$$|{}^3S_1\rangle=\frac{1}{\sqrt{6}}\{|11;++\rangle+|11;--\rangle\}+\frac{1}{\sqrt{3}}\{|11;+-\rangle+|11;-+\rangle\}; \quad (A.7)$$

$$|{}^3D_1\rangle=-\frac{1}{\sqrt{3}}\{|11;++\rangle+|11;--\rangle\}+\frac{1}{\sqrt{6}}\{|11;+-\rangle+|11;-+\rangle\}. \quad (A.8)$$

PHYSICAL REVIEW VOLUME 159, NUMBER 5 25 JULY 1967

Weak-Electromagnetic Decays of Hyperons in a Broken $SU(3)\otimes SU(3)$ Model*

D. S. CARLSTONE AND WALTER A. SIMMONS
Department of Physics, Purdue University, Lafayette, Indiana

(Received 19 January 1967; revised manuscript received 3 April 1967)

The weak (strangeness-changing)–electromagnetic decays of the hyperons of the type $B\rightarrow B\gamma$ are considered in a broken $SU(3)\otimes SU(3)$ model. Various sets of assumptions regarding the transformation properties of the particles, the breaking, and the Hamiltonian are examined and for each a set of sum rules for the decay amplitudes is obtained.

I. INTRODUCTION

SINCE the assumption that the weak hadron currents generate the algebra of $SU(3)\otimes SU(3)$ has recently found wide application in weak-interaction physics, several authors have investigated the possibility that the $SU(3)\otimes SU(3)$ may be an approximate invariance group of the weak Hamiltonian. Iizuka and Miyamoto[1] investigate the possibility that $SU(3)\otimes SU(3)$ may be an exact invariance group of H_w and found that the nonleptonic decays cannot be correctly described in such a scheme. Later, Schechter and Ueda[2] found that broken chiral $SU(3)\otimes SU(3)$ can be used to obtain a new sum rule for the hyperon nonleptonic decays which is in rough agreement with experiment.

The purpose of this paper is to investigate the application of the group structure of broken chiral $SU(3)\otimes SU(3)$ to the weak-electromagnetic processes which have recently been examined with a current algebra approach by Graham and Pakvasa.[3]

II. THE $SU(3)\otimes SU(3)$ MODEL

The space-time structure of the weak-electromagnetic (WE) Hamiltonian is assumed to be given by[4,5]

$$H_{\mathrm{WE}}=\bar{N}(A+B\gamma_5)\sigma_{\mu\nu}NF_{\mu\nu},$$

where N is a baryon 4-spinor. Before examining the $SU(3)\otimes SU(3)$ structure of this Hamiltonian, let us review the model.

According to the model of Marshak, Mukunda, and Okubo[6] the $SU(3)\otimes SU(3)$ algebra is generated by two sets of $SU(3)$ matrices, $A_\nu{}^\mu$ and $B_{\nu'}{}^{\mu'}$, which satisfy the commutation relations

$$[A_\nu{}^\mu,A_\beta{}^\alpha]=\delta_\nu{}^\alpha A_\beta{}^\mu-\delta_\beta{}^\mu A_\nu{}^\alpha,$$
$$[B_{\nu'}{}^{\mu'},B_{\beta'}{}^{\alpha'}]=\delta_{\nu'}{}^{\alpha'}B_{\beta'}{}^{\mu'}-\delta_{\beta'}{}^{\mu'}B_{\nu'}{}^{\alpha'},$$
$$[A_\nu{}^\mu,B_{\beta'}{}^{\alpha'}]=0.$$

For every $SU(3)\otimes SU(3)$ tensor, primes are used throughout this paper to indicate those indices transformed by $B_{\nu'}{}^{\mu'}$ and the unprimed indices are transformed by $A_\nu{}^\mu$. Under parity P and charge conjugation C, the generators are transformed:

$$P:\quad A_\nu{}^\mu\rightarrow B_{\nu'}{}^{\mu'},\qquad B_{\nu'}{}^{\mu'}\rightarrow A_\nu{}^\mu,$$
$$C:\quad A_\nu{}^\mu\rightarrow -B_{\mu'}{}^{\nu'},\quad B_{\nu'}{}^{\mu'}\rightarrow -A_{\mu'}{}^{\nu'}.$$

An irreducible representation of $SU(3)\otimes SU(3)$, (μ_1,μ_2) is transformed according to

$$P:\quad (\mu_1,\mu_2)\rightarrow(\mu_2,\mu_1),$$
$$C:\quad (\mu_1,\mu_2)\rightarrow(\mu_2{}^*,\mu_1{}^*).$$

In the notation of Schechter and Ueda,[7] a four-component baryon spinor $N_b{}^a$ transforms under $SU(3)\otimes SU(3)$ as

$$N_b{}^a=\begin{pmatrix} f_b{}^{a'} \\ -i\sigma_2 g^a{}_{b'}\end{pmatrix},$$

* Work supported in part by an institution grant from the National Aeronautical and Space Administration, Grant No. NGR 15-005-021.

[1] J. Iizuka and Y. Miyamota, Nuovo Cimento **36**, 676 (1965).

[2] J. Schechter and Y. Ueda, Phys. Rev. **148**, 1424 (1966).

[3] S. Pakvasa (private communication).

[4] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

[5] M. Hirooka and M. Hosoda, Progr. Theoret. Phys. (Kyoto) **35**, 648 (1965).

[6] R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. **137**, B698 (1965).

[7] J. Schechter and Y. Ueda, Phys. Rev. **144**, 1338 (1966).

where f and g are two-component spinors, and a $[(3,3^*),(3^*,3)]$ baryon assignment is indicated. Since $\bar{N}_b{}^a=(N_b{}^a)^+\gamma_4$, then

$$\bar{N}_b{}^a(1-\gamma_5)N_d{}^c\sim\bar{f}_{b'}{}^a\sigma_2 g_{d'}{}^c,$$
$$\bar{N}_b{}^a(1+\gamma_5)N_d{}^c\sim\bar{g}_b{}^{a'}\sigma_2 f_d{}^{c'}.$$

It is seen, for the $[(3,3^*),(3^*,3)]$ baryon assignment that $\bar{f}\sim(3^*,3)$ and $g\sim(3^*,3)$. Since $(\bar{f}\times g)=(3,3^*)+(3,6)+(6^*,3^*)+(6^*,6)$, it is impossible [in the limit of exact $SU(3)\otimes SU(3)$ symmetry] to construct a Hamiltonian which transforms as (λ,λ'), where λ and λ' are $SU(3)$ representations of zero triality. The same is true of the $[(3,6),(6,3)]$ baryon assignment. It is necessary then to break the symmetry. The simplest possible breaking is $(3,3^*)+(3^*,3)$,

$$\sum_\mu(T_{\mu'}{}^\mu+T_\mu{}^{\mu'}),$$

but in addition $(6,6^*)+(6^*,6)$ breaking

$$\sum_{\mu,\nu}(T_{\mu'}{}^\mu{}_{\nu'}{}^\nu+T_\mu{}^{\mu'}{}_\nu{}^{\nu'})$$

has been considered.

In the case of an $[(8,1),(1,8)]$ baryon assignment, one can construct, in the unbroken case, only one interaction which, in fact, transforms as (8,8). For the case of breaking as above, no appropriate terms can be constructed. For this assignment, however, the decay $\Sigma^+\to p\gamma$ is forbidden, and since this decay has been observed,[8] this assignment is given no further consideration.

Now let us investigate the transformation properties of the weak-electromagnetic Hamiltonian. The electromagnetic interaction is assumed to transform with respect to $SU(3)\otimes SU(3)$ as[9] $[(8,1),(1,8)]$. This assignment conserves parity P, and CP since

$$P:\quad[(8,1),(1,8)]\to[(8,1),(1,8)],$$
$$CP:\quad[(8,1),(1,8)]\to[(8,1),(1,8)].$$

Similarly the weak interaction is assumed to transform as[2] (8,1), which is seen to conserve CP and to have indefinite transformation properties with respect to parity:

$$P:\quad(8,1)\to(1,8)=\tfrac{1}{2}[(8,1)+(1,8)]+\tfrac{1}{2}[-(8,1)+(1,8)],$$
$$CP:\quad(8,1)\to(8,1).$$

For the over-all transformation properties of the weak-electromagnetic Hamiltonian, everything in the product

$$[(8,1),(1,8)]\otimes(8,1)$$

must be considered, except, of course, (1,1). Such a Hamiltonian does not have as simple transformation properties as we would like. Therefore it is convenient to write the effective weak-electromagnetic Hamiltonian $H_{\rm WE}$ as

$$H_{\rm WE}=H_1+H_2,$$

where

$$H_1\sim(8,8)$$

and

$$H_2\sim(\mu,1),\quad\mu=8,\,10,\,10^*,\,27.$$

Let us assume, then, that this process is dominated by either H_1 or H_2. We would prefer to choose H_1 on the basis of simplicity, since with H_1 we have only one representation to consider. However, it turns out that with a $[(3,3^*),(3^*,3)]$ baryon assignment, H_2 together with CP invariance requires that all parity-violating amplitudes vanish. For this reason and for completeness, the Hamiltonian H_2 will also be given consideration.

Therefore, for the case of H_1 we assign the weak interaction to the unprimed octet and the electromagnetic interaction to the primed octet of $(8,8')$. Note that since the two octets are not identical, $(8,8')$ still has indefinite transformation properties with respect to parity. In terms of tensor indices, then, the Hamiltonian H_1 transforms as $T_{21'}{}^{31'}$. For the Hamiltonian H_2, the term $H_2(8,1)$ will transform as $T_2{}^3$ and the terms $H_2(10,1)$, $H_2(10^*,1)$, and $H_2(27,1)$ will transform as $T_{21}{}^{31}$, with appropriate symmetry of the indices.

Furthermore, we assume $TL(1)$ invariance,[10] or symmetry under the interchange of indices 2 and 3, as in Ref. 2. The use of $TL(1)$ invariance in $SU(3)\otimes SU(3)$ should be valid, because in each case this transformation acts on only one $SU(3)$. That is to say, $TL(1)$ invariance interchanges the unprimed indices 2 and 3, and in no way does it mix primed and unprimed indices. If $TL(2)$ invariance were assumed then, of course, the parity-violating (pv) and parity-conserving (pc) amplitudes would be exchanged.

III. SUM RULES FOR $B\to B_\gamma$

The possible decays of this type are

$$\Sigma^+\to p\gamma,$$
$$\Xi^-\to\Sigma^-\gamma,$$
$$\Lambda\to n\gamma,$$
$$\Sigma^0\to n\gamma,$$
$$\Xi^0\to\Lambda\gamma,$$
$$\Xi^0\to\Sigma^0\gamma.$$

The individual cases for baryon and breaking assignments are given below.

[8] M. Bazin *et al.*, Phys. Rev. Letters **14**, 154 (1964); U. Nauenberg *et al.*, Bull. Am. Phys. Soc. **10**, 466 (1965).

[9] M. Gell-Mann, Physics **1**, 63 (1964). This follows from the assignment of the generators to $(8,1)+(1,8)$.

[10] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

Case I

With a $[(3,3^*),(3^*,3)]$ baryon assignment and $(3,3^*)+(3^*,3)$ breaking the Hamiltonian can be written

$$H_1=a_1[(\bar{f}\sigma_2 g)_{(3,3^*)}\times T(3^*,3)]_{(8,8)}$$
$$+a_2[(\bar{f}\sigma_2 g)_{(3,6)}\times T(3^*,3)]_{(8,8)}$$
$$+a_3[(\bar{f}\sigma_2 g)_{(6^*3^*)}\times T(3^*,3)]_{(8,8)}$$
$$+a_4[(\bar{f}\sigma_2 g)_{(6^*,6)}\times T(3^*,3)]_{(8,8)}+\text{H.c.}+\text{TL-c.},$$
$$H_2=b_1[(\bar{f}\sigma_2 g)_{(6^*,3^*)}\times T(3^*,3)]_{(10^*,1)}$$
$$+b_2[(\bar{f}\sigma_2 g)_{(3,3^*)}\times T(3^*,3)]_{(8,1)}$$
$$+b_3[(\bar{f}\sigma_2 g)_{(6^*3^*)}\times T(3^*,3)]_{(8,1)}+\text{H.c.}+\text{TL-c.},$$

where TL-c. refers to the $TL(1)$ conjugate. For example, the term

$$[(\bar{f}\sigma_2 g)_{(3,6)}\times T(3^*,3)]_{(8,8)}$$

is explicitly given by

$$\epsilon_{\alpha\beta 2}\epsilon^{\gamma'\mu'1'}\delta_{\mu'}{}^3\{\bar{f}_{1'}{}^{\alpha}\sigma_2 g_{\gamma'}{}^{\beta}\},$$

where the brace implies symmetrization in $1'$ and γ', and the breaking $T_{\mu'}{}^{\mu}$ has been written $\delta_{\mu'}{}^{\mu}$. CP invariance is sufficient to show that the constants a_i can be taken real.

The first coupling in the Hamiltonian H_1 does not contribute to these decays, so the twelve amplitudes are written in terms of three constants, and nine sum rules are expected. The sum rules are

$$(\sqrt{6})[A(\Lambda\to n\gamma)-A(\Xi^0\to\Lambda\gamma)]$$
$$=\sqrt{2}[A(\Xi^0\to\Sigma^0\gamma)-A(\Sigma^0\to n\gamma)] \quad (1)$$
$$=A(\Sigma^+\to p\gamma)-A(\Xi^-\to\Sigma^-\gamma), \quad (2)$$
$$B(\Sigma^+\to p\gamma)=B(\Xi^-\to\Sigma^-\gamma)=0, \quad (3)$$
$$\sqrt{3}B(\Lambda\to n\gamma)=\sqrt{3}B(\Xi^0\to\Lambda\gamma)$$
$$=-B(\Sigma^0\to n\gamma)$$
$$=-B(\Xi^0\to\Sigma^0\gamma), \quad (4)$$
$$A(\Sigma^+\to p\gamma)+\sqrt{2}A(\Sigma^0\to n\gamma)=\sqrt{6}B(\Lambda\to n\gamma), \quad (5)$$

where A represents the pc amplitudes and B the pv.

For the ninth sum rule, either of the following may be considered to be independent:

$$\sqrt{3}A(\Lambda\to n\gamma)-A(\Sigma^0\to n\gamma)=2A(\Xi^0\to\Sigma^0\gamma),$$
$$2A(\Lambda\to n\gamma)-A(\Xi^0\to\Lambda\gamma)=\sqrt{3}A(\Xi^0\to\Sigma^0\gamma). \quad (6)$$

The pv amplitudes are consistent with sum rules similar to Eq. (9), but with the sign of the Ξ^0 reversed:

$$\sqrt{3}B(\Lambda\to n\gamma)-B(\Sigma^0\to n\gamma)=-2B(\Xi^0\to\Sigma^0\gamma),$$
$$2B(\Lambda\to n\gamma)+B(\Xi^0\to\Lambda\gamma)=\sqrt{3}B(\Xi^0\to\Sigma^0\gamma). \quad (7)$$

Using H_2, it is found that the pv amplitudes arise only from the term which transforms as (10,1). Note that

$$CP:\quad (10^*,1)\to(10,1).$$

Since (10,1) cannot be constructed with these assignments, if CP invariance is assumed all pv amplitudes vanish. The sum rules for the pc amplitudes are Eq. (6) and

$$A(\Sigma^+\to p\gamma)=\sqrt{2}A(\Sigma^0\to n\gamma),$$
$$A(\Xi^-\to\Sigma^-\gamma)=\sqrt{2}A(\Xi^0\to\Sigma^0\gamma). \quad (8)$$

Finally, if both H_1 and H_2 must be included in the (CP-invariant) Hamiltonian, the surviving sum rules are (1), (3), (4), and (6). Equation (6) is the $SU(3)$ result for both pv and pc amplitudes.[4]

Case II

With the baryons assigned to $[(3,3^*),(3^*,3)]$ and with $[(6,6^*)+(6^*,6)]$ breaking, the Hamiltonian H_1 constructed as in Case I contains four contributing amplitudes. The following sum rules are obtained:

$$\sqrt{3}A(\Lambda\to n\gamma)=-\sqrt{3}A(\Xi^0\to\Lambda\gamma)$$
$$=-A(\Sigma^0\to n\gamma)$$
$$=A(\Xi^0\to\Sigma^0\gamma), \quad (9)$$
$$\tfrac{1}{2}[A(\Sigma^+\to p\gamma)+A(\Xi^-\to\Sigma^-\gamma)]=\sqrt{2}B(\Sigma^0\to n\gamma), \quad (10)$$

and Eqs. (3) and (4). Using H_2, the situation is similar to Case I, in the CP invariance again requires that all pv amplitudes vanish. In addition to Eq. (1), the following are satisfied:

$$A(\Sigma^+\to p\gamma)+A(\Xi^-\to\Sigma^-\gamma)$$
$$=3(\sqrt{6})[A(\Lambda\to n\gamma)+A(\Xi^0\to\Lambda\gamma)]$$
$$=\sqrt{2}[A(\Sigma^0\to n\gamma)+A(\Xi^0\to\Sigma^0\gamma)].$$

Finally, if the full Hamiltonian H_1+H_2 is used the surviving sum rules are (1), (3), and (4) and, of course, (6).

Case III

The baryons, here, are assigned to $[(3,6),(6,3)]$ with the algebra broken by $(3,3^*)+(3^*,3)$ terms. For the Hamiltonian H_1, in addition to Eqs. (1), (3), (6), and (7), the following hold:

$$A(\Xi^-\to\Sigma^-\gamma)=0,$$
$$(\sqrt{6})[B(\Lambda\to n\gamma)-B(\Xi^0\to\Lambda\gamma)]$$
$$=3\sqrt{3}[B(\Sigma^0\to n\gamma)-B(\Xi^0\to\Sigma^0\gamma)],$$
$$A(\Sigma^+\to p\gamma)-3\sqrt{2}A(\Sigma^0\to n\gamma)$$
$$=(8/3)(\sqrt{6})B(\Lambda\to n\gamma)+\sqrt{2}B(\Sigma^0\to n\gamma).$$

With the use of the Hamiltonian H_2, the pv amplitudes do not vanish. In addition to Eqs. (3), (6), and (7), we find

$$A(\Sigma^+\to p\gamma)+\tfrac{3}{2}A(\Xi^-\to\Sigma^-\gamma)=\tfrac{1}{3}(\sqrt{6})A(\Lambda\to n\gamma)$$
$$+\sqrt{2}A(\Sigma^0\to n\gamma),$$
$$A(\Sigma^+\to p\gamma)+\tfrac{1}{2}A(\Xi^-\to\Sigma^-\gamma)=\tfrac{2}{3}(\sqrt{6})B(\Xi^0\to n\gamma),$$
$$(\sqrt{6})A(\Xi^0\to\Lambda\gamma)-\sqrt{2}A(\Xi^0\to\Sigma^0\gamma)=\tfrac{4}{3}(\sqrt{6})B(\Xi^0\to\Lambda\gamma)$$
$$-\tfrac{2}{3}(\sqrt{6})B(\Lambda\to n\gamma).$$

Case IV

With the baryons assigned to [(3,6),(6,3)] and with (6,6*)+(6*,6) symmetry-breaking terms transforming as H_1 only, can be constructed. This may be regarded as one motivation for considering (6,6*)+(6*,6) breaking, since the choice between H_1 and H_2 is already determined. The sum rules are given by Eqs. (3), (6), and (7) and by

$$+(13\sqrt{6})B(\Lambda\to n\gamma)=35A(\Sigma^+\to p\gamma)+\tfrac{3}{2}A(\Xi^-\to\Sigma^-\gamma)-(3\sqrt{6})A(\Lambda\to n\gamma)+(56/3)(\sqrt{6})A(\Xi^0\to\Lambda\gamma),$$

$$(13\sqrt{6})B(\Xi^0\to\Lambda\gamma)=58A(\Sigma^+\to p\gamma)+(15/2)A(\Xi^-\to\Sigma^-\gamma)-(2\sqrt{6})A(\Lambda\to n\gamma)-(85/3)A(\Xi^0\to\Lambda\gamma).$$

IV. THE THREE-BODY DECAYS $B\to B\pi\gamma$

It has been shown[4] that in $SU(3)$ no sum rules can be obtained for decays of this type without introducing various additional assumptions. The same is found to be true in the $SU(3)\otimes SU(3)$ case. For example, if the baryons, mesons, and breaking are each assigned to [(3,3*),(3*,3)] and if the Hamiltonian is assumed to transform as (8,8), then there are twenty possible amplitudes, four unbroken and sixteen broken, in the Hamiltonian. In order to obtain sum rules, it is necessary to introduce additional assumptions.

One possibility is to insist that the Hamiltonian transform as (8,1). This leads to the unlikely result that the sum rules for the weak-electromagnetic decays are identical in form with those obtained by Schechter and Ueda for the hyperon nonleptonic decays.

We might also examine some special cases of more general couplings which are in some sense simple. For instance, the most general form of H_1 is given by

$$H\sim\sum_{\nu\nu'}\sum_{\lambda\lambda'}\{[(\bar{f}\sigma_2 g)_{(\nu,\nu')}\times M(3,3^*)]_{(\lambda,\lambda')}\times T(3^*,3)\}_{(8,8)},$$

where

$$(\nu,\nu')=(3,3^*),\ (3,6),\ (6^*,3^*),\ (6^*,6),$$

and

$$(\lambda,\lambda')=(3^*,3),\ (3^*,6^*),\ \text{etc}.$$

As mentioned above, additional assumptions are still needed. By analogy with "octet dominance" in the nonleptonic decays it might be natural to limit ourselves to that coupling which gives rise only to the **8** representation (aside from the singlet). However, this coupling, the one with $(\lambda,\lambda')=(3^*,3)$, gives no contributions to the three-body amplitudes. A simple (speculative) generalization of this "octet dominance" assumption is to assume that only those representations are present in the three-body Hamiltonian as are present in the corresponding nonleptonic decay Hamiltonian (i.e., the **1**, **8**, and **27**). There are two possible couplings of this type, the one above with $(\lambda,\lambda')=(6,6^*)$ and a similar one with $(\lambda,\lambda')=(3^*,3)$ and with the breaking T taken to transform as (6,6*). The first of these is uninteresting as it is overly restrictive. The second one gives rise to sum rules. Only the unbroken terms give relationships between the pv and pc amplitudes, while the broken terms contribute to the pv amplitudes only:

$$B(\Sigma_-^-)-\frac{1}{\sqrt{2}}B(\Sigma_0^+)+\frac{1}{2\sqrt{3}}[B(\Xi_0^0)-B(\Lambda_0^0)]=-A(\Sigma_-^-)-\frac{3}{\sqrt{2}}[(A(\Sigma_0^+)-2A(\Sigma_+^+)]$$

and

$$\frac{1}{\sqrt{6}}B(\Xi_-^-)+B(\Sigma_+^+)+\frac{1}{\sqrt{2}}B(\Sigma_0^+)=A(\Sigma_-^-)-3A(\Sigma_+^+)-\frac{1}{\sqrt{2}}A(\Sigma_0^+).$$

This three-body decay model cannot be taken too seriously; however, some features of it may be approximately satisfied.

V. DISCUSSION

The results which are common to all cases with the most general Hamiltonian $H=H_1+H_2$ are Eqs. (3), (6), and (7). Equation (3), which says the pv amplitudes for $\Sigma^+\to p\gamma$ and $\Xi^-\to\Sigma^-\gamma$ vanish, is also a result of $SU(3)$ plus $TL(1)$ invariance.[11] Our Eq. (6) holds for the pc amplitudes only, but in $SU(3)$ both pv and pc amplitudes satisfy these relations. Equation (7), which differs from Eq. (6) in phase only, is the analogus result for pv amplitudes. Assuming that the experimental pv amplitudes do not vanish, this phase difference may eventually be tested, but for some time, at least, these results will be indistinguishable.

Also observe that in Case III with H_1, but not H_2, the $\Xi^-\to\Sigma^-\gamma$ decay is forbidden altogether. According to Ref. 4, this mode is not expected to compete with $\Xi^-\to\Lambda\pi^-$, and so this prediction, perhaps, cannot be tested.

As has been seen using the Hamiltonian H_2, and CP invariance, all pv amplitudes vanish in Cases I and II. This result has been obtained by Pakvasa and Graham[3] in a current algebra model, by Toda[12] in a $U(12)$ model, and by Akemollo *et al.*[13] and Matinyar[14] in an $SU(6)$ model. Tanaka[15] has also obtained this result in $SU(3)$ using an octet Hamiltonian. A common feature of all these models is CP invariance.

As an attempt to understand the reason for the phase difference between pc and pv amplitudes in Eqs. (6)

[11] Y. Hara, Phys. Rev. Letters **12**, 378 (1964).
[12] A. Toda, Prog. Theoret. Phys. (Kyoto) **34**, 702 (1965).
[13] M. Ademollo, F. Bacella, and R. Gatto, Nuovo Cimento **30**, 316 (1965).
[14] S. G. Matinyan, Yadernaya Fiz. **2**, 151 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 106 (1965)].
[15] K. Tanaka, Phys. Rev. **140**, B463 (1965).

and (7), consider the consequences of a $|U_f - U_i| = 1$ selection rule. Here, U_i refers to the $SU(3)$ U spin of the initial baryon and U_f is that of the final baryon. Such a selection rule would forbid the transitions

$$|\tfrac{1}{2} - \tfrac{1}{2}\rangle \to |\tfrac{1}{2}\tfrac{1}{2}\rangle,$$
$$|1-1\rangle \to |10\rangle \to |11\rangle,$$

and would allow

$$|1-1\rangle \to |00\rangle \to |11\rangle.$$

The quantum numbers here are, of course, U and U_3. (For our phase choice see the equivalent K-spin assignments of Rosen.[10])

The results obtained with such a selection rule are the same as those given by Eqs. (3) and (4). Therefore, the pv sum rules are consistent with a $|U_f - U_i| = 1$ selection rule in $SU(3)$. However, in $SU(3)$ it is not clear how to define an $SU(3)$ spin with the couplings we have used. Therefore, we cannot show that there is a $|U_f - U_i|$ selection rule as a consequence of the model but we suggest there may be some connection.

It is interesting to note that the major difference between Case I and Case II is in the sum rule relating pv and pc amplitudes. This suggests that it might be of interest to use $(6,6^*)+(6^*,6)$ breaking for the nonleptonic decays. The result is that instead of Eq. (1) of Ref. 2 we obtain

$$A(\Sigma_+^+) + B(\Sigma_+^+) = 0.$$

Since this is obviously not satisfied, the use of the $(6^*,6)+(6,6^*)$ breaking is questionable.

ACKNOWLEDGMENTS

The authors wish to thank Professor S. P. Rosen for several suggestions and for reading the manuscript. We also wish to thank Dr. S. Pakvasa for several useful discussions and Dr. R. H. Graham for a helpful correspondence.

PHYSICAL REVIEW VOLUME 159, NUMBER 5 25 JULY 1967

Some Remarks on Superconvergence Relations

S. RAI CHOUDHURY AND S. NUSSINOV*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 2 February 1967)

Possible superconvergence sum rules are investigated for meson-baryon and baryon-antibaryon scattering. For meson-baryon systems, two simple procedures involving differentiation with respect to t at $t=0$ are presented, and the explicit subtraction of leading Regge-pole contributions is discussed and applied to π-Σ and π-N systems. Finally, we write down superconvergence relations for backward N-$\bar{N}$ scattering and obtain sum rules involving various meson-nucleon coupling constants.

I. INTRODUCTION

IT has been remarked[1] that certain integral relations for invariant amplitudes follow directly from dispersion relations and asymptotic bounds. These "superconvergence relations (SCR's)" usually appear in reactions of particles with spin involving helicity flip in the crossed channel.[2] Using a Regge formalism and conjectured upper bounds for the intercept at $t=0$ for trajectories $\alpha(t)$ with "exotic" quantum numbers, one can find cases where the combined effect of internal quantum numbers and helicity flips suggest a SCR for forward scattering amplitudes.[1,3,4]

The SCR's predict interesting relations if they are saturated by the contributions of a few low-lying states. It has been found in the few cases discussed so far that such a saturation is not unlikely and in particular may reproduce results of various symmetry schemes.[3,4,5] This encourages further investigation of other hopefully superconvergent relations.

By making use of the requirement that superconvergence holds also for negative values of the momentum-transfer variable t, or of the explicit form of amplitudes in the Regge theory, more SCR's can be suggested. Two simple procedures involving differentiation with respect to t at $t=0$ and explicit subtraction of leading Regge-pole contributions are discussed and illustrated in the next two sections.

In Sec. IV we note that a division by the threshold factor leads directly to SCR's for partial-wave amplitudes.

We next consider three SCR's which hold for backward $\bar{N}N$ scattering provided $\alpha_{B=2}(0)<0$, where $\alpha_{B=2}(0)$ is the zero-energy intercept of a Regge trajectory with baryon number $B=2$. The resulting sum

* Work supported by the U. S. Air Force Office of Research, Air Research and Development Command, under Contract No. AF 49(638)-1545.

[1] V. de Alfaro *et al.*, Phys. Letters **21**, 576 (1966).

[2] T. L. Trueman, Phys. Rev. Letters **17**, 1198 (1966).

[3] P. Babu, F. Gilman, and M. Suzuki, Caltech report, 1966 (unpublished).

[4] B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967).

[5] R. Oehme, Phys. Rev. (to be published).