Anomalous Skin Effect in Tin and Indium

K. R. LYALL AND J. F. COCHRAN
Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada
(Received 16 January 1967)

The normal-state reactive and resistive skin depths of very pure polycrystalline tin and indium have been measured as a function of frequency for frequencies between 7 and 20 000 cps. Measurements were made at liquid-helium temperatures, using specimens in the form of flat plates with a thickness of 1 mm. The free-electron anomalous-skin-effect theory of Reuter and Sondheimer was used to extrapolate the data to infinite frequency, and thus to obtain skin depths appropriate to the extreme anomalous limit. Analysis of the data based on the free-electron model and assuming diffuse electron scattering at the metal surfaces yields the values $(\sigma/l)_{\mathrm{Sn}}=(6.9\pm1.5)\times10^{10}\ \Omega^{-1}\ \mathrm{cm}^{-2}$ and $(\sigma/l)_{\mathrm{In}}=(9.0\pm1.6)\times10^{10}\ \Omega^{-1}\ \mathrm{cm}^{-2}$. The skin depths are estimated to deviate from their anomalous limits by approximately 2% at a frequency of 1 Mc/sec.

I. INTRODUCTION

RADIATION incident on the surface of a semi-infinite metal slab is attenuated by currents which flow in such a way as to shield the interior of the slab. The distance δ to which the radiation penetrates can be defined in terms of the flux contained in the metal per unit length of surface:

$$\delta=\frac{1}{H_y(0)}\int_0^\infty H_y(z)dz. \qquad (1)$$

In Eq. (1), z measures distance from the surface of the slab toward the interior of the metal along the normal to the surface, and the magnetic field $H_y(z)$ is taken to be in the y direction. In general, the penetration depth is a complex quantity:

$$\delta=\delta_x+i\delta_R. \qquad (2)$$

The reactive skin depth δ_x is proportional to that part of the flux which is in phase with the magnetic field at the surface of the slab, $H_y(0)$, and the resistive skin depth δ_R is proportional to that part of the flux which is out of phase with $H_y(0)$. The penetration depth is proportional to the component of surface impedance perpendicular to the magnetic field;

$$Z=(4\pi/c)E_x(0)/H_y(0)=-4\pi i\omega\delta/c^2, \qquad (3)$$

where $E_x(0)$ is the amplitude of the electric field at the surface, $i=\sqrt{-1}$, and the time variation of the fields is taken to be $e^{-i\omega t}$. Equation (3) is written in Gaussian units.[1]

The penetration depth depends upon the frequency of the radiation and upon the properties of states at the Fermi surface of the metal. It is useful to distinguish between two limiting cases depending upon whether the mean free paths l of the electrons in the metal are greater than or less than $|\delta|$. In the classical limit $(|\delta|/l)\gg1$, the decay of the fields within the slab is exponential, and the reactive and resistive skin depths are

$$\delta_x=\delta_R=(c^2/8\pi\omega\sigma)^{1/2}, \qquad (4)$$

where σ is the dc conductivity of the metal. In the anomalous limit $(|\delta|/l)\ll1$, the decay of the fields is not exponential, and the theoretical treatment is difficult because the interaction of the electrons with the metal surface must be considered explicitly. Theoretical formalisms have been based on one of two assumptions: (a) that the electrons are specularly reflected at the surface or (b) that they are diffusely reflected at the surface. These two cases are represented by the notations $p=1$ and $p=0$, respectively.

The theory of the skin effect for free electrons has been given by Reuter and Sondheimer.[2] Their results for the anomalous limit are

$$\text{for}\quad p=0,\quad \delta_x^\infty=\sqrt{3}\delta_R^\infty=\left[\frac{9c^2}{64\pi^2\omega(\sigma/l)}\right]^{1/3}, \qquad (5)$$

$$\text{and for}\quad p=1,\quad \delta_x^\infty=\sqrt{3}\delta_R^\infty=\left[\frac{8c^2}{81\pi^2\omega(\sigma/l)}\right]^{1/3}. \qquad (6)$$

Pippard[3,4] has generalized the anomalous-skin-effect theory to include metals having an arbitrary Fermi surface. He has shown that the anomalous limit is characterized by the following three properties:

(i) $\delta_x/\delta_R=\sqrt{3}$,
(ii) δ_x, δ_R are proportional to $\omega^{-1/3}$,
(iii) the skin depth is independent of the purity of the metal.

Thus, for $(|\delta|/l)\ll1$, δ measures a property of the Fermi surface. For free electrons this property is

$$\sigma/l=ne^2/p_F, \qquad (7)$$

where n is the density of charge carriers, e is their charge, and p_F their Fermi momentum. In the general case the penetration depth is proportional to the cube root of the Gaussian curvature of the Fermi surface integrated over a band of "effective" electron states corresponding to charge carriers whose velocities are parallel to the surface of the slab.

[1] If Z is measured in practical units and δ in centimeters, then Eq. (3) becomes $Z=-4\pi i\omega\delta/10^9\ \Omega$.

[2] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **A195**, 336 (1948).
[3] A. B. Pippard, Proc. Roy. Soc. (London) **A224**, 273 (1954).
[4] A. B. Pippard, *The Dynamics of Conduction Electrons* (Gordon and Breach Science Publishers, Inc., New York, 1965).

Original investigations of the anomalous skin effect were carried out at frequencies above 10^9 cps in order that $|\delta|$ be much smaller than l for the then available purity of metals. Metals are now available whose mean free path at liquid-helium temperatures is about 1 mm. Such metals should exhibit anomalous behavior at frequencies less than 1 Mc/sec, i.e., in the low rf range where experimental techniques are very much simpler than in the microwave range. It was the purpose of this investigation to explore the possibility of using the skin effect at frequencies of 1 Mc/sec or less to obtain information about the Fermi surfaces of metals having extreme purity.

II. EXPERIMENTAL

Specimens in the form of thin flat plates were mounted with their surfaces parallel to the oscillating magnetic field generated by a long solenoid. The mutual inductance between this solenoid and a secondary coil wrapped around the specimen was compared with that of a standard mutual inductor by means of a bridge circuit, and provided a measure of flux penetration into the metal.

The normal state penetration depth δ_N was measured relative to the superconducting-state penetration depth δ_S in order to eliminate the necessity of correcting for the dead space between the metal surfaces and the windings of the secondary coil. The bridge was first balanced with the specimen in the superconducting state. The specimen was then allowed to pass into the normal state, and the resulting off-balance signal was measured with a phase-sensitive detector. Components of this signal, one in phase and one out of phase with the primary current, were measured separately and compared with signals derived from a standard mutual inductor. The reactive and resistive skin depths were obtained from the relation

$$\delta_N - \delta_S = K(\Delta M), \tag{8}$$

where ΔM is the equivalent mutual-inductance change corresponding to the appropriate component of the off-balance signal. The calibration constant K was derived from the signal which resulted when the specimen was removed from the secondary coil at low temperatures. The value of K so obtained agreed to within 1% with a calibration constant calculated from the coil constants and specimen dimensions.

The skin depth difference $(\delta_N - \delta_S)$ was measured over the frequency range 10–20 000 cps. Two different secondary coils were used over this range and measurements were not taken at frequencies above 20 kc/sec in order to avoid problems due to self-resonance of the coil system at approximately 200 kc/sec. Measurements were reproducible to within 4% and the systematic error in the skin depths is estimated to be less than 2%.

Tin and indium polycrystalline specimens, 1.25 cm by 10 cm, were cut from sheets of 99.9999% purity metal[5] mechanically pressed to a thickness of 0.1 cm. A similar specimen was cut from 99.999% purity indium foil.[5] The ratio of resistance at 300°K to that at 4.2°K was 2.1×10^4 for the tin specimen, 1.7×10^4 for the higher-purity indium specimen, and 0.55×10^4 for the lower-purity indium specimen. Surfaces of these slabs were smooth to within $\pm10^{-4}$ cm. After measurement they were etched, and examined by means of etch pit reflections. The average grains were approximately 10^{-2} cm² in area. Grain orientation was random, although grains of some orientations could have been larger than those of other orientations, thus producing some anisotropy in the specimens.

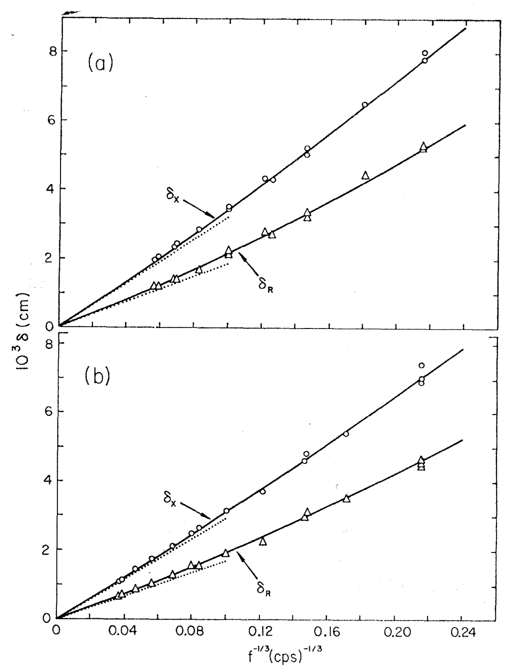

FIG. 1. Frequency dependence of the reactive and resistive skin depths: ○, δ_x; △, δ_R. Solid lines are calculated from the Reuter-Sondheimer theory for $p=0$ [Eq. (10)], with l and S_x^∞ treated as adjustable parameters. (a) Tin; $l=0.065$ cm, $S_x^\infty=0.0321$ cm sec$^{-1/3}$. (b) Indium; $l=0.071$ cm, $S_x^\infty=0.0294$ cm sec$^{-1/3}$. Dotted lines represent the extreme anomalous limit, $\delta_x^\infty=\sqrt{3}\delta_R^\infty=S_x^\infty f^{-1/3}$.

III. RESULTS

The reactive and resistive skin depths of the tin and the higher-purity indium specimen are plotted as a function of (frequency)$^{-1/3}$ in Fig. 1. The data have not been corrected to account for δ_S since the superconducting penetration depth of pure tin and indium at 0°K is approximately 4×10^{-6} cm,[6,7] which corresponds to a correction of less than 1% over the entire frequency range. Because of a size effect which will be discussed

[5] 99.9999% purity tin and indium metals and 99.999% purity indium foil were obtained from the Consolidated Mining & Smelting Company of Canada Limited, Montreal, Quebec, Canada.
[6] R. G. Chambers, Proc. Cambridge Phil. Soc. **52**, 363 (1956).
[7] P. N. Dheer, Proc. Roy. Soc. (London) **A260**, 333 (1961).

below, these skin depths are shown only for frequencies above 100 cps. It is clear from the data that for frequencies greater than 1000 cps ($f^{-1/3}=0.1$) the behavior of these metals closely approaches that expected for the anomalous regime. Straight lines drawn through the origin and fitted to the data points for $f>1000$ cps have slopes S_x, S_R which are very nearly in the ratio $\sqrt{3}$ predicted by the theory. The reactive–skin-depth slope obtained from the straight line, when combined with Eq. (5) and the dc conductivity, yields a ratio of mean free path to skin depth of approximately 10. This ratio is not very large and indicates that S_x, S_R should be corrected to obtain the values S_x^∞, S_R^∞ characteristic of the extreme anomalous limit. In the extreme anomalous limit,

$$\delta_x^\infty=\sqrt{3}\delta_R^\infty=S_x^\infty f^{-1/3}. \tag{9}$$

An extrapolation to obtain the slope S_x^∞ from our data can be conveniently made using the Reuter-Sondheimer theory for free electrons as presented in series form by Dingle.[8] Rewritten in terms of the parameters used in this paper, Dingle's series are

$$\begin{aligned} p=0:\quad &\delta_x=\delta_x^\infty\{1+(\delta_x^\infty/l)[0.5221+0.6219\log_{10}(l/\delta_x^\infty)]+(\delta_x^\infty/l)^2[0.2716+0.6116\log_{10}(l/\delta_x^\infty)]\},\\ &\delta_R=\delta_x^\infty\{0.5774+(\delta_x^\infty/l)[0.3385+1.0773\log_{10}(l/\delta_x^\infty)]\\ &\qquad+(\delta_x^\infty/l)^2[0.1243+1.1932\log_{10}(l/\delta_x^\infty)+1.3461\log_{10}^2(l/\delta_x^\infty)]\}; \end{aligned} \tag{10}$$

$$\begin{aligned} p=1:\quad &\delta_x=\delta_x^\infty\{1+0.7351(\delta_x^\infty/l)+0.6987(\delta_x^\infty/l)^2+0.3356(\delta_x^\infty/l)^3\\ &\qquad-1.3038(\delta_x^\infty/l)^4+(\delta_x^\infty/l)^5[2.0327-6.4415\log_{10}(l/\delta_x^\infty)]+1.5479(\delta_x^\infty/l)^6\},\\ &\delta_R=\delta_x^\infty\{0.5774+1.2732(\delta_x^\infty/l)+(\delta_x^\infty/l)^2[-0.2534+3.0709\log_{10}(l/\delta_x^\infty)]\\ &\qquad-0.5811(\delta_x^\infty/l)^3+0.7532(\delta_x^\infty/l)^4+1.4659(\delta_x^\infty/l)^5+0.8954(\delta_x^\infty/l)^6\}. \end{aligned} \tag{11}$$

Our data have been fitted to Dingle's formulas by means of least squares, letting l and S_x^∞ be free parameters. The values of l and S_x^∞ derived from fitting separately the reactive and resistive skin depths were in agreement within 4%; the curves in Fig. 1 were calculated from the simple average of these two sets. The mean free paths l obtained in this manner are not the same as the mean free paths l_{dc} calculated from S_x^∞ and the dc conductivity,[9] using the relations (5) and (6). This discrepancy indicates that the Reuter-Sondheimer free-electron theory is not strictly applicable to tin and indium, and that other analyses based on the comparison of surface-impedance data with the Dingle formulas, such as that undertaken by Johnson and Johnson[10] for copper, must be treated with some reserve.

A good fit of the data to the Dingle formulas was obtained for both $p=0$ and $p=1$ (Table I). Since the dc size effect is inconsistent with specular surface scattering, and since surface-impedance experiments on several metals[11] indicate that the scattering is diffuse, the Reuter-Sondheimer formula (5) for $p=0$ has been used to obtain values of σ/l. These values are listed in Table II, together with the results of high-frequency anomalous-skin-effect experiments and dc size-effect measurements. All results quoted from the literature

TABLE I. Parameter values obtained from the data analysis.

Metal	S_x[a] (cm sec$^{-1/3}$)	Surface scattering	S_x^∞ [b] (cm sec$^{-1/3}$)	l[c] (10^{-2} cm)	l_{dc}[d] (10^{-2} cm)
Tin	0.0348	$p=0$	0.0321	6.5	2.8
		$p=1$	0.0334	6.6	4.4
Indium	0.0313	$p=0$	0.0294	7.1	2.2
		$p=1$	0.0303	6.9	3.4

[a] The slope of the straight line which best fits the reactive–skin-depth data points (Fig. 1) for frequencies above 1000 cps.
[b] The slope S_x in the limit of infinite frequency (Fig. 1), obtained by fitting the data to the free-electron theory of Reuter and Sondheimer.
[c] The mean free path obtained by fitting the data to the free-electron theory of Reuter and Sondheimer.
[d] The mean free path obtained from S_x^∞ and the dc conductivity.

TABLE II. Values of σ/l obtained using various methods.

Metal	Method	σ/l ($10^{10}\Omega^{-1}$ cm^{-2})	Investigator
Tin	ASE,[a] 10^3-10^4 cps	6.9 +1.5	This work
	ASE, 9×10^6 cps	8.6 ±1.3	Chambers[b]
	ASE, 1.2×10^9 cps	9.5 ±0.8	Chambers[c]
	ASE, 36×10^9 cps	5.9	Fawcett[d]
	ASE, 36×10^9 cps	8.0	Fawcett[e]
	dc size effect	5.0	Andrew[f]
	dc size effect	9.80	Aleksandrov[g]
	Theoretical[h]	22.2	
Indium	ASE, 10^3-10^4 cps	9.0 ±1.6	This work
	ASE, 3×10^9 cps	18.0 ±1.1	Dheer[i]
	dc size effect	7.41	Aleksandrov[g]
	Eddy-current size effect	7.88	Cotti[j]
	Theoretical[k]	18.9	

[a] Anomalous skin effect ($p=0$).
[b] Reference 6.
[c] Reference 11.
[d] E. Fawcett, Proc. Roy. Soc. (London) **A232**, 519 (1955).
[e] Reference d corrected as indicated by E. Fawcett, in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), p. 197.
[f] E. R. Andrew, Proc. Phys. Soc. (London) **A62**, 77 (1949).
[g] B. N. Aleksandrov, Zh. Eksperim. i Teor. Fiz. **43**, 399 (1962) [English transl.: Soviet Phys.—JETP **16**, 286 (1963)].
[h] Calculated assuming four conduction electrons per atom and a spherical Fermi surface.
[i] Reference 7.
[j] P. Cotti, Phys. Kondensierten Materie **3**, 40 (1964).
[k] Calculated assuming three conduction electrons per atom and a spherical Fermi surface.

[8] R. B. Dingle, Physica **19**, 311 (1953).

[9] Values of the dc conductivity at 295°K used in our calculations are $\sigma(\text{tin})=9.1\times10^4$ (Ω cm)$^{-1}$ given by G. T. Meaden, *Electrical Resistance of Metals* (Plenum Press, Inc., New York, 1965), p. 16. $\sigma(\text{indium})=11.4\times10^4$ (Ω cm)$^{-1}$ given by G. K. White and S. B. Woods [Rev. Sci. Instr. **28**, 638 (1957)].

[10] E. W. Johnson and H. H. Johnson, J. Appl. Phys. **36**, 1286 (1965).

[11] R. G. Chambers, Proc. Roy. Soc. (London) **A215**, 481 (1952).

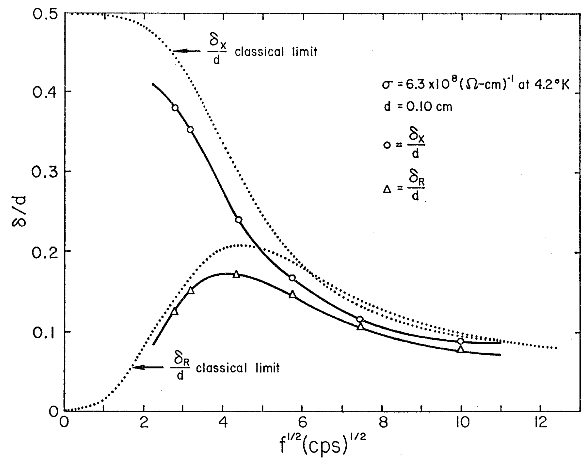

FIG. 2. Size effect exhibited by the lower-purity indium specimen at low frequencies. The dotted curves represent the predictions of the classical-skin-effect theory, $(|\delta|/l)\gg1$, for a specimen of thickness d.

were obtained under the assumption of diffuse electron scattering at the specimen surface.

The uncertainty which has been assigned to our value of σ/l for each metal is an upper limit, calculated from the difference between S_x and S_x^∞. S_x is the slope of the straight line which best fits the data points, and S_x^∞ is the slope obtained by extrapolating the data to the infinite-frequency limit, using the free-electron theory. Although $(S_x-S_x^\infty)/S_x^\infty\approx7\%$, the corresponding uncertainty in σ/l is approximately 20%, since this ratio varies as $(S_x^\infty)^3$.

For clarity, skin depths measured using the lower-purity indium specimen are not shown in Fig. 1(b). These skin depths approached those of the pure indium at high frequencies, but at the highest frequency used $\delta_x/\delta_R=1.4$, which is substantially smaller than the ratio $\sqrt{3}$ characteristic of the extreme anomalous limit.

In Fig. 2 the skin depths observed for the lower-purity indium specimen are shown for frequencies from 7 to 100 cps. These data exhibit the size effect which is expected when the skin depth and the specimen thickness d become comparable. The dotted lines represent the predictions of the classical-skin-effect theory and have been calculated using the dc conductivity of the specimen. The discrepancy between data and theory is attributed to the failure of the classical theory when $|\delta|\approx l$.

In the above discussion we have used the Reuter-Sondheimer theory for a semi-infinite medium to interpret the data even though the mean free path l is comparable with the slab thickness d for our pure tin and indium specimens. From Fig. 2 it is clear that effects due to the interference of the fields penetrating from the two sides of a specimen are only important if $|\delta|$ becomes comparable to d. This result is compatible with the Reuter-Sondheimer theory, since the "effective" carriers—those which are responsible for the surface shielding currents—have velocities which are very nearly parallel with the faces of the slab.

TABLE III. Percentage deviations of the skin depths from their anomalous limits.

Metal	Frequency	$\left(\frac{\delta_x-\delta_x^\infty}{\delta_x^\infty}\right)$	$\left(\frac{\delta_R-\delta_R^\infty}{\delta_R^\infty}\right)$
Tin	1 kc/sec	6.9%	16.7%
	1 Mc/sec	1.0%	2.5%
Indium	1 kc/sec	5.9%	14.4%
	1 Mc/sec	0.8%	2.1%

IV. DISCUSSION

Our value of σ/l for tin is in good agreement with those obtained by other investigators using much higher frequencies. This agreement confirms the predicted frequency dependence of the skin depth over a very wide frequency range. From a comparison of our value of $\sigma/l=6.9\pm1.5\times10^{10}\,\Omega^{-1}$ cm^{-2} at 5×10^3 cps with Fawcett's corrected result, $\sigma/l=8.0\times10^{10}\ \Omega^{-1}$ cm^{-2} at 36×10^9 cps, we obtain $|\delta|\alpha f^{-n}$, where $n=0.34\pm0.01$.

Our result for indium is in better agreement with σ/l obtained from dc size-effect measurements than with that obtained by Dheer from 3 kMc/sec anomalous-skin-effect data. It has been proposed by Bate *et al.*[12] and by Brändli *et al.*[13] that the discrepancy between the indium σ/l values obtained from the high-frequency anomalous-skin-effect measurements and the free-electron theory, on the one hand, and the dc size-effect measurements on the other, is due to a mean free path which varies strongly over the Fermi surface. The prediction of Brändli *et al.* that a low-frequency anomalous-skin-effect measurement in indium would yield results similar to those obtained from dc size-effect experiments is in accordance with our data. It should be noted, however, that Dheer's value is the result of averaging single-crystal measurements over various orientations, and it is possible that part of the discrepancy between his value and ours may be due to some preferred crystal orientation in our indium specimen.

The percentage deviation of the real and imaginary parts of the skin depth from their anomalous limits has been calculated for our tin and indium specimens at 1 kc/sec and 1 Mc/sec and listed in Table III. These estimates, based on a comparison of our data with the free-electron theory, indicate that skin-effect measurements at approximately 1 Mc/sec can be used to investigate the anomalous limit in very pure metals such as those used in this work.

ACKNOWLEDGMENT

The authors would like to thank the National Research Council of Canada for the grants which have supported this work.

[12] R. T. Bate, Byron Martin, and P. F. Hille, Phys. Rev. **131**, 1482 (1963).

[13] G. Brändli, P. Cotti, E. M. Fryer, and J. L. Olsen, in *Proceedings of the Ninth International Conference on Low-Temperature Physics*, edited by J. G. Daunt, D. O. Edwards, F. J. Milford, and M. Yaqub (Plenum Press, Inc., New York, 1965), p. 827.