Dynamical Model for the Hyperon Nonleptonic Decay

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A bootstrap calculation is performed for the nonleptonic hyperon decays. These decays can be reduced to the scattering processes: spurion+baryon → pion+baryon. On the assumption that the spurion has nonvanishing energy and momentum, the partial-wave decomposition is performed. The f/d ratio for the weak and the strong interaction is obtained from the self-consistency condition by using Nishijima and Swank's weak Hamiltonian. By taking the spurion momentum equal to zero, S- and P-wave amplitudes are calculated. The agreement with experiment is satisfactory.

I. INTRODUCTION

THE application of the bootstrap method to the weak and the electromagnetic interaction has been carried through by Dashen and Frautschi[1] by connecting the weak and the electromagnetic processes to the scattering problem. As was shown by Sugawara,[2] the two-body decay process can be formulated in terms of dispersion relations. Actually a nonleptonic hyperon decay can be interpreted as a scattering process: spurion+baryon → pion+baryon, and therefore it can be discussed by the bootstrap method. In nonleptonic hyperon decay the interaction involves both the strong and the weak interactions, so we might expect to obtain from it information about the coupling-constant ratio for the weak and the strong interaction at the same time.

In this article we assume the Hamiltonian which was proposed by Nishijima and Swank.[3] Their Hamiltonian has the following advantages: It is not necessary to assume the spurion explicitly, there is no difficulty about CP invariance as in Feldman, Mathews and Salam's theory,[4] and the $\Delta I=\frac{1}{2}$ rule holds automatically. Using the above-mentioned Hamiltonian, we investigate the bootstrap condition. In our calculation we take the decuplet and the octet of baryons for the u channel and that of the K meson for the t channel. The K^* meson contribution for the t channel is estimated to be negligible. From the f/d ratio for the weak and the strong interaction which is determined through the self-consistency condition, we find that the result is consistent with experiment.

The organization of the material is as follows: In Sec. II we investigate the kinematical properties of spurion+baryon → pion+baryon scattering, where the spurion is assumed to have finite momentum. In Sec. III the dispersion relations for the above processes are discussed. In Sec. IV the bootstrap conditions for the hyperon nonleptonic decay are discussed. In Sec. V the numerical results are obtained by taking the spurion four-momentum to be zero.

II. THE DECAY AMPLITUDES

The S-matrix element for the process $B_i \to B_j+\pi_\beta$ is written as

$$\langle B_j(p'),\pi_\beta(k')|S|B_i(p)\rangle = -i(2\pi)^4\delta(p'+k'-p) \times\langle B_j(p'),\pi_\beta(k')|H_W(0)|B_i(p)\rangle, \quad (2.1)$$

where $H_W(0)$ is the weak Hamiltonian. In this article we adopt the Hamiltonian which has been proposed by Nishijima and Swank, i.e.,

$$H_W{}^{(x)}=\lambda_p\partial_\mu A_\mu{}^{(6)}(x)+\lambda_s\partial_\mu V_\mu{}^{(6)}(x), \qquad (2.2)$$

where $A_\mu{}^{(6)}(x)$ and $V_\mu{}^{(6)}(x)$ are the axial vector and the vector currents, respectively, which transform like the sixth component of the octet.

The matrix element can be interpreted as the scattering process: spurion+baryon → pion+baryon. Although we do not assume the spurion explicitly, we may perform the partial-wave decomposition of the matrix element in the same way as the scattering process.

The matrix element can be expressed in terms of amplitudes $\tilde{A}$ and $\tilde{B}$ as

$$\langle B_j(p'),\pi_\beta(k')|H_W(0)|B_i(p)\rangle = \frac{1}{(2\pi)^{9/2}}\left[\frac{m_im_j}{2\omega_\beta(k')E_i(p)E_j(p')}\right]^{1/2} \times\bar{u}_j(p')[-\tilde{A}+i\mathbf{Q}\tilde{B}]\binom{\gamma_5}{1}u_i(p) \quad (2.3)$$

where $E_i(p)=(\mathbf{p}^2+m_i{}^2)^{1/2}$, $E_j(p')=(\mathbf{p}'^2+m_j{}^2)^{1/2}$, $\omega_k(k')=(\mathbf{k}'^2+m_{\pi\beta}{}^2)^{1/2}$, m_i, m_j, and $m_{\pi\beta}$ being the masses of particles B_i, B_j, and π_β, respectively. Here i, j, β denote the $SU(3)$ suffix for the incoming baryons, the outgoing baryons and π mesons, respectively. Q is defined as $Q=\frac{1}{2}(k+k')$ where $k+p=k'+p'$. γ_5 and 1 in (2.3) should be taken for the parity-conserving and the parity-violating parts, respectively.

The partial-wave amplitudes are now given by

$$\tilde{f}_{l\pm}=\frac{1}{2}\int_{-1}^{1}dx[P_l(x)\tilde{f}_1+P_{l\pm1}(x)\tilde{f}_2], \qquad (2.4)$$

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[1] R. F. Dashen, S. C. Frautschi, and D. H. Sharp, Phys. Rev. Letters **13**, 777 (1964). R. F. Dashen and S. C. Frautschi, Phys. Rev. **143**, 1171 (1966).
[2] M. Sugawara, Phys. Rev. **135**, B252 (1964).
[3] K. Nishijima and L. J. Swank, Phys. Rev. **146**, 1161 (1966).
[4] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).

where $l\pm$ stand for the state with the total and the orbital angular momentum $j=l\pm\frac{1}{2}$ and l, respectively, $x=(\mathbf{p}\cdot\mathbf{p}')/|\mathbf{p}||\mathbf{p}|$ and $P_l(x)$ being a Legendre polynomial. Here $\tilde{f}_i$ $(i=1,2)$ are related to $\tilde{A}$ and $\tilde{B}$ as [5]

$$\tilde{f}_1=\frac{\{[E_i(p)\mp m_i][E_j(p')+m_j]\}^{1/2}}{2w}\times\left[\frac{\tilde{A}}{4\pi}+\left(w+\frac{\pm m_i-m_j}{2}\right)\frac{\tilde{B}}{4\pi}\right], \quad (2.5a)$$

$$\tilde{f}_2=\frac{\{[E_i(p)\pm m_i][E_j(p')-m_j]\}^{1/2}}{2w}\times\left[-\frac{\tilde{A}}{4\pi}+\left(w+\frac{\mp m_i+m_j}{2}\right)\frac{\tilde{B}}{4\pi}\right], \quad (2.5b)$$

where w is the total energy in the center-of-mass system, and the upper and the lower sign before m_i should be taken for the parity-conserving and for the parity-violating part, respectively. To remove the kinematical singularity we introduce the amplitudes $\tilde{g}_{l\pm}$ (w) and $\tilde{h}_{l\pm}$ (w) as follows: For the parity-violating part

$$\tilde{g}_{l+}(w)=[2w/(p')^l(p)^l\{[E_i(p)+m_i][E_j(p')+m_j]\}^{1/2}]\times[\tilde{f}_{l+}(w)], \quad (2.6a)$$

$$\tilde{g}_{l-}(w)=[2w/(p')^{l-1}(p)^{l-1}\{[E_i(p)-m_i]\times[E_j(p')-m_j]\}^{1/2}][\tilde{f}_{l-}(w)], \quad (2.6b)$$

and for the parity-conserving part

$$\tilde{h}_{l+}(w)=[2w/(p')^l(p)^l\{[E_i(p)-m_i][E_j(p')+m_j]\}^{1/2}]\times[\tilde{f}_{l+}(w)], \quad (2.7a)$$

$$\tilde{h}_{l-}(w)=[2w/(p')^{l-1}(p)^{l-1}\{[E_i(p)+m_i]\times[E_j(p')-m_j]\}^{1/2}][\tilde{f}_{l-}(w)]. \quad (2.7b)$$

In this article we assume unsubtracted dispersion relations for $\tilde{g}_{l\pm}(w)$ and $\tilde{h}_l w(_\pm)$, and investigate the self-consistency condition between the left-hand cut and the right-hand cut keeping $k\neq 0$. As was mentioned before, we take the limit $k\to 0$ after the calculation is performed. In this limit we get the following expressions for the S and P amplitudes[6]:

$$S=\left(\frac{2\pi[E_j(p')+m_j]}{m_i}\right)^{1/2}\tilde{g}_{0+}(m_i), \quad (2.8)$$

$$P=\left(\frac{2\pi[E_j(p')-m_j]}{m_i}\right)\tilde{h}_{1-}(m_i). \quad (2.9)$$

[5] See for example I. Umemura and K. Watanabe, Progr. Theoret. Phys. (Kyoto) **29**, 893 (1963).
[6] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. **7**, 407 (1957).

III. DISPERSION RELATIONS

Assuming unsubtracted dispersion relations for the partial-wave amplitudes $\tilde{g}_{l\pm}$ and $\tilde{h}_{l\pm}$, we get the following integral representation[7]:

$$\tilde{g}_{l\pm}{}^{(i\to j)}(w)=\sum_{k,n}\left[\frac{1}{\pi}\int_L dw'\frac{\mathrm{Im}\{\tilde{g}_{l\pm}{}^{(i\to k)}(w')\}D_{l\pm}{}^{(k\to n)}(w')}{w'-w}+\frac{1}{\pi}\int_{\mathrm{inel}} dw'\frac{\mathrm{Im}\{\tilde{g}_{l\pm}{}^{(i\to k)}(w')D_{l\pm}{}^{(k\to n)}(w')\}}{w'-w}\right]\times[D_{l\pm}{}^{-1}(w)]^{(n\to j)}, \quad (3.1)$$

where $D_{l\pm}{}^{(i\to j)}(w)$ is the D function for the pion baryon scattering with the angular momentum $j=l\pm\frac{1}{2}$. Here the suffix i, j, k, and n denote the channel. The contour L runs around the left-hand cuts in $\tilde{g}_{l\pm}{}^{(i\to j)}(w)$. The same equation holds for $\tilde{h}_{l\pm}{}^{(i\to j)}(w)$.

Let us now discuss $P_{1/2}$ state spurion-baryon scattering. For this state $D^{-1}(w)$ has poles corresponding to the octet baryon. To simplify the problem we adopt the following approximations:

(1) Inelastic processes are neglected.

(2) The lowest-order approximation in the symmetry breaking interaction is taken.

(3) $[D_{1-}(w)]^{(i\to j)}$ in the dispersion integral is approximated as $[D_{1-}(w)]^{(i\to j)}=\delta_{ij}\gamma(w-m)$, where γ is a constant and m is the octet baryon mass in the unitary symmetry limit.

Assuming that the dispersion integral in (3.1) does not change appreciably with w, we evaluate them for $w=m$. Then we get the following expressions for $\tilde{g}_{1-}{}^{(i\to j)}$ and $\tilde{h}_{1-}{}^{(i\to j)}$:

$$\tilde{g}_{1-}{}^{(i\to j)}(w)=\gamma\left[\frac{1}{\pi}\int_L dw'\,\mathrm{Im}\tilde{g}_{1-}{}^{(i\to j)}(w')\right]\Big/(w-m_l), \quad (3.2)$$

$$\tilde{h}_{1-}{}^{(i\to j)}(w)=\gamma\left[\frac{1}{\pi}\int_L dw'\,\mathrm{Im}\tilde{h}_{1-}{}^{(i\to j)}(w')\right]\Big/(w-m_l). \quad (3.3)$$

Here m_l denote the mass of the octet baryon which appears in the intermediate state for the process $i\to j$. It must be noticed that no summation over l is necessary because only one octet baryon state appears in the s channel for each process.

Let us now discuss the s, t, u channel contributions for the spurion-baryon scattering.

A. s-Channel Contribution

The lowest-order contribution for $P_{1/2}$ state is the one octet baryon state. As we have adopted Nishijima and Swank's Hamiltonian, the weak vertex part can

[7] J. Bjorken, Phys. Rev. Letters **4**, 473 (1960).

be related to the vector or the axial-vector form factors. The form factors are given by

$$\langle B_l(p')|J_\mu{}^{(\alpha)}(0)|B_i(p)\rangle=\frac{1}{(2\pi)^3}\left(\frac{m_i m_l}{E_i(p)E_l(p')}\right)^{1/2}\bar{u}_l(p')$$

$$\times[\gamma_\mu F_{il\alpha}{}^{(1)}+\sigma_{\mu\nu}q_\nu F_{il\alpha}{}^{(2)}+q_\nu F_{il\alpha}{}^{(3)}]\binom{1}{\gamma_5}u_i(p). \quad (3\ 4)$$

Here 1 and γ_5 should be taken for the vector current and axial-vector current, respectively, $q_\mu=p_\mu-p_\mu'$, and $F_{il\alpha}{}^{(n)}$ are form factors. The spurion-baryon coupling constant is now defined by

$$\langle B_j(q)|H_W{}^{(\alpha)}(0)|B_i(p)\rangle=\frac{1}{(2\pi)^3}\left(\frac{m_i m_j}{E_i(p)E_j(p')}\right)^{1/2}$$

$$\times\lambda(m_i\pm m_j)\sigma_{ij\alpha}\bar{u}_j(q)u_i(p),$$

where the plus and the minus sign before m_j should be taken for the parity-violating part and the parity-conserving part, respectively. Then $\sigma_{ij\alpha}$ are connected with the form factors as

$$\sigma_{il\alpha}=\left(F_{il\alpha}{}^{(1)}-\frac{\mu^2}{m_l\pm m_i}F_{il\alpha}{}^{(2)}\right)\lambda. \quad (3.5)$$

Here μ is the spurion mass which is understood to be taken $\mu^2\simeq m_\pi{}^2$ in discussing the self-consistency condition. From this it follows that f/d ratio for the nonleptonic decays will generally differ from that of the vector and the axial-vector current.

The amplitudes are now given as

$$\tilde{g}_{1-}{}^{(i\to j)}(w)=\frac{\sigma_{il\alpha}G_{jl\beta}{}^{(8)}}{4\pi}\frac{m_l+m_i}{w-m_l}, \quad (3.6)$$

$$\tilde{h}_{-1}{}^{(i\to j)}(w)=\frac{\sigma_{il\alpha}G_{jl\beta}{}^{(8)}}{4\pi}\frac{m_l-m_i}{w-m_l}, \quad (3.7)$$

where $G_{jl\beta}{}^{(8)}$ stand for the octet meson and baryon Yukawa coupling constant.

B. *u*-Channel Contribution

The u channel contribution is approximated by the octet and the decuplet baryons. In evaluating the decuplet contributions we approximate $j=\frac{3}{2}$ resonance by the Rarita-Schwinger field.

C. *t*-Channel Contribution

The lowest-mass contribution for the parity violating and parity-conserving parts are K and K^* mesons, respectively. Let us first discuss the K-meson contribution. Since the vector current is $I=\frac{1}{2}$ current, the matrix element of the Hamiltonian can be related to K_{l3} decay from factors, i.e.,

$$\langle\pi^+(k')|H_W(0)|K^+(k)\rangle=-\lambda_s(m_K{}^2-m_\pi{}^2)$$

$$\times\frac{1}{(2\pi)^3[4\omega_\pi(k')\omega_K(k)]^{1/2}}f_0(s), \quad (3.8)$$

where $f_0(s)$ is the $J=0$ part of the K_{l3} decay form factor, namely $f_0(s)=f_+(s)+sf_-(s)/(m_K{}^2-m_\pi{}^2)$. Assuming the universality for K_{l3} decay, we normalize $f_0(s)$ as $f_0(0)=f_+(0)=1$. The absorptive part is now given as follows:

$$\mathrm{Im}[\tilde{A}]=-\pi\delta(t-m_K{}^2)(m_K{}^2-m_\pi{}^2)\lambda_s G_{ilj}{}^{(8)}f_0(s), \quad (3.9)$$

$$\mathrm{Im}[\tilde{B}]=0. \quad (3.10)$$

For the π^0 decay channel, (3.9) and (3.10) should be divided by $-\sqrt{2}$. The K-meson contribution is about the order of 10% of the u channel.

The K^* contribution for the parity-violating part is now evaluated by relating the matrix element $\langle\pi|\partial_\mu \times A_\mu{}^{(6)}|K^*\rangle$ to $K^*\to K\pi$ decay width by means of the PCAC (partially conserved axial-vector current) theorem. Evaluating the left-hand cut contribution from the experimental K^* width, we find that it is about the order of a few percent of the u-channel contribution.[8] Therefore we neglect K^* contribution in this paper.

IV. THE SELF-CONSISTENCY CONDITION

Equating the right-hand side of (3.2) and (3.3) with (3.6) and (3.7), respectively, we get the eigenvalue equations for the coupling constants. As was discussed in the previous section, we neglect the t-channel contribution in discussing the eigenvalue problem.

Taking the octet and the decuplet contributions for the left-hand cut, we get the following equations:

$$[f(d'-f')(m_N\pm m_\Sigma)-\tfrac{1}{3}d(d'+3f')(m_N\pm m_\Lambda)]A+\tfrac{1}{12}B$$
$$=(d+f)(d'-f')(m_N\pm m_\Sigma)C; \quad (4.1\mathrm{a})$$

$$\sqrt{2}f(d'-f')(m_N\pm m_\Sigma)A+\frac{1}{6\sqrt{2}}B$$
$$=\frac{1}{\sqrt{2}}(d+f)(d'-f')(m_N\pm m_\Sigma)C; \quad (4.1\mathrm{b})$$

$$[-f(d'-f')(m_N\pm m_\Sigma)-\tfrac{1}{3}d(d'+3f')(m_N\pm m_\Lambda)]$$
$$\times A-\tfrac{1}{12}B=0; \quad (4.1\mathrm{c})$$

$$(\sqrt{\tfrac{2}{3}})d(d'-f')(m_N\pm m_\Sigma)A-\frac{1}{2\sqrt{6}}B$$
$$=-\frac{1}{\sqrt{6}}(d+f)(d'+3f')(m_N\pm m_\Lambda)C; \quad (4.1\mathrm{d})$$

[8] This agrees with the argument by J. C. Pati and S. Oneda, Phys. Rev. **140**, B1351 (1965).

$$\frac{1}{\sqrt{3}}d(f'-d')(m_N\pm m_\Sigma)A+\frac{1}{4\sqrt{3}}B$$
$$=\frac{1}{2\sqrt{3}}(d+f)(d'+3f')(m_N\pm m_\Lambda)C; \qquad (4.1\text{e})$$

$$-\frac{1}{\sqrt{6}}(d-f)(d'-3f')(m_\Lambda\pm m_\Xi)A+\frac{1}{2\sqrt{6}}B'$$
$$=(\sqrt{\tfrac{2}{3}})d(d'+f')(m_\Sigma\pm m\Xi)C; \qquad (4.1\text{f})$$

$$\frac{1}{2\sqrt{3}}(d-f)(d'-3f')(m_\Lambda\pm m_\Xi)A-\frac{1}{4\sqrt{3}}B'$$
$$=-\frac{1}{\sqrt{3}}d(d'+f')(m_\Sigma\pm m_\Xi)C, \qquad (4.1\text{g})$$

where f/d and f'/d' stand for the f/d ratio for the strong Yukawa coupling and for the spurion-baryon coupling, respectively. B and B' come from the decuplet contribution. The coefficients A, B, and C are related to $G^{(8)}$, $G^{(10)}$, and $\tilde{G}^{(10)}$ as follows: For the parity-conserving part $C=\sqrt{2}\lambda_s G^{(8)}$, and for the parity-violating part $A=-\sqrt{2}\gamma^{-1}\lambda_p G^{(8)}/3$, $B=\gamma^{-1}K\tilde{G}^{(10)}G^{(8)}/3$ and $C=\sqrt{2}\lambda_p G^{(8)}$, where $G^{(8)}$ is normalized as $(G^{(8)})^2/4\pi=14.5$. $\tilde{G}_{jl\alpha}{}^{(10)}$ is now defined as

$$\langle B_j(p')\,|\,H_W{}^{(\alpha)}(0)\,|\,B_i{}^*(p)\rangle$$
$$=\frac{1}{(2\pi)^3}\left(\frac{m_jM_l}{E_j(p')E_l(p)}\right)^{1/2}\tilde{G}_{jl\alpha}{}^{(10)}\bar{u}_j(p')\begin{pmatrix}1\\ \gamma_5\end{pmatrix}u_\mu{}^{(l)}(p')p_\mu{}',$$

where $u_\mu{}^{(l)}$ is the Rarita-Schwinger field, M_l is the decuplet baryon mass, and 1 and γ_5 should be taken for the parity-violating and the parity-conserving part, respectively. Here $\tilde{G}_{jl\alpha}{}^{(10)}$ is equal to $\tilde{G}^{(10)}$ multiplied by the Clebsch-Gordan coefficient. $G^{(10)}$ is defined in the same way for the strong interaction. K is now given by the following equation:

$$\frac{1}{\pi}\int_L dw'\,\mathrm{Im}[\tilde{g}_{1-}{}^{(i\to j)}(w')]$$
$$=-\frac{1}{3}\left(\sum_l\frac{\sigma_{jl\alpha}G_{il\beta}{}^{(8)}}{4\pi}-\sum_{l'}\frac{\tilde{G}_{jl'\beta}{}^{(10)}G_{il'\beta}{}^{(10)}}{4\pi}K\right). \qquad (4.2)$$

In this calculation we have assumed that $\mu^2/m^2\ll1$ and $m_\pi{}^2/m^2\ll1$. It must be remarked that for the parity-violating part $B=B'$ and for the parity-conserving part generally $B\neq B'$ in the symmetry-breaking interactions.[9]

[9] In order to see this let us expand $\tilde{G}_{il\beta}{}^{(10)}$ as follows:

$$\tilde{G}_{il\beta}{}^{(10)}=\bar{G}_{il\beta}{}^{(10)}+\Sigma_a\,(m_a-m)\frac{\partial}{\partial m_a}[\tilde{G}_{il\beta}{}^{(10)}]_{m_a=m;\,M_b=M}$$
$$+\Sigma_b\,(M_b-M)\frac{\partial}{\partial M_b}[\tilde{G}_{il\beta}{}^{(10)}]_{m_a=m;\,M_b=M},$$

where $\bar{G}_{il\beta}{}^{(10)}$ are the coupling constants in the unitary-symmetry limit. For the parity-conserving part $\bar{G}_{il\beta}{}^{(10)}=0$, because the current is conserved, while for the parity-violating part $\bar{G}_{il\beta}{}^{(10)}\neq0$. Therefore, in the lowest order in the symmetry-breaking interaction, $B=B'$ for the parity-violating part and $B\neq B'$ generally for the parity-conserving part.

V. NUMERICAL RESULTS

We calculate S and P amplitudes which are given by (2.8) and (2.9) by using the f/d and f'/d' ratios which satisfy the self-consistency conditions (4.1).

A. S-Wave Decay Amplitudes

Taking the unitary symmetry limit for the octet baryon mass in (4.1), we get the following solutions for the f/d ratio:

$$f/d=f'/d'=(6\pm\sqrt{21})/3. \qquad (5.1)$$

Assuming that Yukawa interaction between the octet baryons and the pseudoscalar mesons is D dominant, and normalizing f and d as $f+d=1$, $f'+d'=1.14$, we get

$$f=0.32,\quad d=0.68,\quad f'=0.365,\quad\text{and}\quad d'=0.775. \qquad (5.2)$$

The decuplet octet coupling constant is now related to $G^{(8)}$ as follows:

$$\tilde{G}^{(10)}G^{(10)}=8\sqrt{2}\lambda_p m$$
$$\times[3f(d'-f')+d(d'+3f')]G^{(8)}K^{-1}. \qquad (5.3)$$

To evaluate K which is defined by (4.2) we take the unitary symmetry limit as $m=7.59$ and $M/m=1.41$.[10] For these values of the symmetry mass we get

$$K=-3.60m^4/M^2.$$

The S-wave amplitudes are now evaluated by taking $D_{0+}{}^{(i\to j)}=\delta_{ij}$ in (3.1). Then, by taking the spurion momentum $k\to0$ or $w=m_i$, we get

$$\tilde{g}_{0+}{}^{(i\to j)}(m_i)=-\sum_l\frac{\sigma_{jl\alpha}G_{il\beta}{}^{(8)}}{4\pi}+\sum_{l'}\frac{\tilde{G}_{jl'\alpha}{}^{(10)}G_{il'\beta}{}^{(10)}}{4\pi}\frac{1}{6M_{l'}{}^2}$$
$$\times[M_l{}^3+m_i{}^2m_j-(m_i-m_j)(M_l-m_i)M_l$$
$$-m_\pi{}^2(M_l+m_j)]. \qquad (5.4)$$

The numerical results are given in Table I, where we adjust the parameter λ_p from $\Sigma^+\to p\pi^0$, i.e.,[11]

$$\lambda_p/(4\pi)^{\frac{1}{2}}=0.14\times10^{-2}.$$

B. P-Wave Decay Amplitudes

In getting the solution of (4.1) for the parity-conserving part, we fix f/d ratio for the strong interaction by (5.2). The solutions are given as

$$f'/d'=-4.22\quad\text{or}\quad0.894. \qquad (5.5)$$

Assuming that the scalar spurion-baryon coupling is F dominant, we adopt $f'/d'=-4.22$. Since the f'/d' ratio is determined from the self-consistency condition, the P-wave amplitudes can be evaluated from (3.3) by

[10] The result is rather sensitive to m and m/M. Here we determined these values from the condition $g_{0+}(\Sigma^+\to n\pi^+)=0$.

[11] Units in which $\hbar=c=m_{\pi^0}=1$ are adopted throughout this paper.

TABLE I. Calculated values of S- and P-wave amplitudes, and comparison with experiment.

	Theory	Experiment[a]
	$S\times10^2$	
$\Lambda\to p\pi^-$	1.00	1.51 $\pm$0.023
$\Sigma^+\to n\pi^+$	0.00	0.01 $\pm$0.032
$\Sigma^+\to p\pi^0$	-1.48(input)	-1.48 $\pm$0.13
$\Sigma^-\to n\pi^-$	2.08	1.76 $\pm$0.016
$\Xi^-\to\Lambda\pi^-$	-1.99	-1.98 $\pm$0.028
	$P\times10^2$	
$\Lambda\to p\pi^-$	0.93(0.63)	0.57 $\pm$0.025
$\Sigma^+\to n\pi^+$	1.8 (input)	1.81 $\pm$0.033
$\Sigma^+\to p\pi^0$	1.31(1.11)	1.11 $\pm$0.178,1.48$\pm$0.134
$\Sigma^-\to n\pi^-$	0.00(0.29)	-0.015 ± 0.037
$\Xi^-\to\Lambda\pi^-$	0.41(0.47)	0.40 $\pm$0.035

[a] The experimental values taken from J. P. Berge (Ref. 12). We change the sign for Σ_0^+ and Ξ_-^-, and adopt units such that $\hbar=c=m_{\pi^0}=1$. The theoretical values in the bracket for P waves are obtained by taking the t-channel contribution.

taking $w=m_i$. Therefore,

$$\tilde{h}_{1-}{}^{(i\to j)}(m_i)=\frac{\sigma_{il\alpha}G_{jl\beta}{}^{(8)}}{4\pi}. \tag{5.6}$$

The results are summarized in Table I. The t-channel corrections are evaluated by neglecting the final-state interaction. Normalizing $f'+d'=1$, we get the parameter λ_S as

$$\lambda_S/(4\pi)^{\frac{1}{2}}=0.67\times10^{-2},$$

where $\Sigma^+\to n\pi^+$ data are used as the input data.

VI. CONCLUDING REMARKS

As summarized in Table I, the result is consistent with experiment.[12] The f'/d' ratio for the matrix element of the axial-vector current which is given by (5.2) agrees with the recent experimental analysis of the hyperon leptonic decay. According to Brene *et al.*[13], $\alpha=0.665\pm0.018$, while from our calculation we get $\alpha=0.68$. The condition $f/d=f'/d'$ for the parity-violating part indicates the dynamical origin of the Lee-Sugawara relation[14] for the hyperon nonleptonic decay.

For P waves, the adoption of Nishijima and Swank's Hamiltonian is essential to get the F-dominant solution.[15] If we assume the current-current interaction, we get the same f/d ratio for the parity-conserving part as for the parity-violating part. Then Σ_+^+ and Σ_0^+ become very small. The decuplet contributions are very important to get an agreement with experiment.[16] Their result, however, does not satisfy the self-consistency condition.

As compared with the dynamical theory of hyperon nonleptonic decay which has been proposed so far,[8,15,17,18] our theory is an improvement in the following respect: The theory is formulated in a self-consistent way, and therefore it contains only two parameters λ_S and λ_P.

ACKNOWLEDGMENT

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[12] We take the experimental result from J. P. Berge, in *Proceedings of the XIII International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, California, 1967).

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