Interference Effects in Photoproduction of ρ^0 Mesons*

ALLAN S. KRASS
University of California, Santa Barbara, California
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A recently proposed model for the observed shift in the ρ^0 mass in photoproduction experiments is examined in detail. It is shown that if this model is correct, certain observable effects on the ρ^0 helicity density-matrix elements are predicted. Numerical predictions are made for some typical energies and production angles.

THERE has been a great deal of interest lately in the photoproduction of the neutral vector ρ^0 and ω^0. Experiments by the Cambridge Bubble Chamber Group,[1] Lanzerotti *et al.*,[2] and the DESY Bubble Chamber Group[3] have accumulated a fair amount of data on these processes, and recently numerous attempts at theoretical interpretation have been made.

From the data there emerge the following points upon which all authors seem to be in agreement. (1) The ρ^0 production proceeds via a diffraction mechanism as opposed to a one-pion-exchange (peripheral) mechanism. This is seen in the coherence effect from heavy nuclei, the behavior of the total cross section with energy, and the decay angular distributions of the final pions. (2) The ρ^0 mass is shifted downward by anywhere from 20–40 MeV from the mass which is observed in ρ^0's made by pions beams. Granted that because of background effects the mass of the ρ^0 is not a precisely defined number, it is still quite significant that all of the recent experiments have shown a shift, and all shifts have been downward. (3) All one can say at the present time about ω^0 photoproduction is that its total cross section decreases as a function of photon energy above threshold and may or may not be approaching a constant diffraction limit at higher energies.

Since the data on the ρ^0 seem to be the most clear, and since theorists have been reasonably successful in accounting for the general features of the data, we will concern ourselves here with this problem only. We will discuss the proposed mechanism by which the ρ^0 mass is shifted and show that this mechanism also predicts observable effects on the decay angular distribution of the final pions.

The mechanism proposed by Söding[4] to account for the ρ^0 mass shift in photoproduction is the interference between the amplitudes of Figs. 1(a) and 1(b). That the interference between a resonant amplitude and a coherent, essentially constant, background can cause a mass shift is easily seen as follows:

Write the total amplitude as

$$T=\left(-\frac{\gamma}{x-i\gamma}+c\right), \tag{1}$$

where

$$x=(k_+ + k_-)^2-m_\rho{}^2$$

and

$$\gamma=\Gamma_\rho m_\rho.$$

It is easy to show that the maximum of the total amplitude is shifted by an amount

$$\Delta x=-\frac{1}{2}\frac{\gamma}{c}[(1+4c^2)^{1/2}-1], \tag{2}$$

which, in the limit $c\ll\frac{1}{2}$, reduces to

$$\Delta x=-c\gamma, \tag{3}$$

which implies

$$\Delta m_\rho=-\tfrac{1}{2}c\Gamma_\rho. \tag{4}$$

Note that c can be positive or negative, and we have chosen the sign in (1) so that the real part of the resonant amplitude is positive below resonance. One of the bonuses of this model is the connection between the sign of the mass shift and the relative sign of the interfering amplitudes.

In order to calculate the factor c in (4) we must take only the part of Fig. 1(b) in which the two-pion invariant mass is under the ρ^0 peak and also demand that the two pions are in a relative p wave. Any other partial waves in Fig. 1(b) will drop out in the interference term when we integrate over all angles in the ρ^0 rest frame.

In view of the above projections performed on amplitude 1(b) it may strike one as doubtful that a meaningful separation can be made between the p-wave $m=765$-MeV pion pair from Fig. 1(a) and the p-wave, $m=765$-MeV pion pair from 1(b). At first glance it seems that we are counting Fig. 1(b) twice.

We believe this separation to be plausible on the following grounds: Figures 1(b) have poles in the variables $(k-k_\pm)^2$ at μ^2. If we restrict ourselves to events which have $(k-k_\pm)^2$ near this pole, then we can make a distinction between this singularity and the other singularities in the π-π part of Fig. 1(b) which arise from the rescattering of the π's to form a ρ (see Fig. 2).

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[1] Cambridge Bubble Chamber Group, Phys. Rev. **146**, 994 (1966).
[2] L. J. Lanzerotti *et al.*, Phys. Rev. Letters **15**, 210 (1965).
[3] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Nuovo Cimento **41**, 270 (1966).
[4] P. Söding, Phys. Letters **19**, 702 (1966).

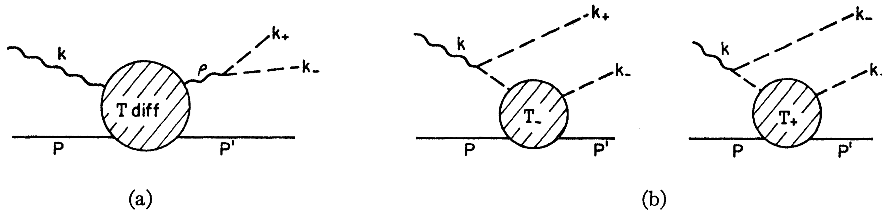

FIG. 1. (a) Diffraction production of ρ^0's by photons. (b) Drell mechanism for pion pair production by photons.

If we think in terms of an Omnes equation for the $\pi\pi$ rescattering, this situation corresponds to the Born term having a quite different singularity structure than the rescattering kernel. In such cases it does not seem unreasonable to separate the Born and resonant amplitudes.[5]

This argument is not intended to be definitive. At present there is no way to rigorously justify the separation theoretically. The real test of this model will be in the experimental verification of its prediction concerning decay angular distributions.

If we now accept the assumption that Figs. 1(a) and 1(b) can be separated, and that the interference of these two amplitudes is responsible for the mass shift, we are led to consider other possible effects which might be attributable to this interference. In particular, we consider the angular distribution of the final pions.

In order to do this we must first postulate a model for the amplitude of Fig. 1(a). We list three possible models which will predict a unique polarization for the final ρ^0:

(1) Söding has used a phenomenological model in which the diffraction amplitude is proportional to

$$\epsilon_\mu^{(\gamma)}(k)\left[-g^{\mu\nu}+\frac{p^\mu k^\nu}{p\cdot k}\right]\epsilon_\nu^{(\rho)}(p)\,, \qquad (5)$$

where $p=k_+ + k_-$. This is the simplest gauge-invariant amplitude one can assume, and it is easy to show that this is the same form as would occur if scalar or pseudoscalar meson exchange were assumed. It is now reasonably well known that this model does not predict the correct polarizations.[1–3]

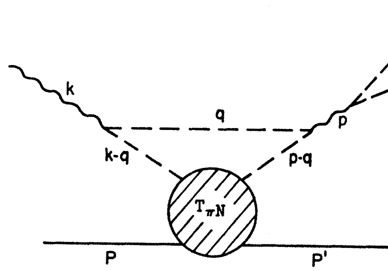

FIG. 2. Amplitude for a real pion pair produced by the Drell mechanism to rescatter as a ρ before leaving the interaction region. This graph should be counted as part of Fig. 1(a).

(2) In their original paper on photoproduction of ρ's Berman and Drell[6] use the Amati-Fubini-Stanghellini (AFS) multiperipheral model and then specialize to the forward direction. One way to get results off the forward direction is to take this model very seriously and calculate the cross section from the Feynman amplitude of Fig. 3.

For the purposes of the loop integration one assumes that the ω is on shell, and that the only nonvanishing π-N scattering amplitude occurs above the physical threshold in the π-N system. This seems a rather artificial assumption, but it is the best one can do within the "spirit" of the AFS model. Finally one assumes a diffraction-type amplitude for the π-N scattering:

$$T_{\pi N}\approx\frac{p_{\pi N}\sqrt{s_{\pi N}}}{M}\sigma_{\pi N}(\mathrm{total})\exp(-\tfrac{1}{2}A\,|t|)\,, \qquad (6)$$

where $p_{\pi N}$ is the c.m. momentum of the π-N system. When the necessary integral is performed (numerically) it is found that the ρ^0 polarization off the forward direction is not very different from that obtained with scalar meson exchange (SME), at least at lab photon energies below 6 BeV. As k_{lab} gets larger ($\gtrsim$10 BeV), the difference between this model and SME increases, and the AFS model tends to give more ρ's with the same state of polarization as the incident γ's than does SME. However, this effect is observed in experiments well below 10 BeV, so the AFS model does not explain this effect.

(3) Eisenberg *et al.*[7] have recently shown that experimental decay angular distribution can be accounted for quite well by the "strong-absorption model." This model parametrizes the nucleon as a semitransparent sphere. Such an optical model can be expected to be reasonably good as long as the momentum transfer to the nucleon is small. Since this model accounts for

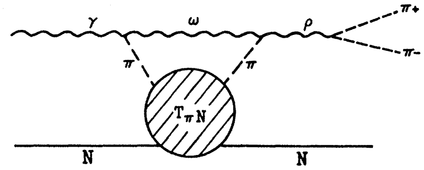

FIG. 3. Berman-Drell diffraction mechanism using AFS multiperipheral model.

[5] See, e.g., Ref. 18 of M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).

[6] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).

[7] Y. Eisenberg *et al.*, Phys. Letters **22**, 217, 223, (1966).

the polarization better than any yet proposed, we will use it to calculate the interference effects.

Before the results are presented, some comment must be made about the gauge invariance of this model. The two graphs of Fig. 1(b) do not lead to a gauge-invariant amplitude. This is a result of the energy dependence of the π-N diffraction scattering amplitude which implies a nonlocal π-N interaction. It is well known that in such nonlocal theories very complicated and arbitrary means must be employed to generate another graph which ensures the gauge invariance of the entire theory.

The amplitude which results from the graphs 1(b) can be written as

$$T\alpha\left[\frac{k_-\cdot\epsilon}{k_-\cdot k}T_+(s_+,t)-\frac{k_+\cdot\epsilon}{k_+\cdot k}T_-(s_-,t)\right], \tag{7}$$

where $k_\pm$ is the momentum of the $\pi_\pm$, $k(\epsilon)$ is the photon momentum (polarization), t is the invariant momentum transfer to the nucleon, and s_π is the square of the center-of-mass energy of the π-N system.

It is clear that the term necessary to ensure gauge invariance of the whole amplitude must have the form

$$-\frac{\epsilon\cdot A}{k\cdot A}[T_+(s_+,t)-T_-(s_-,t)], \tag{8}$$

where A is an arbitrary four vector. Many choices can be made for A, each with its own more or less plausible justification, but none of these is particularly convincing, and different choices for A give different results for the mass shift and polarization.

Another possible approach is to try to build a field theory of charged pions and nucleons which leads to a diffraction type scattering amplitude. Then this (nonlocal) field theory can be "gauged" to determine the necessary modifications in the presence of the electromagnetic field. However, this is not an aesthetically pleasing technique, and it also suffers from problems of non-uniqueness involving path integrals of the electromagnetic potential.

The solution to the gauge question lies in the recognition that the separation into Figs. 1(a) and 1(b) is not gauge invariant, and that whatever is needed to make 1(b) gauge invariant has been "left behind" in 1(a). From this point of view Söding's technique of enforcing gauge invariance on the diffraction amplitude alone would seem to be incorrect. This does not spoil his results, however, since the gauge correction is not large, and the mass shift, as we have seen is really independent of any specific details of the resonant amplitude.

However, the gauge question is extremely important in calculating decay angular distributions, and we need a prescription for handling it. We first note that the strong-absorption model is not gauge invariant. The helicity amplitudes are calculated in the c.m. frame, and the photon is identified simply by restricting its helicity values to ±1. An arbitrary gauge transformation would introduce a zero helicity component for the photon, which would then produce a new helicity amplitude, and there is nothing in the formal structure of the optical model to forbid this. So in effect we have chosen the radiation gauge in the c.m. frame for the diffraction part of the amplitude. This forces us to use the same gauge and Lorentz frame for calculating the Drell process. If for some reason we had chosen to do the calculation in another frame, we would have again picked the radiation gauge in that frame, and then the optical parameters would be changed to fit the data as observed in that frame. In this way the optical model amplitude would absorb the change of gauge in a completely phenomenological way.

The calculation of the helicity amplitudes now proceeds in two parts: first the diffraction amplitudes as given by Eisenberg *et al.*[7] and, second, the projection of the helicity amplitudes from the Drell process.

For the first part we take over directly all the formulas from Ref. 7 except for the following modification: We wish to display explicitly the resonant behavior of the diffraction amplitude in the variable $(k_++k_-)^2=m_\rho^2$. This is accomplished by changing Eq.(1) of Ref. 7 to read

$$\frac{d_\sigma}{d\Omega dm_\rho}=\frac{1}{(2S_a+1)}\frac{1}{(2S_b+1)}\times\sum_{(\lambda i)}|\langle\lambda_c\lambda_d|T'(s,t)|\lambda_a\lambda_b\rangle|^2, \tag{9}$$

where

$$T'(s,t)=\frac{(\Gamma/\pi)^{1/2}}{m_\rho-m_{\rho^0}-i\Gamma/2}T(s,t), \tag{10}$$

and $m_{\rho 0}$, Γ are the mass and width of the ρ^0 meson (765 MeV, 125 MeV, respectively). The multiplying factor is fixed so that an integral of (9) over the ρ_0 peak from $m_{\rho 0}-\Gamma/2$ to $m_{\rho 0}+\Gamma/2$ gives back Eq. (1) of Ref. 7.

The helicity amplitudes for Fig. 1(b) are determined from the expression for the differential cross section:

$$d\sigma=\frac{e^2}{k}\frac{M^2}{(E+P)}\frac{1}{4}\sum_{\text{sp. pol.}}\left|\bar{u}(P')\times\left\{\frac{k_-\cdot\epsilon}{k_-\cdot k}T_+(s_+,t)-\frac{k_+\cdot\epsilon}{k_+\cdot k}T_-(s_-,t)\right\}u(P)\right|^2\times\frac{1}{(2\pi)^5}\delta^4(P+k-P'-k_+-k_-)\frac{d^3k_+}{2\omega_+}\frac{d^3k_-}{2\omega_-}\frac{d^3P'}{2E'}. \tag{11}$$

Defining the new variables

$$p=k_++k_-,$$
$$\Delta=k_+-k_-,$$

we can write

$$\frac{d^3k_+}{\omega_+}\frac{d^3k_-}{\omega_-}=\frac{1}{8}\frac{p_0}{\omega_+\omega_-}d^3\Delta\frac{d^3p}{p_0}, \tag{12}$$

and, using the fact that $d\sigma$ is Lorentz invariant, we can break the expression (11) into two invariant parts:

$$d\sigma=\left\{\frac{e^2}{32(2\pi)^5}\frac{d^3p}{p_0}\delta[(k+P-p)^2-M^2]\sum_{\mathrm{sp}}|\bar{u}(P')u(P)|^2\right\}\left\{\frac{M^2p_0}{k(E+P)\omega_+\omega_-}d^3\Delta\,\tfrac{1}{4}\sum_{\mathrm{pol.}}\left|\frac{k_-\cdot\epsilon}{k_-\cdot k}T_+-\frac{k_+\cdot\epsilon}{k_+\cdot k}T_-\right|^2\right\},\quad(13)$$

where we have already done the integral over final nucleon momenta. Now the expression in the first brackets can be evaluated in the over-all c.m. frame, and the second expression, in which we must integrate over all Δ, is most easily evaluated in the rest frame of the ρ^0.

The integral over Δ becomes

$$\int\Delta^2d\Delta d\Omega_\Delta\,,\quad(14)$$

and from (11), $|\mathbf{\Delta}|\approx m_\rho$ in the ρ^0 rest frame. We assume $v\approx c$ for the pions in this frame. So

$$\Delta^2d\Delta=m_\rho{}^2dm_\rho\,,$$

and (13) becomes:

$$\frac{d\sigma}{d\Omega_pdm_\rho}=\left\{\frac{e^2}{32(2\pi)^5}\frac{|\mathbf{p}|}{2s}\sum_{\mathrm{sp}}|\bar{u}(P')u(P)|^2\right\}_{\mathrm{c.m.}}\left\{\frac{M^2m_\rho}{k'(E'+P')}\int d\Omega_\Delta\sum_{\mathrm{pol.}}{}'\left|\frac{k_-\cdot\epsilon}{k_-\cdot k}T_+-\frac{k_+\cdot\epsilon}{k_+\cdot k}T_-\right|\right\}_{\rho^0}.\quad(15)$$

The primes in the second bracket indicate evaluation in the ρ^0 rest frame. Finally the integral over solid angle in the ρ^0 frame can be converted to a sum over the three helicity states of the ρ^0 if the helicity states are projected out of the amplitude with spherical harmonics.

Two comments are now necessary. First, since the ρ^0 is in the final state, we must use the Y_{lm} instead of the complex conjugate in the projection. Secondly the photon polarization vector must be taken as transverse in the c.m. frame and then Lorentz transformed to the ρ^0 frame without making the usual gauge transformation. So, in the ρ^0 frame the vector ϵ has both timelike and longitudinal components.

The π-N diffraction amplitudes $T_\pm$ in Eq. (15) are given by Eq. (6), with $\sigma_{\pi N}=30$ mb and $A=8$ $(\mathrm{BeV}/c)^{-2}$. If the kinematics of Eq. (6) are now worked out we get:

$$p_\pm\sqrt{s_\pm}=\tfrac{1}{2}\{[s_\pm-(M-\mu)^2][s_\pm-(M+\mu)^2]\}^{1/2},\quad(16)$$

$$\approx\tfrac{1}{2}(s_\pm-M^2).\quad(17)$$

Using $s_\pm=(k_\pm+P')^2=(k+P-k)^2$, and assuming the velocity of the ρ^0 in the c.m. to be c, Eq. (6) can be approximated by,

$$T_\pm\approx\tfrac{1}{2}\sigma_{\pi N}k_l(1\pm\cos\alpha)\,,\quad(18)$$

where k_l is the lab photon energy, and α is the angle between $\mathbf{k}_+$ and the z axis in the ρ^0 rest frame.

The rest of the calculation is straightforward, and we may now list the results. If we define the helicity amplitudes by

$$\frac{d\sigma}{d\Omega dm_\rho}=\tfrac{1}{4}\sum_{(\lambda_i)}|\langle\lambda_c\lambda_d|T_I|\lambda_a\lambda_b\rangle|^2,\quad(19)$$

$$\langle+1\lambda_d|T_I|+1\lambda_b\rangle=+\langle-1\lambda_d|T_I|-1\lambda_b\rangle=GQ_{db}\times(\tfrac{3}{4}+\tfrac{1}{2}\cos\psi-\tfrac{1}{4}\cos2\psi)\,,\quad(20)$$

$$\langle0\lambda_d|T_I|+1\lambda_b\rangle=-\langle0\lambda_d|T_I|-1\lambda_b\rangle=-\sqrt{2}GQ_{db}\sin\psi\times\left(\tfrac{3}{4}\cos\psi-\tfrac{1}{8}-\frac{\beta}{2(1+\beta\cos\psi)}\right),\quad(21)$$

$$\langle-1\lambda_d|T_I|+1\lambda_b\rangle=-\langle+1\lambda_d|T_I|-1\lambda_b\rangle=GQ_{db}(\tfrac{3}{4}-\cos\psi+\tfrac{1}{4}\cos2\psi)\,,\quad(22)$$

where ψ is the angle between the z axis (the ρ^0 momentum direction in the c.m. frame) and the photon momentum in the ρ^0 rest frame, and β is the ρ^0 velocity in the c.m. frame. The other quantities are

$$G=\left(\frac{s-M^2}{s}\right)\left(\frac{m_\rho{}^2}{m_\rho{}^2-t}\right)\left(\frac{s-m_\rho{}^2}{m_\rho{}^2-t}\right)^{1/2}\times\left[\frac{\alpha}{3(2\pi)^3}\right]^{1/2}m_\rho{}^{1/2}\sigma_{\pi N}e^{-1/2A|t|}\,,\quad(23)$$

$$Q_{db}=\begin{array}{c}b\rightarrow\\ d\downarrow\end{array}\begin{pmatrix}\cos(\theta/2) & -(E/M)\sin(\theta/2)\\ (E/M)\sin(\theta/2) & \cos(\theta/2)\end{pmatrix},\quad(24)$$

where E is the nucleon c.m. energy and θ is the c.m. scattering angle. In calculating (24) we have assumed a negligible energy transfer to the nucleon in the c.m. frame.

The differential cross section and spin-density matrix elements are now calculated from

$$\frac{d\sigma}{d\Omega dm_\rho}=\tfrac{1}{4}\sum_{(\lambda)}|\langle\lambda_c\lambda_d|T'+T_I|\lambda_a\lambda_b\rangle|^2,\quad(25)$$

and

$$\rho_{ij}=\frac{1}{N}\sum_{(\lambda)}\langle i\lambda_d|T'+T_I|\lambda_a\lambda_b\rangle\langle j\lambda_d|T'+T_I|\lambda_a\lambda_b\rangle^*,\quad(26)$$

TABLE I. k_{lab}=photon lab momentum, $\theta_{\text{c.m.}}$=center-of-mass angle between photon and ρ^0 momentum, t=invariant momentum transfer to nucleon, Δm=downward shift of ρ^0 mass (we have used the mean of the half-maximum points, which gives twice as large a shift as Eq. (4), ρ^{00} and $\rho^{1,-1}$ are helicity density matrix elements of the final ρ, and $\Delta\rho^{ij}/\rho^{ij}$ is the percentage difference between the values of ρ^{ij} for $m_\rho>765$ MeV and $m_\rho<765$ MeV.

k_{lab} (BeV)	$\theta_{\text{c.m.}}$ (deg.)	$\|t\|$ $(\text{BeV}/c)^2$	Δm (MeV)	ρ^{00}	$\rho^{1,-1}$	$\Delta\rho^{00}/\rho^{00}$ (%)	$\Delta\rho^{1,-1}/\rho^{1,-1}$ (%)
2	10	0.042	14	0.016	0.008	20	14
2	20	0.107	13	0.060	0.029	22	0
2	30	0.208	10	0.126	0.062	16	− 7
4	10	0.057	17	0.016	0.008	27	− 14
4	20	0.202	11	0.058	0.029	19	− 33
4	30	0.428	6	0.124	0.065	9	− 22
6	10	0.080	17	0.016	0.007	29	− 30
6	20	0.306	8	0.054	0.028	12	− 40
10	10	0.133	15	0.015	0.007	22	−200

where

$$N=4\frac{d\sigma}{d\Omega dm_\rho}.$$

The results for some typical energies and angles are given in Table I. It should be emphasized that the spin-density matrix elements are calculated with respect to the ρ^0 momentum direction. The effects on the spin-density matrix elements referred to the photon momentum direction in the ρ^0 rest frame are smaller, roughly by a factor of 3. The interesting density matrix elements are ρ^{00} and $\rho^{1,-1}$,and we see from the table that the model predicts significant variations in these across the resonance peak. The element ρ^{00} is larger for ρ^0's with mass greater than 765 MeV than it is for those with mass less than the peak value. $\rho^{1,-1}$ has the opposite behavior for most of the cases.

The variations in the density matrix elements are significant, but this does not take into account the difficulties in measuring these variations. The matrix elements themselves are quite small for small angles, and the effects tend to disappear at larger angles, because the Drell process falls off somewhat faster with momentum transfer than the diffraction process. It would seem that quite precise measurements will be needed to see these effects. Table I indicates mass shifts ≤ 20 MeV. If the size of the Drell graph is increased (e.g., by increasing the size of $T_{\pi N}$), then the effects on the density matrix elements are increased correspondingly.

Another interesting, and possibly verifiable, prediction is the disappearance of the mass shift with increasing momentum transfer. It also appears that the mass shift increases slowly with photon energy for those ρ^0's produced in very forward directions.

In conclusion it should be emphasized that all of the results presented here are very sensitive to the particular models chosen for the diffraction and Drell processes. The vanishing of the mass shift at high momentum transfer depends on a small difference in diffraction peak widths, and the values of, and variations in, the density matrix elements are both quite model-dependent. This gives some hope that, if measurements can be made precisely enough, one might be able to verify or reject this model with some confidence.[8]

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[8] *Note added in proof.* Since this paper was submitted, the author has received private communications from Dr. H. Spitzer at DESY, which tend to confirm all of the predictions of this paper except for the size of the mass shift. The observed mass shift is larger than our value. Another interesting feature noted by Spitzer is the behavior of the π-π production cross section *outside* the resonance peak. It is in qualitative agreement with the predictions of the Drell process alone. I wish to thank Dr. Spitzer for communicating these results to me before publication.