Determination of the Nucleon-Nucleon Elastic Scattering Matrix. VI. New Results near 50 MeV

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Our previous phase-shift analysis at 50 MeV has been improved by increasing the completeness of the data set. Since no new data have become available at this energy, it was necessary to expand the range of energies. We now obtain reasonable values for the ϵ_1 phase shift and for the pion-nucleon coupling constant g^2. Certain ways of handling the data result in an ambiguity in the $T=0$ phase shifts, which we have explored in detail. We conclude that no precise, reliable values for the $T=0$ phases can as yet be derived from existing data near 50 MeV, although the present solutions must be in the neighborhood of the correct solution.

I. INTRODUCTION

In the previous papers in this series,[1-5] we have evaluated the elastic scattering matrix in six narrow energy bands and by an energy-dependent analysis spanning these six energies. The isotopic spin $T=1$ phase shifts were in general well determined, while the $T=0$ phase shifts in many cases were not. In particular, the ϵ_1 phase showed quite radical fluctuations at 25 and 50 MeV.[3] These fluctuations are readily attributable to the scarcity of scattering data at these energies.

Recently we reanalyzed the data[5] at 25 MeV and, with the aid of new experimental data, were able to obtain reasonable values for ϵ_1 and the pion-nucleon coupling constant g^2. The early data set did not permit the extraction of a meaningful value for g^2. In the present paper, we report a reanalysis of the data near 50 MeV. As at 25 MeV, we have now obtained reasonable values for ϵ_1 and g^2 by using a more complete data set.

Near 50 MeV, unfortunately, the experimental data have been measured at a number of rather widely-spaced energies, and there are as yet only cross section and polarization data for the (n,p) system. Our reanalysis at 50 MeV was occasioned not by the advent of new measurements, as at 25 MeV, but rather by the decision to expand the energy range under consideration in an attempt to extract more information from the existing data. This required including a greater range of energies than is usual in a "single energy" analysis, namely from 40–68 MeV. We have investigated the validity of including such a broad energy range by doing the analysis in two different ways: (1) We used the energy variation of data given by a revision of our previous energy-dependent analysis[4] to shift all of the data to a common energy of 50 MeV, a technique previously used by Signell[6]; (2) we used the data at the experimental energies and assigned a linear energy variation to the phase shifts with the slopes determined from our energy-dependent analysis.[4] Signell[7] has more recently used another technique in which the energy dependence of the phase shifts is taken to be exactly that of an energy-dependent analysis, and just the average values are varied. We decided not to use this method because the former two methods seemed simpler and are probably just as accurate, especially for the $T=0$ phases, which were our principal concern. This conclusion is borne out by the fact that our two methods give closely similar results. Of course, none of these methods are completely accurate when applied to this large an energy span, but the possible errors introduced from the large energy span are not larger than the remaining phase-shift uncertainties and are more than compensated for by the fact that we now have a more complete data set. The principal addition is the inclusion of (p,p) polarization data, which were largely missing in the earlier analysis.[3] The present phase-shift values have small systematic uncertainties in addition to the "standard errors" quoted, but the solutions presented are in the neighborhood of the "correct solution" that will ultimately be obtained when the data set at 50 MeV is complete.

II. DATA SELECTION

The data considered in the present investigation are summarized in Table I. The data used in our previous analysis[3] are indicated. As can be seen, (p,p) polarization measurements are the principal addition to the data set. Expanding the data set did not increase the kinds of (n,p) data, but including more data should improve the accuracy of the $T=0$ phase shifts. Also, the $T=0$ phases are strongly correlated with the $T=1$ phases, making it important to improve the (p,p) data set as well.

Since the 66-MeV (p,p) polarization data constitute the only complete measurement of that type near 50 MeV, it was necessary to expand our energy range this

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TABLE I. Summary of data investigated near 50 MeV. The energies in parentheses show the data used in our previous analysis at 50 MeV (Ref. 3).

Experimental energy	Type	No. data	Angular range	Points deleted in final sel.	Normalization error	Reference
			p-p data			
39.4	σ	27	8°–90°		0.009	a
47.5	A	5	23°–87°	23°	0.050	b
(47.8)	A	5	23°–87°		0.050	c
(47.8)	R	5	23°–87°		0.050	c
(49.9)	P	1	45°			d
(50.0)	D	1	70°			e
(50.0)	σ	1	90°			f
50.2	σ	1	90°			g
(51.5)	σ	10	12°–36°	12°	0.045	h, i
(51.7)	P	1	60°			j, k
(51.8)	σ	9	36°–90°		0.025	h, i
(52.0)	C_{NN}	1	90°			l
(52.0)	C_{KP}	1	90°			l
(53.2)	P	1	75°			j, k
56.0	P	1	45.6°			m, k
56.2	σ	1	90°			g
58.5	P	1	45°			h, i
61.9	σ	1	90°			g
66.0	P	11	20°–71°		0.030	m, k
68.3	σ	26	10°–90°		0.011	n
			n-p data			
47.5	σ	11	7°–102°	102°	0.020	o
47.5	σ	11	78°–173°		0.040	o
48.8	σ_T	1				p, q
(50.0)	P	9	21°–101°		0.047	r
(50.0)	P	7	99°–159°	99°, 149° deleted	0.047	r
50.6	σ_T	1				p, q
52.5	σ_T	1				p, q
(52.5)	σ	12	7°–112°		0.017	o
(52.5)	σ	11	78°–173°		0.038	o
54.5	σ_T	1				p, q
56.6	σ_T	1				p, q
57.5	σ	12	7°–112°		0.020	o
57.5	σ	11	78°–173°	78°	0.040	o
58.8	σ_T	1				p, q
60.0	P	9	21°–101°		0.039	r
60.0	P	7	99°–159°	119°	0.039	r
61.1	σ_T	1				p, q
62.5	σ	12	7°–112°	82°	0.020	o
62.5	σ	11	78°–173°	98°	0.40	o
63.5	σ_T	1				p, q
66.1	σ_T	1				p, q

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far. Therefore it seemed logical to include the 68.3-MeV (p,p) differential-cross-section data as well. However, our past experience with a series of energy-dependent analyses based on several data selections has indicated that the 68.3-MeV (p,p) data points are no more consistent with the other data near 50 MeV than are the 39.4-MeV (p,p) differential-cross-section points. Thus we have also included these latter points. As will be shown in the next section, the phase shifts obtained by using either the 39.4 or 68.3-MeV cross-section data separately are almost identical, in spite of a considerable difference in the goodness-of-fit measure χ^2.

Table I describes the manner in which we have treated the data. Following our usual practice, we have discarded all data points that consistently contribute more than 4 to χ^2, judged not only by the present analysis, but also by our past experience with other analyses which have included this data. The data points removed

TABLE II. Phase-shift solutions at 50 MeV, with $g^2=14$.

Solution	A_1	A_2	A_3	B_3	A_4	A_5	A_6
No. (p,p) data	81	82	82	82	82	82	82
No. (n,p) data	124	124	124	124	124	124	124
Energy range	47–68	39–66	(39–66)	(39–66)	(39–66)	(39–66)	(39–66)
Data energy-shifted	no	no	yes	yes	yes	yes	yes
χ^2	152	132	129	133	129	132	132
Comments	39 MeV, σ del.	68 MeV, σ del.			$^3D_3$ OPEC	$^3D_2$ OPEC	$^3D_2$ EDA
$^1S_0$	38.77±0.28	38.03±0.32	37.59±0.36	37.46±0.38	37.96±0.56	38.10±0.57	38.00±0.59
$^1D_2$	2.07±0.11	2.17±0.09	2.25±0.11	2.29±0.11	2.19±0.13	2.20±0.13	2.23±0.13
$^3P_0$	11.67±0.36	11.70±0.36	12.13±0.38	12.04±0.41	11.76±0.64	11.57±0.66	11.80±0.66
$^3P_1$	−8.22±0.17	−8.09±0.18	−8.13±0.20	−8.17±0.27	−8.12±0.18	−8.08±0.19	−8.02±0.19
$^3P_2$	5.81±0.08	5.84±0.08	5.99±0.09	6.04±0.11	5.91±0.13	5.88±0.14	5.89±0.13
ϵ_2	−1.92±0.13	−2.34±0.14	−2.40±0.15	−2.44±0.15	−2.28±0.22	−2.28±0.22	−2.34±0.23
$^3F_2$	0.22±0.12	0.20±0.12	0.34±0.13	0.31±0.15	0.20±0.20	0.05±0.21	0.10±0.20
$^3F_3$	(−0.75)[a]	(−0.75)[a]	(−0.75)[a]	(−0.75)[a]	−0.53±0.29	−0.40±0.32	−0.50±0.31
$^1P_1$	0.39±0.99	−0.30±0.94	−0.99±1.00	−2.33±0.96	−0.85±0.93	−1.67±0.88	−1.37±0.91
$^3S_1$	59.50±1.51	60.36±1.54	58.14±1.81	65.41±0.93	58.74±0.74	64.33±0.44	62.44±0.42
ϵ_1	3.30±2.06	3.21±2.17	4.72±2.75	−4.45±3.92	3.92±2.66	−2.16±2.20	0.65±2.39
$^3D_1$	−8.64±0.97	−8.04±1.05	−9.63±1.15	−1.42±2.87	−9.35±0.25	−3.92±0.39	−6.35±0.42
$^3D_2$	12.94±1.23	12.33±1.30	13.90±1.40	5.26±2.92	13.45±0.52	(7.69)[a]	(10.11)[b]
$^3D_3$	−0.42±0.64	−0.06±0.66	−1.00±0.71	2.83±0.89	(−0.77)[a]	1.98±0.29	0.88±0.27

Solution	Yale[c]	Dubna[d]	Signell[e]	Harwell[g]	Livermore[h]
No. (p,p) data	Energy-dependent analysis		31	41	Energy-dependent analysis
No. (n,p) data			0	0	
Energy range	At 50 MeV	At 52 MeV	47.5–52	47.5–52	At 52 MeV
Data energy-shifted					
χ^2	YLAM, YLAN3M		18.5	44	
Comments	From graphs	$g^2=22.8±4.8$			
$^1S_0$	38.0	35.52± 1.53	38.1±0.5	38.56±0.43	37.89
$^1D_2$	2.2	2.46± 1.59	2.3±0.2	1.70±0.09	1.83
$^3P_0$	13.0	16.43± 2.39	10.3±0.7	12.22±0.47	12.44
$^3P_1$	−9.0	−6.96± 0.53	−8.0±0.4	−7.81±0.27	−8.08
$^3P_2$	6.0	5.51± 0.63	6.3±0.2	5.92±0.14	5.93
ϵ_2	−2.2	a	−2.2±0.3	a	−1.88
$^3F_2$	0.2	a	(0.28)[f]	a	0.41
$^3F_3$	−0.6	a	0.2±0.3	a	−0.77
$^1P_1$	−6.0	−4.06± 4.49			−3.78
$^3S_1$	60.0	65.11± 4.08			62.03
ϵ_1	2.4	−2.44±29.40			3.20
$^3D_1$	−7.0	−2.82± 9.48			−6.28
$^3D_2$	12.0	5.61±12.50			10.51
$^3D_3$	0.5	3.74± 4.62			0.92

[a] One-pion-exchange-contribution (OPEC) values.
[b] Livermore energy-dependent analysis (EDA) (Ref. 4).
[c] Reference 8.
[d] Reference 9.
[e] Solution $N=7$ from Ref. 6.
[f] Signell energy-dependent analysis, from which unlisted phase shifts were also taken.
[g] Reference 10.
[h] Reference 4.

are listed in Table I. Only 10 points from a total of 242 were removed. As we have stated in detail in previous papers, these data points affect the value of χ^2 considerably, but they have very little effect on the phase-shift values or errors. When we included the 10 deleted data points as a check, the sole significant difference was an increase of about 100 in the value for χ^2.

III. PHASE-SHIFT RESULTS

We are not able to present a precise, unique phase-shift solution. We believe the existing data near 50 MeV do not define "a solution," especially with respect to the $T=0$ phases. The range of solutions we have obtained is summarized in Table II. Solutions A_1 and A_2 show the effect of using the 39.4- or the 68.3-MeV (p,p) differential-cross-section data separately, with a fixed linear energy dependence in the phase shifts. Closely similar results are obtained with data shifted to 50 MeV. Solutions A_2 and A_3 give a comparison between the data treated at the experimental energies (A_2) and the same data all shifted to a common energy of 50 MeV (A_3), as discussed in the previous section. The phase-shift results for these two methods are quite similar, and in fact the particular solutions shown represent the greatest difference due to the two methods among several pairs of solutions that we tested in this manner (using different data or parameter sets).

With the data shifted to 50 MeV, we found an ambiguity in the $T=0$ phases leading to a separate minimum in χ^2, solution B_3. The nature of this ambiguity is displayed in Fig. 1. This ambiguity disappeared when either $^3D_2$ or $^3D_3$ was held fixed, as in solutions A_4, A_5, and A_6. The unshifted data did not show this ambiguity in the $T=0$ phases. They gave only the type A solution.

When $^3F^3$ and $^3F_4$ were added as searched phases, it was found that $^3F_4$ did not differ significantly from its one-pion-exchange contribution (OPEC) value. $^3F_3$ also remains close to its OPEC value (solutions A_4, A_5, and A_6). It should be noted that including $^3F_3$ in the search results in much larger errors for some of the $T=1$ phases, indicating the onset of too much freedom in the phases.

Varying the value of g^2, we obtained from $\chi^2(g^2)$ curves the values $g^2=11\pm3$ for solutions A_3 and B_3, and 12 ± 5 for solution A_4. These results are a little low, but not unreasonable. Solutions in which $^3F_3$ and $^3F_4$ were also searched give $g^2\sim16\pm7$, with the large error indicating too much freedom in the phases. Our earlier analysis[3] at 50 MeV gave no reasonable determination of g^2.

IV. DISCUSSION

As discussed in some detail in our previous papers,[1–5] a simple listing of phase shifts and errors, as shown in Table II, does not accurately describe the physical content of a phase-shift solution. A full error matrix is necessary to describe the correlations between the various phase-shift uncertainties. Also, systematic errors can occur due to inaccuracies or incompleteness in the data selection, and these are often not fully reflected in the error matrix. One way to test for these systematic errors is to see if $\chi^2(\delta)$ for a particular phase shift δ is parabolic in the neighborhood of a solution when δ is assigned a series of values, at each of which the other phases are allowed to readjust. Examples of such "parameter studies" were given in our recent analysis[5] at 25 MeV. When we repeated these parameter studies at 50 MeV, we found that for the $T=1$ phases, $\chi^2(\delta)$ is parabolic over a range of 3 to 4 standard deviations from the minimum, while for the $T=0$ phases, $\chi^2(\delta)$ is parabolic over a range of only 1 or 2 standard deviations. Thus the data are adequate to describe the $T=1$ phases but are barely sufficient to delimit the $T=0$ phases.

It is interesting to compare different phase shift solutions at 50 MeV. In Table II we have listed, in addition to the solutions already discussed, solutions from Yale,[8] Dubna,[9] Michigan State,[6] Harwell,[10] and Livermore[4] analyses.

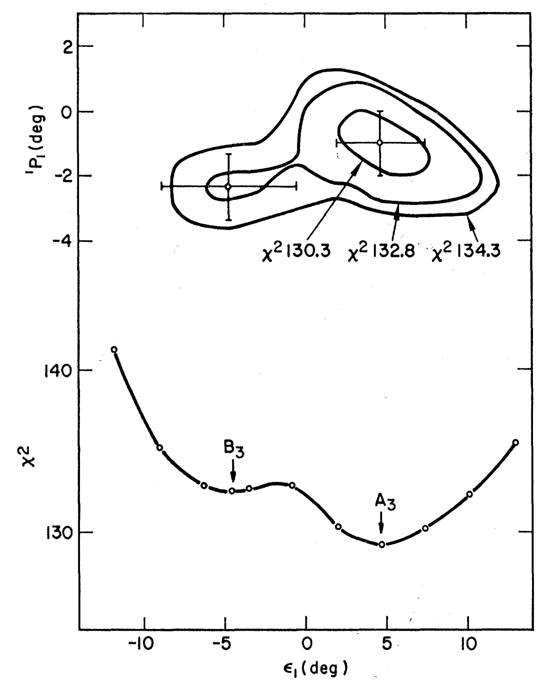

FIG. 1. The nature of the ambiguity in the T=0 phases, showing the minimum χ^2 values as ϵ_1 alone, and as ϵ_1 and $^1P_1$ together, are varied through fixed values. Positions of the solutions A_3 and B_3 are indicated by arrows. Irregularities in the upper curves are intentional.

The $T=1$ phases from all of the solutions are very similar, although the differences between phases are not small when compared to the quoted errors. The $T=0$ phases from the Yale solution[8] are quite similar to our solution type A. The Dubna $T=0$ phases,[9] as judged by the ^3D waves, are like our type B, although the errors on their phases imply that their analysis gives little information about the $T=0$ amplitudes. They used a more restricted data set than ours, and their value for g^2 was considerably different. The Livermore energy-dependent analysis (EDA) $T=0$ phases are generally like the solution type A, although the fact that free parameters were used up through H waves or $T=1$ and up through G waves for $T=0$ in the EDA makes an exact comparison difficult.

Since the difficulties at 50 MeV are due to the incompleteness of the data selection, we have investigated the observable predictions for solution types A_3 and B_3 in Table II. Figure 2 shows the corridor of errors for observables D_T, R, A, and C_{NN} for these two solutions. As can be seen, accurate measurements near 90° c.m. for any of these observables would be helpful. The other observables gave substantially the same results for both solutions A_3 and B_3 and hence would not be as useful in reducing the solution ambiguities.

The present phase-shift results are in substantial agreement with our previous energy-dependent analyses,[4] as shown in the last column of Table II. Where a data set at a single energy is reasonably complete and

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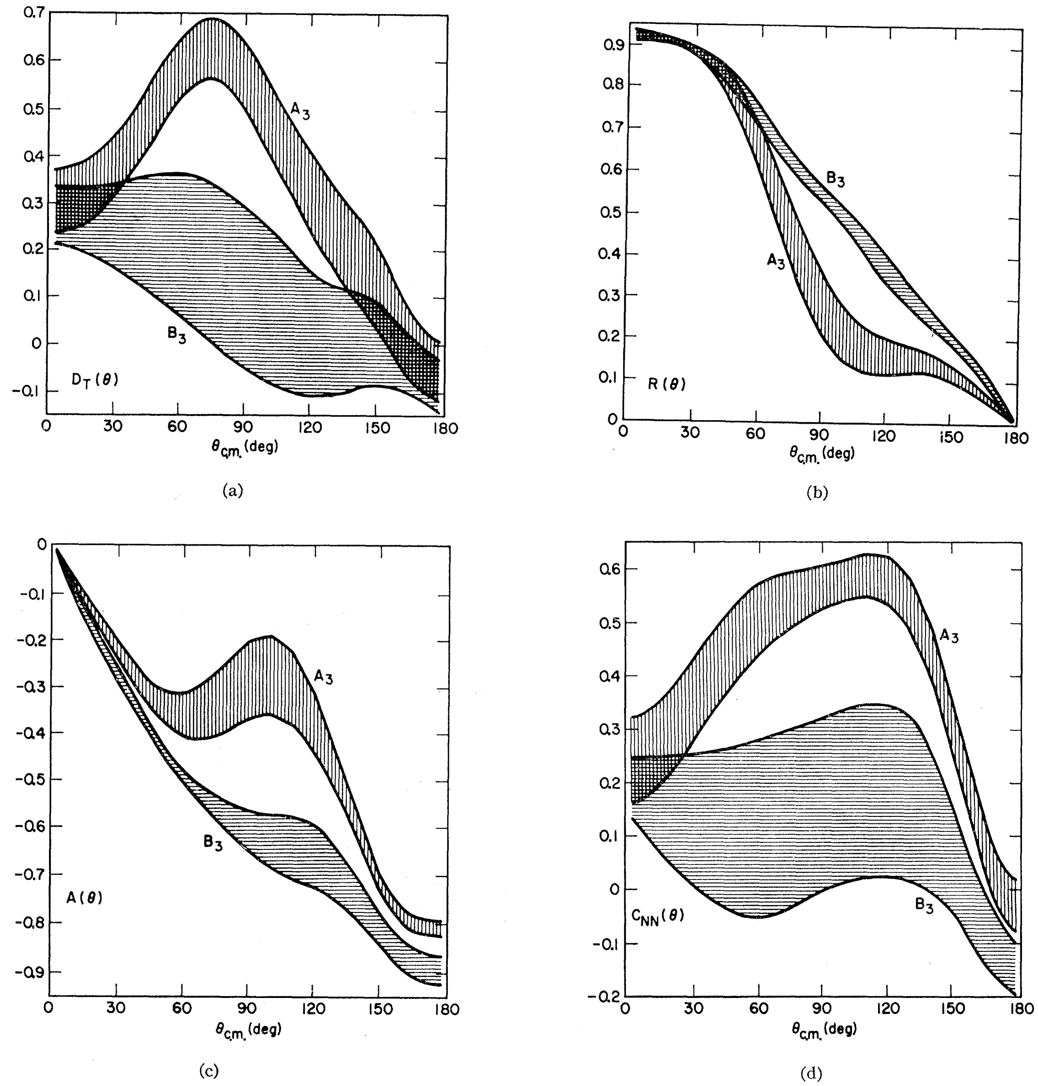

FIG. 2. The (n,p) observables D_T, R, A, and C_{NN} [(a), (b), (c), and (d), respectively] for solutions A_3 and B_3. Other observables are much more nearly the same for the two solutions.

reliable (for example, at 140 MeV), an energy-dependent analysis (EIA) may give more reliable results than an EDA, since the former contains no model dependence. However at 50 MeV the most reliable results at present are probably obtained from an EDA. The present work is somewhat intermediate between the two types of analysis.

Note added in proof. Two new and very accurate (p,p) differential cross-section measurements at 50 MeV (from Harwell and Tokyo) have recently become available.