

Note that we do not thereby violate unitarity, since it is a *fixed* pole only to lowest order in the photon-nucleon coupling and presumably becomes a moving pole (or essential singularity) in higher orders. (b) The existence of moving cuts in the angular-momentum plane (in this case generated by multiple Pomeranchuk exchange) could provide a contribution to $f_{ab,ab}(\theta=0)$, making σ^{tot} at least comparable to σ^{el} as $s \rightarrow \infty$. (c) Another possibility is some "conspiracy" of moving poles which are related at $t=0$ in such a way as to allow the Pomeranchuk to contribute.²⁵ (d) If $\alpha_P(0) < 1$, we avoid the vanishing of the forward nonflip elastic amplitudes.

²⁵ Such a mechanism has been suggested in NN scattering. For the present case see V. D. Mur, *Zh. Eksperim. i Teor. Fiz.* **45**,

(3) Finally, it should be noted that the prediction given by Eq. (16) is essentially independent of the difficulty discussed above in (2) and will provide a further experimental test of the Regge-pole hypothesis.

ACKNOWLEDGMENTS

I am grateful to Professor Raymond Sawyer for his interest and encouragement and for many discussions concerning this work. Conversations with Dr. Ling-Lie Wang, Professor R. Blankenbecler, and Professor A. Krass have also been helpful.

1051 (1963) [English transl.: *Soviet Phys.—JETP* **18**, 727 (1964)].

Boson Masses. II

D. C. PEASLEE

Research School of Physical Sciences, The Australian National University, Canberra, Australia

(Received 30 January 1967)

The baryon-antibaryon model is employed to attempt a complete empirical assignment for the boson nonets with orbitals 1S_0 , 3S_1 , 1P_1 , 3P_0 , 3P_1 , 3P_2 . Only one state seems to be missing experimentally, and a search area is indicated by a simple degeneracy in the model. Some substantial level shifts are proposed to result from open channels for multiboson decay; with these can be associated the possibility of reduced ω - φ mixing. The octet SU_3 mass formula appears to be valid for only the 1S_0 nonet; a generalized formula reflects major R_7 and minor G_2 mixing with the basic SU_3 . For K - and η -type mesons the triplet orbitals display strong tensor as well as spin-orbit splitting. As a consequence the $K^*(1400)$ should be a mixture of $2^+({}^3P_2)$ and $1^-({}^3D_1)$ resonances, with respective dominant modes $K\pi\pi$ and $K\pi$. Some remarks are added about D states and the validity of A parity.

I. INTRODUCTION AND SUMMARY

RECENTLY augmented data¹ allow us to extend and improve earlier considerations² on the baryon-antibaryon model for bosons. We first note that the model itself implies a likely degeneracy for bosons of a given 3L_J nonet: between the charge singlet **1** and the $I=1$ member of the charge octet **8**. This is observed in the approximate degeneracies of ρ and ω , A_2 and f , and leads to the prediction of other resonances—in particular, a **1** state of 3P_1 at ~ 1090 MeV to accompany the A_1 .

Measured mass differences in these degenerate states can be semiquantitatively interpreted as level shifts due to open channels. This interpretation leads away from the idea that deviations from the SU_3 octet mass formula can be attributed to strong mixing of the ω - φ type. Accordingly, we attempt to analyze the four

probably established nonets 1S , 3S , 1P , 3P in terms of clashing symmetries: $SU_3 + gR_7 + fG_2$. The empirical result is that f is very small, while variation in g is responsible for significant changes in pattern.

Comparison of the ${}^3P_{0,1,2}$ nonets suggest strong spin-orbit coupling throughout; in general there is also a tensor-type force, but this surprisingly vanishes just for the degenerate states **8** $I=1$ and **1**. Knowledge of this structure is sufficient to identify some D states among the fragmentary data at higher energies and to predict regions for other D states.

In conclusion are added a few remarks about A parity. Its validity is enhanced by our abandonment of ω - φ mixing. The chief experimental difficulty is then the comparable decay rates for $(K\pi)$ and $(K\pi\pi)$ modes of the $K^*(1400)$. We suggest that the reported state is in fact an accidental conjunction of 3P_2 and 3D_1 resonances induced by a strong tensor addition to spin-orbit splitting.

II. DEGENERACY ARGUMENT

The crux of the baryon-antibaryon model is avoidance of parastatistics in favor of ordinary Fermi sta-

¹ G. Goldhaber and R. H. Dalitz in, *Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, California, 1967).

² R. W. King and D. C. Peaslee, *Phys. Rev.* **143**, 1321 (1966). Equations (4) and (4') of this reference are now seen to hold only for S orbitals. Exploration of D and F resonances will be needed to settle the question of even-odd alternation.

tistics, whence

$$P^B P^C P^L P^S = -1. \quad (1)$$

Here the fermion exchange operators are P^L, P^S, P^C for space, spin, and charge; while P^B exchanges fermion and antifermion character, and we take it equal to the empirical quantum number A . The argument is now simple: P^B and P^C are both formal operators, and we may assume the energy levels of the fermion-antifermion system to be unaffected by $(P^B, P^C) \rightarrow (-P^B, -P^C)$ without change of any other factors, *provided* that no known symmetry features are violated. This proviso greatly limits the number of states involved; for

(i) symmetries based on ordinary isospin (SU_2) are much better kept than others in SU_3 , where mass deviations are very apparent;

(ii) despite mass breaking the dominant symmetry is still SU_3 , so that $P^C \rightarrow -P^C$ must involve a shift between $\mathbf{8}$ and $\mathbf{1}$;

(iii) even the limited success of SU_6 indicates that singlet-singlet states ($SU_3 = \mathbf{1}$ and real spin $S=0$) should be considered separate from the others.

When restrictions (i)–(iii) are taken into account, degeneracy remains only between states of the same 3L_J and charge character $\mathbf{1}$ or $\mathbf{8}$ $I=1$.

Experimentally this implies degeneracy of the $\rho(760)$ and $\omega(783)$, and of the $A_2(1320)$ and $f^0(1255)$. Failure of exact degeneracy is most readily ascribed to multi-boson channels, in which the virtual bosons mainly carry high kinetic energies and produce a negative level shift $\sum_m |H_{0m}|^2 / (E_0 - E_m)$. The shift magnitude from a given boson channel is difficult to estimate, but for orientation we may take it to be of the same order as the width for free decay into that channel. Resonances with large, two-pion decay widths will accordingly have the largest downward shifts; qualitatively $m(\rho) < m(\omega^0)$, $m(f^0) < m(A_2)$, as observed.

The most striking case of this sort arises in comparison of the $\delta(964)$ and $\epsilon(\sim 750)$ mesons, which we take to be $\mathbf{8}$ $I=1$ and $\mathbf{1}$ of the 3P_0 nonet. This identification is quite straightforward despite experimental uncertainties. Because of its inhibitions, the $I=1$ member of the 3P_0 nonet must be narrow²; but its most likely decay mode will be 2π through I violation. The δ meson seems to have just these properties.^{3,1} The $\mathbf{1}$ 3P_0 has the least inhibited 2π decay of all resonances; it should be enormously broad and considerably shifted downwards relative to the δ , exactly the features of the ϵ .

The best direct measurement yet available on the ϵ appears to be that of Jones *et al.*,⁴ who find a δ_0^0 for $\pi\pi$ scattering that increases smoothly over the region 300–600 MeV; they estimate $\delta_0^0 = \pi/2$ between 700 and 800

MeV. More graphic is a plot of average forward-backward symmetry in ρ^0 production.⁵ This displays a resonance-like structure centered at ~ 800 MeV with half-width ~ 450 MeV. If this represents the interference between a broad resonance and a relatively narrower resonance of opposite parity, both centered at about the same energy, it is easy to show that the observed envelope is substantially the Breit-Wigner shape of the broader resonance, in this case the s wave.

If we momentarily accept these estimates of position and width for the ϵ , the resonance shift is $\Delta \approx -160$ MeV, so that

$$(\Delta/\Gamma)_\epsilon \approx -\frac{1}{3}. \quad (2a)$$

The situation here is reminiscent of the K_1 and K_2 , where the current experimental quotation is

$$(\Delta/\Gamma)_K = -\frac{1}{2}. \quad (2b)$$

It has been argued⁶ that the K_1 – K_2 mass difference arises predominantly from the 2π decay channel of the K_1 ; but the numerical details of that argument depend strongly on the ϵ resonance as an intermediate state. On the other hand, the agreement between (2a) and (2b) suggests that (Δ/Γ) may be somewhat more universal; we return to this remark below.

The postulated degeneracy has been discussed for all triplet S and P states except 3P_1 , where no experimental candidate now presents itself for the $\mathbf{1}$; call it the F meson. It should be approximately degenerate with the A_2 , but its decay modes are even more restricted: 2π forbidden by spin and parity, and $\pi\rho$ inhibited by G conservation. Decay by 4π emission is allowed but will not be enhanced by cascade processes through intermediate resonances. The J is therefore very narrow and perhaps a little above the A_2 in energy, say 1080–1100 MeV. Experimental location of this meson would test our degeneracy hypothesis.

III. POSSIBLE REDUCTION IN NONET MIXING ANGLE

Once substantial shifts are admitted in boson-resonance energies, it is difficult to be dogmatic about the degree of ω – φ mixing displayed by various nonets. As an example, we show how the mixing angle in the 3S_1 and 3P_2 nonets could be taken of the same order as in 1S_0 : namely, $\sim 10^\circ$.

Suppose the same ratio Δ/Γ for the two resonances with dominant 2π channels, f^0 and ρ . Then the Δ are approximately equal, and the shifts in (mass)² in the ratio of the masses, $-b$ and $-0.6b$ for f^0 and ρ , respectively. For a constant singlet-octet mixing angle θ the shifts in (mass)² are $-0.7 \sin^2\theta$ for f^0 and $-0.43 \sin^2\theta$ for ω . Neglecting interference between these two terms

³ D. D. Allen, G. P. Fisher, G. Godden, L. Marshall, and R. Sears, *Phys. Letters* **22**, 543 (1966).

⁴ L. W. Jones, D. O. Caldwell, B. Zacharov, D. Harting, E. Bleuler, W. C. Middelkoop, and B. Elsner, *Phys. Letters* **21**, 390 (1966).

⁵ G. Goldhaber, in *Second Coral Gables Conference on Symmetry Principles at High Energy*, edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman and Company, San Francisco, 1965), Fig. 12–2, p. 114.

⁶ S. H. Patil, *Phys. Rev. Letters* **13**, 454 (1964).

for the f^0 and assuming other shifts to be small, we have in units of BeV^2

$$m^2(A_2) - m^2(f^0) = b + 0.7 \sin^2\theta \approx 0.12, \quad (3)$$

$$m^2(\omega) - m^2(\rho) = 0.6b - 0.43 \sin^2\theta \approx 0.035.$$

These yield $\theta \approx 12^\circ$ and $b = 0.09 \text{ BeV}^2$, corresponding to $\Delta/\Gamma \approx -0.4$, consistent with estimates (2a) and (2b). The uncertainties make this result only illustrative, but it at least suggests that singlet-octet mixing may have a fairly universal and relatively small value.

Of course a small mixing angle θ implies failure of the SU_3 octet mass formula for 3S_1 and 3P_2 ; but we shall see that this is very much the rule, since three out of four known nonets exhibit this failure. It seems more likely that the pseudoscalar octet is anomalous in following the ideal SU_3 relation.

IV. SPIN-ORBIT AND TENSOR COUPLINGS

Enough data are now on hand to permit assignment of the six lowest boson nonets: those with orbitals 1S_0 , 3S_1 , 1P_1 , ${}^3P_{0,1,2}$. From the last three we can deduce spin-orbit and tensor coupling terms in the masses. Table I shows our assignments, taking into account the preceding discussion; they generally agree with the current consensus,¹ and any peculiarities are noted.

Inclusion of spin-orbit and tensor terms in the expression for (mass)² yields

$$m^2({}^3L_{L+1}) = m^2({}^3L) + sL - t(L/2L+3),$$

$$m^2({}^3L_L) = m^2({}^3L) - s + t, \quad (4)$$

$$m^2({}^3L_{L-1}) = m^2({}^3L) - s(L+1) - t(L+1/2L-1),$$

where $m^2({}^3L)$ is the central value for the triplet, s the spin-orbit, and t the tensor coefficient. For 3P states as in Table I we obtain the parameters shown in Table II.

The tensor coefficient in Table II shows an interesting behavior: It is entirely absent for the most accessible mesons, strongly present for the more recondite.

TABLE I. Boson nonet assignments.

Orbital	Charge state			
	1	8 I=1	8 I=0	8 I=½
3P_2	$f^0(1255)$	$A_2(1320)$	$f^*(1500)$	$K^*(1430)$
3P_1	$F(1090?)^a$	$A_1(1080)$	$E(1420)^b$	$K^*(1320)$
3P_0	$\epsilon(750)$	$\delta(964)$	$\lambda(1068)$	$\kappa(730)^c$
1P_1	$H(1000)$	$B(1224)$	$D(1286)^b$	$K_B(1270)^d$
3S_1	$\omega(783)$	$\rho(760)$	$\varphi(1018)$	$K^*(890)$
1S_0	$X^0(959)^e$	$\pi(140)$	$\eta(550)$	$K(495)$

^a As discussed above.

^b The experimental parameters for D and E are not yet conclusive, but they must both be in column 3 because the absence of multipion decay shows that $A = -G$. For 1P_1 the decay mode $\rightarrow \eta + \omega$ (threshold 1332 MeV) would be allowed, for 3P_1 it is A -forbidden; its comparative absence suggests the ordering (D, E) as (${}^1P_1, {}^3P_1$).

^c This is a remarkably durable resonance, in spite of all objections; e.g., N. M. Cason, S. Mikamo, and A. Subramanian, Phys. Rev. Letters 17, 838 (1966).

^d As in (b) decay into $(\omega + K)$ would be allowed for $m \gtrsim 1280$ MeV.

^e By elimination, this is the only spot left for the X , unless it should turn out to be what we have called the F . The X and F are identical in all charge and isospin quantum numbers, differing only in J^P .

TABLE II. Mass parameters in BeV^2 .

Parameter	Charge state			
	1	8 I=1	8 I=0	8 I=½
$m^2({}^3P)$	a	1.43	2.05	1.75
$s({}^3P)$	a	0.25	0.25	0.30
$t({}^3P)$	a	0	0.31	0.46

^a Errors on all entries of order ± 0.03 ; the first column is postulated to be the same as the second but in practice is subject to large shifts due to open 2π channels.

Perhaps the first of these two statements is the greater surprise.

Maintenance of the SU_3 octet mass formula for each J value in this triplet has as a necessary condition the constancy of s and t across the columns of Table II. Within uncertainties this is true for s , definitely not for t . Thus, we have a second argument against universality of the SU_3 octet formula, this one independent of singlet-octet mixing.

The exceptional strength of the tensor term in the last column is associated with an outstandingly low value for the mass of the K -type meson of lowest J value in any triplet, in this case, with the identification of the $\kappa(730)$ as 3P_0 . It would be desirable to have corroborating evidence on this point, and we suggest the following. The $K^*({}^3D_1)$ will by this argument be much the lowest of the mesons with D orbitals, with estimated mass in the neighborhood of 1400 MeV. This would imply that the $K^*(1400)$ is in fact a double resonance, with $J^P = 2^+$ and 1^- . The 1^- component would behave just like the $K^*(890)$ and should have a dominant ($K\pi$) decay mode, while the 2^+ component favors ($K\pi\pi$) in analogy with the A_2 . It is notable that these two decay modes seem to peak at different energies¹—although still within overlapping experimental errors, namely, ~ 1395 MeV for ($K\pi$) and ~ 1430 MeV for ($K\pi\pi$). Moreover, although the favored J^P for $K^*(1400)$ considered as a single resonance is 2^+ , the choice 1^- generally obtains honorable mention and is in fact preferred by at least one group⁷ who analyzed ($K\pi$) decays at 1390 MeV. The extreme variability of the decay ratios ($K\pi$)/($K\pi\pi$) found in different experiments could be readily interpreted as differences in excitation of the 1^- and 2^+ components.

In summary, it seems already very plausible that the $K({}^3D_1)$ has an unusually low mass value, confirming the large coefficient t in Table II. Conclusive evidence would be the experimental resolution of $K^*(1400)$ into two components as described.

V. ATTEMPT AT A BOSON MASS FORMULA

Doubts concerning the octet mass formula make it seem worthwhile to attempt a nonet generalization.

⁷ F. Schweingruber, J. Simpson, A. Cooper, M. Derrick, T. Fields, L. Hyman, J. Loken, R. Ammar, R. Davis, C. Hwang, W. Kropac, and J. Mott, Report at XIIIth International Conference on High Energy Physics, Berkeley, 1966 (unpublished).

One possibility is to consider⁸ admixtures of terms with G_2 and R_7 symmetries to the basic SU_3 , which we take to be pure F coupling for baryon-antibaryon interactions.⁹ Lowest-order perturbation diagrams (single loops) suffice to indicate the charge space character of mass-splitting terms.

Consider first boson δm^2 with an intermediate baryon loop:

$$\delta m_A^2 \sim \text{Tr}(\lambda_A + g'\Gamma_A + e'\epsilon_A)^2, \quad (5)$$

with $\lambda_A, \Gamma_A, \epsilon_A$ given in Ref. 8; the charge subscripts are as follows: $A=8$ is the $\mathbf{1}$, $A=1-3$ the $\mathbf{8} \ I=1$, $A=4-7$ the $\mathbf{8} \ I=\frac{1}{2}$, $A=0$ the $\mathbf{8} \ I=0$. It turns out that mass splitting in Eq. (5) arises solely from the term $e'\text{Tr}\{\lambda_A, \epsilon_A\}$ and separates the K meson from degenerate π, η mesons. Also $\delta m_8^2=0$, so that there is complete breakdown of the nonet into $\mathbf{8}+\mathbf{1}$; this situation is approached empirically only for $J^P=0^-$.

Moreover, one can argue that even the K splitting in Eq. (5) is probably very small. The same coefficients occur in the single loop for δm of the $\frac{1}{2}^+$ baryons. The octet mass formula, which is known to have high validity for this case, arises only from λ_A, Γ_A interference, the $\{\lambda_A, \epsilon_A\}$ pattern being quite different. It is at once clear that $|e| < |g| \ll 1$; assuming the total $|\delta m(\frac{1}{2}^+)|$ to be of order 0.5 BeV we have $|e| \sim 1\%$. The corresponding K -meson shift is of order 10 MeV, which is less than our empirical limits of error.

Accordingly, we turn to intermediate boson loops, having in mind the model of the (vector)³ vertex. Here

$$\delta m_A^2 \sim \text{Tr}(\Lambda_A + g''G_A + e''E_A)^2 \equiv \left[\sum_B (\Lambda_B + g''G_B + e''E_B)^2 \right]_{AA}. \quad (6)$$

Here the Λ_A, G_A, E_A are 9×9 matrices operating on column vectors of the meson charge nonet. Λ_A is defined by Eq. (8) of Ref. 8, with the added conventions of Eq. (10) and the understanding that $\sum_{AB} = -i \times (1_{AB} - 1_{BA})$, where 1_{AB} is the unit element in the A th row and B th column. For convenience we take E_A to be 2 times the E_A in Eq. (31) of Ref. 8, with the replacement $\frac{1}{2}\Gamma_A \rightarrow \sum_{8A}$. This step already assures coupling of the mesons into a full nonet that does not decompose into $\mathbf{8}+\mathbf{1}$. The quantity G_A is obtained from E_A in the same way as Γ_A from ϵ_A : by dropping all terms in Eq. (31) but the first, so that $G_A = \sum_{8A}$.

In both Eqs. (5) and (6) the successive terms are of symmetry SU_3, R_7 and G_2 . The boson equation (6) has a double form, however, because the (VVV) loop is the same whether we treat it as analogous to a baryon loop for boson self-energy (first form) or to a baryon self-energy loop with boson emission (second form). For the

Λ_A this equivalence is automatically satisfied:

$$\text{Tr}\Lambda_A^2 = 3(1 - \delta_{A8}) = \left[\sum_{B=0}^8 (\Lambda_B)^2 \right]_{AA}, \quad (7)$$

where we have defined $\Lambda_8 \equiv 0$. In other cases it is not always automatic and provides a limiting condition on the definition of quantities like E_8, G_8 , which did not occur in the spinor case.⁸

For the G_A with $A=1-7$

$$\text{Tr}G_A^2 = 2(1 - \delta_{A8})(1 - \delta_{A0}), \quad (8a)$$

$$\left[\sum_{B=1}^7 G_B^2 \right]_{AA} = (1 - \delta_{A0})(1 + 6\delta_{A8}). \quad (8b)$$

These are made compatible by adding

$$G_0=0, \quad G_8 = \begin{pmatrix} 0 & & & & & & & & 0 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & 1 \\ & & & & & & & & & - \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & z \end{pmatrix}. \quad (9)$$

Then

$$\text{Tr}G_A^2 = 2(1 - \delta_{A0})[1 + (6+z^2)\delta_{A8}] = \left[\sum_{B=0}^8 G_B^2 \right]_{AA}. \quad (10)$$

The value of z^2 is not determined, but the general property of this interaction is that the $A=8$ meson has 7 times as many interactions as those with $A=1-7$; to preserve this ratio, we take $z^2=7$. In any case it is true that

$$\text{Tr}\{\Lambda_A, G_A\} = 0 = \left[\sum_{B=0}^8 \{\Lambda_B, G_B\} \right]_{AA}. \quad (11)$$

Instead of dealing with E_A , we take

$$F_A = E_A - G_A, \quad (12a)$$

for $A=1$ to 7 because for these A values

$$\text{Tr}\{F_A, G_A\} = 0 = \left[\sum_{B=1}^7 \{F_B, G_B\} \right]_{AA}. \quad (12b)$$

It seems natural to complete this with

$$F_0 = F_8 = 0. \quad (12c)$$

Then

$$\text{Tr}F_A^2 = 6(1 - \delta_{A0})(1 - \delta_{A8}) = \left[\sum_{B=0}^8 F_B^2 \right]_{AA} \quad (13)$$

and

$$\text{Tr}\{\Lambda_A, F_A\} = -6\delta_{AK} = \left[\sum_{B=0}^8 \{\Lambda_B, F_B\} \right]_{AA}, \quad (14)$$

⁸ D. C. Peaslee, J. Math. Phys. 4, 910 (1963).

⁹ C. Y. Chien, J. Lach, J. Sandweiss, H. D. Taft, N. Yeh, Y. Oren, and M. Webster, Phys. Rev. 152, 1171 (1966).

TABLE III. Nonet charge-splitting parameters.

Orbital \ Parameter	$(X-\pi)$ (BeV ²)	$(\eta-\pi)$ (BeV ²)	$(K-\pi)$ (BeV ²)	I (BeV ²)	g^2	f
¹ S	0.90	0.28	0.23	0.86	0.16	0.04
³ S	~0 ^a	0.46	0.22	0.92	0.25	0.04
³ P	~0 ^a	0.62	0.32	1.24	0.25	0.04
¹ P	-0.5	0.17	[0.13] ^b	0.17	1.0	[0.1] ^b

^a This is the hypothesis of degeneracy discussed above.

^b These entries seem especially uncertain.

where $\delta_{AK} = \delta_{A4} + \delta_{A5} + \delta_{A6} + \delta_{A7}$. Since the $\{\Lambda, F\}$ contribution comes entirely from $\{\Lambda, E\}$ according to Eq. (11), the result in Eq. (14) is just the analog of that found for the baryon loop in Eq. (5). Here, however, there are no apparent restrictions on the magnitudes involved.

In these terms we write the interaction form as $(\Lambda_A + gG_A + fF_A)$ and find

$$\begin{aligned} X &= m_0^2 - 14g^2I, \\ \pi &= m_0^2 - (3 + 2g^2 + 6f^2)I, \\ \eta &= m_0^2 - 3I, \\ K &= m_0^2 - (3 + 2g^2 + 6f^2 - 6f)I. \end{aligned} \quad (15)$$

Here X, π, η, K are generic names for different charge states whatever the orbital, and I is a (cut off) integral that is seen to be positive in sign from the fact that for all bosons $0 < \eta - \pi = (2g^2 + 6f^2)I$. Comparison with the data shows at once that f is very small, just as in the baryon coupling. Dropping terms in f^2 from Eq. (15) yields

$$\begin{aligned} I &= \frac{1}{3}(X - \pi) + 2(\eta - \pi), \\ g^2 &= (\eta - \pi)/2I, \\ f &= (K - \pi)/6I. \end{aligned} \quad (16)$$

These (mass)² differences for the four orbitals considered, and the derived quantities (16) are presented in Table III.

Although it would be premature to take literally all details of Table III, the following qualitative features may be of interest:

- (i) The pattern magnitude I is comparable for all cases except ¹P, where it is particularly small;
- (ii) The SU_3 octet formula requires $f/g^2 = \frac{1}{4}$, and this obtains only in the ¹S case;
- (iii) Variation among the patterns seems to reflect mainly that in g^2 , while f remains small. Note that the magnitude of g indicates a generally appreciable admixture of R_7 to the dominant SU_3 .

VI. HIGHER RESONANCES

The apparent adequacy of the present phenomenology for S and P orbitals emboldens us to try an extension to at least the ³D system with what fragmentary data are available. Chief among these are the missing-mass spectrometer results¹⁰ that $Y=0$, $I_z = -1$

¹⁰ M. N. Focacci, W. Kienzle, B. Levrat, B. C. Maglić, and M. Martin, Phys. Rev. Letters 17, 890 (1966).

(presumably $I=1$) states occur at $m=1.63, 1.70, 1.75, 1.93$ BeV. Higher resonances are quoted, but the listing is less likely to be exhaustive. Resonances decaying by $(\pi^+\pi^0)$ have been reported¹¹ in bubble chamber work at 1.62 and 1.91 BeV. If we assume a pattern similar to ³P, the assignments are immediate: ³D₁ to 1.63 and ³D₃ to 1.92, these being the only D states with allowed 2π decay modes. If the tensor coupling is absent for $\mathbf{8}$ $I=1$ as before, the ³D₂ state is predicted at 1.75 BeV, in good agreement with observation. This leaves the 1.70-BeV state to be ¹D₂. The spin-orbit coefficient here is $s(^3D) = 0.21$ BeV², a reduction of about 15% from $s(^3P)$, which seems reasonable.

If the $K^*(^3D_1)$ is really at 1395 MeV, as suggested above, then with s and t values comparable to those already known the three $K^*(D)$ states should all cluster in the region 1.75–1.90 BeV. There have certainly been a number of K^* resonances observed in this region, but their resolution is not yet clear.

Our initial postulate requires the ³D $\mathbf{1}$ states to be almost coincident with the corresponding $\mathbf{8}$ $I=1$ resonances—say at $\sim 1.65, 1.75, 1.95$ BeV. If the ¹D pattern is like the ¹P, the $\mathbf{1}$ state will be somewhat lower, at ~ 1.60 BeV. In fact an $I=0$ resonance has been reported at this energy¹² with a 4π decay mode, which would be allowed for ¹D.

There is at present a paucity of data on $(\bar{K}K\pi^n)$ resonances for masses in excess of 1.8 BeV, where practically all the D -state resonances of η type ($\mathbf{8}$ $I=0$) should be.

VII. A PARITY

The concept of A parity¹³ as distinct from G parity has had empirical vicissitudes. At any given time there is an average of two outstanding data in contradiction with A conservation, while all others agree. The chief positive indication is that the outstanding objections change continuously with time. As an example of this process, we note that the main current objections to A conservation disappear quite incidentally to the arguments presented here. First, the A -violating $(K\pi/K\pi\pi)$ ratio for $K^*(1400)$ could be resolved if there are really two different resonances. Second, abandonment of the SU_3 octet mass formula means that $\omega - \varphi$ mixing angles of 40° are no longer required: about 10° is not incompatible with the usual degree of A failure, say 3–5%.

¹¹ M. Deutschmann, R. Schulte, R. Steinberg, H. Weber, W. Woischnig, C. Grote, J. Klugow, W. Meyer, S. Nowak, S. Brandt, V. T. Cocconi, O. Czyzewski, P. F. Dalpiaz, E. Flaminio, G. Kellner, and D. R. O. Morrison, Phys. Letters 18, 351 (1965).

¹² W. J. Kernan, D. E. Lyon, and H. B. Crawley, Phys. Rev. Letters 15, 803 (1965). It should also be remarked, however, that a $(\rho\rho)$ resonance has been reported at 1.41 BeV [A. Bettini, M. Cresti, S. Limentani, A. Loria, L. Peruzzo, R. Santangelo, L. Bertanza, A. Bigi, R. Carrara, R. Casali, E. Hart, and P. Loriccia, Nuovo Cimento 42, 695 (1966)].

¹³ J. B. Bronzan and F. E. Low, Phys. Rev. Letters 12, 522 (1964); D. C. Peaslee, Phys. Rev. 117, 873 (1960).