High-Spin Baryons. IV. Decay Modes and Possible Regularities*

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Effective coupling constants for decays of the type baryon → baryon plus pseudoscalar meson are derived from experiment. In the absence of a reliable theory of the structure (barrier-penetration factor), we are unable to attribute an intrinsic meaning to these couplings. However, for the pion-nucleon system we tentatively suggest the existence of several approximate rules. With appropriate definitions of the couplings, these are as follows: (1) The πBB coupling is independent of the baryon B; (2) decays down a trajectory have a strength independent of trajectory type and rung on the trajectory; (3) a scaling law holds for decays from one trajectory to another. An interesting result of this law, which is independent of the barrier factor, is that the decay $N_2(1688) \to \Delta_1(1238)\pi$ is strongly inhibited relative to other p-wave decays. In the Appendix a simple derivation is given of the most general expression for the decay of an arbitrary-spin baryon into a spin-$\frac{1}{2}$ final baryon and a spin-zero meson.

I. INTRODUCTION

MANY new features of the energy spectrum of excited states of baryons have recently appeared. The most striking result is the existence of a sequence of high-mass, high-spin states having relatively narrow widths. The difficulty of obtaining an understanding of the many-channel dynamics suggests a more phenomenological approach. In previous papers,[1,2] dynamical models leading to Regge trajectories in the baryon system were discussed. However, experience shows the value of a "spectroscopic" approach in which model-independent features of the data are emphasized.

Here we discuss the transitions among baryon states

$$B_i \to B_j + P_k \qquad (1.1)$$

involving the emission of a pseudoscalar meson. We assume that these decays can be described by an effective vertex, in which the unstable baryons are described by Rarita-Schwinger fields.[3] The effective coupling constant is then regarded as a measure of the intrinsic strength of the decay (1.1).

The effective couplings can then be used to test or discover regularities in the decays. The most common use is to test $SU(3)$ multiplet assignments.[4] Since a recent study of this subject has been published by Goldberg *et al.*,[5] we shall instead emphasize a newer and more speculative aspect, the possible systematics in transitions between Regge trajectories. These possibilities are discussed in Sec. IV.

In Sec. II the formulas are stated and discussed. A serious limitation to results of this sort is the lack of understanding of the structure effects.[6] In Sec. III numerical results are given for all reactions of type (1.1) we could think of. For some cases the decay widths are poorly known, if at all. In such cases we simply assume a value; the reader can adjust the coupling constants as more data (or theories) become available. Section IV is given to speculations on possible regularities among couplings, related to Regge recurrences.

II. DECAY RATES; EFFECTIVE COUPLING CONSTANTS

We shall use the formulas for decay rates given in paper I of this series.[7] Many of the formulas we shall use were first derived by more complicated means by Brudnoy[8] and Rushbrooke.[9] Without isospin factors the simplest effective coupling of baryons with spins $s=k+\frac{1}{2}$, $s'=k'+\frac{1}{2}$ to a pseudoscalar meson ϕ is $(k'\geqslant k)$

$$gm^{k-k'}\bar{\psi}^{\mu'}\cdots{}^{\mu k'}(x)\Gamma\psi_{\mu_1\cdots\mu_k}(x)\partial_{\mu_{k+1}}\cdots\partial_{\mu_{k'}}\phi+\text{H.c.} \qquad (2.1)$$

Here $\psi_{\mu_1\cdots\mu_k}$ is the Rarita-Schwinger field.[3] The mass m is required to make g dimensionless. Γ is 1 or $i\gamma_5$ depending on whether the relative γ parity[7] of baryons s and s' is odd or even. The γ parity is $+1$ for baryons of $J^P=\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, \cdots$ and -1 for $\frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+, \cdots$. (In Ref. 7 the opposite sign convention was used; we now prefer that the nucleon have $\gamma=+1$.) In I an explicit formula was given for the matrix elements of (2.1) when one baryon is at rest.

The decay rates were found to be, for relative γ parity $\gamma_R=\pm1$,

$$\Gamma_\pm(B_{s'}\to B_s+P)=\frac{g^2}{4\pi}\frac{1}{2s'+1}\frac{p^{2(s'-s)+1}}{m^{2(s'-s)}}\frac{E_s\mp M_s}{M_{s'}}$$
$$\times\sum_\lambda(\epsilon_{s\lambda}{}^{\pm})^2\Big[\prod_{k=1}^{s'-s}C(s+k-1,1,s+k;\lambda,0)\Big]^2. \qquad (2.2)$$

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[1] P. Carruthers, Phys. Rev. 154, 1399 (1967). This paper is referred to as II.
[2] P. Carruthers and M. M. Nieto, Phys. Rev. (to be published). This paper is referred to as V. Also, Phys. Rev. Letters 18, 297 (1967).
[3] W. Rarita and J. Schwinger, Phys. Rev. 60, 21 (1941).
[4] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).
[5] M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, Nuovo Cimento 45, 169 (1966).
[6] In potential scattering one speaks of "barrier penetration factors."
[7] P. Carruthers, Phys. Rev. 152, 1345 (1966). This is paper I of the series.
[8] D. Brudnoy, Phys. Rev. Letters 14, 273 (1965); Phys. Rev. 145, 1229 (1966).
[9] J. G. Rushbrooke, Phys. Rev. 143, 1345 (1966).

p is the c.m. decay momentum, M_s and $M_{s'}$ the baryon masses, and E_s the total energy of the final baryon. The $\epsilon_{s\lambda}^{\pm}$ are polynomials in the variable E_s/M_s, given in Ref. 7. [λ is the helicity, running from $-s$ to $+s$ in the sums of Eq. (2.2). C is the Clebsch-Gordan coefficient in the notation of Rose.[10]]

In the important special case in which the meson P is a pion, it was found convenient to use generalized isospin matrices $\mathbf{T}$ which treat the baryons in the vertex in a symmetrical way; the modification in formula (2.2) is to multiply the right-hand side by $2T+1$, where T is the isospin of the final baryon. In Sec. IV we shall use this modification.

The main energy dependence of the decay widths is given by the p^{2L+1} factor where L is the orbital momentum of the emitted meson. In addition the polynomials $\epsilon_{s\lambda}^{\pm}$ are mildly energy-dependent. However, from resonance theory in potential scattering[11] one knows of significant modifications due to the structure of the decaying state. Clearly our effective interaction (2.1) is a point interaction.

The experimental evidence on the energy dependence of resonance widths indicates that the essentially p^{2L+1} dependence of Γ is considerably too rapidly varying. An excellent discussion of such questions has been given by Roper.[12] The reader should bear in mind that all results may depend sensitively on the "barrier penetration factor" of which there is no real understanding. Our results are given for structureless particles; the reader should modify our coupling constants according to the penetration factor of his choice. The calculations of Goldberg *et al.*[5] show convincingly that comparison of transitions involving the same L and comparable momenta are rather insensitive to the radius of the state. However, many of the applications of Sec. IV depend sensitively on the choice of radius.

It should be stressed that when the final baryon in Eq. (1.1) has spin greater than $\frac{1}{2}$, couplings other than (2.1) can be formed. Although these more complicated couplings contain only higher multipoles in the Breit frame[13] (two more units of angular momentum), it is not inconceivable that they could be important for transitions involving large momentum. Rushbrooke[9] has derived formulas for transitions (1.1) when B_j has spin $\frac{3}{2}$. Because the explicit methods[7] of I are so much simpler than covariant techniques for high spin, we have thought it worthwhile to rederive his result in the Appendix. However, all quoted coupling constants have been computed from formula (2.2).

[10] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).
[11] J. M. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1950).
[12] L. D. Roper, University of California Report No. UCRL-14193 (unpublished).
[13] L. Durand, III, P. DeCelles, and R. B. Marr, Phys. Rev. **126**, 1882 (1922).

III. NUMERICAL RESULTS FOR EFFECTIVE COUPLING CONSTANTS

We have used formula (2.2) to compute effective point couplings involving known decays of baryons of spin $\leqslant\frac{7}{2}$. Higher-spin baryons probably exist, but present experimental evidence is fragmentary. Results are given in Table I for decays of the type spin $J\rightarrow$ spin-$\frac{1}{2}$+meson and spin $J\rightarrow$ spin-$\frac{3}{2}$+meson. Precise information on modes of the latter is presently very scarce.

We emphasize once again that at present the only safe application of these results involves *relative* magnitudes of couplings involving transitions of *comparable* momenta and the same orbital angular momentum. Nevertheless comparison of decays involving quite different linear and orbital momenta is of great interest.

The masses of the decay products used in our calculations were taken to be the average over the isotopic spin multiplet.[14] The unit of mass m is now taken to be the pion mass μ.

IV. SPECULATIONS ON REGULARITIES IN DECAY MODES OF EXCITED STATES OF THE NUCLEON

In this section we attempt to guess a pattern relating various decays of the nucleon resonances. While experimental uncertainties, and especially ignorance of the barrier factor, afflict many of our results we hope to make a convincing case for the importance of such studies. In particular we wish to point out that patterns in the πN_iN_j vertices (N_i arbitrary nucleon isobars) may be indicative of algebraic relations among various Regge residues.[15] In addition we find the interesting result, independent of interaction radius, that the p-wave decay $N_2(1688)\rightarrow\Delta_1(1236)+\pi$ is strongly suppressed relative to other p waves. Finally, these results suggest an invariance of certain coupling constant ratios under simultaneous replacement of particles by their Regge recurrences. This principle, which can be supplemented by further speculations, leads to many new experimental predictions.

The most prominent pion-nucleon resonances can be grouped into two "principal series" S^+ and S^-. The groups of objects S^+ is made up of the members of two positive-parity Regge trajectories N_i[$i=1$, 2, $\cdots$ corresponds to states of isospin-spin-parity $(T,J^P)=(\frac{1}{2},\frac{1}{2}^+)$, $(\frac{1}{2},\frac{5}{2}^+)\cdots$] and Δ_i [$i=1$, 2, $\cdots$ corresponds to states $(\frac{3}{2},\frac{3}{2}^+)$, $(\frac{3}{2},\frac{7}{2}^+)\cdots$]. N_1 and Δ_1 are, respectively, the nucleon and the 3-3 resonance (1236). The series S^- is comprised of the negative-parity states $N_i^-[(\frac{1}{2},\frac{3}{2}^-)$, $(\frac{1}{2},\frac{7}{2}^-)\cdots]$ and $N'^-_i[(\frac{1}{2},\frac{5}{2}^-),(\frac{1}{2},\frac{9}{2}^-)\cdots]$. The last resonance has not yet been seen but is predicted[2] by a systematic dynamical theory of the series S^-. In Fig. 1

[14] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).
[15] N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters **22**, 336 (1966).

TABLE I. Phenomenological coupling constants. $g^2/4\pi$ is given, assuming that the width is due to the simplest possible coupling. Masses of decay products are taken as the average mass of the isospin multiplet. Masses and widths are from Ref. 14, except values shown with an asterisk, which are arbitrarily chosen. The reader may adjust these according to his taste.

Mass (MeV)	Resonance J	P	$2T$	Y	Meson	Baryon	J	P	$2T$	Momentum (μ)	Γ (MeV)	$g^2/4\pi$
1400	$\frac{1}{2}$	1	1	1	π	N	$\frac{1}{2}$	1	1	2.662	140.0	0.768e 01[a]
1525	$\frac{3}{2}$	-1	1	1	π	N	$\frac{1}{2}$	1	1	3.328	68.2	0.576e 00
1525	$\frac{3}{2}$	-1	1	1	π	Δ	$\frac{3}{2}$	1	3	1.663	21.0	0.553e-01
1570	$\frac{1}{2}$	-1	1	1	π	N	$\frac{1}{2}$	1	1	3.559	39.0	0.623e-01
1570	$\frac{1}{2}$	-1	1	1	η	N	$\frac{1}{2}$	1	1	1.762	91.0	0.307e 00
1670	$\frac{5}{2}$	-1	1	1	π	N	$\frac{1}{2}$	1	1	4.057	56.0	0.227e-02
1670	$\frac{5}{2}$	-1	1	1	π	Δ	$\frac{3}{2}$	1	3	2.590	1.0*	0.861e-01
1670	$\frac{5}{2}$	-1	1	1	K	Λ	$\frac{1}{2}$	1	0	1.474	1.0*	0.577e-02
1670	$\frac{5}{2}$	-1	1	1	η	N	$\frac{1}{2}$	1	1	2.673	1.0*	0.341e-03
1688	$\frac{5}{2}$	1	1	1	π	N	$\frac{1}{2}$	1	1	4.144	71.5	0.333e-01
1688	$\frac{5}{2}$	1	1	1	π	Δ	$\frac{3}{2}$	1	3	2.697	10.0*	0.713e-02
1688	$\frac{5}{2}$	1	1	1	K	Λ	$\frac{1}{2}$	1	0	1.693	1.0*	0.271e 00
1688	$\frac{5}{2}$	1	1	1	η	N	$\frac{1}{2}$	1	1	2.812	1.0*	0.676e-02
1700	$\frac{1}{2}$	-1	1	1	π	N	$\frac{1}{2}$	1	1	4.202	240.0	0.344e 00
2190	$\frac{7}{2}$	-1	1	1	π	N	$\frac{1}{2}$	1	1	6.429	60.0	0.103e-03
2190	$\frac{7}{2}$	-1	1	1	K	Λ	$\frac{1}{2}$	1	0	5.152	1.0*	0.138e-04
1236	$\frac{3}{2}$	1	3	1	π	N	$\frac{1}{2}$	1	1	1.673	120.0	0.360e 00
1670	$\frac{1}{2}$	-1	3	1	π	N	$\frac{1}{2}$	1	1	4.057	72.0	0.105e 00
1920	$\frac{7}{2}$	1	3	1	π	N	$\frac{1}{2}$	1	1	5.233	100.0	0.106e-03
1920	$\frac{7}{2}$	1	3	1	π	Δ	$\frac{3}{2}$	1	3	3.985	20.0*	0.387e-01
1920	$\frac{7}{2}$	1	3	1	K	Σ	$\frac{1}{2}$	1	2	3.081	1.0*	0.375e-04
1405	$\frac{1}{2}$	-1	0	0	π	Σ	$\frac{1}{2}$	1	2	1.074	35.0	0.138e 00
1520	$\frac{3}{2}$	-1	0	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	1.739	6.2	0.128e 01
1520	$\frac{3}{2}$	-1	0	0	π	Σ	$\frac{1}{2}$	1	2	1.912	8.2	0.134e 01
1670	$\frac{1}{2}$	-1	0	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	2.985	1.0*	0.206e-02
1670	$\frac{1}{2}$	-1	0	0	η	Λ	$\frac{1}{2}$	1	0	0.473	1.0*	0.114e-01
1700	$\frac{3}{2}$	-1	0	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	3.189	8.0	0.929e-01
1700	$\frac{3}{2}$	-1	0	0	π	Σ	$\frac{1}{2}$	1	2	3.002	1.0*	0.195e-01
1820	$\frac{5}{2}$	1	0	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	3.934	58.1	0.418e-01
1820	$\frac{5}{2}$	1	0	0	π	Σ	$\frac{1}{2}$	1	2	3.662	9.1	0.132e-01
1820	$\frac{5}{2}$	1	0	0	π	(1385)	$\frac{3}{2}$	1	2	2.629	14.9	0.112e-01
1820	$\frac{5}{2}$	1	0	0	η	Λ	$\frac{1}{2}$	1	0	2.536	0.8	0.140e-01
2100	$\frac{7}{2}$	-1	0	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	5.429	46.4	0.338e-03
2100	$\frac{7}{2}$	-1	0	0	π	Σ	$\frac{1}{2}$	1	2	5.086	1.0*	0.158e-04
1385	$\frac{3}{2}$	1	2	0	π	Λ	$\frac{1}{2}$	1	0	1.510	33.7	0.130e 00
1385	$\frac{3}{2}$	1	2	0	π	Σ	$\frac{1}{2}$	1	2	0.896	3.3	0.575e-01
1660	$\frac{3}{2}$	-1	2	0	π	(1405)	$\frac{1}{2}$	-1	0	1.432	50.0	0.442e 02
1660	$\frac{3}{2}$	-1	2	0	π	Σ	$\frac{1}{2}$	1	2	2.773	1.0*	0.282e-01
1660	$\frac{3}{2}$	-1	2	0	π	Λ	$\frac{1}{2}$	1	0	3.185	1.0*	0.133e-01
1660	$\frac{3}{2}$	-1	2	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	2.914	1.0*	0.176e-01
1770	$\frac{5}{2}$	-1	2	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	3.635	43.6	0.329e-02
1770	$\frac{5}{2}$	-1	2	0	π	Λ	$\frac{1}{2}$	1	0	3.773	15.1	0.809e-03
1770	$\frac{5}{2}$	-1	2	0	π	(1520)	$\frac{3}{2}$	-1	0	1.411	16.9	0.396e 02
1770	$\frac{5}{2}$	-1	2	0	π	(1385)	$\frac{3}{2}$	1	2	2.318	10.7	0.191e 01
1770	$\frac{5}{2}$	-1	2	0	η	Σ	$\frac{1}{2}$	1	2	1.061	1.8	0.535e-01
1770	$\frac{5}{2}$	-1	2	0	π	Σ	$\frac{1}{2}$	1	2	3.392	0.9	0.778e-04
1910	$\frac{5}{2}$	1	2	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	4.442	4.8	0.157e-02
1910	$\frac{5}{2}$	1	2	0	π	Λ	$\frac{1}{2}$	1	0	4.484	6.0	0.214e-02
1910	$\frac{5}{2}$	1	2	0	π	Σ	$\frac{1}{2}$	1	2	4.135	1.8	0.119e-02
2035	$\frac{7}{2}$	1	2	0	$\bar{K}$	N	$\frac{1}{2}$	1	1	5.102	25.6	0.347e-04
2035	$\frac{7}{2}$	1	2	0	π	Λ	$\frac{1}{2}$	1	0	5.092	40.0	0.477e-04
2035	$\frac{7}{2}$	1	2	0	π	Σ	$\frac{1}{2}$	1	2	4.767	1.0*	0.180e-05
1530	$\frac{3}{2}$	1	1	-1	π	Ξ	$\frac{1}{2}$	1	1	1.083	7.3	0.720e-01
1815	$\frac{3}{2}$	-1	1	-1	$\bar{K}$	Λ	$\frac{1}{2}$	1	0	2.843	10.4	0.266e 00
1815	$\frac{3}{2}$	-1	1	-1	π	Ξ	$\frac{1}{2}$	1	1	2.982	1.6	0.378e-01
1815	$\frac{3}{2}$	-1	1	-1	π	(1530)	$\frac{3}{2}$	1	1	1.653	3.2	0.820e-02

[a] The number following e specifies the exponent in the power of 10 involved. Thus 0.768e 01 means 0.768×10^1.

TABLE II. Coupling constants related by our systematics. L is the angular momentum of the outgoing pion. $\gamma_R=+1$ if γ_5 occurs in the coupling.

Decay mode	(L,γ_R)	Γ (MeV)	p (μ)	$g^2/4\pi$	$G(a=0)$	$G(a=1/3\mu)$
$N_1 \rightarrow N_1\pi$	(1, +)	...	...	22.5	0.122	...
$\Delta_1 \rightarrow N_1\pi$	(1, −)	120	1.673	0.180	0.180	0.236
$N_2 \rightarrow \Delta_1\pi$	(1, −)	10	2.697	1.78×10^{-4}	1.78×10^{-4}	3.22×10^{-4}
$N_2 \rightarrow N_1\pi$	(3, +)	71	4.144	1.67×10^{-2}	5.01×10^{-5}	1.23×10^{-3}
$\Delta_2 \rightarrow \Delta_1\pi$	(3, +)	27(?)	3.985	1.31×10^{-2}	2.62×10^{-5}	5.51×10^{-4}
$\Delta_2 \rightarrow N_1\pi$	(3, −)	70[a]	5.233	3.73×10^{-5}	3.73×10^{-5}	2.46×10^{-3}
$\Delta_2 \rightarrow N_2\pi$	(1, −)	25(?)	1.268	0.269	0.269	0.317
$N_1^- \rightarrow N_1\pi$	(2, +)	60	3.328	0.288	9.58×10^{-4}	4.77×10^{-3}
$N_2^- \rightarrow N_1\pi$	(4, +)	25[b]	6.429	2.15×10^{-5}	5.00×10^{-7}	4.88×10^{-5}

[a] Calculated from Ref. 21.
[b] Calculated from Ref. 24.

these states have been depicted in a manner convenient for discussion of decay modes.[2] There are other minor effects, secondary trajectories,[16] and probably even higher spin states,[17] but for clarity we do not discuss those here. The motivation for the introduction of Fig. 1 was obtained from dynamical considerations. However, the results obtained here can be regarded as independent of detailed models.

In Fig. 1, we have dropped the isospin designation and added an extra quantum number, the "γ parity" of the baryon, which was shown[7] to determine the structure of the $\pi B_i B_j$ vertex. Notice that members of the top (bottom) horizontal plane have positive (negative) γ parity. The back vertical plane (S^+) has positive ordinary parity and the front vertical plane (S^-) has negative ordinary parity. Hence couplings within each horizontal plane have γ_5 in the vertex, while those between members of the top and bottom planes do not. The matrices T are not ordinary isospin matrices but have been defined[7] in a way designed to treat the particles symmetrically and to give systematically the statistical weight $2T+1$ in the decay rate.[18]

The presence of γ_5 in those vertices of positive relative γ parity reduces them in magnitude by a factor of order $1/M^2$. For example, when we compare the $\pi N_1 N_1$ coupling to the $\pi N_1 \Delta_1$ coupling the proper measure of the former is $f^2=(g/2M)^2/4\pi$. Then the coupling constants are of the same order of magnitude. In the present case we define reduced coupling constants for the $\pi N_a N_b$ vertex by

$$G(ab)=g^2(N_aN_b)/16\pi M_aM_b\,,\quad \gamma_R=+1\,,\quad (4.1)$$

$$G(ab)=g^2(N_aN_b)/4\pi\,,\qquad \gamma_R=-1\,.\quad (4.2)$$

It should be emphasized that the detailed choice taken here is perhaps ambiguous and may well be changed by a more illuminated theoretical interpretation of the semiquantitative relations we wish to point out. Note that we are using the pion mass μ as the unit.

[16] For example, the P_{11} resonance near 1550 MeV probably signals the beginning of another "nucleon" trajectory N_i'.

[17] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913, 1135(E) (1966) and Phys. Rev. **155**, 1792 (1967).

[18] For example, $\mathbf{T}_{11}=(2/3)^{1/2}\boldsymbol{\tau}$, so that the πN coupling constant s $g^2(N_1N_1)/4\pi=22.5$ rather than 15.

We adopt the modified potential-scattering barrier penetration factor used by Glashow and Rosenfeld[4] and others.[5] Whenever possible we compare decays with comparable momenta to reduce the sensitivity. The functional form is $F(a,p,l)=(1+a^2p^2)^{-l}$, where a is the radius of the state, p the decay momentum, and l the orbital angular momentum.

In Table II we list some pertinent parameters derived from experiment or predicted by the rules suggested below. In many cases the decay widths are known only very approximately, with many discrepancies among various experimenters. In our calculations we used a wide variety of radii but report here results for the most satisfactory value, $1/a=3\mu$, which is close to that used by other authors to test $SU(3)$ predictions.[4,5]

The first result is

$$\frac{G(N_1N_1)}{G(N_1\Delta_1)}=\frac{G(N_1N_2)}{G(N_1\Delta_2)}\,. \quad (4.3)$$

Figure 2 shows the variation of the two sides of this expression with the radius.[19] This equality shows that replacement of one N_1 in the $N_1N_1\pi$ vertex by its recurrence is balanced by replacing Δ_1 in the $N_1\Delta_1\pi$ vertex by its recurrence. Note that the individual

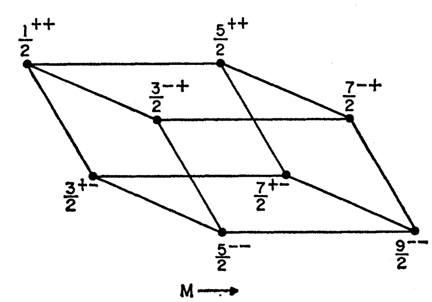

FIG. 1. The four main Regge trajectories in the pion-nucleon system are shown, labeled by $J^{P\gamma}$. Vertical planes have fixed parity and horizontal planes fixed γ parity. The figure may be extended to the right.

[19] This equality is not a sensitive function of a, even though the individual terms on the right-hand side are. As $1/a$ increases from 2 to 6μ the left-hand side varies from 0.39–0.60 while the right-hand side increases from 0.40–0.77.

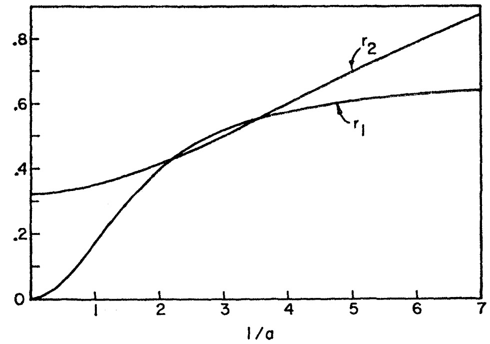

FIG. 2. Graph of the two sides of Eq. (4.3) as a function of $1/a$. $r_1=G(N_1N_1)/G(N_1\Delta_1)$; $r_2=G(N_1N_2)/G(N_1\Delta_2)$.

coupling constants on the left and right sides of (4.3) differ by two orders of magnitude.

We might also replace both left-hand particles by their recurrences and check the equality

$$\frac{G(N_1N_1)}{G(N_1\Delta_1)}=\frac{G(N_2N_1)}{G(N_2\Delta_1)}. \tag{4.4}$$

This relation then gives $G(N_2\Delta_1)=2.38\times10^{-3}$ and $\Gamma(N_2\to\Delta_1\pi)=7.39$ MeV, for $1/a=3\mu$.

This relation is remarkable in that it requires the p-wave transition $N_2\to\Delta_1\pi$ to have a coupling constant characteristic of an f-wave decay, hence at least an order of magnitude smaller than typical p-wave couplings ($G\cong0.1$ to 0.2). Thus our invariance principle says that the transition $N_2\to\Delta_1\pi$ is strongly inhibited. This result fits well with the fact that the first nucleon recurrence is quite elastic compared with other resonances in this energy region. It also agrees with dynamical considerations[2] which suggest that elastic forces are mainly responsible for the existence of this resonance.

Unfortunately it is difficult to test the prediction (4.4) precisely because this particular result is sensitive to the choice of interaction radius. However the most important conclusion—that the decay $N_2\to\Delta_1\pi$ is inhibited—is not dependent on this factor. For $1/a=3\mu$ our predicted $\Gamma(N_2\to\Delta_1\pi)=7.4$ MeV is quite compatible with experiment[20] if the inelastic modes of $N_2(1688)$ have a partial width of 20 MeV, as given by some experiments.[21,22] However, recent phase-shift analyses[23] indicate a somewhat greater inelasticity for this resonance. For $1/a=2\mu$ Eq. (4.4) gives $G(N_2\Delta_1)=2.18\times10^{-2}$, or $\Gamma(N_2\to\Delta_1\pi)=42$ MeV. Although $G(N_2\Delta_1)$ is one order of magnitude smaller than typical p-wave couplings [$G(N_1N_1)=0.125$, $G(N_1\Delta_1)=0.25$], the corresponding width is somewhat greater than allowed by experiment. Further increase of $G(N_1\Delta_1)$ accompanying the decrease of $1/a$ is therefore prohibited by experiment. Thus for any plausible value of $1/a$ the coupling constant $G(N_2\Delta_1)$ is very small compared with the other $L=1$ transitions.

Rearrangement of Eq. (4.3) leads to

$$\frac{G(N_1N_1)}{G(N_1N_2)}=\frac{G(N_1\Delta_1)}{G(N_1\Delta_2)}, \tag{4.5}$$

which suggests that $G(N_1B_i)/G(N_1B_{i+1})$ is independent of the isobar B. The ratio (4.5), which compares transitions having quite different momenta, is sensitive to the barrier factor. Nevertheless we can see whether $N_1^-(1525)$ and $N_2^-(2190)$ fit into this pattern. For the parameters $\Gamma(N_1^-\to N_1\pi)=68$ MeV, $\Gamma(N_2^-\to N_1\pi)=25$ MeV, and $1/a=3\mu$, agreement is obtained. All the latter numbers are subject to some doubt, but the correlation of such widely differing numbers is perhaps not accidental.

If Eq. (4.4) is true, rearrangement yields

$$\frac{G(N_1N_1)}{G(N_1N_2)}=\frac{G(\Delta_1N_1)}{G(\Delta_1N_2)}, \tag{4.6}$$

which suggests that

$G(B_iB_j')/G(B_iB'_{j+1})$ is independent of B and B'. (A)

It is very tempting to speculate that

$$G(\Delta_2N_2)=G(\Delta_1N_1). \tag{4.7}$$

This is motivated by taking the Δ_2 to be made up of an N_2 and a π in the same way as a Δ_1 is of an N_1 and a π. Ignoring difficulties due to the proximity of the Δ_2 and the N_2 and their large widths, this equality would predict $\Gamma(\Delta_2\to N_2\pi)=18.6$ MeV.

Conjecture (A) gives $G(N_2N_1)/G(N_2N_2)=G(\Delta_2N_1)/G(\Delta_2N_2)$ which, when multiplied by (4.5), gives

$$\frac{G(N_1N_1)}{G(N_2N_2)}=\frac{G(N_1\Delta_1)}{G(N_2\Delta_2)}. \tag{4.8}$$

Using (4.7) and (4.8) we then find

$$G(N_1N_1)=G(N_2N_2) \tag{4.9}$$

or

$$g^2(N_2N_2)=(M_2/M_1)^2g^2(N_1N_1)=3.24g^2(N_1N_1).$$

This value is conveniently near that found in Ref. 2.

Conjecturing that the coupling strength down the trajectory is independent of the trajectory gives

$$G(B_iB_{i+1})=G(B_i'B_{i+1}') \tag{B}$$

[20] J. P. Merlo and G. Valladas, Proc. Roy. Soc. (London) **289A**, 489 (1966).

[21] P. J. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, J. J. Thresher, H. H. Atkinson, C. R. Cox, and K. S. Heard, Phys. Rev. Letters **15**, 468 (1965).

[22] One can make a rough estimate from the data of Ref. 20. The partial width $\Gamma(N_2\to\Delta_1\pi)$ ranges from 9-23 MeV depending on which experiments are used.

[23] P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965).

and in particular

$$G(\Delta_1\Delta_2)=G(N_1N_2). \tag{4.10}$$

Here we may argue that the Δ_2 is composed of an N_2 and π, and the decay $\Delta_2\to\Delta_1\pi$ is due to the decay of the $N_2\to N_1\pi$, with the spectator π remaining bound to the N_1. Eq. (4.10) predicts $\Gamma(\Delta_2\to\Delta_1\pi)=66$ MeV. Note that the width of the Δ_2 is now

$$\begin{aligned}\Gamma(\Delta_2)&=\Gamma(\Delta_2\to N_1\pi)+\Gamma(\Delta_2\to\Delta_1\pi)+(\Delta_2\to N_2\pi)+\cdots\\&=70\text{ MeV}+66\text{ MeV}+19\text{ MeV}=155\text{ MeV},\end{aligned}$$

out of 170–200 MeV (experimental).

A particular consequence of conjecture (A) is that $G(B_iB_i)/G(B_iB_{i+1})$ is independent of B. The denominator is also independent of B from conjecture (B), so we find $G(B_iB_i)$ is independent of B. In particular, $G(N_1N_1)=G(\Delta_1\Delta_1)$. In our notation the $SU(6)$ prediction for $g^2(\Delta_1\Delta_1)$ is 20.25, which is very nearly $g^2(N_1N_1)=22.5$. In that theory $M_1=M_2$, so our result seems compatible with $SU(6)$.

Conjecture (A) also tells us that

$$\frac{G(B_iB_i)}{G(B_{i+1}B_{i+1})}=\frac{G(B_iB_i)}{G(B_iB_{i+1})}\frac{G(B_{i+1}B_i)}{G(B_{i+1}B_{i+1})}$$

is independent of B. Finally (4.9) tells us that for $i=1$, the ratio is 1. Thus we are led to conjecture

$$G(B_iB_i)\text{ is independent of }B\text{ and }i. \tag{C}$$

Thus we surmise the following relations among coupling constants of the particles on the Regge trajectories of the pion nucleon system:

(A) $G(B_iB_j')/G(B_iB'_{j+1})$ is independent of B and B',
(B) $G(B_iB_{i+1})$ is independent of B and i,
(C) $G(B_iB_i)$ is independent of B and i.

In view of the uncertainties in our "derivation" and the large number of guesses, these conjectures will only be convincing if systematically derived from some theory and the present calculation used to show that experiment offers no contradiction.

V. CONCLUSIONS

The "effective coupling constants" for the vertices PB_iB_j are of great importance in developing a unified theory to the baryon energy spectrum. Models based on standard dynamical approximations are generally unsuccessful in giving meaningful widths. The present paper is an attempt to study the same problem from a more phenomenological point of view. However, in the last analysis the new attempt fails to give convincing results because of ignorance of structure effects associated with the "barrier-penetration factor" of potential theory. However, an optimist can perceive some vague patterns through the veil of uncertainties. The truths of any or all the predictions of Sec. IV are susceptible to direct experimental test if careful measurements of the branching ratios of high-spin resonances into lower-lying resonances are made. An interesting byproduct of our calculation is the observation that the decay $N_2\to\Delta_1\pi$ is strongly inhibited for any reasonable choice of the barrier-penetration factor. This fact follows from the scaling law proposed in Sec. IV, but of course may also follow from some other theory.

We have several suggestions for profitable research along these lines. (1) A theory of the barrier-penetration factor for decays of unstable particles is urgently needed. The lack of such a theory casts some doubt on all attempts to correct for $SU(3)$ symmetry breaking.[5,24,25] (2) Careful experiments on branching ratios on the decay modes of high-spin baryons are extremely useful. Photon-decay modes are also important though not discussed here. (3) Implications for possible algebraic relations among Regge residues of equations such as (A) should be examined. (4) Transitions among baryons having strangeness should be scrutinized for relations such as those found in Sec. IV.

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APPENDIX

In I the B^*-B-π vertex was calculated assuming that only the term with the fewest derivatives in the interaction Hamiltonian contributed. In general, there are $s-\frac{1}{2}$ terms which can contribute, where s is the lesser of the baryon spins. Here we work out the special case $s=\frac{3}{2}$, including both possible terms in the Hamiltonian.

The interaction Hamiltonian is

$$H'=\int d^3x\,\bar{\psi}_\mu(x)\Gamma\psi_{\nu_1\cdots\nu_k}(x)(G_1g^{\mu\nu_1}-G_2\partial^\mu\partial^{\nu_1})\times(-i\partial^{\nu_2})\cdots(-i\partial^{\nu_k})\phi(x)+\text{H.c.}, \tag{A1}$$

where $\Gamma=1$, $i\gamma_5$ for $\gamma=-1$, 1, respectively.

If q is the final $s=\frac{3}{2}$ baryon momentum and k the pion momentum, M the resonance mass, then in the rest frame of the resonance

$$\langle q\lambda_q,k|H'|\lambda\rangle=\left(\frac{m_q}{2\omega_kE_qV}\right)^{1/2}\delta^3{}_{-q,k}\bar{u}_\mu(q,\lambda_q)\Gamma u_{\nu_1\cdots\nu_k}(\lambda)\times(G_1g^{\mu\nu_1}+G_2k^\mu k^{\nu_1})k^{\nu_2}\cdots k^{\nu_k}. \tag{A2}$$

This can be evaluated recursively as in I. If θ is the angle

[24] A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters 10, 222 (1963).
[25] P. Carruthers, *Introduction to Unitary Symmetry* (Interscience Publishers, Inc., New York, 1966), Chap. 4.

between q and the λ quantization axis,

$$\langle q\lambda_q,k|H'|\lambda\rangle=\left(\frac{m_q}{2\omega_k E_q V}\right)^{1/2}\delta^3{}_{-q,k}q^{k-1}d^{s'}{}_{\lambda,\lambda_q}(\theta)\times f_\gamma(|q|,\lambda_q)C_{\lambda_q}\,,\quad \text{(A3)}$$

where

$$C_{\pm1/2}=\left(\frac{3(k+1)!}{2(2k+1)!!}\right)^{1/2},\quad C_{\pm3/2}=\left(\frac{(k-1)!(k+2)!}{2(2k+1)!!k!}\right)^{1/2},$$

$$f_\gamma(|q|,\lambda_q)=\bar{u}_\mu(q',\lambda_q)\Gamma u_\nu(\lambda_q)(G_1g^{\mu\nu}+G_2k'^\mu k'^\nu)\,,\quad \text{(A4)}$$

and

$$q=|\mathbf{q}|\,,\quad q'^\mu=(q^0,0,0,q)\,,\quad\text{and}\quad k'^\mu=(k^0,0,0,-q)\,.$$

$$f_+(q,\lambda_q)=i\sinh(\tfrac{1}{2}\omega_q)(-1)^{\lambda_q+1/2}\times\left\{\frac{2G_2Mq^2}{3m_q}\delta_{|\lambda_q|,1/2}+G_1\epsilon^+{}_{3/2,\lambda_q}\right\},\quad \text{(A5)}$$

$$f_-(q,\lambda_q)=\cosh(\tfrac{1}{2}\omega_q)\left\{\frac{2G_2Mq^2}{3m_q}\delta_{|\lambda_q|,1/2}+G_1\epsilon^-{}_{3/2,\lambda_q}\right\},\quad \text{(A6)}$$

$$\Gamma_\gamma=2\pi\sum_{q,\lambda_q}\delta(q_0+k_0-M)|\langle q\lambda_q k|H'|\lambda\rangle|^2=\frac{1}{2\pi}\frac{m_q}{M}\frac{q^{2k-1}}{2k+2}\sum_{\lambda_q}|f_\gamma(q,\lambda_q)|^2C^2{}_{\lambda_q}\,,\quad \text{(A7)}$$

$$\Gamma_\pm=\frac{1}{4\pi}\frac{q^{2k-1}}{6M}(E_q\mp m_q)\frac{(k-1)!}{(2k+1)!!}\times\left\{k\left[\pm\frac{2G_2Mq^2}{m_q}+G_1\left(1\mp\frac{2E_q}{m_q}\right)\right]^2+3(k+2)G_1{}^2\right\}.\quad \text{(A8)}$$

This result agrees with that found by a more complicated calculation by Rushbrooke.[9]

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External Mass Continuation and Commutation Relations in π-π Scattering*

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Single-variable, subtracted dispersion relations in the external mass and energy are used to calculate the difference between the π-π amplitude evaluated at physical threshold and at the unphysical zero point which occurs in the application of current-algebra techniques. All differences are relatively small. For one of the amplitudes this is in disagreement with the result obtained using expansion techniques. Further, the pseudoscalar density commutator, which lies outside the $SU_3\times SU_3$ algebra, is evaluated from a model using only conventional fields. The resulting current-algebra relations differ from those given by the quark model. However, the scattering lengths obtained, $a_0=+0.18$ and $a_2=-0.04$, are consistent with those obtained by Weinberg using expansion techniques together with the quark model. This appears to be nontrivial, since our mass extrapolation and quark-model commutation relations for the pseudoscalar density give a different result, $a_0=+0.05$, $a_2=-0.09$. The π-π sum rule involving an unsubtracted dispersion relation is evaluated. The ρ meson gives the dominant contribution, in contrast to the subtracted dispersion integrals where it gives only minor contributions. Crossing symmetry implies large background contributions from the ρ in the crossed channel and when these are taken into account, the sum rule is approximately satisfied with our scattering lengths.

I. INTRODUCTION

THE extraction of information from the low-energy theorems which follow from the $SU_3\times SU_3$ current algebra of Gell-Mann[1] involve the extrapolation of scattering amplitudes from zero to physical values of the external mass and energy. If the target mass M is much larger than the pion mass (here set equal to 1), then approximation schemes[2] based upon $1/M$ as an expansion parameter to affect this extrapolation are appealing and seem to agree with experiment, when available. As emphasized earlier,[3,4] such arguments do not apply to π-π scattering, since all particles have the same mass. A related feature is that cuts in the external mass squared begin at 3 rather than 9 as expected if $1/M$ is small. Furthermore, the pseudoscalar density commutator $[\partial^\mu A_\mu{}^a,A_0{}^b]$, which is not given by the $SU_3\times SU_3$ algebra, becomes important even though it seems unimportant when $1/M$ is small.

There have been essentially three approaches to the mass and energy extrapolations which start from our definition of the off-mass-shell π-π scattering amplitude.[5]

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[2] For a summary, see N. Fuchs, Phys. Rev. **155**, 1785 (1967).
[3] F. T. Meiere and M. Sugawara, Phys. Rev. **153**, 1709 (1967).
[4] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).
[5] For an alternate definition see A. Balachandran, M. Gundzik, and F. Nicodemi, Nuovo Cimento **44A**, 1257 (1966).