The angles in Sec. IV are given by

$$\cos\psi_{12}=(2q_aq_1)^{-1}\left[-w_bw_1'-\frac{m_b^2}{\sqrt{(s_{ab}t_{3a})}}\times(s_{12}-m_b^2+m_a^2-m_3^2)\right],$$

$$\cos\psi_b=(2q_3q_1)^{-1}\left[w_3(\sqrt{s_{ab}}-w_1')-\frac{s_{12}}{\sqrt{(s_{ab}t_{3a})}}\times(s_{12}-m_b^2+m_a^2-m_3^2)\right],$$

$$\cos\psi_3=(2q_aq_a)^{-1}\left[w_aw_a'-\frac{m_a^2}{\sqrt{(s_{ab}t_{3a})}}\times(s_{12}-m_b^2+m_a^2-m_3^2)\right],$$

$$\cos\psi_a=(2q_3q_a)^{-1}\left[-w_3w_b'-\frac{m_3^2}{\sqrt{(s_{ab}t_{3a})}}\times(s_{12}-m_b^2+m_a^2-m_3^2)\right].$$

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Intrinsic Orbital Angular Momentum and W Spin

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The W-spin classification is extended to include states of quarks and antiquarks with finite intrinsic oribtal angular momentum (orbital excitation). The conservation of this extended W spin is shown to be automatically incorporated in many quark-model treatments, and some applications are discussed.

I. EXTENDED DEFINITION OF W SPIN

THE general W-spin classification has been given for all states which are constructed from quarks and antiquarks in a relative s state.[1] In this paper we consider the extension of the W-spin classification to states having intrinsic orbital angular momentum. This is not only an abstract mathematical problem of defining a logical extension of W spin to cases of finite orbital angular momentum. There must also be some physical reason for the application of W spin to such systems, such as the requirement of W-spin conservation in a more inclusive theory or specific model. The W spin as defined here is not only a straightforward extension; its conservation is relevant in many specific quark models for transitions. This is explicitly demonstrated in Sec. II. In Sec. III some implications of W-spin conservation are discussed.

Consider hadron states at rest, constructed in the quark model by adding orbital excitation. The L-S coupling scheme has always been used in constructing models of orbital excitation.[2] The spins of all the quarks $\mathbf{S}_q$ and antiquarks $\mathbf{S}_{\bar{q}}$ are coupled to a total quark spin $\mathbf{S}$. The orbital angular momentum $\mathbf{L}$ is then coupled to $\mathbf{S}$ to give the total angular momentum $\mathbf{J}$ which is the spin of the physical particle.

$$\mathbf{S}_q+\mathbf{S}_{\bar{q}}=\mathbf{S}, \tag{1a}$$

$$\mathbf{L}+\mathbf{S}=\mathbf{J}. \tag{1b}$$

We now define the W spin of such a state in the conventional fashion,[1] in terms of the quark spin variables S_q and $S_{\bar{q}}$. In the language of group theory we are using[3] the group $U(6)\times U(6)\times O(3)$, where the $U(6)\times U(6)$ is the same group which was used in I. The spin variables in $U(6)\times U(6)$ are the quark spins $\mathbf{S}_q$ and $\mathbf{S}_{\bar{q}}$ and are independent of $\mathbf{L}$. The $O(3)$ group is just the orbital rotations in three-dimensional configuration space, *excluding spins*.

The W-spin classification for any state of quarks and antiquarks defined in the L-S coupling scheme is now given directly by the method described in I. However, because $\mathbf{S}$ is no longer equal to $\mathbf{J}$, an additional transformation involving Clebsch-Gordan coefficients is required between the states used in I which are eigenstates of S_z and the polarization states of physical particles. The latter are functions of J and J_z, and usually not of S_z.

$$|L,S_q,S_{\bar{q}},S,J,M\rangle=\sum_{M_L,M_S}(LSM_LM_S|JM)\times|L,S_q,S_{\bar{q}},S,M_L,M_S\rangle, \tag{2}$$

where M, M_L, and M_S are eigenvalues of J_z, L_z, and S_z,

[1] H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, Phys. Rev. 146, 1052 (1966), hereafter referred to as I. A detailed list of earlier references are given in this paper.
[2] For a general review of the quark model see R. H. Dalitz, in *High Energy Physics*, edited by C. De-Witt and M. Jacob (Gordon and Breach, Science Publishers, Inc., New York, 1965) p. 253.
[3] K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 163 (1965); R. Gatto, H. Maiani, and G. Preparata, Nuovo Cimento 39, 1192 (1965).

respectively, and $(LSM_LM_S|JM)$ is a Clebsch-Gordan coefficient.

II. W-SPIN CONSERVATION IN QUARK MODELS

The use of this definition of W spin in physical problems is of interest in any theory or model where W spin is conserved or where interactions which break the symmetry have simple transformation properties under W spin or the larger group $SU(6)_W$. It has been shown that W-spin conservation follows in a restricted form from general arguments based on the collinear little group g_W, which is a subgroup of the improper Lorentz group.[4] Here we consider a few explicit examples and see that the general arguments are unaffected by the addition of orbital excitation.

Consider transitions of the form

$$A \to B+M\,, \tag{3}$$

where A and B are hadrons containing quarks and antiquarks with arbitrary orbital angular momentum and M is a pseudoscalar or vector boson. In a large number of quark-model treatments the transition matrix element describing the process (3) is given by the sum of independent single-quark transition operators,[2] corresponding to the emission of the boson by a single quark

$$\langle BM|T|A\rangle = \langle B|\sum_i t_i(M)e^{i\mathbf{k}\cdot\mathbf{r}_i}|A\rangle\,, \tag{4}$$

where $\hbar\mathbf{k}$ is the momentum of the emitted boson, $\mathbf{r}_i$ is the coordinate of the ith quark in the hadron and $t_i(M)$ is an operator depending on the dynamical variables of the ith quark and the emitted boson.

The specific form of the operator $t_i(M)$ is severely restricted by the known conservation laws. We consider here a nonrelativistic description appropriate to the nonrelativistic quark model.[2] Translational invariance requires $t_i(M)$ to be independent of the spatial coordinate $\mathbf{r}_i$. Gallilean invariance restricts the momentum dependence of $t_i(M)$ to be a function of only of the momentum transfer $\mathbf{k}$ and to be otherwise independent of the quark momentum in the initial state. Thus $t_i(M)$ can only be a function of the quark spin variables $\boldsymbol{\sigma}_i$ and the wave and polarization vectors $\mathbf{k}$ and $\boldsymbol{\varepsilon}$ of the boson. The requirements of angular momentum and parity conservation then fix the following forms for t_i as the most general allowed by conservation laws.

$$t_i(P) = (\boldsymbol{\sigma}_i\cdot\mathbf{k})f_a(k^2)\,, \tag{5a}$$

$$t_i(V_\pm) = (\boldsymbol{\sigma}_i\cdot\mathbf{k}\times\boldsymbol{\varepsilon})f_b(k^2)\,, \tag{5b}$$

$$t_i(V_0) = (\boldsymbol{\varepsilon}\cdot\mathbf{k})f_c(k^2)\,, \tag{5c}$$

where P, $V_\pm$, and V_0 denote pseudoscalar, transverse-vector, and longitudinal-vector boson states and f_a, f_b, and f_c are arbitrary functions.

Let us now consider the transformation properties of the matrix elements (4) under W-spin transformations as defined above. Since W spin acts only on the quark spin variables and not on spatial degrees of freedom, the only operators appearing in Eqs. (4) and (5) which are affected by W-spin transformations are the quark-spin operators $\boldsymbol{\sigma}_i$. We choose the conventional collinear coordinate system used in W-spin calculations in which all momenta are in the z direction.[1] The operators (5a), (5b), and (5c) transform under rotations in quark-spin space, respectively, like the z component of a vector, the x and y components of a vector, and a scalar. These spin transformation properties of the operators $t_i(M)$ are just the ones used in the conventional W-spin classification for pseudoscalar and vector particles. The states P and $V_\pm$ form a W-spin vector and the V_0 state is a W-spin scalar.

Since the transformation properties of $t_i(M)$ are the same as those given to the emitted meson M in the conventional W-spin classification, the matrix elements for any particular state M satisfy the same relations as required by W-spin conservation. The matrix elements between any state A in a given W-spin multiplet and a state B in another given W-spin multiplet are given by the Wigner-Eckart theorem, using the appropriate W-spin property for the transition operator $t_i(M)$ which is the same as that of the emitted meson M. The transition matrix elements for *different states M belonging to the same W-spin multiplet* are not related at all by this general argument. However, the additional assumption of full W-spin conservation is compatible with the model.

The same argument can be applied with the inclusion of internal degrees of freedom such as isospin and hypercharge for the quarks. One then finds that the assumption of $SU(3)$ symmetry for the transition operator (4) is sufficient together with the conservation laws considered above to require a restricted $SU(6)_W$ invariance.[4] The matrix elements for a single state M and all states A and B in two given $SU(6)_W$ supermultiplets are related as required by $SU(6)_W$ invariance. The matrix elements for different boson states M normally classified in different $SU(3)$ multiplets within the same $SU(6)_W$ supermultiplet are not related without additional assumptions, but full $SU(6)_W$ invariance is compatible with the model.

III. APPLICATIONS OF W-SPIN CONSERVATION WITH ORBITAL EXCITATION

Let us now consider the application of W-spin conservation to particular types of transitions (3) where A is a state which has orbital excitation but the states B and M have $L=0$. These represent all cases of practical interest at present as they include all final states con-

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[4] H. J. Lipkin, in *Proceedings of the Third Coral Gables Conference on Symmetries at High Energies, University of Miami, 1966* (W. H. Freeman and Company, San Francisco, California, 1966), p. 97.

sisting only of combinations of the low-lying pseudoscalar and vector nonets, the spin-$\frac{1}{2}$ baryon octet and the spin-$\frac{3}{2}$ baryon decuplet. Since $W_z=S_z$ is conserved and J_z is conserved, L_z must also be conserved. The final state has $L_z=L=0$; thus only the $L_z=0$ component of the initial state can contribute to decay. The transition matrix element for the decay of a given polarization state $|J_A,M_A\rangle$ of the particle A is then

$$\begin{aligned}\langle BM|T|L_A,S_{qA},S_{\bar{q}A},S_A,J_A,M_A\rangle \\ =\langle BM|T|L_A,S_{qA},S_{\bar{q}A},S_A,\, m_S=M_A,\, m_L=0\rangle \\ \times(L_AS_SOM_A|J_AM_A). \quad (6)\end{aligned}$$

The matrix elements of T on the right-hand side are now expressed in terms of eigenstates of S_z and can be treated as described in I without further consideration of the orbital angular momentum.

An interesting example of selection rules obtained from W-spin conservation in the presence of orbital excitation is found in the decays of axial-vector mesons[5] in a model where they are composed of a quark-antiquark pair in a p state.[2,3] There are two such states, the triplet and singlet spin states which have opposite behavior under charge conjugation. The singlet spin state has $s_z=S=0$ and can decay only into a final state having $S_z=0$ by conservation of S_z. The triplet spin state is constructed by coupling $S=1$ with $L=1$ to obtain $J=1$. The $J_z=0$ state has no contribution from $L_z=0$ because of the vanishing of the appropriate Clebsch-Gordan coefficient. Thus only the $J_z=\pm1$ states are allowed to decay by W-spin conservation and these decay to final states having $J_z=S_z=\pm1$. Thus opposite polarization behavior in the final states is predicted for the decays of the two kinds of axial-vector mesons having opposite behavior under charge conjugation. If for example the A_1 and B mesons[6] are the triplet and singlet axial-vector mesons, the following polarizations are predicted for the vector-meson final states.

$$A_1\rightarrow\rho+\pi;\quad \rho \text{ is in state } m=\pm1\,; \quad (7a)$$

$$B\rightarrow\omega+\pi;\quad \omega \text{ is in state } m=0. \quad (7b)$$

These predictions can easily be checked by observing the angular distributions of the ρ and ω decay products relative to the momentum of the decay pion. Similar predictions hold for other axial-vector decays in this model.

Other selection rules found in specific-model calculations are also easily obtained from $SU(6)$ conservation. For example, we obtain Moorhouse's selection rule[7] forbidding the photoproduction of N^{*+} resonances composed of three quarks coupled to $S=\frac{3}{2}$ with arbitrary L and J. This state is in the **70** supermultiplet of $SU(6)_W$, and the selection rule follows from the vanishing of $SU(6)$ Clebsch-Gordan coefficient coupling this state, that of the proton, and the particular states in the **35** supermultiplet representing the photon.

An additional selection rule is obtained by examining the "anomalous parity" operator[8]

$$P_x^{\text{an}}=Pe^{i\pi(J_x-W_x)} \quad (8)$$

where P is the ordinary space-inversion operator. In the absence of orbital excitation, P_x^{an} reduces to the identity for all quark-model states having momenta only in the yz plane.[8] When orbital excitation is present this is no longer true, and

$$P_x^{\text{an}}\underset{yz}{=}P_{\text{int}}e^{i\pi(L_x+S_x-W_x)}, \quad (9)$$

where $\underset{yz}{=}$ denotes equality for states in which all momenta are in the yz plane, P_{int} is the intrinsic parity and $\mathbf{L}$ is the total *intrinsic* orbital angular momentum. This includes only the orbital angular momentum of the quarks *within* a hadron and does *not* include the relative orbital angular momentum between different hadrons. The operator P_x^{an} is a conserved quantity if P, $\mathbf{J}$, and $\mathbf{W}$ are conserved. Thus states of "anomalous parity" corresponding to the negative eigenvalue of (9), cannot decay into normal particles in a collinear process. Since only the $L_z=0$ component can contribute to such a decay, we consider

$$P_x^{\text{an}}(L_z=0)\underset{yz}{=}P_{\text{int}}(-1)^Le^{i\pi(S_x-W_x)}. \quad (10)$$

An example of this selection rule is the metastable baryon state discussed by Morpurgo[9] which has $P_{\text{int}}=1$, $L=1$ and $\mathbf{S}=\mathbf{W}$, and therefore has

$$P_x^{\text{an}}(L_z=0)=-1.$$

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[5] J. Uretsky (to be published) has found these selection rules in a specific model. We are indebted to him for calling these results to our attention.

[6] For a review of the experimental situation, see G. Goldhaber, in *Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, California, 1967).

[7] R. G. Moorhouse, Phys. Rev. Letters **16**, 772 (1966). Note that the present derivation depends upon the vanishing of a particular Clebsch-Gordan coefficient and shows that the selection rule *does not hold* for the photoproduction of the corresponding neutral N^{*0} on neutrons. This point was clarified in a discussion with R. G. Moorhouse and S. Meshkov. It may have interesting experimental implications.

[8] H. J. Lipkin and S. Meshkov, Phys. Rev. **143**, 1269 (1966).

[9] G. Morpurgo, Phys. Letters **22**, 214 (1966).