Microwave Photon Interaction with Superconducting Tunneling Currents

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(Received 1 February 1967)

The photon-assisted tunneling, originally observed by Dayem and Martin, has been remeasured for Al-In junctions and extended to Sn-Pb junctions. Particular attention has been devoted to obtaining detailed quantitative measurements of the current I and the conductance dI/dV versus voltage and for various microwave power levels. In addition to the steps in the tunneling current at $eV_0=\Delta_1+\Delta_2\pm n\hbar\omega$ observed by Dayem and Martin, we have observed an additional set at $eV_0=\Delta_2-\Delta_1\pm n\hbar\omega$. We can conclude that the phase shift in peak position above the tunneling edge is from an almost exact superposition of these two sets of peaks, a consequence of the near coincidence of the microwave photon energy and one-half the energy gap of indium. A detailed comparison of our results with the theory of Tien and Gordon gives good agreement as to peak positions and shapes while the dependence of peak amplitude on microwave power level is poor, particularly at higher power levels. We have extended the Tien and Gordon theory to provide for a modulation of the density of states in both superconductors and have recalculated the tunneling current. The agreement with experiment is good, in particular, the nonvanishing of the conductance for any value of the microwave power is accounted for.

I. INTRODUCTION

THE discovery by Giaever *et al.*[1] of the unique properties of tunneling between a normal metal and a superconductor or between two superconductors has given rise to the subsequent investigation of the variation of the energy gap with temperature for different superconducting metals. It has also proved to be possible, by investigation of the details of the tunneling current and its first and second derivatives versus voltage, to obtain information regarding the density-of-states function[2,3] and in particular, to observe effects due to the interaction between phonons and electrons.[4] Subsequent to this work, investigations of the properties of phonons have been carried out by subjecting the tunneling junction to microwave phonons[5] and observing the effects induced on the tunneling current.

For a tunneling experiment between a normal and superconducting metal, the current I versus voltage V_0 response shows no current at temperatures low compared to the transition temperature until eV_0 is equal to one-half the energy gap Δ for the superconductor. At this "tunneling edge" the current rises very rapidly and then increases nearly linearly with voltage as the voltage is increased beyond this value. For tunneling between two dissimilar superconductors, the response is similar with the tunneling edge at $\Delta_2+\Delta_1$, where Δ_2 and Δ_1 are equal to half the energy gaps for the two superconductors, respectively. In addition to this edge, there is a peak in the tunneling current followed by a decrease at $\Delta_2-\Delta_1$, where $\Delta_2>\Delta_1$. This peak arises because of those electrons thermally excited across the energy gap in No. 1 tunneling into No. 2. This peak can be made very small if the thermal excitation is reduced by lowering the temperature. The expected effects of a microwave field on the tunneling would be two: (1) For voltages such that eV_0 is less than $\Delta_1+\Delta_2$ the electrons could, by absorbing one photon of microwave energy in No. 1 have an energy suitable for tunneling into No. 2 at a reduced voltage, i.e., $eV_0+\Delta_1+\Delta_2-\hbar\omega$. (2) Since the density-of-states function is highly peaked in the vicinity of the energy gap, an electron with energy greater than $\Delta_1+\Delta_2$ could, by stimulated emission, emit one photon and thereby tunnel into a higher density-of-states region. One would therefore expect stimulated emission to change the tunneling current at $eV=\Delta_1+\Delta_2+\hbar\omega$.

The initial investigation of the effect of microwave photons was by Dayem and Martin[6] who observed the interaction of 35-GHz photons with the tunneling between In-Al films. Their data, obtained by photographing the I-V characteristics displayed on an oscilloscope, indicated steps in the current both above and below the edge of the gap at $eV=\Delta_1+\Delta_2\pm\hbar\omega$. Their results, however, indicated more than just two steps, rather they found a series of steps which could be almost exactly represented by an equation of the form

$$eV_0=(\Delta_2+\Delta_1)\pm n\hbar\omega, \tag{1}$$

where n is an integer. The small phase shift for steps above $\Delta_2+\Delta_1$, as will be shown in Sec. III, is the result of a superposition of two sets of steps, there being no phase shift for each one taken separately. The striking

[1] I. Giaever, Phys. Rev. Letters **5**, 147 (1960).
[2] I. Giaever, H. R. Hart, Jr., and K. Megerk, Phys. Rev. **126**, 941 (1962).
[3] J. M. Rowell, P. W. Anderson, and D. E. Thomas, Phys. Rev. Letters **10**, 334 (1963).
[4] J. M. Rowell, A. G. Chynoweth, and J. C. Phillips, Phys. Rev. Letters **9**, 59 (1962).
[5] Y. Goldstein, B. Abeles, and R. Cohen, Phys. Rev. **151**, 349 (1966).
[6] A. Dayem and R. J. Martin, Phys. Rev. Letters **8**, 246 (1962).

feature of their results is that steps corresponding to n as large as five could be observed, leading to the conclusion that multiple photon absorption and emission processes were being observed.

Following the work of Dayem and Martin a theoretical treatment of this effect was given by Tien and Gordon.[7] They considered two possible formulations of the problem, one based on the vector potential **A**, and the other assuming that the total effect of the microwave field was to induce a time-dependent potential difference between the two metals. Since the two treatments lead to essentially the same qualitative results, we will summarize the results for just the latter case. The approach is to assume that one side of the barrier is held at a fixed potential, the other side being modulated with respect to it by an amount

$$eV_M(t)=eV_M\cos\omega t. \quad (2)$$

This additional potential energy is added to the original Hamiltonian, and the new wave functions are calculated. From these wave functions the new density-of-states function is determined to be

$$\rho'(E)=\sum_{n=-\infty}^{n=+\infty}\rho(E+n\hbar\omega)\,J_n^2(\alpha), \quad (3)$$

where $\rho(E)$ is the original density of states in the absence of a microwave field, $J_n(\alpha)$ is the usual Bessel function of order n, and α is given by $\alpha=eV_M/\hbar\omega$. Inserting this into the expression for the tunneling current, they get

$$I=C\sum_{n=-\infty}^{n=+\infty}J_n^2(\alpha)\int_{-\infty}^{+\infty}[f(E-eV_0)-f(E+n\hbar\omega)]\times\rho_A(E-eV_0)\rho_B(E+n\hbar\omega)dE \quad (4)$$

$$=C\sum_{n=-\infty}^{n=+\infty}J_n^2(\alpha)I_0(eV_0+n\hbar\omega), \quad (4')$$

where $I_0(eV_0)$ is just the usual tunneling current in the absence of a microwave voltage. Qualitatively, this predicts a tunneling current that is the superposition of a series of current steps occurring at $eV_0=(\Delta_2+\Delta_1)\pm n\hbar\omega$, each step having an amplitude proportional to $J_n^2(\alpha)$. The agreement with the experimentally determined positions of the steps is excellent, the quantitative agreement, however, is unsatisfactory. The theory predicts explicitly the magnitude of the current steps. Tien and Gordon have adjusted α to give the approximate magnitude of the steps for particular microwave power levels and found that the values of the microwave fields required would be much larger than those actually used in the experiment. In particular, a reasonable fit to most of one set of experimental data is obtained with $\alpha=2$. This results in $J_0^2(\alpha=2)\approx0$ predicting that the main tunneling edge at $\Delta_2+\Delta_1=eV_0$ should go to zero, a result that is not observed experimentally.

[7] P. K. Tien and J. Gordon, Phys. Rev. **129**, 647 (1963).

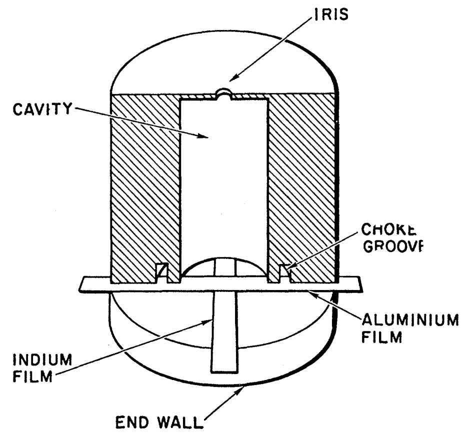

FIG. 1. A sectional view of the microwave cavity showing tunneling junction in position against the end wall. The superconducting strips are insulated from the cavity.

The results of Tien and Gordon can be summarized as accurately predicting the positions of the current steps, qualitatively describing the relative amplitudes but proving inadequate in describing either the detailed qualitative or quantitative results.

We have repeated these experiments but with the equipment necessary to make more detailed and precise measurements of the shape and power-level dependence of these current steps. Following a description of the experimental details in Sec. II, our results are presented in Sec. III. In Sec. IV is presented an extension of the Tien-Gordon theory and a comparison with experiments is made.

II. EXPERIMENTAL DETAILS

The films were prepared using standard evaporation techniques and were in the form of a cross, each film being approximately 1 mm wide. In all cases, the starting material was at least 99.999% pure and special attention was given in the design of the source holder to eliminate the possibility of contamination of one metal with the other. To facilitate mounting in the microwave cavity, a metal sample holder was prepared over which was stretched a sheet of Mylar that was approximately 6 μ thick and onto which were deposited the metal films. This structure served as both the sample holder and the end wall of the resonant cavity, as shown in Fig. 1. In all cases the evaporation was not started until the pressure had reached 3×10^{-7} Torr; outgassing during the deposition was minimized by a suitable premelting and preheating cycle. In all cases, the evaporation source was brought to its operating temperature and held for several moments until a suitable rate had been established before the shutter was opened for the deposit. For all of the films prepared, the thicknesses were between 300 and 500 Å as measured interferometrically and deposited in a time of between 1 and 2 sec.

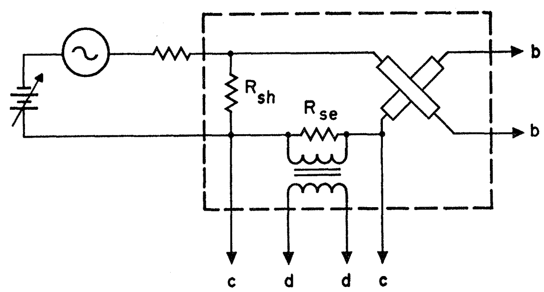

FIG. 2. Simplified diagram of the measuring circuit. Components inside the dotted box are within the Dewar. Conductance is computed from the voltage between the leads (d, d).

The sample holder was held in place in the evaporation chamber on the end of a thin-walled stainless-steel finger and surrounded by four additional fingers. Between the sample holder and its finger was sandwiched a thin washer of indium to ensure good thermal contact. For the Al-In junctions, the substrate was initially held at room temperature, the surrounding fingers being filled with liquid nitrogen to further reduce the pressure in the vicinity of the substrate. The aluminum film was deposited, followed immediately, for the production of the tunneling barrier, by the admission of pure oxygen to a pressure of about 10 μ. After a few seconds allowed for oxidation the chamber was evacuated. At this point the substrate was cooled to liquid-nitrogen temperatures for the deposition of the indium. This cooling, necessary to prevent indium migration, was avoided during the initial evaporation and oxidation to minimize the absorption of ambient gas. For the Sn-Pb junctions the tin was deposited first and oxidized for 16 h at 1 atm at 50°C. The junctions were warmed to room temperature in a vacuum, transferred to the resonant cavity, inserted in the Dewar, and immediately cooled to liquid-nitrogen temperatures to minimize the increase in barrier resistance that takes place with time. It should be mentioned that when special attention was paid to minimizing the possibility of mechanical and diffusion-pump oil in the chamber, this aging process was greatly reduced.

The sample was mounted as the end plate of a right circular cylindrical resonator operating at $\nu=36.000$ GHz in a TE_{111} mode as shown in Fig. 1, and the entire assembly was suspended in a helium Dewar. The junction being immersed directly in the helium promoted good thermal contact with the bath. Operation in the vicinity of 1°K and thus well below the λ point of helium eliminated cavity noise associated with boiling helium. For some films it was observed that at high microwave power levels, heating resulted which was sufficient to drive them normal in spite of their thinness and contact with superfluid helium. We have not included any data in which this type of behavior was observed. The microwave power from the reflex Klystron was measured using a standard dry calorimeter, all subsequent power levels being set by means of a calibrated attenuator.

The electronic measuring equipment was all of standard design, use being made of low-noise microvoltmeters for measuring voltage and (with a series resistor) the current, while small-amplitude voltage modulation and a phase sensitive detector were used to measure the conductance dI/dV of the junction. The low-level voltages encountered in measuring dI/dV necessitated using a special low-noise preamplifier with suitable impedance matching transformers mounted in the Dewar as close to the sample as possible. In all cases, the junction current I and conductance dI/dV were displayed continuously as a function of junction voltage on an x-y recorder. Junctions between dissimilar superconductors that were sufficiently thin to exhibit a large interaction with the microwave field also had a high conductance which was characteristically of the order of 1 to 5 Ω at voltages greater than $\Delta_2+\Delta_1$. A constant voltage source for the investigation of the negative-resistance region between $\Delta_2-\Delta_1$ and $\Delta_2+\Delta_1$ required using a shunting resistor R_{sh} across the junction, and a small resistor R_{se} in series with the junction to measure the actual current, these being mounted in the helium and as close to the junction as possible, as illustrated in Fig. 2. Characteristic values of R_{se} and R_{sh} required for stable operation were between 0.05 and 0.25 Ω. The voltage, being measured directly across the junction with a high-impedance microvoltmeter, was used to drive the x axis of an x-y recorder. In this manner the I-V characteristics were determined directly. The conductance dI/dV is not linearly related to the output of the phase-sensitive detector, because the modulation amplitude across the junction is not constant with changing conductance. However, from I-V characteristics, a calibration curve was determined and used to correct all of the data in which the conductance versus microwave power is plotted. The figures in which dI/dV versus V is plotted are actual recorder plots to which the correction has not yet been applied.

The temperature, measured with a manostat to a precision greater than 0.1%, was observed to depend on the level of the incident microwave power. In the case of Al-In junctions operating at about 0.90°K, this resulted in a maximum temperature rise of the helium bath, at full power, of as much as 0.04°K. Measurements were subsequently made in which the temperature was adjusted at the extremes of microwave power used, to the same values. The current and conductance dependence so determined was identical—to within experimental uncertainty—with that obtained when the temperature was not held constant. This indication that the energy gaps are only very weakly temperature-dependent at this temperature is consistent with the transition temperature of about 1.6°K for Al films of this thickness. No subsequent efforts were made to control the temperature. In the case of the Sn-Pb junctions at 1.4°K, possible temperature effects were not specifically investigated, but an effect due to temperature will be discussed in Sec. V.

III. EXPERIMENTAL RESULTS

In Fig. 3 is displayed a typical tunneling curve for a junction of aluminum and indium. In this case both the current I and the conductance dI/dV are displayed as a function of the voltage V across the junction. The usual features of a maximum in the current at $\Delta_2-\Delta_1$ followed by a region of negative resistance and subsequent tunneling edge at $\Delta_2+\Delta_1$ are readily apparent. (It is necessary to remark parenthetically that these junctions also display a Josephson[8] current at zero voltage, leading to an instability in our measuring system at zero voltage.) The recorder traces were begun at a few μV above zero to eliminate this problem. In the case of the Sn-Pb junctions the instability was not severe and the complete tracings are displayed. The effect on the conductance when the junction is subjected to a microwave field is shown in Fig. 3. In addition to the normal peaks, there are two additional peaks (steps in the current) above the gap and two below. The separation between these extra peaks is just $\Delta V=\hbar\omega/e$, indicating that the photon interaction is just with the single-particle tunneling current. This particular curve, at relatively low microwave power, illustrates the nature of the interaction without the added complication of many peaks found at higher power levels. There are several qualitative conclusions that can be drawn from these curves. It has been shown,[2] for the case of tunneling between a superconductor and a normal metal, that the conductance dI/dV at temperatures low compared to T_c is proportional to the density of states in the superconductor. In particular, based on the BCS theory,[9] the density of states has the form

$$\rho_s(E)=\rho_n(E)\,|E|\,(E^2-\Delta^2)^{-\frac{1}{2}},\qquad |E|>\Delta, \quad (5)$$

$$\rho_s(E)=0,\qquad |E|<\Delta, \quad (5')$$

where $\rho_n(E)$ is the density of states in the normal metal and is essentially constant in the energy range of interest, Δ is half the energy gap, and E is the energy measured from the Fermi energy. The rounding of the peak in the density of states observed is the usual result of experiment which is attributed to such effects as finite temperatures, averaging of the anisotropy of the energy gap, etc. The first qualitative feature of our results is that the photon-induced peak at 0.9 mV has a shape that is the same as that at the tunneling edge $\Delta_2+\Delta_1$, indicating that the effect of the microwave field has been to induce an additional tunneling edge at $eV=\Delta_2+\Delta_1+\hbar\omega$. Although not readily apparent in Fig. 4, as the microwave power is increased and the higher-order peaks become more prominent, this characteristic shape is reproduced for each succeeding higher-order peak. This result is in exact agreement with the theory of Tien and Gordon [Eq. (4′)], which predicts a set of identical tunneling edges at $eV=\Delta_2+\Delta_1+n\hbar\omega$, but with an amplitude proportional to $J_n^2(\alpha)$. It has most recently been shown by Werthamer[10] that the tunneling current between two superconductors is obtainable from the individual density-of-states functions as given in Eq. (5), the result being expressible in terms of elliptic integrals. However, the qualitative dependence of dI/dV is very similar to that obtained when only one metal is superconducting.

The peak in the tunneling current at $\Delta_2-\Delta_1$, at 0.4 mV for these junctions being also associated with the peaking of the density of states at the edge of the gap, should reflect the splitting due to the microwave field. That this is happening can be seen by comparing the shape of the $\Delta_2-\Delta_1$ peak with the photon-induced peak below it at 0.25 mV. The correspondence is very good and supports this conjecture. Finally, the peak at 0.6 mV is midway between $\Delta_2-\Delta_1$ and $\Delta_2+\Delta_1$, a consequence of the photon energy being almost exactly equal to Δ_1. This peak is thus a superposition of two

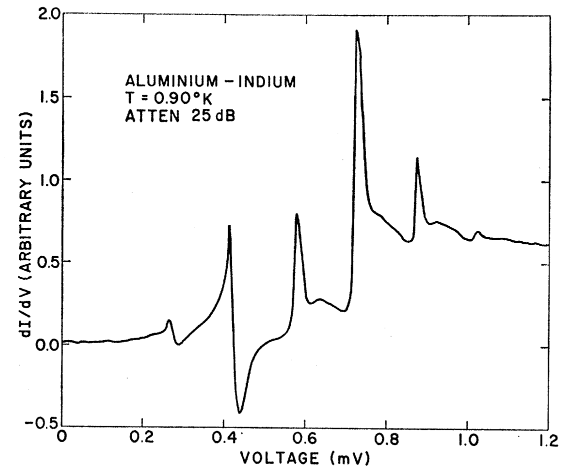

FIG. 4. Conductance versus voltage for the junction of Fig. 3 with 25 dB relative attenuation. This corresponds to a microwave power of approximately 2μW dissipated in the cavity.

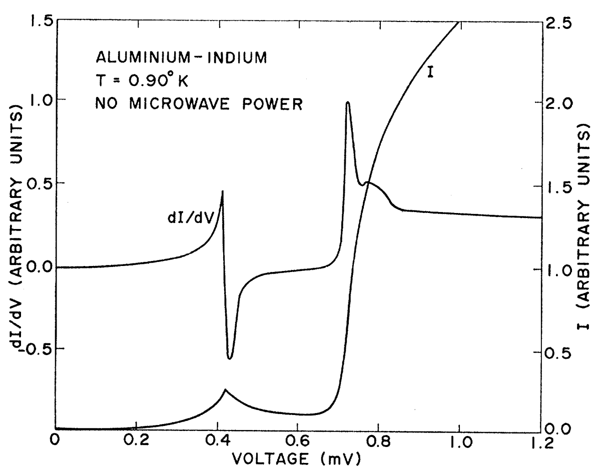

FIG. 3. Current I and conductance dI/dV versus voltage for an Al-In junction at 0.9° K. The zeros of I and dI/dV have been displaced for clarity.

[8] B. D. Josephson, Advan. Phys. **14**, 419 (1965).

[9] J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[10] N. Werthamer, Phys. Rev. **147**, 255 (1966).

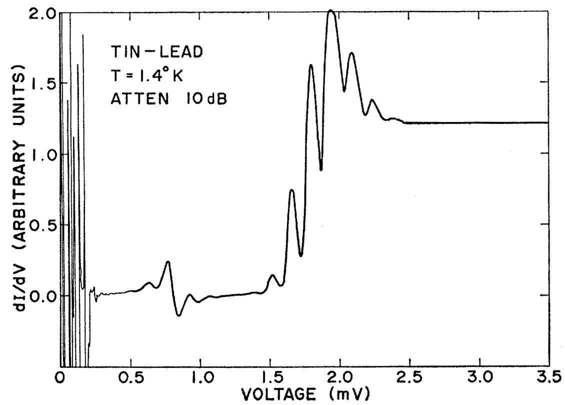

FIG. 5. Conductance versus voltage for a Sn-Pb junction. The power for this curve was chosen to demonstrate the additional multiphoton process associated with the peak at (a) and (b), as discussed in the text.

contributions, one from the edge at $\Delta_2-\Delta_1$ and the other from $\Delta_2+\Delta_1$. To eliminate this small separation compared with the photon energy, the same experiments were repeated on Sn-Pb junctions. In Fig. 5 is shown the result obtained when the microwave power level is carefully adjusted to avoid the overlap in the two sets of peaks. The main edge at 1.9 mV has associated with it the multiple-photon processes both above and below the edge. The peak $\Delta_2-\Delta_1$ at 0.7 mV also shows the photon-induced contributions associated with this point. The microwave power has been set high enough to display both processes but at the same time low enough to keep the two sets from overlapping. This demonstrates that the two processes are separate, with the response becoming superimposed at high microwave power levels. The phase shift observed by Dayem and Martin[6] for tunneling in Al-In junctions at 35 GHz is the result of this superposition, the peak position of each process separately being accurately given by $eV_0=\Delta_2+\Delta_1\pm n\hbar\omega$ and $eV_0=\Delta_2-\Delta_1\pm n\hbar\omega$.

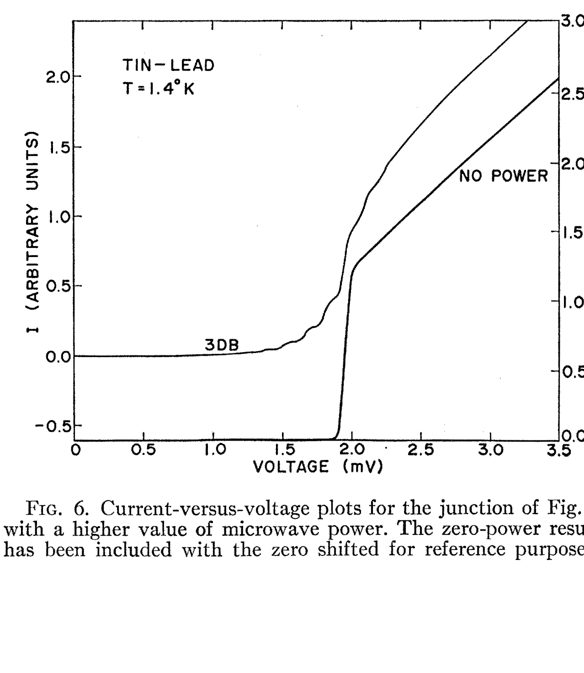

FIG. 6. Current-versus-voltage plots for the junction of Fig. 5 with a higher value of microwave power. The zero-power result has been included with the zero shifted for reference purposes.

The characteristics of the tunneling current I and the conductance dI/dV, at high microwave power levels for Sn-Pb junctions, are shown in Figs. 6 and 7, respectively. The steps in the current associated with the photon-induced edges are clearly discernible but become more prominent when the conductance is plotted. In each case the results with no microwave power have been included, with the zero displaced for clarity. In Figs. 8 and 9 are shown the behavior of a second Sn-Pb junction in which the barrier was thinner, so that the steps in the tunneling current became prominent at much lower microwave power levels. Figure 8 indicates the type of limiting behavior observed at high power levels. At progressively higher microwave power levels, the number of observable peaks continues to increase, with the amplitude of any particular peak tending to reach a limiting value and then to slowly decrease as the power is further increased. For all of

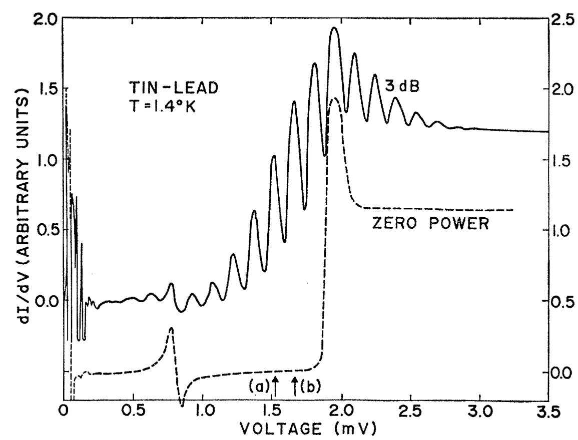

FIG. 7. Conductance versus voltage for the junction and conditions of Fig. 6. The arrows (b) and (a) indicate the position of the peaks $N=2$ and $N=3$, respectively, occurring below the tunneling edge for which the detailed comparison with theory has been made.

the Sn-Pb junctions measured, a strong Josephson current is present at zero bias and no applied microwave power. With increasing microwave power, the interaction with these currents becomes more prominent, as has been previously reported by Shapiro *et al.*[11,12] This behavior was always present in our Sn-Pb junctions and has been displayed to illustrate the nature of this interaction with a microwave field and the completely different character of this contribution, and to differentiate it from the single-particle tunneling interaction. This interaction is currently under further investigation.

A quantitative comparison of our results with the T-G theory can be made by differentiating Eq. (4′) to obtain the conductance

$$\frac{dI'}{dV_0}=C\sum_{n=-\infty}^{n=+\infty}J_n^2(\alpha)\,\frac{d}{dV_0}I_0(eV+n\hbar\omega). \qquad (6)$$

[11] S. Shapiro, A. Janus, and S. Holly, Rev. Mod. Phys. **36**, 223 (1964).

[12] D. Coon and M. Fiske, Phys. Rev. **138**, A744 (1965).

The theoretical conductance is proportional to $J_n^2(\alpha)$ and in particular for sufficiently large values of α, first the $J_0^2(\alpha)$ term, then the J_1^2 terms, etc., should decrease to zero and then rise again. In Fig. 11 (below) we have plotted the functions $J_0^2(\alpha)$ and $J_2^2(\alpha)$ to illustrate this expected behavior and the measured maximum in the conductance for $N=2$ and $N=3$, the peaks at 1.6 and 1.5 mV, respectively, versus relative microwave power on a log-log scale. These experimental points are identified by the arrows (a) and (b) in Fig. 7. The extensive use of these two for quantitative comparison was convenient because of the wide range in power levels over which meaningful measurements could be made and because the zero-power conductance is zero, no background corrections being necessary. At low power levels, the amplitude does diminish rapidly towards zero. For the limiting case of α small, the amplitude in the general case should decrease as $P^{n/2}$, where P is the microwave power and n is the order of the peak. This expected behavior is not observed. At high power levels, sufficiently large values of α, the amplitude should go through a maximum and then decrease towards zero, a result which has not been observed experimentally. The amplitudes observed tend towards a limiting value in the range of powers available to us. Finally, if we were to assume that the limiting behavior of the peak $n=3$ were due to α becoming large enough for $J_3^2(\alpha)$ to approach a maximum, the main peak at $n=0$ ($eV=\Delta_2+\Delta_1$) should have gone through zero and risen to a near second maximum value as illustrated. This has not been observed experimentally, even though it has been carefully looked for.

We can summarize our experimental results by noting that the positions and shapes of the photon-induced peaks in the conductance are in good agreement with the T-G theory. The question of a phase shift in the position of the peaks relative to the tunneling edge has been shown to be associated with the superposition of two sets of peaks arising from the same modulation of the density of states but associated with different groups of electrons. It has been shown that the quantitative peak-amplitude dependence on microwave power level does not agree with theory, that in fact the vanishing predicted at certain levels is never observed. In the next section we will modify the T-G theory by assuming that the action of the microwave field is to modulate the density of states in the two superconductors in an essentially equivalent way and to calculate the resulting power dependence of the conductance.

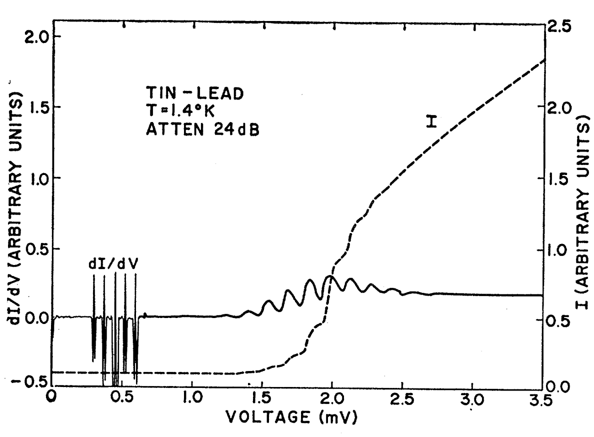

FIG. 8. Conductance and current for a Sn-Pb junction with films thinner than those of Figs. 5 and 6, showing increased interaction. Note that the structure in the conductance around 0.5 mV is due to the microwave Josephson-current interaction.

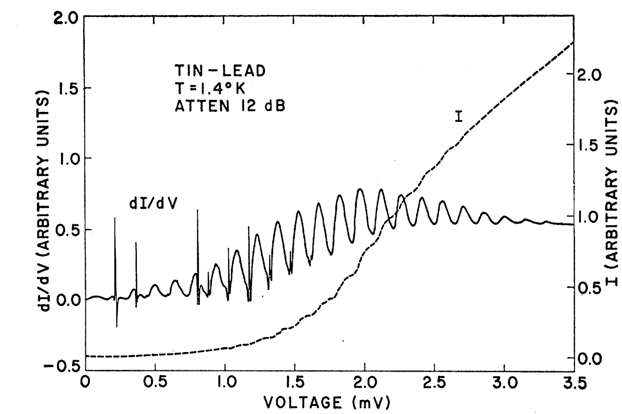

FIG. 9. Conductance and current for the diode of Fig. 7, showing the saturation of peak heights at increased power. In this plot the gain is 2.5 greater than that of Fig. 8.

IV. THEORETICAL MODEL

The T-G model for the microwave interaction with the tunneling current assumes that one of the superconductors can be regarded as being held at a fixed potential while the other has a time-dependent potential difference measured relative to the first. As indicated in Sec. I, this leads to a modulation in the density of states for this superconductor and is given by Eq. (3), with the current given in Eq. (4). This modulation applies to both the occupied and unoccupied states, but in the T-G theory it is assumed that only one superconductor, without specifying which one, is so affected. Since this reference to zero potential is somewhat arbitrary, we have elected to assume that both superconductors feel an oscillating potential, the reference being chosen to make this amplitude equal in each metal. As a result, the density of states in both superconductors is modulated and has the form of Eq. (4). The resulting form of the density of states for dissimilar superconductors is illustrated schematically in Fig. 10 by the solid lines, the dashed lines representing the situation at zero applied microwave power. This represents the behavior at zero bias and at $T=0$°K. (We have elected to illustrate the peaks in the density of states as being sharp with well-defined edges but finite in their magnitudes for convenience in illustration and to reflect the finite conductance observed experimentally.) Inserting this density of states into the

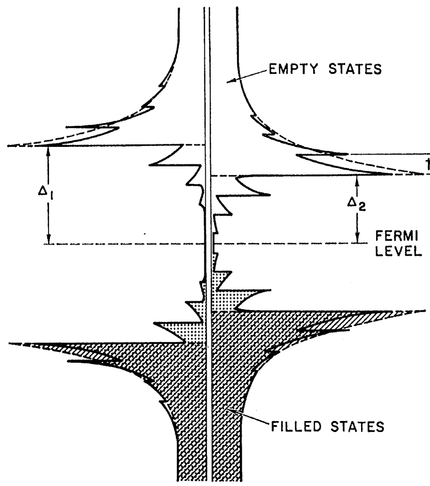

FIG. 10. Modified density-of-states function shown by solid lines, and the density-of-states function at zero power shown by dashed lines.

usual integral for the tunneling current, we obtain

$$I'=C\sum_{m,n=-\infty}^{m,n=+\infty} J_m^2(\alpha)\,J_n^2(\alpha)$$
$$\times\int_{-\infty}^{+\infty}[f(E+m\hbar\omega-eV_0)-f(E+n\hbar\omega)]$$
$$\times\rho_A(E+m\hbar\omega-eV_0)\rho_p(E+n\hbar\omega)dE. \quad (7)$$

This result is more convenient when transformed and rewritten in terms of one integral representing the current as

$$I'=\sum_{n,m=-\infty}^{n,m=+\infty} J_n^2(\alpha)I[eV_0+(n-m)\hbar\omega], \quad (8)$$

where

$$I=C\int_{-\infty}^{+\infty}\{f[E'-eV_0+(n-m)\hbar\omega]-f(E')\}$$
$$\times\rho_A[E'-eV_0-(n-m)\hbar\omega]\rho_B(E')dE. \quad (9)$$

The differences $n-m=N$ now correspond to all of the N photon processes. The total current can be expressed as a sum of these individual processes

$$I_{AB}=\sum_N I_N, \quad (10)$$

where I_N is given by

$$I_N=\{\sum_{m=-\infty}^{m=+\infty} J_m^2(\alpha)\,J_{N+m}^2(\alpha)\}I(eV_0+N\hbar\omega), \quad (11)$$

or alternatively as

$$I_N=\{J_0^2(\alpha)\,J_N^2(\alpha)$$
$$+\sum_{m=-\infty}^{m=+\infty} J_m^2(\alpha)[J_{n+m}^2(\alpha)+J_{n-m}^2(\alpha)]\}$$
$$\times I(eV_0+N\hbar\omega). \quad (12)$$

For N positive we have a process associated with absorption, and for N negative with the emission of N photons. For small values of α, the behavior is dominated by the first term in Eq. (12), giving for the current

$$I_N\cong J_N^2(\alpha)I(ev+N\hbar\omega), \quad (13)$$

the result of Tien and Gordon. If we call the tunneling at $eV=\Delta_2+\Delta_1$ the $n=0$ edge, this result indicates that when the modulation of the density of states is weak, i.e., at low microwave powers, the N-photon process is dominated by the first-order process, namely transitions from $n=0$ in superconductor A to $n=N$ in superconductor B, and $n=N$ in A to $n=0$ in B. The other contributions, being proportional to $J_m^2J_{N\pm m}^2$, do not contribute measurably. As α increases, the zero-order Bessel function does decrease towards zero, but at the same time, the other contributions increase and keep the total current associated with the N-photon process from ever going to zero. While this is the behavior obtained by considering all of the possible contributions to the N-photon process, the total current is given by the sum of the various multiple processes as given by Eq. (10). We have computed coefficients of the various functions I_N, as they appear in Eqs. (11) and (12). A further computation of the general function $I(eV_0+N\hbar\omega)$ and its derivative dI/dV_0 can be obtained from Werthamer,[10] with the usual result that at 0°K, the conductance at $eV_0=\Delta_1+\Delta_2$ is discontinuous. For the contribution due to the other terms we have made use of the measured dependence of dI/dV such as is shown in Fig. 6 for zero microwave power. In particular, at one photon energy above the edge, the conductance is essentially a constant. The total conductance was then computed as follows: The term under consideration [I_N of Eq. (12)] was calculated taking $I(eV_0+N\hbar\omega)$ as unity, to this was added the term appropriate to $N+1$, $N+2$, etc., but multiplied by the ratio of the conductance at one photon energy above the edge to that at the edge as determined experimentally. This ratio, being essentially constant for all of the terms, is not determined with great precision owing to the extremely nonlinear response of our measuring system for very large values of conductance. It does lie between 0.05 and 0.20. Using this ratio as an adjustable parameter, the best fit to our data was obtained with a value of 0.15. This function has been indicated for $N=0$, 1, 2, 3, 4 in Fig. 11. Also plotted for purposes of comparison, are $J_0^2(\alpha)$ and $J_2^2(\alpha)$, the expected results of T-G. For the specific case of $N=2$ the essential feature predicted by Eq. (12) is that it rises to a maximum value largely as a result of J_2^2, goes through a maximum for a 40% lower value of α than predicted by T-G, followed by a slow decrease with some evidence of very shallow maxima and minima as α continues to increase. For $N=1$ and higher values of N, the behavior is very similar, the main maximum in the conductance occurring at a lower value of α for $N=1$ and at progressively higher values for increasing values of N. The total response of the system is, for any particular peak associated with an N-photon process, to rise rapidly like $J_N^2(\alpha)$ for small values of α, to go through a broad maximum, and then to decrease slowly as the power is further increased. The detailed quantitative comparison shown in Fig. 12 indicates that the agreement with theory is good except at the very highest power levels used, where the conductance shows a tendency to saturate but then continues to rise slowly. We believe that this rise is a consequence of finite-temperature effects. With increasing microwave power, we have observed in some films that they can be driven normal due to microwave heating. This extra rise in the conductance is most likely due to the extra conductance below the tunneling edge that results when the temperature of the junction is caused to rise towards the transition temperature. This effect should initially affect the $N=3$ peak less strongly than the one at $N=2$. The relative microwave power $\alpha^2=(eV_m/\hbar\omega)^2$ necessary to produce saturation as predicted will always produce a heating effect. However, it should be possible to cover a wider range in α with less heating by decreasing the frequency to 24 GHz, while not having two individual peaks so close together as to make the interpretation difficult.

The remaining problem associated with this theory is related to the actual magnitudes of α and hence the electric fields required to produce these interactions. In their paper, T-G found that the values of α required for an approximate fit to the size of the tunneling steps were approximately two orders of magnitude larger than would be possible from considerations of the magnitude of the microwave power, the cavity Q, and the location of the sample in the cavity. Werthamer,[10] on the other hand, has pointed out that the relevant voltage is the actual voltage in the tunneling junctions, and not that in an equivalent section of waveguide. He conjectures that, because of the large impedance mis-

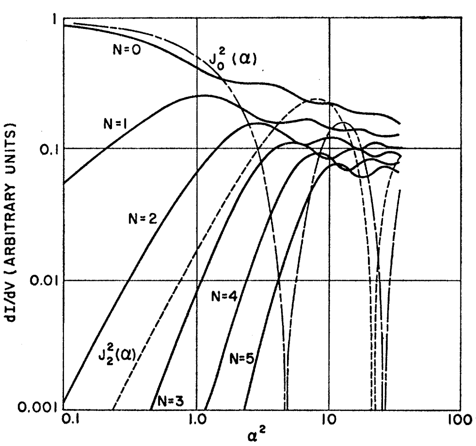

FIG. 11. Relative conductance as a function of relative microwave power shown by solid lines, and J_n^2 shown in broken lines. The index N refers to the order of the multiphoton process.

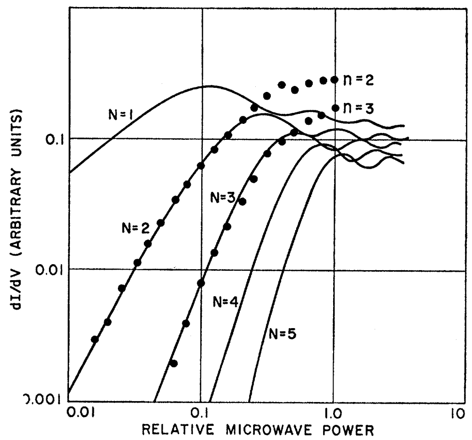

FIG. 12. Computed relative conductance versus relative microwave power for $N=0$, 1, 2, 3, 4, and 5, with experimental points for $N=2$ and 3 from the peaks indicated by arrows [(a) and (b)] in Fig. 7.

match between the ends of the junctions and the cavity, the rf voltage in the junction may be much larger than previously estimated and the apparent quantitative disagreement may be spurious.

V. DISCUSSION AND CONCLUSIONS

We have carried out a set of measurements on the microwave photon interaction in superconducting Al–In and Sn–Pb junctions with the tunneling current. We find that the apparent phase shift in the positions of the peaks in the conductance is the result of the superposition of two sets of peaks at relatively high microwave power levels, and that there is no phase shift in the individual processes. The shape of the photon-induced peaks in the conductance should be the same as for the peak at the tunneling edge at $\Delta_2+\Delta_1$, while those peaks associated with $eV=\Delta_2-\Delta_1$ should have that characteristic shape. These consequences of the T-G theory are all confirmed from our experimental data. The qualitative behavior of the amplitude of the conductance peaks with microwave power and, in particular, their behavior at high power levels is in good semiquantitative agreement with the theory proposed in Sec. IV. The premise of this theory, that the density of states in the two superconductors are modulated in the same way and to essentially the same degree, while apparently somewhat arbitrary in that the zero or reference potential difference can be assigned to any point in the measuring circuit, appears adequately justified by the substantial agreement with experiment.

For the films used in these experiments, the thickness, approximately 300 Å, is small compared to the coherence length and consequently the penetration depth is much larger than the film thickness. The microwave electric field E and hence the vector potential $\mathbf{A}$ should be essentially uniform in each film. The semiquantitative agreement of our theory leads us to believe that the interaction is a consequence of the density-of-states modulation in each superconductor separately owing to the electric field or equivalently, the vector potential, and not just the potential difference V_m, or the vector potential difference $\Delta\mathbf{A}$. A test of this conjecture will result from an investigation of the tunneling behavior for the case when one of the superconducting strips is thick compared to the coherence length (one strip must be thin to facilitate microwave penetration into the region of the barrier) resulting in an electric field that is finite in its spatial extent. A second experiment will be to examine the photon interaction when one of the metals is normal. In this case the density of states in the normal metal, being essentially constant in energy, should remain so under the action of a microwave field; the resulting behavior should then very nearly approximate the result obtained by T-G. These two lines of investigation are currently in progress.

ACKNOWLEDGMENTS

The authors wish to show appreciation for the help and encouragement of Frederick Essig and for the assistance of James Gulley and Ivan Rainwater in performing the experiments.

PHYSICAL REVIEW VOLUME 159, NUMBER 2 10 JULY 1967

Consequences of Demagnetizing Field Inhomogeneity on the Shape of Ferromagnetic Resonance Lines*

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(Received 21 February 1967)

The asymmetry of the magnetic resonance line in bulk ferromagnetic metals is of one sign when the magnetic field is parallel to the sample surface and is of opposite sign when the field is perpendicular. All known relaxation theories lead to the type of asymmetry which is observed in the parallel case. It is demonstrated here that for both parallel and perpendicular resonance a contribution to asymmetry as well as to linewidth comes from the inhomogeneity of the internal field and this contribution to asymmetry is of reversed sign in the perpendicular case. Elimination of edge effects reduces it to zero.

THE inhomogeneity of the demagnetizing field which is present in a ferromagnetic platelet of finite thickness contributes both to the width[1] and to the asymmetry of the resonance line. Associated with the second of these two effects is also a reversal in sign of the asymmetry when one passes from the case of parallel resonance (H_0 parallel to the surface of the platelet) to the case of perpendicular resonance (H_0 perpendicular to the surface). Since relaxation theories predict an asymmetry of a single sign (steeper rise to a maximum on the low-field side than on the high-field side) for both parallel[2] and perpendicular[3] resonance, the wrong sign of the asymmetry frequently observed in the case of perpendicular resonance had to be found in some other effect. It is the purpose of this paper to demonstrate in terms of a particular example that the asymmetry observed in perpendicular resonance can be eliminated or at least reduced by a reduction in the inhomogeneity of the demagnetizing field.

The inhomogeneity of the demagnetizing field can be reduced over the active volume of the sample by shielding the outer portions of the platelet from the radio-frequency field.[1] This has been accomplished in these experiments by pressing gold foil over those portions of the sample surface which are to be excluded. Two

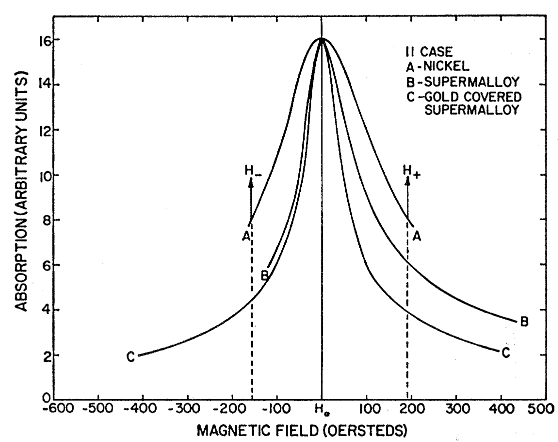

FIG. 1. Absorption with magnetic field parallel to surface of sample.

samples which are otherwise identical are then compared, one with the gold foil shield and one without.

We will here present data on three samples so as to include within the unshielded category two ferromagnetic samples of different material and different size. Thus we will demonstrate for different samples the

* Research supported by the National Science Foundation.
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