new "charge symmetry" or symmetry under the transformation $V_1 \rightleftarrows V_2$. This symmetry was also derived previously from the $Z=0$ condition. Thus the two approaches, as well as the bootstrap approach of looking at eigenvalues of the crossing matrix, yield identical results. This is a further confirmation that $Z=0$ is indeed a bootstrap condition. It is hoped that this conclusion can also be obtained for the other models considered in the strong-coupling theories. If so, we may be led to a method which can be used in relativistic theory, where g^2 is expected to have a bound, and the $Z_3 \rightarrow 0$ limit may be more convenient.

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Quark Model for Baryon-Antibaryon Processes

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A recently proposed quark (Q) model treating positive-parity and negative-parity baryonic states as $3Q$ composites belonging to the $(\mathbf{56}, 1)^+$ and $(\mathbf{70}, 3)^-$ representations of $SU(6)\times O(3)$, respectively, is extended to baryon-antibaryon processes. Both one and two units of ΔS, ΔQ, ΔI exchange processes are considered through single and double pair scattering of quarks by antiquarks. Apart from reproducing the $SU(3)$ results, a number of new relations similar to those obtainable from higher symmetries are derived for the cross sections of inelastic processes, in fair agreement with available experimental data.

1. INTRODUCTION

RECENTLY a nonrelativistic quark (Q) model has been proposed according to which the positive-parity and negative-parity baryonic states are identified as $3Q$ composites belonging to the $(\mathbf{56}, 1)^+$ and $(\mathbf{70}, 3)^-$ representations of $SU(6)\times O(3)$, respectively. The applications of this model to meson-baryon processes,[1,2] photoproduction,[3] and baryon-baryon processes[4] yield a good number of predictions [over and above $SU(3)$] which are similar to those obtainable from higher symmetries. In this paper, we wish to investigate the consequences of this theory for baryon-antibaryon ($B\bar{B}$) processes. As has been considered earlier for BB processes, we regard the $B\bar{B}$ interaction as a consequence of the existence of two distinct types of Q-Q ($\bar{Q}$-$\bar{Q}$) and $Q\bar{Q}$ forces with nonoverlapping domains of validity. The QQ (or $\bar{Q}\bar{Q}$) interaction is strong and short-ranged, and acts between the quarks (or antiquarks) of the same baryon (or antibaryon). The $Q\bar{Q}$ interaction arising from different hadrons which contributes to the amplitude of the $B\bar{B}$ process, is taken to be weaker and long-ranged. This enables us to regard the entire $B\bar{B}$ system in terms of two distinct three-body symmetries which are mainly determined by the QQ, $\bar{Q}\bar{Q}$ interactions within the $3Q$ or $3\bar{Q}$ systems, respectively. We now consider the $B\bar{B}$ amplitude as a sum of the amplitudes for individual $Q\bar{Q}$ scattering as well as for the direct scattering of two quarks with two antiquarks, one quark and one antiquark being spectators. The inclusion of such effects in an $SU(3)$ invariant manner enables the $\Delta S, \Delta Q$, or $\Delta I=2$ processes to be accommodated in the simple quark model. Such processes are forbidden in the Lipkin model.[5] The processes which involve $\Delta S=3$ exchange (Ω^- production in $p\bar{p}$ collisions) can be considered through the simultaneous scattering of three $Q\bar{Q}$ pairs. The $Q\bar{Q}$ amplitude, taken to be $SU(3)$ invariant must have the forms, 1; $\lambda_i^{(1)}\cdot\lambda_j^{(2)}$; $\lambda_i^{(1)}\cdot\lambda_j^{(2)}\lambda_{i'}^{(1)}\cdot\lambda_{j'}^{(2)}$ where the indices $i, i'(j,j')$ denote the $i, i'(j,j')$ quarks (antiquarks) of baryon 1 (antibaryon 2), and λ's are the eight 3×3 Gell-Mann matrices.[6] We neglect the relatively small effect of the $\Delta S=3$ channel as evident from the small production cross section of Ω^- in $p\bar{p}$ collisions.[7] From a comparison with $B\bar{B}$ processes in the collinear $U(3)\times U(3)$ group of Volkov and Ruegg,[8] we find an analogous structure of the $B\bar{B}$ amplitude with four terms contributing to the elastic, ΔS or $\Delta Q=1$ exchange, $\Delta S, \Delta Q=2$ exchange and $\Delta S=3$ exchange channels for the $l=0$ case. The dynamical aspects of the problem are touched only qualitatively and no quantitative evaluation of the $B\bar{B}$ amplitude is made. In this

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[5] H. J. Lipkin, F. Scheck and H. Stern, Phys. Rev. **152**, 1375 (1966).
[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[7] C. Y. Chien *et al.*, Phys. Rev. **152**, 1171 (1966).
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paper we consider the physically interesting cases like production of $B\bar{B}$, $\bar{B}B^*$, $B\bar{B}^*$, and $B^*\bar{B}^*$ pairs in $p\bar{p}$ and $n\bar{p}$ collision processes. In Sec. 2 the basic $Q\bar{Q}$ amplitudes are defined taking account of $SU(3)$ degrees of freedom, and a general method of evaluating the $B\bar{B}$ amplitude matrix element is outlined. Section 3 deals with the calculations of the above processes. The results are obtained in terms of sum rules for the amplitudes for these processes. Section 4 discusses the experimental status of the results, their comparison with predictions from other models, as well as the possibility of extending the model to include the production of negative-parity baryonic states.

2. GENERAL STRUCTURE AND EVALUATION OF $B\bar{B}$ AMPLITUDE MATRIX ELEMENT

Under the present scheme, the complete $SU(3)$ invariant $B\bar{B}$ matrix elements are obtained as

$$\sum_{\substack{i,i'=1\\ i\neq i'}}^{3}\sum_{\substack{j,j'=1\\ j\neq j'}}^{3}(\psi(1)\psi(2)|a^{(+)ij}+a^{(-)ij}\lambda_i^{(1)}\cdot\lambda_j^{(2)} +b^{ii'jj'}\lambda_i^{(1)}\cdot\lambda_j^{(2)}\lambda_{i'}^{(1)}\cdot\lambda_{j'}^{(2)}|\psi(1)\psi(2)), \quad (2.1)$$

where the coefficients $a^{(\pm)ij}$, $b^{ii'jj'}$ have appropriate spin and spatial structures so as to give the correct matrix element. $\psi(1)$, $\psi(2)$ are nonrelativistic $3Q$ (baryon) and $3\bar{Q}$ (antibaryon) wavefunctions of Refs. (1) and (2). Under parastatistics,[9,9a] over-all symmetric states for $(\mathbf{56},1)^+$ representations of $SU(6)\times O(3)$ that can be constructed are

$$\psi_B=\psi^s(\chi'\phi'+\chi''\varphi'')(2)^{-1/2},$$
$$(8) \qquad \psi_{\bar{B}}=\psi^s(\chi'\bar{\phi}'+\chi''\bar{\phi}'')(2)^{-1/2}, \quad (2.2)$$
$$\psi_{B^*}=\psi^s\phi^s\chi^s,$$
$$(10) \qquad \psi_{\bar{B}^*}=\psi^s\bar{\phi}^s\chi^s, \quad (2.3)$$

where ψ^s, $\chi^{\prime,\prime\prime,s}$, $\phi^{\prime,\prime\prime,s}(\bar{\phi}^{\prime,\prime\prime,s})$ are the spatial, spin and $SU(3)$ wave functions, and superscripts s, prime, and double-prime indicate permutation symmetry.[10]

We evaluate the $B\bar{B}$ matrix elements which are of the twenty-four types

$$((\varphi^{\prime,\prime\prime,s})_{(1)}(\varphi^{\prime,\prime\prime,s})_{(2)}|1;\lambda_i^{(1)}\cdot\lambda_j^{(2)};\ \lambda_i^{(1)}\cdot\lambda_j^{(2)}\lambda_{i'}^{(1)}\cdot\lambda_{j'}^{(2)}|(\phi^{\prime,\prime\prime})_{(1)}(\varphi^{\prime,\prime\prime})_{(2)}), \quad (2.4)$$

for appropriate initial and final states, considering only the $SU(3)$ degree of freedom. While further dynamical assumptions are necessary to consider the spin and spatial degrees of freedom, a mere evaluation of (2.4) is sufficient for our purpose to give a number of relations for the cross sections of the $B\bar{B}$ processes.

3. SUM RULES FOR $B\bar{B}$ PROCESSES

(a) Octet-Octet Processes $N\bar{N}\rightarrow B\bar{B}$

The amplitudes for $N\bar{N}\rightarrow B\bar{B}$ are obtained by substituting ψ_B, $\psi_{\bar{B}}$ of (2.2) for $\psi(1)$, $\psi(2)$ of (2.1). The reduced matrix element consists now of sixteen terms for each (ij) and $(ii'jj')$ values of $Q\bar{Q}$ pairs. For processes involving two units of ΔS, ΔQ or ΔI exchange, one can look for $SU(3)$-type sum rules by comparing these terms, confining oneself to each $(ii'jj')$ value of $Q\bar{Q}$ pairs separately. For processes involving one unit of ΔS, ΔQ or ΔI exchange, a separate comparison of the sixteen terms for each (ij) value and for each $(ii'jj')$ value of $Q\bar{Q}$ pairs give the same set of $SU(3)$-type relations for the amplitudes of these processes. Because of the totally symmetric wave functions, such relations are independently satisfied for all possible values of (ij) and $(ii'jj')$ quark-antiquark pairs. The use of the additivity assumption for both these contributions is now sufficient to give the following relations between the $B\bar{B}$ amplitudes $A(B\bar{B}|n\bar{p})$ or $A(B\bar{B}|p\bar{p})$ which we abbreviate simply as $(B\bar{B})$:

$$(\bar{\Sigma}^+\Sigma^-)=(\bar{\Xi}^0\Xi^0), \quad (3.1)$$
$$(\bar{p}p)=(\bar{p}n)+(\bar{n}n), \quad (3.2)$$
$$2(\bar{\Sigma}^0\Sigma^0)=(\bar{\Sigma}^+\Sigma^-)+(\bar{\Sigma}^-\Sigma^+), \quad (3.3)$$
$$(\Lambda\bar{\Sigma}^0)=(\bar{\Lambda}\Sigma^0), \quad (3.4)$$
$$(\Lambda\bar{\Sigma}^0)=(2)^{-1/2}(\bar{\Lambda}\Sigma^-), \quad (3.5)$$
$$(2)^{-1/2}(\bar{\Sigma}^-\Sigma^0)=(\bar{\Sigma}^-\Sigma^+)-(\bar{\Sigma}^0\Sigma^0), \quad (3.6)$$
$$(\bar{\Xi}^0\Xi^0)=(\bar{\Xi}^+\Xi^-)-(\bar{\Xi}^0\Xi^-), \quad (3.7)$$

and

$$3(\bar{\Lambda}\Lambda)+(\bar{\Sigma}^0\Sigma^0)=2(\bar{n}n)+2(\bar{\Xi}^0\Xi^0)+2(3)^{1/2}(\Lambda\bar{\Sigma}^0). \quad (3.8)$$

These results which also follow from $SU(3)$ invariance[11] may be taken to provide merely a consistency check on this formulation. Equations (3.3) and (3.4) have been checked with experimental data by Chien *et al.*[7]

(b) Octet-Decuplet and Decuplet Pair Production Processes

The amplitudes for $N\bar{N}\rightarrow\bar{B}B^*$ and $N\bar{N}\rightarrow B^*\bar{B}^*$ are obtained by replacing in (2.1) either (or both) of $\psi(1)$, $\psi(2)$ by ψ_{B^*}, $\psi_{\bar{B}^*}$ in the final state. Thus eight (or four) types of terms are obtained in the reduced matrix elements for each (ij), $(ii'jj')$ values of $Q\bar{Q}$ pairs. These give a set of relations for different $\bar{B}B^*$, $B^*\bar{B}^*$ processes which are independently satisfied for any set (ij), $(ii'jj')$ values of $Q\bar{Q}$ pairs because of the over-all symmetry as indicated before. The additivity

[9] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[9a] While the structures of the wave functions have been considered under parastatistics, it may be noted that the same results are obtainable under Fermi statistics by making the replacement of the spatial functions $S\rightarrow A$, $M''\rightarrow M'$, $M'\rightarrow M''$, where M' and M'' are the two independent functions of mixed symmetry.

[10] M. Verde in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. **39**, p. 170.

[11] K. Tanaka, Phys. Rev. **135**, B1186 (1964).

assumption for these two types of contributions now gives the $SU(3)$ prediction

$$(\bar{N}^{*--}N^{*++})-(\bar{N}^{*+}N^{*-})=3[(\bar{N}^{*-}N^{*+})-(\bar{N}^{*0}N^{*0})], \quad (3.9)$$

and also the following new relations involving higher symmetry, which are true for any direction,

$$(\bar{n}N^{*0})=(\bar{p}N^{*+}), \quad (3.10)$$

$$(\bar{\Sigma}^{+}Y^{*-})=(\bar{\Xi}^{+}\Xi^{*-}), \quad (3.11)$$

$$2(\bar{\Sigma}^{0}Y^{*0})=(\bar{\Sigma}^{-}Y^{*+})-(\bar{\Sigma}^{+}Y^{*-}), \quad (3.12)$$

$$(\bar{Y}^{*+}Y^{*-})=2(\bar{\Xi}^{*+}\Xi^{*-}), \quad (3.13)$$

$$(\bar{N}^{*+}N^{*-})=3(\bar{\Xi}^{*+}\Xi^{*-}), \quad (3.14)$$

$$2(\bar{Y}^{*0}Y^{*0})=(\bar{Y}^{*-}Y^{*+})+(\bar{Y}^{*+}Y^{*-}), \quad (3.15)$$

and

$$(\bar{N}^{*--}N^{*++})=3[(\bar{Y}^{*-}Y^{*+})-(\bar{\Xi}^{*0}\Xi^{*0})]. \quad (3.16)$$

These give the corresponding relations and inequalities for the cross-sections $\bar{\sigma}=|A|^2$,

$$\bar{\sigma}(\bar{n}N^{*0})=\bar{\sigma}(\bar{p}N^{*+}), \quad (3.17)$$

$$\bar{\sigma}(\bar{\Sigma}^{+}Y^{*-})=\bar{\sigma}(\bar{\Xi}^{+}\Xi^{*-}), \quad (3.18)$$

$$2\sqrt{\bar{\sigma}}(\bar{\Sigma}^{0}Y^{*0})\leq\sqrt{\bar{\sigma}}(\bar{\Sigma}^{-}Y^{*+})+\sqrt{\bar{\sigma}}(\bar{\Sigma}^{+}Y^{*-}), \quad (3.19)$$

$$\bar{\sigma}(\bar{Y}^{*+}Y^{*-})=4\bar{\sigma}(\bar{\Xi}^{*+}\Xi^{*-}), \quad (3.20)$$

$$\bar{\sigma}(\bar{N}^{*+}N^{*-})=9\bar{\sigma}(\bar{\Xi}^{*+}\Xi^{*-}), \quad (3.21)$$

$$2\sqrt{\bar{\sigma}}(\bar{Y}^{*0}Y^{*0})\leq\sqrt{\bar{\sigma}}(\bar{Y}^{*-}Y^{*+})+\sqrt{\bar{\sigma}}(\bar{Y}^{*+}Y^{*-}), \quad (3.22)$$

and

$$\sqrt{\bar{\sigma}}(\bar{Y}^{*-}Y^{*+})\leq\tfrac{1}{3}\sqrt{\bar{\sigma}}(\bar{N}^{*--}\bar{N}^{*++})+\sqrt{\bar{\sigma}}(\bar{\Xi}^{*0}\Xi^{*0}). \quad (3.23)$$

4. EXPERIMENTAL VERIFICATION AND DISCUSSION OF RESULTS

The relations (3.1)–(3.8) have been obtained by $SU(3)$ symmetry alone and are in fair agreement with available experimental data. Equation (3.8) gives an inequality for the scattering cross-sections $\bar{\sigma}=|A|^2$,

$$2\sqrt{\bar{\sigma}}(\bar{n}n)\leq 3\sqrt{\bar{\sigma}}(\bar{\Lambda}\Lambda)+\sqrt{\bar{\sigma}}(\bar{\Sigma}^{0}\Sigma^{0})+2\sqrt{\bar{\sigma}}(\bar{\Xi}^{0}\Xi^{0})+2(3)^{1/2}\sqrt{\bar{\sigma}}(\bar{\Lambda}\Sigma^{0}). \quad (4.1)$$

The relations (4.1), (3.3), (3.4) have been successfully compared with the experimental data[12,13] after including

TABLE I. References 12[a] and 13[b] are considered for $p\bar{p}$ data at $\bar{p}$ lab momentum 3 BeV/c and $E_{c.m.}=2.7$ BeV and $\bar{p}$ lab momentum 3.7 BeV/c, $E_{c.m.}=2.97$ BeV.

Process	p_f (BeV/c)	ρ (BeV^2)	σ (mb)	$\sqrt{\bar{\sigma}}=\sqrt{(\rho\sigma)}$ (μb)
$p\bar{p}\rightarrow\bar{N}^{*--}N^{*++}$	0.849	12.05	1.8	4.67
$\bar{Y}^{*-}Y^{*+}$	0.513	19.94	8×10^{-3} [b]	0.4
$\bar{Y}^{*+}Y^{*-}$	0.513	19.94	5×10^{-3} [b]	0.316
$\Sigma^{-}Y^{*+}$	0.495	15.7	11×10^{-3} [a]	0.4157
$\bar{\Sigma}^{0}Y^{*0}+\Sigma^{0}\bar{Y}^{*0}$	0.495	15.7	$<5\times10^{-3}$ [a]	<0.2802
$\bar{\Sigma}^{+}Y^{*-}$	0.495	15.7	1×10^{-3} [a]	0.1253

[a] Reference 12.
[b] Reference 13.

the kinematic correction for phase space [14]

$$\bar{\sigma}=|A|^2=(E_{c.m.}{}^2\times p_{in}/p_f)\sigma=\rho\sigma, \quad (4.2)$$

where p_{in}, p_f are the initial and final c.m. momenta of the $B\bar{B}$ system and $E_{c.m.}$ is the total c.m. energy. From Table I,[8] we find the new predictions (3.19), (3.23) to be fairly well satisfied. For negative-parity $(\mathbf{70},3)^-$ states of $SU(6)\times O(3)$, the model predicts the relations (3.1)–(3.8) for the members of the same octet $(\mathbf{8},J)^-$. Specific results for $(\mathbf{70},3)^-$ and $(\mathbf{20},3)^+$ states are obtainable from spin calculations. Since the same radial integrals are involved in the amplitudes for $N\bar{N}\rightarrow B\bar{B}$, $\bar{B}B^*$, $B^*\bar{B}^*$ through the spatial wave function ψ^s, such considerations would also enable us to relate the amplitudes for the processes. Thus our results, arising from a preliminary inclusion of single and double pair scattering of quarks by antiquarks, seem to be in fair agreement with existing experimental data.

Most of our results are similar to those obtained under collinear $U(3)\times U(3)$, $SU(6)_W$,[15] and $U(12)$ symmetries as may be expected from the use of $SU(6)\times O(3)$ type of structures for the wave functions. This is as far as we can go without making any additional assumptions regarding the dynamics of quarks and antiquarks.

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