Regge-Pole Theory and Spin Independence of the Total Cross Sections*

KEH YING LIN
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 23 February 1967)

The general conditions under which the total cross sections are independent of spin are discussed. It is shown that the forward elastic scattering matrix is proportional to the unit matrix in the helicity space if and only if the crossed channels either do not flip helicities, or do not flip helicities more than 1 and have the quantum number $P=(-1)^J$. This theorem follows directly from the Trueman-Wick crossing relations. In particular, the theorem implies that the total cross sections are spin-independent at the high-energy limit if the forward elastic-scattering amplitudes are dominated by the Pomeranchuk trajectory.

RECENTLY, Hara[1] suggested that at very high energy, the total cross sections for the scattering of particles of definite helicity are independent of the helicity states and are factorizable. (In order to avoid confusion, let us emphasize that the total cross sections mentioned in this paper are different from the spin-averaged total cross section.) His theory is based on the following assumptions: (1) At very high energy, the contribution of the Pomeranchuk trajectory dominates the forward elastic-scattering amplitudes. Since the complex angular momentum $\alpha(t)$ associated with this trajectory is 1 at $t=0$, the crossed channels (the particle and its own antiparticle) do not flip helicities by more than 1.[1] (2) The residue functions of the Regge trajectories are factorizable.[2-4] His proof depends essentially on the theorem that if one of the incident particles has spin zero, then the forward (spin-nonflip) elastic-scattering amplitudes are spin-independent if the crossed channels do not flip helicities. Hara's result is well known for spin 0, spin $\frac{1}{2}$ and spin $\frac{1}{2}$, spin $\frac{1}{2}$ scattering, and in a stronger form: $\alpha(0)$ need not be unity for the leading even signature trajectory. It is natural to ask the following questions. Is $\alpha(0)=1$ really necessary for higher spins? Under what kind of general condition are the total cross sections independent of spin? These and other related questions are answered in this paper. It would be clear later that the condition $\alpha(0)=1$ is really necessary for the cases of higher spins and that the spin independence of total cross sections does not require the factorization theorem of the residue functions.

According to the optical theorem, the total cross sections are proportional to the imaginary part of the corresponding (spin-nonflip) forward elastic-scattering amplitudes. Therefore, the spin independence of the forward elastic-scattering amplitudes automatically implies the spin independence of the total cross sections.

Let us now define our notations. The energy variable will be suppressed throughout this paper. We consider a two-body reaction $a+b\rightarrow a+b$ where the incoming (outgoing) particles a and b have the helicity states α (α^*) and β (β^*), respectively. The forward scattering amplitudes are denoted by $F(\alpha^*\beta^*,\alpha\beta)$. The crossed channels correspond to $a+\bar{a}\rightarrow \bar{b}+b$ where $\bar{a}$ means the antiparticle of a. The crossed-channel scattering amplitudes (corresponding to the forward scattering) are denoted by $T(\alpha'\alpha'',\beta'\beta'')$ where α', α'', β', and β'' are the corresponding helicity states. We shall base our discussions on the following two theorems.

Theorem 1. The forward elastic-scattering matrix (in the helicity space) is proportional to the unit matrix if and only if one of the following two conditions is satsified: (a) the crossed channels do not flip helicities (i.e., $\alpha'=\alpha''$, $\beta'=\beta''$). (b) The crossed channels do not flip helicities by more than 1 (i.e., $|\alpha'-\alpha''|\leq 1$, $|\beta'-\beta''|\leq 1$) and have the quantum number P (parity) $=(-1)^J$ where J is the total angular momentum.

Theorem 2. If the crossed channels do not flip helicities by more than 1, then we have $\sum_\beta F(\alpha^*\beta,\alpha\beta)=\delta_{\alpha^*\alpha}C$ where C is a function of the energy.

The proofs of these theorems depend entirely on the Trueman-Wick crossing relations.[5] They are valid at all energies (including the high-energy limit). The details are given in the Appendix. A special case of the first theorem was discussed by Peierls and Trueman.[6] They proved that if the crossed channels have the quantum number $GP=+1$ and do not flip helicities, then the forward elastic-scattering matrix is a multiple of the unit matrix.

Let us now discuss the particular case where the spins of both particles are less than 1. This case is particularly simple because the crossed channels can not flip helicities by more than 1. Therefore the total cross sections are spin-independent if the crossed channels are dominated by Regge trajectories with positive parity. This result is not new. However, in the case with higher spins, the crossed channels in general would flip helicities by more than 1 for any finite energy. This is precisely the reason why the condition $\alpha(0)=1$ for the leading Regge trajectory is really necessary for higher spins. It has been pointed out by Hara[1] that the Regge trajectories with $\alpha(0)=1$ do not flip helicities

* Supported in part by the Office of Naval Research.
[1] Y. Hara, Phys. Letters 23, 696 (1966).
[2] M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962).
[3] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).
[4] Y. Hara, Progr. Theoret. Phys. (Kyoto) 28, 711 (1962).
[5] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, 322 (1964).
[6] R. F. Peierls and T. L. Trueman, Phys. Rev. 134, B1365 (1964).

by more than 1. According to the second theorem, if we assume that the elastic-forward-scattering amplitudes at high energy are dominated by exchanging Regge trajectories with $\alpha(0)=1$ (they may have different parities, G-parities, etc.), then the total cross sections averaged over the helicities states of the particle b are independent of the helicity states of the particle a at the high-energy limit. If we further assume that these Regge trajectories have positive parity, then the total cross sections are independent of the helicities of both particles.

It is clear that our conclusion does not depend on the factorization theorem of the residue functions of the Regge trajectories. However, if we do use this theorem and further assume that the residues of different trajectories are not related to each other, then we have to associate a kinematic factor $t^{1/2}$ for each residue function which flips odd-helicity states [i.e., $T(\alpha'\alpha'',\beta'\beta'')=0$ if $\alpha'-\alpha''=\text{odd}$ and/or $\beta'-\beta''=\text{odd}$].[7,8] This means that the trajectories with $\alpha(0)=1$ actually do not flip any helicities. This argument was used by Hara to show that the total cross sections are factorizable and spin-independent at high energy if the leading trajectories have $\alpha(0)=1$.

ACKNOWLEDGMENT

The author would like to thank Dr. Hyman Goldberg for helpful discussions and Professor T. L. Trueman for an interesting correspondence. The author was informed by Professor T. L. Trueman that he and A. Mueller had performed the same generalization of Hara's result using a different approach. Their result, however, was not available to the author during the course of this work.

APPENDIX

The proofs of the Theorems 1 and 2 are given in this Appendix. The scattering amplitudes satisfy the following crossing relations[5]:

$$T(\alpha'\alpha'',\beta'\beta'')=\sum_{\alpha,\beta,\alpha^*,\beta^*} d(\alpha''\alpha)d(\beta''\beta)d(\alpha'\alpha^*)d(\beta'\beta^*)\times F(\alpha^*\beta^*,\alpha\beta)\,,$$

where $d(\alpha''\alpha)=d^J{}_{\alpha''\alpha}(\pi/2)$, $d(\beta''\beta)=d^{J'}{}_{\beta''\beta}(\pi/2)$, and J (J') is the spin of the particle a (b). Let us define $\mu=\alpha'-\alpha''$, $\lambda=\beta'-\beta''$. If $\lambda+\mu$ is an odd integer, then $T(\alpha'\alpha'',\beta'\beta'')$ vanishes identically for all energies, because of the kinematic factor $t^{1/2}$.[7,8] Conservation of angular momentum gives $F(\alpha^*\beta^*,\alpha\beta)=F(\alpha^*\beta^*,\alpha\beta)\delta_{\alpha-\alpha^*,\beta-\beta^*}$. The condition

$$T(\alpha'\alpha'',\beta'\beta'')=(-1)^{\lambda-\mu}T(-\alpha'-\alpha'',-\beta'-\beta'')$$

follows from the parity conservation.[9] Besides, we have

$$d(\alpha'\alpha)=(-1)^{J-\alpha}d(-\alpha'\alpha)=(-1)^{J+\alpha'}d(\alpha'-\alpha)$$

and

$$\textstyle\sum_\alpha d(\alpha\alpha')d(\alpha\alpha'')=\delta_{\alpha'\alpha''}\,.$$

Using the orthonormal properties of the d functions, the crossing relations can be rewritten in the form

$$\sum_{\alpha'',\beta''} T(\alpha'\alpha'',\beta'\beta'')d(\alpha''\alpha)d(\beta''\beta) =\sum_{\alpha^*,\beta^*} d(\alpha'\alpha^*)d(\beta'\beta^*)F(\alpha^*\beta^*,\alpha\beta)\delta_{\alpha-\alpha^*,\beta-\beta^*}\,.$$

Applying $\sum_{\alpha'}d(\alpha',\alpha+j)$ to both sides, we have

$$F(\alpha+j,\beta+j,\alpha,\beta)d(\beta',\beta+j) =\sum_{\alpha',\alpha'',\beta''} T(\alpha'\alpha'',\beta'\beta'')d(\alpha''\alpha)d(\beta''\beta)d(\alpha',\alpha+j)\,. \quad (1)$$

The first part of Theorem 1 can be expressed in terms of these notations:

Theorem 1(*a*). If $T(\alpha'\alpha'',\beta'\beta'')=M(\alpha'\beta')\delta_{\alpha'\alpha''}\delta_{\beta'\beta''}$, where $M(\alpha'\beta')\equiv T(\alpha'\alpha',\beta'\beta')$, then we have

$$F(\alpha^*\beta^*,\alpha\beta)=F(JJ',JJ')\delta_{\alpha\alpha^*}\delta_{\beta\beta^*} \quad (2)$$

and

$$T(\alpha'\alpha'',\beta'\beta'')=F(JJ',JJ')\delta_{\alpha'\alpha''}\delta_{\beta'\beta''}\,. \quad (3)$$

Proof. From Eq. (1), we have

$$F(\alpha+j,\beta+j,\alpha,\beta)d(\beta',\beta+j) =[\textstyle\sum_{\alpha'} M(\alpha'\beta')d(\alpha'\alpha)d(\alpha',\alpha+j)]d(\beta'\beta)\,.$$

Because of the fact that $d(\beta',J'+|j|)\equiv 0$ if $j\neq 0$ and $d(\beta',J')\neq 0$ if $|\beta'|\leq J'$,[10] we can write either

$$\sum_{\alpha'} M(\alpha'J')d(\alpha'\alpha)d(\alpha'\alpha^*)=\delta_{\alpha\alpha^*}F(\alpha\beta,\alpha\beta) \quad (4)$$

or

$$\sum_{\alpha'} M(\alpha'\beta')d(\alpha'\alpha)d(\alpha'\alpha^*)=\delta_{\alpha\alpha^*}F(\alpha J',\alpha J')\,. \quad (5)$$

Equation (4) implies $F(\alpha\beta,\alpha\beta)=M(\alpha J')$, while Eq. (5) implies $F(\alpha J',\alpha J')=M(\alpha\beta')$. Therefore, $M(\alpha\beta)$ is independent of β. The fact that $M(\alpha\beta)$ is independent of α can be proved in the same way.

The second part of Theorem 1 is:

Theorem 1(*b*). If the crossed channels correspond to $P=(-1)^L$ (L is the total angular momentum) and $T(\alpha'\alpha'',\beta'\beta'')=0$ if $|\lambda|>1$ and/or $|\mu|>1$, then Eqs. (2) and (3) are true.

Proof. Let us rewrite the crossing relations in the form

$$T^\pm(\alpha'\alpha'',\beta'\beta'')\equiv T(\alpha'\alpha'',\beta'\beta'')\pm T(-\alpha'-\alpha'',\beta'\beta'') =\sum_{\alpha,\beta,\alpha^*,\beta^*}[1\pm(-1)^{\alpha-\alpha^*}]d(\alpha''\alpha)d(\beta''\beta)\times d(\alpha'\alpha^*)d(\beta'\beta^*)F(\alpha^*\beta^*,\alpha\beta)\,,$$

which means [see Eq. (1)]

$$F(\alpha+j,\beta+j,\alpha,\beta)d(\beta',\beta+j)[1\pm(-1)^j] =\sum_{\alpha',\alpha'',\beta''} T^\pm(\alpha'\alpha'',\beta'\beta'')d(\alpha''\alpha)d(\beta''\beta)d(\alpha',\alpha+j)\,. \quad (6)$$

[7] Y. Hara, Phys. Rev. **136**, B507 (1964).
[8] L. L. C. Wang, Phys. Rev. **142**, 1187 (1966).
[9] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[10] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

The following linear combination of T's corresponds[11] to $P=(-1)^{L+1}$ and therefore vanishes: ($\lambda+\mu=$even)

$$\cos(\theta/2)^{|\lambda+\mu|}\sin(\theta/2)^{|\lambda-\mu|}T(\alpha'\alpha'',\beta'\beta'')-\cos(\theta/2)^{|\lambda-\mu|}\times\sin(\theta/2)^{|\lambda+\mu|}T(-\alpha'-\alpha'',\beta'\beta'')=0\,, \quad (7)$$

where θ is the scattering angle in the c.m. system of the crossed channels. In particular, we have $T^{-}(\alpha\alpha,\beta\beta)=0$.

Using the relation $d(\beta,J'+|j|)=0$ if $j\neq 0$, we have

$$\sum_{\alpha',\alpha'',\beta''} T^{-}(\alpha'\alpha'',\beta'\beta'')d(\alpha''\alpha)d(\beta''J')d(\alpha'\alpha^{*})=0\,,$$

which means $T^{-}(\alpha\alpha',\beta\beta+1)d(\beta+1,J')+T^{-}(\alpha\alpha',\beta\beta-1)\times d(\beta-1,J')=0$. Putting $\beta=J'$ in the above equation, we get $T^{-}(\alpha\alpha',J',J'-1)=0$. From Eq. (6), we obtain the desired result $F(\alpha+j,\beta+j,\alpha,\beta)=0$ if $j=$odd, which implies that all T^{-} vanish. These results together with Eq. (7) give $T(\alpha'\alpha'',\beta'\beta'')=0=T^{+}(\alpha'\alpha'',\beta'\beta'')$ for $|\lambda|=|\mu|=1$. The rest of the proof is the same as those of Theorem 1(a).

The proof of Theorem 2 follows immediately from the following relation:

$$\Big[\sum_{\beta} F(\alpha^{*}\beta,\alpha\beta)\Big]\delta_{\alpha^{*}\alpha}=\sum_{\alpha',\alpha''} d(\alpha''\alpha)d(\alpha'\alpha^{*})\Big[\sum_{\beta'} T(\alpha'\alpha'',\beta'\beta'')\Big].$$

[11] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

PHYSICAL REVIEW VOLUME 159, NUMBER 5 25 JULY 1967

Quark Model and Quadratic Mass Formulas for Mesons*

SHIN ISHIDA† AND P. ROMAN
Department of Physics, Boston University, Boston, Massachusetts

(Received 21 November 1966; revised manuscript received 2 March 1967)

Within the framework of the nonrelativistic quark model, a simple field-theoretical prescription is given for using quadratic mass formulas for all mesons and linear mass formulas for all baryons. It is postulated that the unperturbed part of the quark-system Hamiltonian possesses $SU(6)$ symmetry with respect to the spin and unitary spin of the constituent quarks. Quadratic mass relations within meson multiplets as well as quadratic electromagnetic mass relations are derived, and it is shown that the parameters can be uniquely fitted to experiment, avoiding the serious discrepancy which exists when such a fit is attempted in the standard quark-model framework which uses linear mass relations.

I. INTRODUCTION

RECENTLY many interesting results in hadron physics have been derived from adopting the framework of the nonrelativistic quark model.[1] In this note we are especially interested in the mass splitting of particles from this point of view. In the standard quark model[2] the particle masses are given by the expectation values of the Hamiltonian for the quark system in the relevant particles states, taken in their rest frame. This necessarily leads to the use of linear mass formulas for both mesons and baryons. Assuming, however, that the quark model leads effectively to the usual Lagrangian field theory of hadrons, we shall give in the present paper a prescription for using a quadratic mass formula[3] for mesons and a linear mass formula for baryons. We shall show that this procedure improves the results considerably when compared with the usual treatment within the framework of the quark model with linear mass relations for both mesons and baryons.

Let us consider the fundamental Hamiltonian for the quark system,

$$H=H_0+\lambda H_\delta\,, \quad (1)$$

where H_0 has the relevant symmetry (to be specified later), and H_δ corresponds to the symmetry breaking. Here we introduced the perturbational parameter λ which will be finally taken to $\lambda=1$. Corresponding to the viewpoint that all hadrons are composed of quarks, H_0 is expected to be effectively represented by the particle fields ϕ_i^0 (bosons) and ψ_j^0 (fermions) which span representations of the relevant symmetry. Thus, in the

* Research supported by the U. S. Air Force under Grant No. AF-AFOSR-385-65.

† On leave of absence from Nihon University, Department of Physics, Chiyoda-ku, Tokyo, Japan.

[1] For a review, see, for example, the following articles: R. H. Dalitz, in *Proceedings of the Oxford Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966); in *Proceedings of the High Energy Physics Conference*, Berkeley, 1966 (University of California Press, Berkeley, 1967); H. J. Lipkin, in Proceedings of the Yalta International School, 1966 (unpublished); O'Hara, J. Iizuka, M. Namiki, S. Ishida and S. Tanaka; Soryushiron Kenkyu 33, 167 (1966); M. Namiki, Waseda University, Report, 1966 (unpublished); G. Morpurgo, Physics 2, 95 (1965).

[2] G. Zweig, CERN Report, 1964 (unpublished), or S. Ishida, Progr. Theoret. Phys. (Kyoto) 32, 922 (1964); 34, 64 (1965); Soryushiron Kenkyu 30, 372 (1964).

[3] The question as to whether a linear or a quadratic mass formula should be used for mesons, is a well-known controversial issue. See for example, F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. 135, B467 (1964), or S. Okubo and S. Ryan, Nuovo Cimento 34, 64 (1965).