Possible CP Violation in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\gamma$†

N. CHRIST
Department of Physics, Columbia University, New York, New York
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A comparison of the decays $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\gamma$ may show relatively large CP-violating effects. A complete phenomenological analysis of these decays is given, including precise specification of the implications of T, CP, and CPT. A simple but reasonable model is developed which allows both prediction and theoretical interpretation of CP violation in these decays.

I. INTRODUCTION

As is well known, any asymmetry between the decays

$$K^{+} \rightarrow \pi^{+}+\pi^{0}+\gamma \tag{1}$$

and

$$K^{-} \rightarrow \pi^{-}+\pi^{0}+\gamma \tag{2}$$

is an indication of CP nonconservation, and such CP-violating effects may be large in these decays, even when CPT invariance is assumed.[1-3] In the discussion to follow, a detailed analysis of reactions (1) and (2) is presented. CPT symmetry is assumed throughout. Decays (1) and (2) can proceed through two mechanisms whose contributions may be of comparable size[1]: inner bremsstrahlung and direct radiation. The presence of these two processes would make a comparison of decays (1) and (2) sensitive both to electromagnetic CP violation[4] and to CP nonconservation by a small component of the weak interactions allowing $|\Delta I|>\frac{1}{2}$.[5]

These considerations are divided into two parts. Section II contains a complete phenomenological analysis of the decays (1) and (2) and a precise specification of the implication of the symmetries T, CP, and CPT. In Sec. III a reasonable model is presented which provides a prediction of the angular distribution of decays (1) and (2) in terms of 4 parameters and permits the theoretical interpretation of an asymmetry between the Dalitz plots of reactions (1) and (2).

The model described in Sec. III is compatible with asymmetries as large as 10% in particular regions of the Dalitz plot and in the energy spectrum of the charged pion.[6] In the case of large electromagnetic CP violation, effects of this size can be expected, provided: (a) the difference between the s-wave and p-wave $\pi^{\pm}$-π^{0} phase shifts, $|\delta_1-\delta_0|\geqslant 10°$; (b) the contributions of inner bremsstrahlung and direct electric dipole radiation to decays (1) and (2) are of roughly the same size. [The latter uncertainty can of course be removed by more precise measurement of the energy spectrum of the charged pion in reaction (1) or (2).] If there is no large electromagnetic CP violation, then a CP violation in these decays provides a measurement of the CP-violating phase difference between the $|\Delta I|>\frac{1}{2}$ amplitudes $A_2^{\pm}$, responsible for the decays $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}$.[7] Under this circumstance, effects one-tenth the size of those mentioned above may be expected if conditions (a) and (b) are satisfied.

II. PHENOMENOLOGICAL ANALYSIS

In this section we will undertake a complete phenomenological analysis of the decays (1) and (2), concentrating on the implications of the discrete symmetries T, CP, and CPT. These decays are determined by the matrix elements

$$M^{\pm}(p_c,p_0,q,\kappa,\epsilon)=\langle\pi^{\pm}(p_c)\pi^{0}(p_0)\gamma(q,\epsilon)^{\text{out}}|K^{\pm}(\kappa)\rangle, \tag{3}$$

where p_c, p_0, q, and κ are the momenta of the charged pion, neutral pion, photon and K meson, respectively. ϵ is the 4-vector polarization of the γ ray. The initial state $|K^{\pm}(\kappa)\rangle$ is an eigenstate of the strong and electromagnetic interactions containing a single charged K meson, while the final state

$$|\gamma(q,\epsilon)\pi^{0}(p_0)\pi^{\pm}(p_c)^{\text{out}}\rangle \tag{4}$$

is an eigenstate of the total Hamiltonian. Asymptotically the *outgoing* part of state (4) corresponds to two pions and a γ ray, each with definite linear momentum. CPT symmetry requires

$$CPT|\gamma(q,\epsilon),\pi^{0}(p_0)\pi^{\pm}(p_c)^{\text{out}}\rangle = |\gamma(q,\epsilon),\pi^{0}(p_0)\pi^{\mp}(p_c)^{\text{in}}\rangle, \tag{5}$$

where the states on the right-hand side of (5) are identical with those in (4) except that their asymptotic *incoming* part is that of a plane wave.

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[1] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966).
[2] D. Cline, Nuovo Cimento **36**, 1055 (1965).
[3] S. Barshay, Phys. Rev. Letters **18**, 515 (1967).
[4] For these decays, electromagnetic CP violation can include the violation of C and T by the electromagnetic interaction of the hadrons [as suggested by J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965); S. Barshay, Phys. Letters **17**, 78 (1965)] and a CP violation arising from the simultaneous action of both weak and electromagnetic forces [as proposed, for example, by F. Salzman and G. Salzmann, Phys. Letters **15**, 91 (1965)].
[5] T. N. Truong, Phys. Rev. Letters **13**, 358 (1964).
[6] These percentages represent a comparison of the differences of quantities measured identically in decays (1) and (2) with their sum.
[7] The amplitudes $A_2^{\pm}$ are defined by $A_2^{\pm}\delta^3(p_f-\kappa)=\langle\pi^{\pm}\pi^{0}|H_{\text{wk}}|K^{\pm}(\kappa)\rangle(2\pi)^{-3}$ where H_{wk} is the weak Hamiltonian, κ the K-meson's four-momentum, and $|\pi^{\pm}\pi^{0}\rangle$ a stationary eigenstate of the strong interaction with total four-momentum p_f which asymptotically is two s-wave pions.

In the case of time-reversal invariance

$$T|\gamma(q,\epsilon),\pi^0(p_0)\pi^{\pm}(p_c)^{\mathrm{out}}\rangle = |\gamma(q_T,\epsilon_T),\pi^0(p_{0T})\pi^{\pm}(p_{cT})^{\mathrm{in}}\rangle, \quad (6)$$

where for a 4-vector, $u=(\mathbf{u},iu_0)$, $u_T=(-\mathbf{u},iu_0)$. The consequences of T, CP, and CPT for the matrix elements given in (3) follow easily. With standard phase conventions: CPT implies

$$(M^{\pm})^*=\sum_{\Gamma}\langle\pi^{\mp}(p_c)\pi^0(p_0)\gamma(q,\epsilon)^{\mathrm{in}}|\Gamma(\mathrm{out})\rangle \times\langle\Gamma(\mathrm{out})|K^{\mp}(\kappa)\rangle. \quad (7)$$

T implies

$$(M^{\pm})^*=\sum_{\Gamma}\langle\pi^{\pm}(p_{cT})\pi^0(p_{0T})\gamma(q_T,\epsilon_T)^{\mathrm{in}}|\Gamma(\mathrm{out})\rangle \times\langle\Gamma(\mathrm{out})|K^{\pm}(\kappa_T)\rangle, \quad (8)$$

where we have inserted a sum over a complete set of outgoing states $|\Gamma(\mathrm{out})\rangle$. CP requires $M^+(p_c,p_0,q,\kappa,\epsilon) =M^-(p_{cT},p_{0T},q_T,\kappa_T,\epsilon_T)$. All matrix elements in (7) and (8) are, in principle, measurable since they involve either scattering or K meson decay. However, because the sum over Γ includes the state (4), Eqs. (7) and (8) are rather complicated conditions on the matrix elements $M^{\pm}$.

Considerable simplification is obtained if one introduces the multipole moments

$$\delta^4(P_f-\kappa)\alpha_{l\sigma}{}^{\pm}=N_\sigma\frac{1}{2}\frac{2l+1}{l(l+1)}\int_{-1}^{1}P_l'(x)\,dx \times\langle\pi^{\pm}(p_c)\pi^0(p_0)\gamma(q,\epsilon_\sigma)^{\mathrm{out}}|K(\kappa)\rangle e^{-i\delta_l}, \quad (9)$$

$$\delta^4(P_f-\kappa)\beta_{l\sigma}{}^{\pm}=N_\sigma\sum_{\Gamma\neq\pi^{\pm}\pi^0\gamma}\frac{1}{2}\frac{2l+1}{l(l+1)}\int_{-1}^{1}P'_l(x)\,dx \times\langle\pi^{\pm}(p_c)\pi^0(p_0)\gamma(q,\epsilon_\sigma)^{\mathrm{in}}|\Gamma(\mathrm{out})\rangle \times\langle\Gamma(\mathrm{out})|K(\kappa)\rangle e^{i\delta_l}, \quad (10)$$

where $\sigma=+$ or $-$, $\boldsymbol{\epsilon}_+=\hat{p}_c-\hat{q}\hat{p}_c\cdot\hat{q}$, $\boldsymbol{\epsilon}_-=\hat{p}_c\times\hat{q}$,[7a] $N_+ =m(\frac{1}{4}m_{\pi\pi}{}^2-\mu^2)^{-1/2}$, and $N_-=N_+m^2/km_{\pi\pi}$. The function P_l' is the first derivative of the Legendre polynomial of order l. The matrix elements are to be evaluated in the center-of-mass system of the two pions. x is the cosine of the angle between the three-momentum of the charged pion ($\mathbf{p}_c$) and that of the γ ray ($\mathbf{q}$) as measured in that system.

$$x=\mathbf{p}_c\cdot\mathbf{q}/|\mathbf{p}_c||\mathbf{q}|, \quad\text{and}\quad k=|\mathbf{q}|.$$

δ_l is the phase shift for $\pi^{\pm}$-π^0 scattering in the channel with angular momentum l, m is the K-meson's mass, and μ that of the π meson. During the integration over x, the invariant mass of the $\pi^{\pm}\pi^0$ system, $m_{\pi\pi}$, and the 4-vectors q and κ are to be kept fixed—consequently the total four-momentum of the final state P_f is a constant. Our states (4) and $|K(\kappa)\rangle$ are given invariant normalization. $\alpha_{l\pm}$ and $\beta_{l\pm}$ are thus functions of the invariants, $m_{\pi\pi}$ or $q\cdot(p_c+p_0)$. These expressions are inverted in the Appendix.

In terms of these parameters, T, CP, and CPT have simple implications[8]:

$$T \text{ implies}\quad s_\sigma\alpha_{l\sigma}{}^{\pm*}-\alpha_{l\sigma}{}^{\pm}=\beta_{l\sigma}{}^{\pm}, \quad (11)$$

$$CP \text{ implies}\quad s_\sigma\alpha_{l\sigma}{}^{+}=\alpha_{l\sigma}{}^{-}, \quad (12)$$

$$CPT \text{ implies}\quad \alpha_{l\sigma}{}^{\pm*}=\alpha_{l\sigma}{}^{\mp}+\beta_{l\sigma}{}^{\mp}, \quad (13)$$

where $s_+=+1$ and $s_-=-1$. These implications can be stated in a manner independent of phase conventions and the radiative $K^{\pm}$ decays described in a charge symmetric way if we introduce the parameters

$$\xi_{l\sigma}=-i\left[\frac{\alpha_{l\sigma}{}^{+}}{\beta_{l\sigma}{}^{+}}-\frac{\alpha_{l\sigma}{}^{-*}}{\beta_{l\sigma}{}^{-*}}\right], \quad (14)$$

and

$$\beta_{l\sigma}=\beta_{l\sigma}{}^{+},$$

then

$$\alpha_{l\sigma}{}^{+}=\tfrac{1}{2}\beta_{l\sigma}(i\xi_{l\sigma}-1),\quad \alpha_{l\sigma}{}^{-}=-\tfrac{1}{2}\beta_{l\sigma}{}^*(i\xi_{l\sigma}{}^*-1),$$
$$\beta_{l\sigma}{}^{+}=\beta_{l\sigma},\qquad \beta_{l\sigma}{}^{-}=-\beta_{l\sigma}{}^*, \quad (15)$$

and T or CP requires

$$\mathrm{Im}\,\xi_{l\sigma}=0,$$

$$\mathrm{Im}\,[\beta_{l+}\beta_{l'+}{}^*]=\mathrm{Im}\,[\beta_{l-}\beta_{l'-}{}^*]=\mathrm{Re}[\beta_{l+}\beta_{l'-}{}^*]=0. \quad (16)$$

Consequently, even when CPT invariance is assumed, two complex numbers, $\xi_{l\sigma}$ and $\beta_{l\sigma}$, and one phase shift δ_l are needed to specify the amplitudes for transition from $K^{\pm}$ to a $\pi^{\pm}\pi^0\gamma$ state of definite multipolarity.

Let us conclude this section by noting that for low γ-ray energy, $k\ll m_\pi$, the form of $\alpha_{l\sigma}{}^{\pm}$ and $\beta_{l\sigma}{}^{\pm}$ is known.[9,10] For $\alpha_{l\sigma}{}^{\pm}$ we have the formulas

$$\alpha_{l+}{}^{\pm}=\frac{2ymA_2{}^{\pm}(4\pi\alpha)^{1/2}e^{+i(\delta_0-\delta_l)}}{m_{\pi\pi}k} \times[Q_{l+1}(y)-Q_{l-1}(y)]+O(k), \quad (17)$$

$$\alpha_{l-}{}^{\pm}=O(k),$$

where $y=(1-4\mu^2/m_{\pi\pi}{}^2)^{-1/2}$ and $\alpha\simeq1/137$ is the fine structure constant. The first term in the expression for $\alpha_{l+}{}^{\pm}$ is the usual inner-bremsstrahlung contribution. If we neglect all but the states $\Gamma=\pi^{\pm}\pi^0$ in the sum $\sum_{\Gamma\neq\pi\pi\gamma}$ appearing in (10), then the low-energy limit[11]

[7a] *Note added in proof.* It is the spacial part of the four-vectors $\epsilon_{\pm}$ which is specified here; their fourth components are zero. The author is indebted to Professor Oreste Piccioni for calling attention to errors in the original manuscript.

[8] We will neglect all but lowest-order weak and electromagnetic effects.

[9] F. Low, Phys. Rev. **110**, 974 (1958).

[10] H. Chew, Phys. Rev. **123**, 377 (1961).

[11] A simple estimate of the term arising from $\Gamma=\pi\pi\pi$ suggests that this approximation introduces an error $\lesssim10\%$ in $\beta_{l\sigma}$.

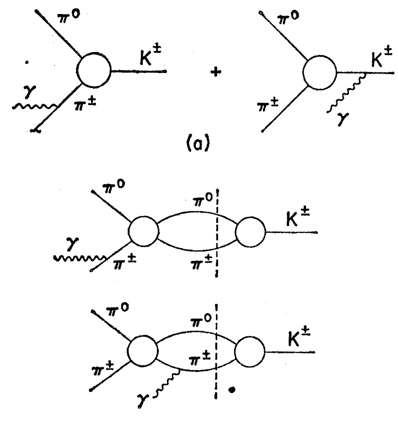

FIG. 1. Diagrams representing the inner bremsstrahlung contribution to $\alpha_{l\sigma}^\pm$ [Fig. 1(a)] and $\beta_{l\sigma}^\pm$ [Fig. 1(b)]. Intermediate lines cut by a dotted line refer to particles on the mass shell. The circle on the left-hand side of each of the diagrams in (b) represents π-π scattering without radiative corrections.

of $\beta_{l\sigma}^\pm$ can also be simply stated:

$$\beta_{l+}{}^\pm=\frac{2ymA_2{}^\pm(4\pi\alpha)^{1/2}e^{+i(\delta_0+\delta_l)}}{km_{\pi\pi}}(e^{-2i\delta_0}-e^{-2i\delta_l})\times[Q_{l+1}(y)-Q_{l-1}(y)]+O(k)\,. \quad (18)$$

$$\beta_{l-}{}^\pm=O(k)\,.$$

$Q_l(z)$ is the Legendre polynomial of the second kind of order l.[12] The terms $O(k)$ represent quantities approaching zero at least linearly in k. Note that in this approximation time reversal invariance (11) requires $A_2{}^{\pm *}=A_2{}^\pm$ as is expected. Expressions (17) and (18) are obtained by applying (9) and (10) to the inner-bremsstrahlung amplitudes represented by the diagrams shown in Figs. 1(a) and 1(b), respectively.

III. A SIMPLE MODEL

We will now formulate a 5-parameter model for the decays (1) and (2), consistent with CPT symmetry but non-invariant under CP and T. Clearly, CP and T violation must involve direct amplitudes in addition to the inner-bremsstrahlung contribution shown in Fig. 1. In similar decays (e.g., $K_1\rightarrow\pi^+\pi^-\gamma$) dimensional arguments suggest that such direct amplitudes, those represented by $O(k)$ in (17) and (18), are much smaller than the inner bremsstrahlung. As is well-known,[1] however, the smallness of $A_2{}^\pm$, as required by the $\Delta I=\frac{1}{2}$ rule in weak interactions, makes possible roughly equal direct and inner bremsstrahlung contributions to the decays (1) and (2). This suggests that the comparison of decays (1) and (2) may be a particularly sensitive test of CP invariance.

These arguments apply to the decay multipole moments $\alpha_{l\sigma}^\pm$ but not to the $\beta_{l\sigma}^\pm$ introduced in Sec. II. If, in the sum $\sum_{\Gamma\neq\pi\pi\gamma}$ of Eq. (10), we neglect all but the states[11] $\Gamma=\pi\pi$ (as will be done throughout this section), both the direct and the inner-bremsstrahlung contribution to $\beta_{l\sigma}^\pm$ require a change of isospin by $\frac{3}{2}$ before the photon is emitted. Consequently, $\beta_{l\sigma}^\pm$ will be approximated by the inner-bremsstrahlung contribution alone. If we also neglect the direct contribution to transitions of higher order than dipole and approximate the direct electric and magnetic dipole moments by constants, our model is completely specified.[13]

The consequences of these assumptions are as follows:

$$M^+(p_c,p_0,q,\kappa,\epsilon)=\delta^4(p_c+p_0+q-\kappa)(4\pi\alpha)^{1/2}\epsilon^\mu\times\left\{A_2{}^+e^{i\delta_0}\left[\frac{p_c{}^\mu}{p_c\cdot q}-\frac{\kappa^\mu}{\kappa\cdot q}\right]-e^{i\delta_1}\frac{E_1}{m^4}(\kappa\cdot q p_c{}^\mu-p_c\cdot q\kappa^\mu)-e^{i\delta_1}\frac{M_1}{m^4}p_c{}^\nu p_0{}^\rho\kappa^\sigma\epsilon_{\mu\nu\rho\sigma}\right\}, \quad (19)$$

and

$$M^-(p_c{}'p_0,q,\kappa,\epsilon)=\delta^4(p_c+p_0+q-\kappa)(4\pi\alpha)^{1/2}\epsilon^\mu\times\left\{A_2{}^{+*}e^{i\delta_0}\left[\frac{p_c{}^\mu}{p_c\cdot q}-\frac{\kappa^\mu}{\kappa\cdot q}\right]-\frac{e^{i\delta_1}E_1{}^*}{m^4}(\kappa\cdot q p_c{}^\mu-p_c\cdot q\kappa^\mu)+\frac{e^{i\delta_1}M_1{}^*}{m^4}p_c{}^\nu p_0{}^\rho\kappa^\sigma\epsilon_{\mu\nu\rho\sigma}\right\}, \quad (20)$$

where m is the K-meson's mass. Since an over-all phase is not measurable, the relevant unknown parameters are $|E_1|$, $|M_1|$, $\delta_1-\delta_0$, and the phase difference between $A_2{}^+$ and E_1 and between $A_2{}^+$ and M_1. Let

$$Ee^{i\phi}=E_1|A_2{}^+|/(A_2{}^+|A_0|),\quad 0\leqslant\phi<\pi\,,$$
$$Me^{i\phi'}=M_1|A_2{}^+|/(A_2{}^+|A_0|),\quad 0\leqslant\phi'<\pi\,, \quad (21)$$

where E and M are real numbers and A_0 is the amplitude for K° decay into an $I=0$ two-pion state.[14] CP invariance requires $\phi=\phi'=0$. Because of the presence of the constant $|A_2{}^+|/|A_0|$ in (21), dimensional arguments suggest E and M are of order 1.[15] All of these five quantities have been assumed constant.

If the photon polarization is not observed, the magnetic amplitude interferes with neither E_1 nor the inner-bremsstrahlung contribution; consequently only four parameters are required to describe the angular distribution of decays (1) and (2): E, M, δ_1-δ_0, and ϕ. Let us work in the rest system of the K meson. If the decay is described by two variables: $\omega_c{}'$, the charged pion's energy and ω_γ, the photon's energy, the Dalitz

[12] $Q_l(z)=\frac{1}{2}\int_{-1}^1 P_l(x)/(z-x)\,dx$, where $P_l(x)$ is a Legendre polynomial of order l.

[13] The model is merely a simple extension of the description of the decay $K^+\rightarrow\pi^+\pi^0\gamma$ given by J. D. Good, Phys. Rev. **113**, 352 (1959).

[14] C. N. Yang and T. T. Wu, Phys. Rev. Letters **13**, 380 (1964).

[15] We use the value $|A_0|/|A_2{}^+|=18.4$ in the computations described at the end of Sec. III.

plot distributon for $K^{\pm}$ decay is given by

$$\frac{d^2R^{\pm}}{d\omega_\gamma d\omega_c}=\frac{\alpha}{(4\pi)^2}\frac{p^2}{m^3}\sin^2\vartheta\left\{\frac{|A_2{}^+|^2}{(\frac{1}{2}m-\omega_0)^2}+\frac{2\cos(\pm\phi+\delta_1-\delta_0)|A_2{}^+||A_0|E\omega_\gamma}{m^2(\frac{1}{2}m-\omega_0)}+(|E_1|^2+|M_1|^2)\omega_\gamma{}^2/m^4\right\}, \quad (22)$$

where $p=(\omega_c{}^2-\mu^2)^{1/2}$, $\omega_0=m-\omega_c-\omega_\gamma$, and ϑ is the angle between the photon's momentum and that of the charged pion

$$\cos\vartheta=(p\omega_\gamma)^{-1}[\tfrac{1}{2}m^2-m(\omega_c+\omega_\gamma)+\omega_\gamma\omega_c].$$

Clearly, if $\phi\neq 0$, the distributions for K^+ and K^- decay given by (22) differ (provided δ_1-$\delta_0\neq 0$ and $E_1\neq 0$), violating the prediction of *CP* invariance. This expression is so normalized that

$$\frac{d^2R^{\pm}}{d\omega_\gamma d\omega_c}\Delta\omega_\gamma\Delta\omega_c\times(K^{\pm}\text{ lifetime})$$

is the fraction of all $K^{\pm}$ decays which leads to a final state of two pions and one photon with charged-pion energy lying between ω_c and $\omega_c+\Delta\omega_c$ and photon energy between ω_γ and $\omega_\gamma+\Delta\omega_\gamma$, for small $\Delta\omega_c$ and $\Delta\omega_\gamma$. If this expression is integrated over ω_γ, the energy spectrum of the charged pion is obtained[16]:

$$\frac{dR^{\pm}}{d\omega_c}=\frac{\alpha}{(2\pi)^2}\left\{\frac{2|A_2{}^+|^2}{m(m-2\omega_c)}\left[\frac{\omega_c}{m}\ln\left(\frac{\omega_c+p}{\mu}\right)-\frac{p}{m}\right]+\tfrac{1}{2}\cos(\pm\phi+\delta_1-\delta_0)|A_2{}^+||A_0|E\frac{(m-2\omega_c)}{m^3}\times\left[\frac{1}{2}\left(1-\frac{\mu^2}{m^2}\right)\ln\left(\frac{m-\omega_c+p}{m-\omega_c-p}\right)-\frac{\mu^2}{m^2}\ln\left(\frac{\omega_c+p}{\mu}\right)-p/m\right]+(|E_1|^2+|M_1|^2)\frac{1}{24}\frac{m^{-4}p^3(m-2\omega_c)^3}{[(m-\omega_c)^2-p^2]^2}\right\}. \quad (23)$$

CP violation is indicated by the observation of a nonvanishing asymmetry between decays (1) and (2) either in the Dalitz plot,

$$a_2(\omega_\gamma,\omega_c)=[(d^2R^+/d\omega_\gamma d\omega_c)-(d^2R^-/d\omega_\gamma d\omega_c)]\times[(d^2R^+/d\omega_\gamma d\omega_c)+(d^2R^-/d\omega_\gamma d\omega_c)]^{-1}, \quad (24)$$

or in the energy spectrum of the charged pion,

$$a_1(\omega_c)=[(dR^+/d\omega_c)-(dR^-/d\omega_c)]\times[(dR^+/d\omega_c)+(dR^-/d\omega_c)]^{-1}. \quad (25)$$

[16] J. D. Good (Ref. 13).

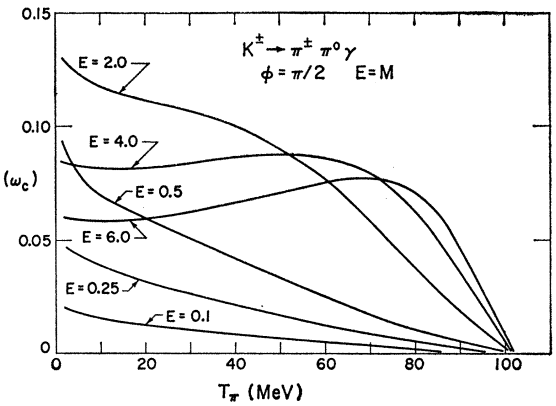

FIG. 2. The asymmetry $a_1(\omega_c)$ between the energy spectrum of the charged pion in decays (1) and (2), plotted as a function of the charged pion's kinetic energy T_π as measured in the rest system of the K meson. The electric and magnetic dipole moments have been set equal; $|E|=|M|\leq 2$ is consistent with our present knowledge of radiative K decay.

Either $a_1\neq 0$ or $a_2\neq 0$ indicates an unequivocal violation of *CP* symmetry.

A prediction of the quantities a_1 and a_2 requires a knowledge of ϕ, δ_1-δ_0, E, and $|E|^2+|M|^2$. An accurate measurement of $dR^+/d\omega_c$ could determine $|E|^2+|M|^2$ and set a lower limit on $|E|$.[17] The value of δ_1-δ_0 is essentially unknown and can only be guessed at. If one sets $\delta_0=0$ and obtains δ_1 by a Breit-Wigner extrapolation from the ρ resonance, δ_1-$\delta_0=10°$ is obtained.[18] The final unknown ϕ is of course the quantity of interest. $\phi\neq 0$ may arise from an electromagnetic *CP* violation in the direct electric dipole transition which allows $\phi\sim\frac{1}{2}\pi$ or from a small $\Delta I=\frac{3}{2}$ or $\frac{5}{2}$ *CP*-violating part of the weak interactions affecting the amplitude $A_2{}^+$ appearing in the inner-bremsstrahlung contribution yielding $\phi\sim 1$. The asymmetries a_1 and a_2 can easily be computed for reasonable values of the unknown parameters. In Fig. 2, $a_1(\omega_c)$ is plotted for δ_1-$\delta_0=10°$ and $\phi=\frac{1}{2}\pi$ for various values of $E=M$. Presently available data is compatible with $|E|=|M|\leqslant 2$. In Fig. 3, contours of equal asymmetry, $a_2(\omega_\gamma,\omega_c)$, are shown on the Dalitz plot for the $K\to\pi\pi\gamma$ decay. $\delta_1-\delta_0=10°$, $\phi=\pi/2$ and $E=M=2$ and 0.1 are used. As can be seen in Figs. 2 and 3, the choice of $\delta_1-\delta_0=10°$, $\phi=\frac{1}{2}\pi$, and an optimum value $E=M=2$ predicts a maximum value of 0.12 for a_1 and a_2. Asymmetries roughly one-tenth that size are found for $\phi=0.1$ rad. These values are intended to be the largest that our present knowledge of decays (1) and (2) can reasonably predict.[19]

[17] If $|E|^2+|M|^2$ is found to be considerably less than 1, the direct contribution to β_{1+} can no longer be expected to be negligible, adding an additional unknown parameter to the analysis.

[18] In fact, such an extrapolation predicts a variation of δ_1 with the invariant mass of the $\pi\pi$ system, $m_{\pi\pi}$: $\delta_1=13°$ for $m_{\pi\pi}=m_K$, while for $m_{\pi\pi}=2m_\pi$ $\delta_1=7°$. This suggests that the approximation of E, M, ϕ, ϕ', and $\delta_1-\delta_0$ by constants involves errors of the order of 30%.

[19] Values of a_1 and a_2 much larger than these cannot be rigorously ruled out. If, for example, $E=2$ but $M=0$, the maximum values of a_1 and a_2 roughly double. If also, $\delta_1-\delta_0=\pi/2$, values of a_1 and a_2 near one could be expected.

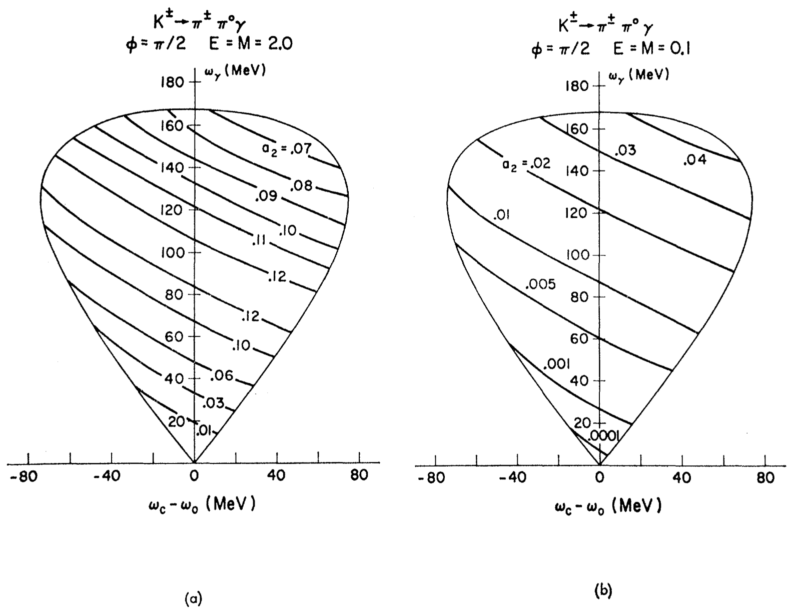

FIG. 3. Contours of equal asymmetry $a_2(\omega_c,\omega_\gamma)$ drawn on the Dalitz plot for reactions (1) and (2). In (a) values of $E=M=2.0$ are used while for (b) E and M are set equal to 0.1. The ordinate is the energy of the γ ray, ω_γ, and the abscissa is the difference between the energy of the charged pion and that of the neutral pion, $\omega_c-\omega_0$. All energies are measured in the K-meson's rest system.

We can conclude that a comparison of the radiative two-pion decays of the K^+ and K^- mesons is a particularly sensitive test of CP violation. In contrast to a search for π^+-π^- asymmetry in the decays[1]

$$K_2 \to \pi^+\pi^-\gamma \quad \text{or} \quad \eta \to \pi^+\pi^-\gamma\,,$$

only lowest-order direct multipole radiation is required for the presence of CP-violating effects. Except for the unknown π-π phase shifts, a detailed experimental study of the energy spectrum of the charged pion in the K^+ decay allows a reasonably unambiguous theoretical interpretation of either negative or positive results in such a search for CP violation.

ACKNOWLEDGMENTS

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APPENDIX

Given the multipole moments $\alpha_{l\sigma}{}^{\pm}$, one can easily reconstruct the matrix elements $M^{\pm}(p_c,p_0,q,\kappa,\epsilon)$ of (3):

$$M^{\pm}(p_c,p_0,q,\kappa,\epsilon)=\sum_{l=1}^{\infty} P_l'\left(\frac{q\cdot(p_0-p_c)}{q\cdot\kappa(1-4\mu^2/m_{\pi\pi}{}^2)^{1/2}}\right)e^{i\delta_l}$$
$$\times\left\{\epsilon^\mu\left(p_c{}^\mu-\kappa^\mu\frac{p_c\cdot q}{\kappa\cdot q}\right)\alpha_{l+}{}^{\pm}/m-i\epsilon^\mu\frac{p_c{}^\nu q^\rho p_0{}^\sigma}{m^3}\epsilon_{\mu\nu\rho\sigma}\alpha_{l-}{}^{\pm}\right\}$$
$$\times\delta^4(p_c+p_0+q-\kappa)\,,$$

where μ is the pion's mass, m is the K meson's mass, $\epsilon_{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor, $\epsilon_{1234}=1$, and P_l' is the first derivative of the Legendre polynomial of order l. We have expressed in terms of invariants the cosine of the angle between the momentum of the charged pion and that of the photon, $\hat{q}\cdot\hat{p}_c=\cos\theta$, as measured in the center-of-mass system of the two pions,

$$\hat{q}\cdot\hat{p}_c=q\cdot(p_0-p_c)(q\cdot\kappa)^{-1}(1-4\mu^2/m^2{}_{\pi\pi})^{-1/2}.$$