \quad \text{(A2)}$$

The potential, $V(t')$ is given by

$$V(t')=V_0-(1/C_d)\int_0^{t'} J(t')dt'. \quad \text{(A3)}$$

Converting the right side of this expression to dimensionless quantities by the prescription of Eqs. (8), one obtains

$$\omega(t)=(C_s/C_d)\int_0^t j(t)dt. \quad \text{(A4)}$$

Equation (A2) for the current can now be written

$$j(t)=\chi\int_0^1 j_c(x,t)dx, \quad \chi=C_d/(C_s+C_d), \quad \text{(A5)}$$

which replaces Eq. (17). The rest of the analysis is unchanged.

PHYSICAL REVIEW VOLUME 159, NUMBER 3 15 JULY 1967

Trapping Processes in Amorphous Selenium*

R. M. BLAKNEY†
Institute of Optics, University of Rochester, Rochester, New York

AND

H. P. GRUNWALD‡
Department of Physics, University of Rochester, Rochester, New York

(Received 15 December 1966)

The theory of small-signal current transients is applied to the study of electron trapping processes in amorphous selenium. In the experiment, a 10^{-8}-sec light pulse illuminated one side of the sample and produced free carriers near this surface. The free electrons were drawn across the sample in an applied electric field, and the shape of the current transients thus produced was studied. The shape of these transients indicated that electron trapping processes in vitreous selenium involve three distinct species of trap: those which control the mobility (m traps), a deep trapping level (d traps), and a shallow trapping level (s traps). Magnitudes are given for the ratio of the m-trap density to the density of states in the conduction band (N_m/N_c) and for the ratio of the s-trap density to the density of states in the conduction band (N_s/N_c). For a selenium film which has been evaporated onto a substrate held at 38°C during the evaporation, these ratios are $N_m/N_c=5.2\times10^{-4}$ and $N_s/N_c=2.4\times10^{-6}$, respectively. It is further shown that the magnitude of these ratios decreases as the substrate temperature at which the samples are prepared is increased. The energy separation of the s level and the m level from the conduction-band edge depends also on the sample preparation, the separation increasing as the substrate temperature is increased. For the film prepared at 38°C, the levels are at $E_s=0.39$ eV and $E_m=0.29$ eV, respectively, below the conduction-band edge. The capture probability for both the s and d traps was measured and was found to increase exponentially with $1/T$ with a characteristic energy. It is suggested that the electron trapping processes in vitreous selenium are closely connected with the structural properties of the material. This is strongly indicated by a decrease in the shallow-trap density as the measuring temperature approaches the glass transition temperature.

I. INTRODUCTION

IN this paper, we present the results of an experimental investigation of electron trapping processes in amorphous selenium. The theoretical basis for these studies was given in the preceding paper.[1] There it was shown that under certain well-defined and experimentally realizable conditions, the equations governing the transient flow of current in an insulator with several species of traps could be solved exactly. These conditions are: (1) A small quantity of charge is injected into the insulator such that the injected space charge field produces a negligible perturbation on the externally applied electric field. (2) The time for injection is much shorter than the free-carrier transit time across the sample, so that in the absence of trapping, the current induced in the external circuit is the result of the drift motion of a thin sheet of charge along the potential gradient in the insulator. (3) Blocking electrodes are applied to the sample to prevent further injection of

* Research supported in part by the National Aeronautics and Space Administration.

† Present address: E. H. Plessett Associates, Incorporated, Santa Monica, California.

‡ This work is based in part on a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Rochester, Rochester, New York. Present address: Department of Physics, Union College, Schenectady, New York.

[1] R. M. Blakney and H. P. Grunwald, preceding paper, Phys. Rev. **159**, 658 (1967).