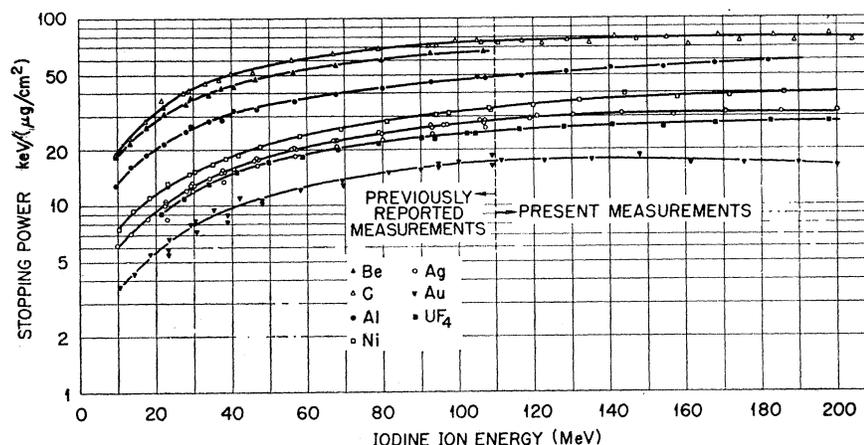


FIG. 1. Stopping power of several solid materials for iodine ions.



the stopping power is expected to decrease with velocity. It is expected that  $dE/dx$  should reach a maximum value at or below  $v_0 Z^{2/3}$  ( $0.1c$  for  $^{127}\text{I}$ ). From the results in the figure it can be seen that this maximum has been reached in the case of Au at about 125 MeV corresponding to a velocity of  $0.046c$ . For the other materials measured the maxima appear to occur at some-

what higher velocities but still well below the value  $v_0 Z^{2/3}$ .

#### ACKNOWLEDGMENTS

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### Paramagnetic Resonance and Relaxation of Some Rare-Earth Ions in $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ \*

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The paramagnetic resonance at  $\nu = 9.3$  Gc/sec has been observed at helium temperatures for  $\text{Ce}^{3+}$ ,  $\text{Nd}^{3+}$ ,  $\text{Dy}^{3+}$ ,  $\text{Er}^{3+}$ , and  $\text{Yb}^{3+}$  diluted in single crystals of  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ . The principal values  $g_x$ ,  $g_y$ , and  $g_z$  of the  $g$  tensor were measured and showed marked anisotropy. The spin bath relaxation rate  $T_b^{-1}$  was measured for  $\text{Nd}^{3+}$  and  $\text{Er}^{3+}$  over the temperature range  $0.2 \leq T \leq 5^\circ\text{K}$  and for  $\text{Ce}^{3+}$  and  $\text{Dy}^{3+}$  over the range  $1.3 \leq T \leq 5^\circ\text{K}$ , by observing the transient recovery of the microwave resonance following a saturating pulse. The temperature dependence of the Orbach process,  $T_b^{-1} \propto \exp(-\Delta/T)$ , is displayed for all four ions, with values of  $\Delta$  in agreement with optical measurements in concentrated crystals. There is some evidence of phonon bottlenecking of the Orbach process for  $\text{Dy}^{3+}$ . The Raman process  $T_b^{-1} \propto T^9$  was observed for  $\text{Ce}^{3+}$  and  $\text{Dy}^{3+}$ . The direct process  $T_b^{-1} \propto \coth(h\nu/2kT)$  was observed for all four ions, and displayed a very large anisotropy, of order  $10^3$  to  $10^4$ , for  $\text{Nd}^{3+}$  and  $\text{Er}^{3+}$ , which can be approximately understood in terms of the matrix elements of the Zeeman perturbation between ground and excited states. For  $\text{Nd}^{3+}$  there was some evidence for a phonon bottleneck of the direct process,  $T_b^{-1} \propto T^2$ .

#### I. INTRODUCTION

In a previous paper,<sup>1</sup> referred to as I, we reported an experimental study of spin-lattice relaxation of some rare-earth ions in the host crystal  $\text{LaF}_3$ , by observing the transient recovery of the microwave paramagnetic resonance absorption following a saturating pulse. In this paper we present very similar studies for  $\text{Ce}^{3+}$ ,  $\text{Nd}^{3+}$ ,  $\text{Er}^{3+}$ , and  $\text{Dy}^{3+}$  in the host crystal  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ ,

in the temperature range  $0.2$ – $5^\circ\text{K}$  in fields of a few thousand Oersteds, corresponding to a microwave frequency of  $\nu = 9.3$  Gc/sec. We refer to I for notation, theory, description of the apparatus, and experimental procedures. Our interest in studying rare-earth ions in the hydrated chlorides arose in connection with nuclear-spin refrigerators,<sup>2</sup> in which the magnetization of a system of paramagnetic ions is cyclically transferred by crystal rotation to a nuclear-spin system, by virtue of extreme anisotropies in the paramagnetic relaxation

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<sup>1</sup> M. B. Schulz and C. D. Jeffries, *Phys. Rev.* **149**, 270 (1966).

<sup>2</sup> C. D. Jeffries, *Cryogenics* **3**, 41 (1963); A. Abragam, *ibid.* **3**, 42 (1963).

TABLE I. Paramagnetic resonance results for rare-earth ions in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ .

Ion	$g_x$	$g_y$	$g_z$	$\chi_0$	Hyperfine constants
1.0% $\text{Ce}^{3+}$	$2.67 \pm 0.01$	$2.305 \pm 0.010$	$0.782 \pm 0.005$	$69 \pm 2^\circ$	
0.1% $\text{Nd}^{3+}$	$0.26 \pm 0.01$	$0.33 \pm 0.02$	$3.50 \pm 0.02$	$63 \pm 2^\circ$	$A_z^{143} = 0.0356 \pm 3 \text{ cm}^{-1}$ $A_z^{146} = 0.0222 \pm 2 \text{ cm}^{-1}$
0.1% $\text{Dy}^{3+}$	$2.00 \pm 0.02$	$1.52 \pm 0.01$	$16.52 \pm 0.10$	$-23 \pm 2^\circ$	$A_z^{161} = 0.0471 \pm 2 \text{ cm}^{-1}$ $A_z^{168} = 0.0657 \pm 2 \text{ cm}^{-1}$
0.1% $\text{Er}^{3+}$	see text	see text	$13.74 \pm 0.10$	$77 \pm 2^\circ$	$A_z^{167} = 0.0475 \pm 3 \text{ cm}^{-1}$
1.0% $\text{Yb}^{3+}$	$<0.5$	$<0.5$	$5.57 \pm 0.05$	$71 \pm 2^\circ$	

rate and the  $g$  factor. The only substance yet reported which has such extreme anisotropies is  $(\text{Yb}, \text{Y}) \times (\text{C}_2\text{H}_5\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$ ; sizeable proton polarizations have been obtained in this crystal.<sup>3,4</sup> As part of a program to find other suitable substances we selected the hydrated chlorides; although little was known about them, it appeared that they may be quite anisotropic. It has turned out that the direct relaxation process is exceedingly anisotropic, by three orders of magnitude, as discussed in Sec. III, but that the  $g$  factor is only moderately anisotropic. The anisotropy in the relaxation is the largest yet reported for paramagnetic crystals.

## II. CRYSTAL STRUCTURE AND SAMPLE PREPARATION

The hydrated trichlorides of yttrium as well as the rare earths above praeosdymium crystallize in a monoclinic structure with six waters of hydration,  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ , whereas La, Ce, and Nd trichlorides crystallize with seven waters in a triclinic structure. We are concerned only with small fractions of rare-earth ions in the monoclinic structure, shown in Fig. 1, for  $\text{GdCl}_3 \cdot 6\text{H}_2\text{O}$  from x-ray data.<sup>5</sup> There are two Gd atoms per unit cell, each on a twofold symmetry axis parallel to the  $b$

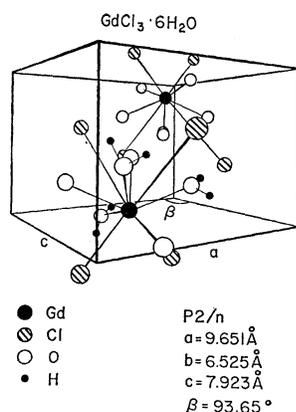


FIG. 1. The crystal structure of  $\text{GdCl}_3 \cdot 6\text{H}_2\text{O}$  suggested by Marezio *et al.* (Ref. 5). The Gd atoms are on twofold axes parallel to the  $b$  axis. The closest Gd-Gd distance is 6.525 Å, along the  $b$  axis. Only a few of the hydrogen atoms are shown.

axis. The two sites are electrically and magnetically equivalent. The closest distance between Gd atoms is 6.525 Å in the  $b$  direction.

Although no paramagnetic resonance has been previously reported on rare-earth ions in these monoclinic crystals, extensive optical work, including Zeeman studies, has been done.<sup>6-20</sup> Crystal-field parameters have been calculated for  $\text{ErCl}_3 \cdot 6\text{H}_2\text{O}$ .<sup>21</sup> The optical work shows that the maximum Zeeman splitting for the ground state occurs with the magnetic field perpendicular to  $b$ ; the principal axes ( $x, y, z$ ) of the  $g$  tensor are shown in Fig. 2, in relation to the crystal axes ( $a, b, c$ ) and the macroscopic form of a typical crystal. The angle  $\chi_0$  depends on the rare-earth ion; for a given ion, it may further vary from one Kramers doublet to the next. The optical data suggest that  $g_x \approx g_y \neq g_z$ ; it may turn out that the waters of hydration are so arranged about the ion that the  $z$  axis is approximately an axis of sixfold symmetry.<sup>19</sup> It has also been suggested that the crystals may be ferroelectric or antiferroelectric and that the internal field created by the ferroelectric alignment causes the axis of quantization to shift from the crystallographic  $b$  axis to a plane perpendicular to this axis.<sup>22</sup> Harrop<sup>21</sup> used  $C_2$  site symmetry for Er in finding the crystal-field parameters; however we will use in this paper the approximation that the rare-earth site symmetry in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  is hexagonal, as suggested by the EPR data.

We have made microwave paramagnetic resonance measurements on the lowest Kramers doublet for several

<sup>6</sup> F. H. Spedding and H. F. Hamlin, *J. Chem. Phys.* **5**, 429 (1937).

<sup>7</sup> H. Ewald, *Ann. Physik* **34**, 209 (1939).

<sup>8</sup> F. H. Spedding and R. S. Bear, *Phys. Rev.* **42**, 58 (1932); **42**, 76 (1932).

<sup>9</sup> H. Lammernann, *Z. Physik* **150**, 551 (1958).

<sup>10</sup> K. H. Hellwege and H. G. Kahle, *Z. Physik* **129**, 62 (1951).

<sup>11</sup> H. G. Kahle, *Z. Physik* **155**, 157 (1959).

<sup>12</sup> G. H. Dieke and L. Leopold, *J. Opt. Soc. Am.* **47**, 944 (1957).

<sup>13</sup> H. G. Kahle and H. Kalbfleisch, *Z. Physik* **166**, 184 (1962).

<sup>14</sup> G. Gramberg, *Z. Physik* **159**, 125 (1960).

<sup>15</sup> G. H. Dieke and S. Singh, *J. Opt. Soc. Am.* **46**, 495 (1956).

<sup>16</sup> H. G. Kahle, *Z. Physik* **145**, 347 (1956).

<sup>17</sup> H. G. Kahle, *Z. Physik* **145**, 361 (1956).

<sup>18</sup> K. H. Hellwege, S. Hufner, and H. G. Kahle, *Z. Physik* **160**, 162 (1960).

<sup>19</sup> G. H. Dieke and H. M. Crosswhite, *J. Opt. Soc. Am.* **46**, 885 (1956).

<sup>20</sup> J. B. Gruber and J. G. Conway, *J. Chem. Phys.* **32**, 1178 (1960).

<sup>21</sup> I. H. Harrop, *J. Chem. Phys.* **42**, 4000 (1965).

<sup>22</sup> J. C. Eisenstein, *J. Chem. Phys.* **35**, 2097 (1961).

<sup>3</sup> K. H. Langley and C. D. Jeffries, *Phys. Rev. Letters* **13**, 808 (1964); *Phys. Rev.* **152**, 358 (1966).

<sup>4</sup> J. R. McColl and C. D. Jeffries, *Phys. Rev. Letters* **16**, 316 (1966).

<sup>5</sup> M. Marezio, H. A. Plettinger, and W. H. Zachariasen, *Acta Cryst.* **14**, 234 (1961).

ions in dilute concentrations (0.1–5 at. %) in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ ; the resonance data were fitted by the spin Hamiltonian

$$\mathcal{H} = \beta(g_x H_x S_x + g_y H_y S_y + g_z H_z S_z), \quad (1)$$

for even-even isotopes. For odd isotopes with nuclear spin, it is necessary to add the hyperfine term

$$\mathcal{H}_{hfs} = A_x I_x S_x + A_y I_y S_y + A_z I_z S_z. \quad (2)$$

These equations, together with Fig. 2, seem to describe the data for all ions except  $\text{Er}^+$ , as described below. The observed resonance parameters are summarized in Table I, and display large anisotropies. The resonance lines had linewidths of typically 10–30 Oe in low magnetic fields; at high fields, they broadened to several hundred Oe.

The crystals used in the resonance and relaxation experiments were prepared from 99.999% purity  $\text{Y}_2\text{O}_3$  and 99.9% purity rare-earth oxides obtained from Lindsay Chemical Co. The oxides were slowly dissolved in a slight excess of reagent grade HCl. A saturated aqueous solution was put into small dishes in a desiccator at room temperature to grow seeds; from these, larger crystals were grown in a few days in the form of plates, approximately  $5 \times 5 \times 1$  mm as in Fig. 2, the best developed edge being the  $a$  axis. The crystals are quite hygroscopic.

### III. RESONANCE AND RELAXATION MEASUREMENTS AND INTERPRETATION

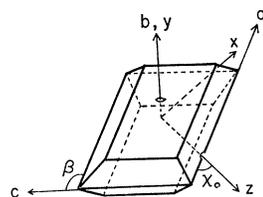
#### A. $\text{Nd}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$

Optical studies<sup>6,7</sup> on  $\text{NdCl}_3 \cdot 6\text{H}_2\text{O}$  have established two of the excited states in the  $^4I_{9/2}$  ground multiplet at 62 and 250  $\text{cm}^{-1}$  above the ground state. Pfeffer<sup>23</sup> suggested that the other two states are in the neighborhood of 135  $\text{cm}^{-1}$ . We have made resonance and relaxation measurements at  $\nu = 9.36$  Gc/sec on two  $\text{Nd}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  crystals grown from 0.1 and 1 atomic percent solutions. The resonance results are given in Table I. In Fig. 3 are shown some of the relaxation data over the range  $0.2 \leq T \leq 5^\circ\text{K}$  with  $z \parallel H = 1910$  Oe. In all cases the recovery of the saturated signal appeared to be exponential, with a well-defined rate. The data are well fit by the expression for the spin bath relaxation rate

$$T_b^{-1} = 7 \times 10^{-3} \coth(0.23/T) + 4 \times 10^{11} \times \exp(-89/T) + 4.8 \times 10^{-4} T^9 \text{ sec}^{-1}. \quad (3)$$

The longest relaxation times, between 10 and 100 sec,

FIG. 2. The orientation of the principal directions,  $x$ ,  $y$ ,  $z$ , of the  $g$  tensor for rare-earth ions in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  for which  $\beta = 92^\circ$ ; see Ref. 19.



<sup>23</sup> W. Pfeffer, Z. Physik 164, 295 (1961).

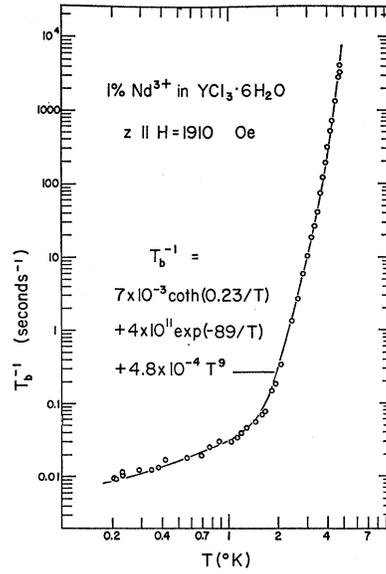


Fig. 3. Observed relaxation rate  $T_b^{-1}$  versus  $T$  for 1%  $\text{Nd}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  with  $z \parallel H$ , in the range  $0.2 < T < 5^\circ\text{K}$ .

were measured by modulation of the magnetic field at 80 cps and displaying the derivative of the resonance signal on a chart recorder, which traced out the recovery of the signal. We interpret the terms in Eq. (3) as representing the direct Van Vleck–Kronig process, the Orbach process, and the Raman process, respectively. The direct process is unusually weak; the Orbach rate determines  $\Delta = 89^\circ$  for the level of the first crystal-field excited state, in good agreement with the value  $\Delta = 62 \text{ cm}^{-1} = 89^\circ\text{K}$ , determined for  $\text{NdCl}_3 \cdot 6\text{H}_2\text{O}$  from optical spectroscopy.<sup>6,7</sup> Spin-lattice relaxation in polycrystalline  $\text{NdCl}_3 \cdot 6\text{H}_2\text{O}$  has been measured by Weber,<sup>24</sup> whose data agree with ours for  $T > 1.5^\circ$ ; the lower-temperature data would not be expected to agree because the polycrystalline measurement conceals the large anisotropy in the direct process.

This anisotropy, more than three orders of magnitude, is shown in Fig. 4 for 1%  $\text{Nd}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  at  $T = 1.45^\circ\text{K}$  for various orientations  $\theta$  between  $H$  and the  $z$  axis, with  $H$  in the  $ac$  plane. The microwave frequency was held fixed at  $\nu = 9.37$  Gc and the external magnetic field rotated with respect to the crystal and cavity. The small angular range  $86^\circ < \theta < 94^\circ$  could not be measured because the maximum field obtainable was only 19 kOe. The data shown with open circles indicate the region over which  $T_b^{-1}$  was also measured with a 0.1% Nd sample; no dependence on concentration was observed. The signal recovery traces were not quite exponential for  $T_b^{-1} > 40 \text{ sec}^{-1}$ , possibly because of the onset of a phonon bottleneck.

We can roughly explain the  $g$ -factor data as well as the angular dependence of the direct process in the following way. Assuming that  $J = \frac{9}{2}$  is a good quantum

<sup>24</sup> G. Weber, Naturwissenschaften 49, 323 (1962); Z. Physik 171, 335 (1963).

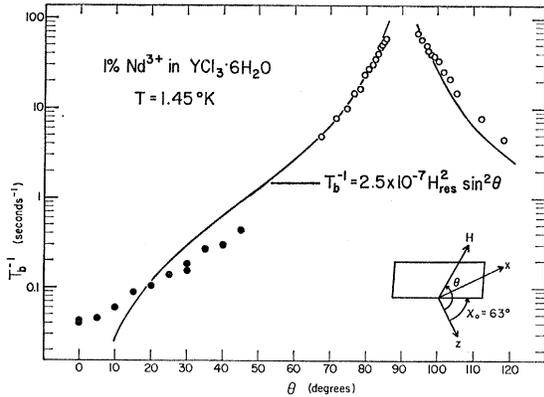


FIG. 4. Observed relaxation rate  $T_b^{-1}$  versus angle between  $H$  and  $z$  axis for 1%  $\text{Nd}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  at  $T = 1.45^\circ\text{K}$ . The magnetic-field direction was rotated about the normal to the crystal plane. The data shown with open circles indicate the region over which data were also taken on a 0.1% sample. No dependence on concentration was found.

number for the ground manifold, the five Kramers doublets must be composed of linear combinations of  $|J_z = \pm \frac{9}{2}\rangle$ ,  $|\pm \frac{7}{2}\rangle$ ,  $|\pm \frac{5}{2}\rangle$ ,  $|\pm \frac{3}{2}\rangle$ , and  $|\pm \frac{1}{2}\rangle$ . However, because the  $\text{Nd}^{3+}$  site symmetry is nearly hexagonal, (e.g.,  $C_6$  or  $C_{3h}$ ) only functions differing by  $\Delta J_z = \pm 6$  are admixed, leading to these combinations:  $\cos\alpha |\pm \frac{9}{2}\rangle + \sin\alpha |\mp \frac{3}{2}\rangle$ ;  $\cos\beta |\pm \frac{7}{2}\rangle + \sin\beta |\mp \frac{5}{2}\rangle$ ;  $|\pm \frac{1}{2}\rangle$ . The Zeeman interaction  $\mathcal{H}_z = \Delta\beta \mathbf{H} \cdot \mathbf{J}$  when applied to the ground Kramers doublet  $|a\rangle$ ,  $|b\rangle$  leads to the usual expressions for the spectroscopic splitting factors:

$$g_z = 2\Lambda |\langle a | J_z | a \rangle|, \quad (4a)$$

$$g_x = \Lambda |\langle a | J_+ + J_- | b \rangle|, \quad (4b)$$

$$g_y = \Lambda |\langle a | J_+ - J_- | b \rangle|, \quad (4c)$$

where  $\Lambda = 8/11$  is the Landé  $g$ -factor for  $\text{Nd}^{3+}$ . From Eq. (4) and the observed values  $g_z = 3.5$ ,  $g_x \approx g_y \approx 0.25$ , it follows that the ground doublet cannot be  $|\pm \frac{1}{2}\rangle$ . Although either of the other two combinations is possible, as a first approximation it is tempting to take the ground doublet as a pure  $|\mp \frac{5}{2}\rangle$  state, which yields  $g_z = 40/11 = 3.63$ ,  $g_x = g_y = 0$ , roughly in agreement with measurements. Clearly there must be admixed in a small amount of  $|\pm \frac{7}{2}\rangle$  to prevent  $g_x$  and  $g_y$  from vanishing. The angular dependence of the direct process at constant frequency is seen to come mostly from the factor  $\langle a | \mathbf{H} \cdot \mathbf{J} | i \rangle = H \langle a | J_x \sin\theta + J_z \cos\theta | i \rangle$  in Eq. (8) of I, where  $|i\rangle$  is an excited state. In the rough approximation above, we see that the matrix element of  $J_x$  will dominate over that of  $J_z$ , and thus we expect

$$T_{1a}^{-1}(\theta) \propto H^2(\theta) \sin^2\theta. \quad (5)$$

In Fig. 4 the solid line is the curve

$$T_b^{-1} = 2.5 \times 10^{-7} H^2(\theta) \sin^2\theta \text{ sec}^{-1}, \quad (6)$$

which does fit the data except at small angles, where the additional admixtures into  $|\mp \frac{5}{2}\rangle$  become more im-

portant. Further relaxation data at  $\theta = 84.5^\circ$  where the direct process is strong, showed a low-temperature dependence  $T_b^{-1} = 25T^2 \text{ sec}^{-1}$  for both 0.1% and 1% crystals. This temperature dependence suggests a phonon bottleneck; its apparent independence of concentration is not consistent with this interpretation, however, unless the linewidths  $\Delta H \approx 250 \text{ Oe}$  for 0.1% and  $\Delta H \approx 1 \text{ kOe}$  for 1% somehow compensate for the spin-phonon heat-capacity ratio. The higher-temperature data at this orientation again displayed an Orbach process  $T_{10}^{-1} = 7 \times 10^{11} \exp(-89/T) \text{ sec}^{-1}$ , which is nearly twice as strong as observed for  $H \parallel z$ , Eq. (3). This apparent angular variation of the Orbach process has been observed before, e.g., for  $\text{Nd}^{3+}$  in lanthanum magnesium nitrate,<sup>25</sup> and may be caused by a large anisotropy of the excited state through which the relaxation is proceeding.

### B. Dy:YCl<sub>3</sub>·6H<sub>2</sub>O

The levels of the ground multiplet  ${}^6\text{H}_{15/2}$  of  $\text{DyCl}_3 \cdot 6\text{H}_2\text{O}$  have been fairly well determined from optical experiments, the first and second excited states being at  $34 \text{ cm}^{-1}$  and at  $63 \text{ cm}^{-1}$ .<sup>14,15</sup> Gramberg<sup>14</sup> also reports Zeeman studies which indicate these parameters for the ground doublet:  $g_x = 1.8 \pm 0.1$ ,  $g_y = 1.66 \pm 0.2$ ,  $g_z = 18.06 \pm 0.2$ ,  $\chi_0 = -23.8 \pm 1^\circ$ . He also found that the principal axes of the  $g$  tensors of the excited states did not coincide with one another nor with that of the ground state.

We have observed resonance and relaxation on crystals of  $\text{Dy:YCl}_3 \cdot 6\text{H}_2\text{O}$  grown from solutions of 5%, 1%, and 0.1%  $\text{Dy}^{3+}$ , respectively. The resonance parameters found, Table I, are close to those of Gramberg for the concentrated salt. Figure 5 shows some relaxation data taken on a 1% crystal at two different orientations and measured at  $\nu = 9.36 \text{ Gc}$ . The solid lines are the expressions

$$\begin{aligned} \mathbf{H} \parallel z, \quad T_b^{-1} &= 2.25T + 5 \times 10^{-3} T^3 + 8 \times 10^9 \\ &\times \exp(-49/T) \text{ sec}^{-1}, \quad (7) \end{aligned}$$

$$\begin{aligned} \mathbf{H} \parallel x, \quad T_b^{-1} &= \frac{225 \times 250 T^2}{225 T + 250} + 5 \times 10^{-3} T^3 \\ &+ 8 \times 10^9 \exp\left(-\frac{49}{T}\right) \text{ sec}^{-1}. \quad (8) \end{aligned}$$

The Orbach process determines  $\Delta = 49^\circ\text{K} = 34 \text{ cm}^{-1}$ , in good agreement with the optical data. Weber<sup>24</sup> has found  $T_{10}^{-1} = 1.7 \times 10^{10} \exp(-48/T) \text{ sec}^{-1}$  in concentrated  $\text{DyCl}_3 \cdot 6\text{H}_2\text{O}$  for  $\mathbf{H} \parallel z$ . The first term in Eq. (7) we interpret as the direct process; measurements on a 0.1% crystal at  $H \parallel z$  yielded the same results as Eq. (7).

<sup>25</sup> G. H. Larson and C. D. Jeffries, Phys. Rev. **141**, 461 (1966); **145**, 311 (1966).

In Eq. (8) the form required to fit the data for  $H \parallel x$ ,  $T_b^{-1} = ADT^2/(DT+A)$  is that expected<sup>26</sup> for a partial phonon bottleneck. The value of  $D=225$ , the observed linewidth  $\Delta H=56$  Oe, and an estimated sound velocity  $v=2 \times 10^5$  cm/sec lead to a phonon mean free path  $\bar{l}=0.2$  mm. In order to investigate this further we took the additional data of Fig. 6 for 0.1% and 5% concentrations. The 0.1% crystal data are fit by

$$T_b^{-1} = 250T + 5 \times 10^{-3} T^9 + 2.7 \times 10^{10} \exp(-49/T) \text{ sec}^{-1} \quad (9)$$

The first term being linear in  $T$  suggests the not surprising result that the bottleneck is broken for this

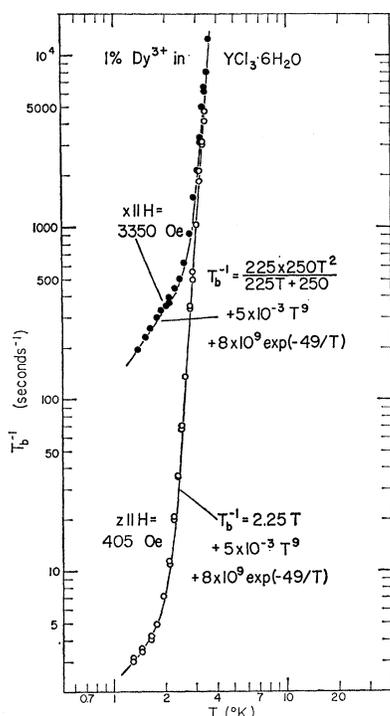


FIG. 5. Observed relaxation rate  $T_b^{-1}$  versus temperature for 1%  $\text{Dy}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  for  $H \parallel z$  and for  $H \parallel x$ .

lower concentration. However the exponential term in Eq. (9) is also larger than in Eq. (8). This is very unusual and suggests that the Orbach process is also bottlenecked in Eq. (8); this possibility has been briefly considered earlier.<sup>26</sup> The data for a 5% crystal are also shown on Fig. 6 and seem anomalous. Why should the 5% crystal not be *more* bottlenecked than the 1%, rather than less? This is possibly because of the resonance linewidth  $\Delta H=350$  Oe for the 5% crystal, compared to  $\Delta H=56$  Oe for the 1% crystal, allowing the spins to relax to a larger bandwidth of phonons in the 5% crystal. This may more than compensate for the increased spin concentration in the 5% sample.

<sup>26</sup> P. L. Scott and C. D. Jeffries, Phys. Rev. **127**, 32 (1962).

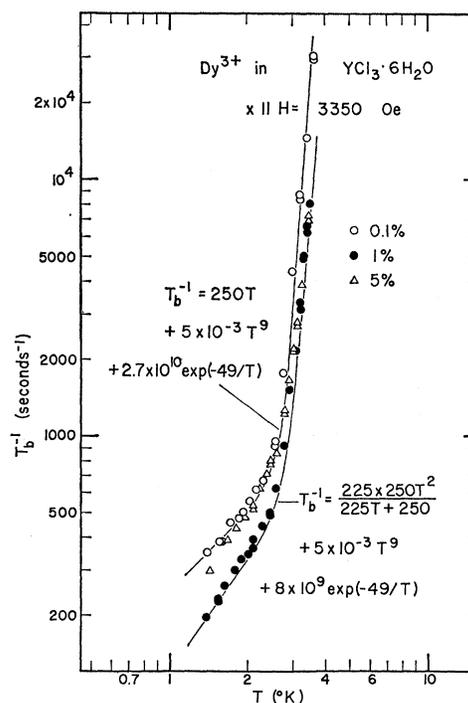


FIG. 6. Observed relaxation rate  $T_b^{-1}$  versus temperature for three concentrations of  $\text{Dy}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  for  $H \parallel x$ .

Further relaxation data for the 1% crystal at  $T=1.4^\circ\text{K}$  are shown in Fig. 7, as a function of the angle  $\theta$  between  $H$  and  $z$ . The data display the surprising result that the rate is not symmetric with respect to the principal axes of the  $g$  tensor. However from Eq. (8) of I we recall that the direct process comes about through admixture with the excited states. But since the  $g$  tensors of the excited doublets will generally not be in the same orientation, this will make the direct

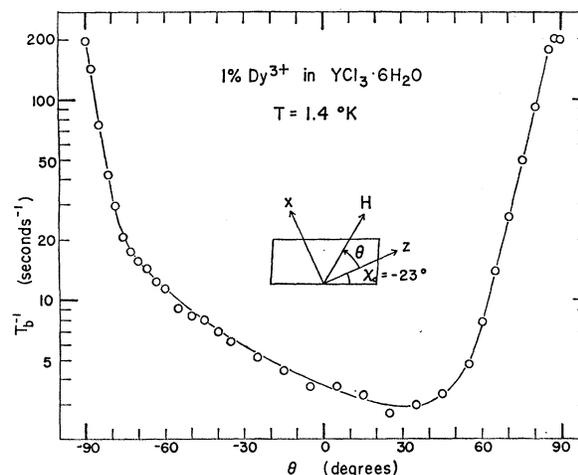


FIG. 7. Observed relaxation rate  $T_b^{-1}$  at resonance as a function of orientation for 1%  $\text{Dy}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  at constant temperature  $T=1.4^\circ\text{K}$  and frequency  $\nu=9.36$  GHz. The magnetic field was rotated about the crystal  $b$  axis ( $y$  axis).

process a complicated function of orientation. In Eq. (8) of I, the term  $\langle a | \mathbf{H} \cdot \mathbf{J} | i \rangle$  will become

$$H \langle a(\theta) | J_x \sin\theta + J_z \cos\theta | i(\theta) \rangle,$$

where  $a(\theta)$  and  $i(\theta)$  will not necessarily be the same function of  $\theta$  nor have the same symmetry. This is undoubtedly the reason for the skewed behavior of  $T_b^{-1}(\theta)$  in Fig. 7.

### C. Ce:YCl<sub>3</sub>·6H<sub>2</sub>O

Our resonance and relaxation measurements were made on crystals grown from 0.1% and 1% Ce<sup>3+</sup> solutions. The measured parameters are given in Table I. The  $g$  factors are explained by assuming a nearly axial field, with the axis in the crystal plane parallel to  $z$ . For hexagonal site symmetry the  ${}^2F_{5/2}$  ground multiplet of Ce<sup>3+</sup> would split into pure  $J_z$  states  $|\pm\frac{1}{2}\rangle$ ,  $|\pm\frac{3}{2}\rangle$ , and  $|\pm\frac{5}{2}\rangle$ . The  $|\pm\frac{1}{2}\rangle$  state has  $g_x = \Lambda = 0.86$ ,  $g_z = g_y = 3\Lambda = 2.57$ , reasonably close to the observed values for the ground doublet.

Relaxation data are shown in Fig. 8. For the 1% crystal the data are fit by

$$H \parallel x, \quad T_b^{-1} = 0.27T + 2.5 \times 10^{-5}T^9 + 2 \times 10^7 \times \exp(-60/T) \text{ sec}^{-1}, \quad (10)$$

$$H \parallel z, \quad T_b^{-1} = 0.14T + 2.5 \times 10^{-5}T^9 + 2 \times 10^7 \times \exp(-60/T) \text{ sec}^{-1}. \quad (11)$$

There seems to be a slight concentration dependence of the first term; however, the recovery was not strictly exponential, especially at low microwave powers, indicating some spectral diffusion. Measurements of  $T_b^{-1}(\theta)$  at 1.4°K for the 0.1% crystal showed an anisotropy of 10:1, with a sharp skewed dip at  $\theta = 10^\circ$ . The Orbach terms in Eqs. (10) and (11) indicate an excited state at  $\Delta = 60 \pm 10^\circ\text{K} \approx 42 \text{ cm}^{-1}$ ; there are no optical data to confirm this

### D. Er:YCl<sub>3</sub>·6H<sub>2</sub>O

The splittings of the  ${}^4I_{15/2}$  ground multiplet of Er<sup>3+</sup> in YCl<sub>3</sub>·6H<sub>2</sub>O have been partially determined,<sup>17</sup> the first two excited states being at 16.91 and 37.07 cm<sup>-1</sup>, respectively. Optical Zeeman studies<sup>18</sup> on the ground state yield the following parameters:  $g_x = 0$ ,  $g_y = 1.26$ ,  $g_z = 13.26$ ,  $\chi_0 = 76^\circ$ .

Our resonance and relaxation measurements were made on crystals grown from solutions containing 1% and 0.1% Er<sup>3+</sup>, respectively. The resonance results indicated that one of the principal directions of the ground-state  $g$  tensor is in the crystal plane but that neither of the other two were, and that the orientation of these other two axes seemed to vary from sample to sample. This may have been caused by an imperceptible twinning of our samples; or possibly the Er<sup>3+</sup> ion somehow reduces the site symmetry of the

YCl<sub>3</sub>·6H<sub>2</sub>O host crystal. The resonance results on a 0.1% sample were:  $g_z = 13.74 \pm 0.10$ ,  $g_{\perp \text{ min}} = 0.58 \pm 0.02$ ,  $g_{\perp \text{ max}} = 0.92 \pm 0.02$ ,  $\chi_0 = 77 \pm 2^\circ$ . The  $g_{\perp}$  factors were determined in the plane perpendicular to the  $z$  axis. For this 0.1% sample the direction corresponding to  $g_{\perp \text{ min}}$  seemed to be approximately  $9^\circ$  out of the crystal plane, but was as much as  $20^\circ$  for another sample.

Figure 9 shows some relaxation measurements as a function of temperature for two concentrations of Er<sup>3+</sup> in YCl<sub>3</sub>·6H<sub>2</sub>O. The results are fit by the expressions

$$1\%, z \parallel H, \quad T_b^{-1} = 0.016 \coth(0.23/T) + 3.22 \times 10^6 \times \exp(-24.5/T) \text{ sec}^{-1}, \quad (12)$$

1%,  $z \perp H$  in crystal plane,

$$T_b^{-1} = 125T + 7.31 \times 10^6 \exp(-24.5/T) \text{ sec}^{-1}, \quad (13)$$

0.1%  $z \perp H$  in crystal plane,

$$T_b^{-1} = 165T + 8.8 \times 10^6 \exp(-24.5/T) \text{ sec}^{-1}. \quad (14)$$

It is seen that in all three cases an Orbach process is demonstrated with the first excited energy level at  $\Delta = 24.5^\circ\text{K} = 17 \text{ cm}^{-1}$ , in excellent agreement with the optically determined value. However, the strength of

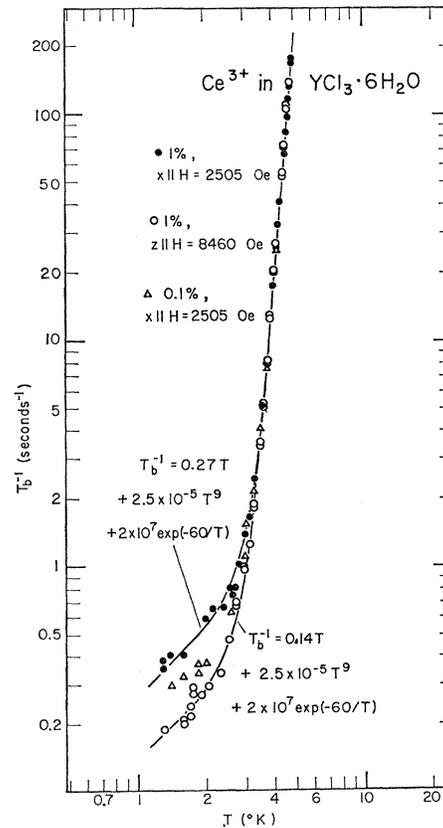


FIG. 8. Observed relaxation rate  $T_b^{-1}$  versus temperature for two concentrations of Ce<sup>3+</sup> in YCl<sub>3</sub>·6H<sub>2</sub>O.

the Orbach process seems to depend rather strongly on sample orientation. This is probably for the same reason as in  $\text{Nd}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ , and suggests that the excited Zeeman splitting is very anisotropic. Relaxation measurements on concentrated  $\text{ErCl}_3 \cdot 6\text{H}_2\text{O}$  have been reported by Weber,<sup>24</sup> who finds for  $\mathbf{H} \parallel z$  an Orbach process  $T_{10}^{-1} = 3.64 \times 10^6 \exp(-24.6/T) \text{ sec}^{-1}$  in good agreement with our results.

The data in Fig. 9 indicate that the Raman process is very weak since no term in  $T^9$  is needed to fit the data. From the observed relaxation data we may estimate that the Raman process is  $CT^9 < 10^{-3} T^9 \text{ sec}^{-1}$ . The term  $0.016 \coth(0.23/T)$  in Eq. (12) indicates a relatively weak direct process for  $z \parallel H$ . For  $kT \gg h\nu$ , this term may be replaced by  $0.07T$ . The concentration dependence was not checked for this orientation, but the observed fit to the coth form indicates that the true direct process is probably being observed here. The terms linear in  $T$  for the orientation  $z \perp H$  are probably also manifesting a direct process. Since the observed rate is so fast for this orientation, a bottleneck of the direct process might be expected. However, the observed linewidths for the two samples for this orientation were 100 Oe for the 0.1% crystal, and 800 Oe for the 1% crystal. The effective bandwidth of phonons involved in the direct process relaxation is therefore large enough that the specific heat of the phonons remains large compared to the specific heat of the spin system. The slight apparent concentration dependence of  $T_b^{-1}$  for  $z \perp H$  is probably due to a slight misorientation

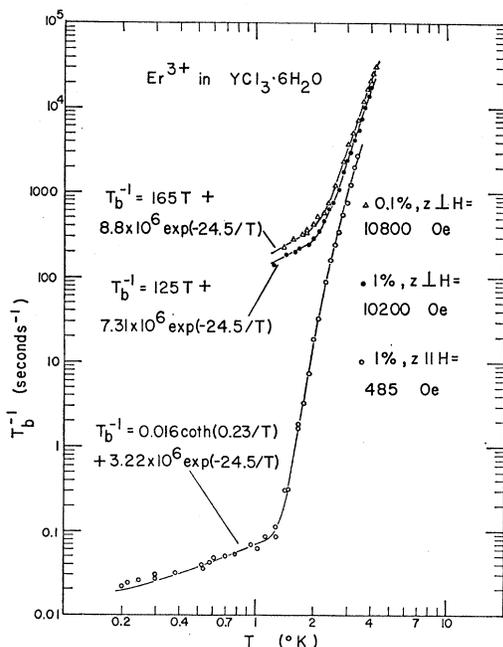


FIG. 9. Observed relaxation rate  $T_b^{-1}$  as a function of temperature for two concentrations of  $\text{Er}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ . The discrepancy for  $z \perp H$  is due to a slight misalignment of the samples.

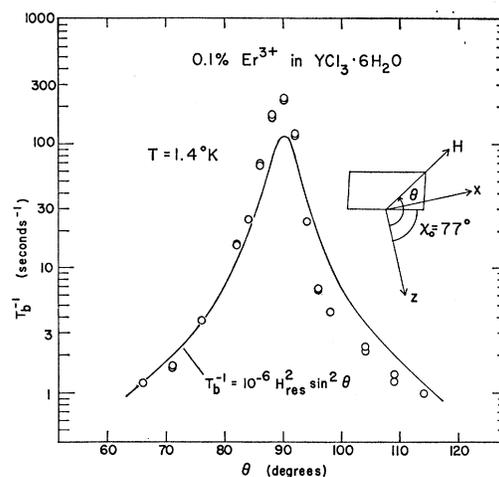


FIG. 10. Observed relaxation rate  $T_b^{-1}$  as a function of orientation between the magnetic field and the  $z$  axis at the constant temperature  $T = 1.39^\circ\text{K}$  for 0.1%  $\text{Er}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ . The magnetic field was rotated about the normal to the crystal plane.

of the crystal, and it will be seen in Fig. 10 that the relaxation rate is very sensitive to angle for this orientation. Figure 10 shows some relaxation rate measurements for a sample of 0.1%  $\text{Er}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  at constant temperature  $T = 1.39^\circ\text{K}$  as a function of the angle  $\theta$  between  $H$  and  $z$ . The results are not quite symmetric with respect to the  $z$  axis; the maximum relaxation rate seems to occur at  $\theta \approx 89^\circ$ , thus indicating that the excited-state  $g$  tensor is only slightly misaligned with respect to that of the ground state. The data can be nearly fitted with

$$T_b^{-1} = 10^{-6} H^2(\theta) \sin^2 \theta \text{ sec}^{-1}, \quad (15)$$

indicating that the ground state is only weakly connected to the excited states by the operator  $J_z$ .

### E. $\text{Yb}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$

Magnetic resonance was observed for a 1%  $\text{Yb}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  crystal with the results of Table I. The  $g_x$  and  $g_y$  values are not well determined because the resonance signal was quite weak and the linewidth broadened from 17 Oe with  $H \parallel z$  to more than 100 Oe with  $\theta = 50^\circ$ . The signals were too weak to determine the hyperfine constants. Attempts to measure the relaxation rate also failed, probably because of inability to saturate the resonance. The  $^2F_{7/2}$  ground multiplet of  $\text{Yb}^{3+}$  would split in a hexagonal field into these combinations of basis functions:  $|\pm \frac{1}{2}\rangle$ ,  $|\pm \frac{3}{2}\rangle$ , and  $\cos\phi |\pm \frac{5}{2}\rangle \sin\phi |\mp \frac{7}{2}\rangle$ . If the lowest doublet were predominantly  $|\pm \frac{5}{2}\rangle$ , this would predict  $g_x = g_y = 0$ ,  $g_z = 40/7 = 5.71$ , not too far from those observed. The first excited state<sup>19</sup> is at  $132 \text{ cm}^{-1}$  so that the third-order Zeeman splitting will be quite small. Optical Zeeman studies<sup>19</sup> in  $\text{Yb}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  yield  $g_x = g_y = 0$ ,  $g_z = 5.65$ ,  $\chi_0 = 71^\circ$ . Although our data indicates only that  $g_x < 0.5$ ,

$g_{\parallel} < 0.5$ , it was thought that this crystal might actually have  $g_{\perp}$  as small as a nuclear  $g$  factor, and thus would be suitable for use as a spin refrigerator.<sup>4</sup> However, McColl<sup>27</sup> did not observe any proton polarization enhancement upon rotation of a large field ( $\sim 10$  kOe) in the  $ac$  plane.

No paramagnetic resonance was found for  $\text{Sm}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  and for  $\text{Pr}:\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  in fields up to 19 kOe at  $\nu = 9.3$  Gc.

#### IV. SUMMARY AND CONCLUSIONS

Magnetic resonance and spin bath relaxation has been measured for the Kramers ions  $\text{Ce}^{3+}$ ,  $\text{Nd}^{3+}$ ,  $\text{Dy}^{3+}$ , and  $\text{Er}^{3+}$  diluted into  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$ . Resonance was also observed for  $\text{Yb}^{3+}$  in  $\text{YCl}_3 \cdot 6\text{H}_2\text{O}$  but the relaxation rate could not be measured, probably because of an inability to saturate the resonance line. The resonance results indicate the general feature that the rare-earth ions experience an effective crystalline field which is nearly  $C_6$  or  $C_{3h}$  in character and that this pseudoaxis is perpendicular to the crystal twofold axis. Angular dependence of the relaxation rate and published optical Zeeman studies indicate, however, that the axis is not quite in the same direction for the excited states, even within a multiplet, as it is for the ground state. The direction of the pseudoaxis also depends on the ion under consideration. This latter result is in agreement with the x-ray structure since a  $C_2$  site symmetry requires only that one principal direction of the  $g$  tensor be along the  $C_2$  crystal axis. The other two principal directions of the  $g$  tensor will be determined by the crystal-field eigenfunctions. However, neither of these other two directions should display a  $C_6$  or  $C_{3h}$  axial symmetry. The reason for the observed pseudo-hexagonal axis is not really understood.

The relaxation measurements show the same general features as similar measurements in hydrated crystals<sup>25,26</sup> and do not give evidence for cross relaxation as observed for anhydrous crystals such as  $\text{LaF}_3$  in I. Usually a Raman process was found, of order  $T_{1R}^{-1} \sim 10^{-4} T^9 \text{ sec}^{-1}$ . An exponential Orbach process was observed for all four ions. Except for  $\text{Ce}^{3+}$ , for which no optical data has been reported, the observed values of  $\Delta$  are in excellent agreement with the optically determined values in the concentrated salts. The Orbach process displayed some angular dependence for  $\text{Nd}^{3+}$  and  $\text{Er}^{3+}$ , varying by nearly a factor of two for  $\text{Nd}^{3+}$

and approximately a factor of three for  $\text{Er}^{3+}$ . This is probably due to a large anisotropy in the excited state  $g$  factor. There was also evidence of a possible bottlenecked Orbach process with  $\text{Dy}^{3+}$ , which showed an increase by approximately a factor of three in the Orbach relaxation rate  $T_{10}^{-1}$  when the concentration of  $\text{Dy}^{3+}$  was decreased from 1% to 0.1%. The direct process was displayed for all cases and was observed to be unusually anisotropic for  $\text{Nd}^{3+}$  and  $\text{Er}^{3+}$ , varying by  $10^3$  to  $10^4$ . This is the largest anisotropy yet reported for paramagnetic relaxation. For orientations for which the direct process became most rapid, there was evidence of a phonon bottleneck for  $\text{Nd}^{3+}$  and  $\text{Dy}^{3+}$ , displayed by a  $T^2$  temperature dependence. Attempts to check the concentration dependence of the bottleneck were somewhat thwarted by the fact that the magnetic resonance linewidth increased drastically with increasing concentration, probably due to crystal strains. The increased linewidth increases the effective bandwidth of phonons involved in the direct process and therefore increases their specific heat. Calculations of the mean free path for phonon scattering from the observed values of  $D$  yielded inconclusive results since it appeared that for  $\text{Nd}^{3+}$ , the hot phonons were being reflected once or twice before reaching the helium bath whereas for  $\text{Dy}^{3+}$ , they seemed to be scattered internally. The relaxation traces were usually exponential except for  $\text{Nd}^{3+}$  for the orientation at which a phonon bottleneck was observed. The angular dependence of the direct process relaxation rate for  $\text{Ce}^{3+}$  and  $\text{Dy}^{3+}$  displayed the unusual result that the rate was not symmetric with respect to the principal directions of the ground-state  $g$  tensor. However, this is probably because the principal directions of the  $g$  tensor of the excited state, which must be admixed into the ground state for Kramers ion relaxation, do not coincide with those of the ground state.

Since accurate wave functions and crystal-field parameters are not known, no attempts to estimate the relaxation rate were made using the phenomenological theory discussed in I. However, the experimental resonance and relaxation data should nevertheless be useful for further work with these crystals.

#### ACKNOWLEDGMENTS

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<sup>27</sup> J. R. McColl (private communication).