by Amann, Freund, Oehme, and Rotelli that higher intermediate states can be included in our superconvergence relations without changing the results obtained with the low-lying states alone. One possibility is to introduce these states in sets corresponding to multiplets of the rest symmetry $U(6)\otimes U(6)\otimes O(3)$. It turns out that every irreducible representation $(6,\bar{6};l)$ with mass $m(l)$ *separately* satisfies the superconvergence relations for the reactions $PV\rightarrow PV$, $PV\rightarrow VV$, and $VV\rightarrow VV$ considered in this paper, provided the vertices are invariant under the collinear $U(6)\otimes O(2)$ group. One may try to use the infinite sequence of particles corresponding to the representations $(6,\bar{6};l)$, $l=1, 2, \cdots\infty$ in order to saturate the nonforward superconvergence relations. It appears, however, that no nontrivial solution exists for mass spectra $m^2(l)$ with accumulation points $m^2(\infty)>4m^2(0)$. [P. G. O. Freund, R. Oehme, and P. Rotelli (to be published)].

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Quark Model for Baryon-Baryon Processes

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A quark (Q) model recently proposed by the authors for high-energy meson-baryon and photoproduction processes is extended to high-energy baryon-baryon scattering and production of negative-parity baryons. The positive- and negative-parity baryons are assumed to belong, respectively, to the representations (**56**,1) and (**70**,3) of the group $SU(6)\times O(3)$. Sum rules resembling those of $SU(3)$ and $SU(6,6)$, but differing in details, are obtained for the baryon-baryon processes within the **56** representation. An interesting sum rule which is obtained for negative-parity baryon production involves only nonstrange particles and could be within fairly early reach of experiment. The model is primarily characterized by an impulse approximation for Q-Q scattering and thus disallows processes involving exchanges of more than one unit of charge, hypercharge, and even spatial symmetry, among the initial and final $3Q$ states representing the baryons. This last condition precludes the high-energy peripheral production of metastable baryon states like the Roper resonance, which we believe to have an internal orbital structure of $L^P=1^+$.

1. INTRODUCTION

RECENTLY, a quark model was proposed for meson-baryon processes[1] and photoproduction,[2] based on a sort of impulse approximation characterized by the scattering of a pseudoscalar-meson octet by a quark triplet ($Q+\Pi\rightarrow Q+\Pi$) in the first case and the photoproduction of a meson on a quark ($Q+\gamma\rightarrow Q+\Pi$) in the second. The various degrees of freedom involved in the basic amplitudes were averaged over the initial and final $3Q$ wave functions appropriate to the baryon (B) states of interest. The $3Q$ structures of the baryons of positive and negative parities are in turn given by the (**56**,1) and (**70**,3) representations, respectively, of the group $SU(6)\times O(3)$, as has been shown elsewhere.[1,3] This procedure gave a set of sum rules broadly resembling $SU(6)$ and allied symmetries,[4] but differing in detail. One distinguishing feature of this model is that, unlike other contemporary ones,[5,6] it regards the mesons as elementary and baryons as composite. The physical motivation behind this "asymmetric" treatment between baryons and mesons stems from a comparison of their respective binding energies taking two-body forces (Q-Q or Q-$\bar{Q}$) as the basic dynamics. Since the Q-Q system (diquark) has a much greater mass (mass $\sim M_Q$)[7] than a Q-$\bar{Q}$ system (mass $\sim$ a few pion masses), it seems to make sense to regard the mesons (as Q-$\bar{Q}$ composites) as "more elementary" than baryons regarded as $3Q$ systems. Thus a "parametrization" was sought in terms of quarks and mesons[1,2] rather than in terms of quarks and antiquarks, taking advantage of the idea that any intimate relation between particles and antiparticles is an essentially relativistic concept, so that in the nonrelativistic limit, Q and $\bar{Q}$ are really distinct entities with little relation to each other.

Even such an asymmetric treatment between baryons and mesons did provide a good number of sum rules, many of which agreed with the results of the additivity model of Lipkin.[5] Of course the latter, being a more comprehensive assumption, is capable of yielding a richer variety of results. Our weaker assumption of ignoring the compositeness of meson structures, however, was largely compensated by the assumption of

[1] G. C. Joshi, V. S. Bhasin, and A. N. Mitra, Phys. Rev. **156**, 1572 (1967).
[2] S. Das Gupta and A. N. Mitra, Phys. Rev. **156**, 1581 (1967).
[3] A. N. Mitra, Phys. Rev. **151**, 1168 (1966); Ann. Phys. (N. Y.) (to be published).
[4] D. A. Akyeampong and R. Delbourgo, Phys. Rev. **140**, B1013 (1965); V. Barger and M. H. Rubin, *ibid.* **140**, B1366 (1965).
[5] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966); H. J. Lipkin and F. Scheck, *ibid.* **16**, 71 (1966).
[6] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters **17**, 412 (1966).
[7] A. N. Mitra, Phys. Rev. **142**, 1119 (1966).

$SU(6)\times O(3)$ group structures of the baryons as $3Q$ states.[3]

In this paper we propose an extension of this approach to baryon-baryon processes (scattering and production). Such calculations have been already performed in the Lipkin model[5] by Levin and Frankfurt.[8] In a sense, this case is somewhat simpler since only quarks and *not* their antiparticles are involved. The main assumption here is that a quark from the first baryon scatters a quark from the second, and that this scattering can be taken in the impulse approximation. The justification for such an assumption has been given by Levin and Frankfurt,[8] through their ideas of a "deep vector well" for each baryon state. Such a model does not necessarily contradict the idea of nonrelativistic quark motions within the same well, but one must assume that the quarks in different wells (as in two baryons) have in general highly relativistic velocities with respect to one another in an over-all frame. One should perhaps add a second condition that the bindings of these quarks to their respective wells do not appreciably distort their motions.[9] In any event, for a successful application of the impulse approximation techniques, one needs to assume two distinct types of Q-Q interaction, with nonoverlapping domains of validity, viz. (i) within a baryon well, the Q-Q forces must be very strong and short ranged, and (ii) between two quarks in different wells, there exist weaker but long-range forces. Quarks in different wells have little chance of coming within a range short enough to experience the strong Q-Q forces of type (i), if we assume that strong repulsive barriers separate these entities from one another.[10]

Under such an assumption one can think of a Q-Q amplitude generated by the "long-range force." This amplitude, regarded as an operator, plays the basic role in generating the baryon-baryon amplitudes of interest through the evaluation of its matrix elements between suitable two-baryon wave functions, each baryon regarded as a $3Q$ system as in Ref. 1 or 2. This Q-Q amplitude, which may be taken in an $SU(3)$ invariant form, must have the unitary-spin dependence restricted to 1 or $\lambda_1\cdot\lambda_2$, where λ is the octet of 3×3 Gell-Mann matrices for each quark.[11] Thus in the impulse approximation all processes involving more than one unit of ΔQ, ΔS, or ΔI must vanish. Using such an amplitude one can consider not only processes like B-B scattering and B-B^* production within the **56** of (positive-parity) baryons but even the production of negative-parity baryons belonging to (**70**,3). For processes involving particles within the same representation, (**56**) or (**70**,3), one would expect a set of sum rules, but the correlation of one set with another is in general a dynamical question. While many of the sum rules would involve B-B amplitudes which cannot be of immediate physical interest, it may be possible to pick at least some with comparatively better chances of confrontation with experiment. We shall find that while most of the results within the **56** are largely conventional,[4,5] an interesting sum rule within experimental reach in the not too distant future is obtained for the negative-parity baryons.

One simplifying feature of the model stems from the (assumed) physical impossibility of quarks in different baryons from coming within the domain of strong short-range interaction. This prevents the quarks in each baryon from exchanging positions with their counterparts in the other baryon. The situation is analogous to a diatomic molecule in which the chances of the *nuclear* constituents in each atom interchanging positions (physically) are almost negligible. This feature makes it possible to consider the problem, not in terms of a full "six-body" symmetry but in such a way that the symmetry can be confined to each $3Q$ system separately. By this "separation" of one $3Q$ system from another, one can easily incorporate the usual Fermi statistics for baryons by invoking the antisymmetry requirement when the two $3Q$ composites fully interchange places. This, however, does not prevent one from invoking a different form of statistics within each $3Q$ system. In particular, the formalism can be consistent with the assumption of, say, parastatistics[12] for the quarks within a baryon, characterized by the appearance of symmetrical $3Q$ functions.[3]

In Sec. 2 the basic Q-Q scattering amplitude is defined taking account of spin, $SU(3)$, and spatial degrees of freedom, and the general method of evaluating its matrix elements (with respect to the first two degrees of freedom) between various B-B states is outlined. Section 3 deals with the calculation of amplitudes for scattering and production processes within the **56** of baryons. In Sec. 4, we obtain results in terms of sum rules for these B-B processes and discuss their experimental status. Section 5 is concerned with the production of negative-parity states, and a sum rule involving only nonstrange particles within reasonable experimental access is obtained. Finally, Sec. 6 is a short discussion of the results in relation to other models and the possibility of production of certain positive-parity states like the Roper resonance within this model.

2. STRUCTURE OF THE B-B AMPLITUDE

The $SU(3)$-invariant amplitude for the scattering of a quark i in the first baryon and a quark j in the

[8] E. M. Levin and L. L. Frankfurt, JETP Pis'ma Redaktsiya **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)].

[9] See, e.g., P. B. James and H. D. D. Watson, Phys. Rev. Letters **18**, 179 (1967).

[10] See, e.g., M. Gell-Mann, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

[11] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[12] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

second is of the form

$$A_{ij}=A_{ij}^{(+)}+A_{ij}^{(-)}\boldsymbol{\lambda}_i^{(1)}\cdot\boldsymbol{\lambda}_j^{(2)}, \tag{2.1}$$

where we use the convention that the first and second indices in A_{ij} refer to the first and second baryons, respectively (in an arbitrary enumeration). To write down the spatial structures of $A_{ij}^{(\pm)}$, one has to distinguish between several different coordinate frames. One is the six-body c.m. frame in which the momenta of the two sets of quarks are $(\mathbf{P}_1^{(1)},\mathbf{P}_2^{(1)},\mathbf{P}_3^{(1)})$ and $(\mathbf{P}_1^{(2)},\mathbf{P}_2^{(2)},\mathbf{P}_3^{(2)})$, which are related to the c.m. baryon momenta $\mathbf{K}$, $-\mathbf{K}$ by

$$\mathbf{P}_1^{(1)}+\mathbf{P}_2^{(1)}+\mathbf{P}_3^{(1)}=\mathbf{K}=-(\mathbf{P}_1^{(2)}+\mathbf{P}_2^{(2)}+\mathbf{P}_3^{(2)}). \tag{2.2}$$

A similar relation holds in the final states where the various quantities are denoted by primes. There is a second coordinate frame in which the center of mass of the quarks within each baryon is at rest. If we denote the two independent relative 3-momenta within baryon i by

$$\mathbf{Q}_1^{(i)}=6^{-1/2}(\mathbf{P}_2^{(i)}+\mathbf{P}_3^{(i)}-2\mathbf{P}_1^{(i)}), \tag{2.3}$$

$$\mathbf{q}_1^{(i)}=2^{-1/2}(\mathbf{P}_3^{(i)}-\mathbf{P}_2^{(i)}), \tag{2.4}$$

it is clear that for nonrelativistic quark motion within each baryon, the $3Q$ wave function will depend only on these two quantities for each quark and that the bodily motions will be represented by the vectors (2.2). Similar considerations apply to the final state as well. The 3-momentum of any quark is now expressible in terms of $\pm\mathbf{K}$, $\mathbf{Q}_1^{(i)}$, and $\mathbf{q}_1^{(i)}$ according to (2.2), (2.3), and (2.4). Of course, since the quarks in different baryons can be relativistic, there still remains the problem of calculating their 4-momenta. One possibility is to use the Kokkedee–Van Hove model[13] in which the 4-momenta $P_i^{(1)}$ and $P_j^{(2)}$ of two such quarks in baryons (1) and (2) are expressible as $c_i^{(1)}p^{(1)}$ and $c_j^{(2)}p^{(2)}$, where $p^{(1)}$ and $p^{(2)}$ are the 4-momenta of the bodily motions of the baryons and $c_i^{(1)}$ and $c_j^{(2)}$ are certain *internal* structure constants. While such considerations are necessary for writing down the completely relativistic structures of the amplitudes, they clearly do not affect Eq. (2.2) for the 3-vector parts of the various momenta. On the other hand, as we are merely interested in sum rules between certain amplitudes and not in their quantitative structures, the relativistic details will not be pursued further.

Finally, we have a third coordinate system in which the c.m. of the two quarks, one in each baryon, is at rest. This is the frame most suitable for the Q-Q scattering amplitude, on lines analogous to N-N scattering. In terms of the usual c.m. invariants (s,t,u) of the initial and final 4-momenta of the quarks, amplitudes $A_{ij}^{(\pm)}$ are expressible in the over-all c.m. frame of six particles as

$$A_{ij}^{(\pm)}=\delta^4(P_i^{(1)}+P_j^{(2)}-P_i^{(1)\prime}-P_j^{(2)\prime})a_{ij}^{(\pm)},$$
$$a_{ij}^{(\pm)}=\sum_{\mu,\nu=0,1,2,3}a_{ij}^{(\pm)\mu\nu}\sigma_{i\mu}^{(1)}\sigma_{j\nu}^{(2)}, \tag{2.5}$$

where σ_0 is a 2×2 unit matrix and $\sigma_{i\mu}^{(1)}$ $(i=1,2,3)$ are the usual Pauli spin matrices for quark i in baryon (1), and the coefficients on the right-hand side of (2.5) depend on the (s,t,u) variables. This form leaves open the question of inclusion of the relativistic energies, in addition to the c.m. 3-momenta defined by

$$2\mathbf{p}_{ij}=\mathbf{P}_i-\mathbf{P}_j,\quad 2\mathbf{p}_{ij}'=\mathbf{P}_i'-\mathbf{P}_j',$$

in the arguments of the coefficients $a_{ij}^{(\pm)\mu\nu}$. The complete Q-Q amplitude is now

$$\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}, \tag{2.6}$$

which we must regard as an operator whose matrix elements should be evaluated between appropriate baryon-baryon states. The details of the nonrelativistic $3Q$ wave function representing a baryon can be obtained from Ref. 1 both for the (**56**,1) and (**70**,3) representations of $SU(6)\times O(3)$. Under parastatistics, the over-all symmetric states that can be constructed are

$$(8):\quad \Psi=2^{-1/2}\psi^s(\chi'\phi'+\chi''\phi''), \tag{2.7}$$

$$(10):\quad \Psi=\psi^s\chi^s\phi^s. \tag{2.8}$$

For the (**70**,3) states the spatial function is of mixed symmetry with $L^P=1^-$. Thus Ψ contains terms like

$$\Big(\sum_{m_s m_l}\langle Lm_l Sm_s|Jm\rangle\psi_{L,m_l}{}^{\alpha}\chi_{S,m_s}{}^{\beta}\Big)\phi^{\gamma},$$

which we abbreviate as[3,14]

$$[\psi^{\alpha}\chi^{\beta}]_J\phi^{\gamma}, \tag{2.9}$$

where α,β,γ are the subscripts (each being of the type prime, double prime, and s) which indicate the permutation symmetry.[15] Since such functions must be written for *each* baryon, we are required to evaluate matrix elements of the type

$$M_{ij}=\langle\Psi_f^{(1)}\Psi_f^{(2)}|A_{ij}|\Psi_i^{(1)}\Psi_i^{(2)}\rangle, \tag{2.10}$$

where each Ψ is an expression of the form (2.7) or (2.8), as the case may be.[1] The antisymmetry between identical baryons is taken care of in a pragmatic way.[16]

[13] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966).

[14] See, e.g., A. N. Mitra and M. H. Ross, Phys. Rev. **58**, 1630 (1967).

[15] M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170.

[16] However, the antisymmetry in the wave function of two baryons has very little significance for high-energy scattering, since there is always the relative angular momentum l to take care of this requirement. It is only for low-energy scattering corresponding to, say, $l=0$ or 1 that this requirement is a pressing one.

We now consider separately the cases of octet baryon-baryon scattering, decuplet production, and the production of negative-parity baryons.

3. CALCULATIONAL TECHNIQUES

A. Octet-Octet Scattering

We start by evaluating the $SU(3)$-spin matrix elements, which, in this case, are of the 16 types,

$$\langle(\phi',\phi'')^{(1)}\times(\phi',\phi'')^{(2)}|1;\ \lambda_i^{(1)}\cdot\lambda_j^{(2)}|(\phi',\phi'')^{(1)}\times(\phi',\phi'')^{(2)}\rangle,\quad(3.1)$$

for each of nine possible pairs of (ij) values. The physically interesting cases always include a proton (target) as one of the initial particles, while the other is a nucleon or at most a Λ^0 or $\Sigma^\pm$. The evaluation of (3.1) has therefore been confined only to cases which are in keeping with the above requirement. For each pair of initial and final baryons, one has a set of 16 number arrays each of which can be displayed as a 3×3 matrix whose elements correspond to the nine pairs of (ij) values distinguishing the quarks of the two baryons. The advantage of such a representation is that one can easily look for $SU(3)$-type sum rules for B-B processes confining oneself to *each pair of* (ij) *values separately*. This last requirement is important if one notes that the spatial and spin-dependent coefficients multiplying the $SU(3)$ operators 1 or $\lambda_i^{(1)}\cdot\lambda_j^{(2)}$ in the total $QQ\to QQ$ amplitude may be quite different for the different i and j indices.

For a particular octet-octet scattering process, one has to take a linear combination of the sixteen terms. A sum rule or an $SU(3)$ relation is now obtained for a set of amplitudes if one finds the *same* $SU(3)$ relation for *each* of the nine pairs of (ij) indices.

The spin degrees of freedom can next be taken into account, by evaluating matrix elements like

$$\langle(\chi',\chi'')^{(1)}\times(\chi',\chi'')^{(2)}|O_{ij}|(\chi',\chi'')^{(1)}\times(\chi',\chi'')^{(2)}\rangle,\quad(3.2)$$

where O_{ij} is one of the following operator types:

$$\delta_{ij};\quad \boldsymbol{\sigma}_i^{(1)}\pm\boldsymbol{\sigma}_j^{(2)};\quad \boldsymbol{\sigma}_i^{(1)}\cdot\boldsymbol{\sigma}_j^{(2)};\quad \boldsymbol{\sigma}_i^{(1)}\times\boldsymbol{\sigma}_j^{(2)};\quad \sigma_{i\mu}^{(1)}\sigma_{j\nu}^{(2)}+\sigma_{i\nu}^{(1)}\sigma_{j\mu}^{(2)}-\tfrac{2}{3}\delta_{\mu\nu}\boldsymbol{\sigma}_i^{(1)}\cdot\boldsymbol{\sigma}_j^{(2)}.\quad(3.3)$$

Here μ, ν are the three-dimensional tensorial indices. For each pair of (i,j) indices one can again evaluate these matrix elements on similar lines. The spin-flip (SF) and non-spin-flip (NSF), amplitudes can be evaluated separately according to the usual definitions:

$$\text{NSF}:\quad \chi_{\pm1/2}^{(1,2)}\to\chi_{\pm1/2}^{(1,2)},$$
$$\text{SF}:\quad \chi_{\pm1/2}^{(1,2)}\to\chi_{\mp1/2}^{(1,2)}.\quad(3.4)$$

Thus for NSF amplitudes we finally have a matrix element of the form

$$M_{ij}^{(\text{NSF})}=\langle\psi_s^{(1)}\psi_s^{(2)}|B_{ij}^{\text{NSF}(+)}+B_{ij}^{\text{NSF}(-)}|\psi_s^{(1)}\psi_s^{(2)}\rangle;\quad(3.5)$$

$$B_{ij}^{\text{NSF}(\pm)}=\sum_{\mu,\nu=0,3}\alpha_{\mu\nu}^{(\pm)}a_{ij}^{\mu\nu}.\quad(3.6)$$

The quantities $\alpha_{\mu\nu}^{(\pm)}$ are purely geometrical numbers dependent on the spin and unitary spin matrix elements corresponding to the type of baryons concerned. In particular $\alpha_{\mu\nu}^{(+)}$ is an isotropic term in the $SU(3)$ indices (*not spin*!) contributing only to elastic scattering. Note that the α's are independent of the (ij) indices, a fact that has been mentioned already in connection with the $SU(3)$ matrix elements. This fact really provides the clue to sum rules since no special assumption is being made on the spatial structures of the coefficients $B_{ij}^{(\pm)}$. For SF amplitudes we have similarly

$$M_{ij}^{\text{SF}}=\langle\psi_s^{(1)}\psi_s^{(2)}|B_{ij}^{\text{SF}(+)}+B_{ij}^{\text{SF}(-)}|\psi_s^{(1)}\psi_s^{(2)}\rangle,\quad(3.7)$$

where

$$B_{ij}^{\text{SF}(\pm)}=\sum_{\mu,\nu=1,2}\beta_{\mu\nu}^{(\pm)}a_{ij}^{(\pm)\mu\nu}.\quad(3.8)$$

The following types of sum rules can now be obtained. First, the NSF amplitudes, taken in the forward direction, give the relations between total cross sections. Separate relations for SF and NSF amplitudes in any direction can also be written down, with the help of which one obtains sum rules for certain differential cross sections. Finally, one can distinguish between charge-exchange and charge-nonexchange processes. However, since the only case of physical interest is that of n-p, we shall not discuss this last possibility any further, except to note that we get results identical to those of DeSouza *et al.*[17] in this respect. We simply refer to their paper for further details on this point. The other sum rules for octet-octet scattering are given in Sec. 4, in relation to the experimental situation.

B. Decuplet Production in B-B Processes

As in Sec. 3 A, we can in principle evaluate the $SU(3)$ spin matrix elements for the processes in which a decuplet is produced in the final state, by replacing, in (3.1), $(\phi',\phi'')_f^{(1)}$ and/or $(\phi',\phi'')_f^{(2)}$ by $\phi_s^{(1)}$, $\phi_s^{(2)}$, and correspondingly the spin functions (χ',χ'') by χ^s in the appropriate baryon. The cases of practical interest are, however, those in which only one member of the decuplet is involved in the final state. With this physical restriction, one obtains a total of 16 different terms for a given initial pair of baryons (8 each for the replacement of a final octet by a decuplet). These 16 terms, because of the spatially symmetric baryon wave functions, give identical decuplet-production matrix elements in the joint space of spin and unitary spin for each quark pair (ij). For NSF processes for the relevant baryon, characterized by $(\chi_{1/2}',\chi_{1/2}'')\to\chi_{1/2}^s$, the matrix ele-

[17] P. D. DeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, B565 (1964).

ments can finally be expressed as

$$M_{ij}^{\mathrm{NSF}}=\langle\psi_s^{(1)}\psi_s^{(2)}|D_{ij}|\psi_s^{(1)}\psi_s^{(2)}\rangle, \quad (3.9)$$

$$D_{ij}=\sum_{\mu,\nu=0,3}\delta_{\mu\nu}{}^{(-)}a_{ij}{}^{(-)\mu\nu}. \quad (3.10)$$

Here, $\delta_{\mu\nu}$ are geometrical coefficients independent of the indices (ij) as stated above, so that identical relations between production amplitudes can again be obtained individually for each pair of quarks (ij). Note that the coefficients in (3.10) are the same as (3.6) given earlier. This is because the complete production amplitude is expressible in terms of the *same* radial integrals as those found for *B-B* scattering.[18] This enables us to find, in principle, relations (listed in Sec. 4) between the two processes, though the latter are as yet mere predictions because of the lack of suitable experimental data.

4. SUM RULES WITHIN 56

We now list certain sum rules for *B-B* scattering, which are true *separately* for the superscripts NSF and SF denoting, respectively, the non-spin-flip and spin-flip amplitudes in any general direction. In these relations the amplitudes for the allowed charge (or hypercharge) exchange processes are listed together, e.g., $\langle\Lambda N+N\Lambda|\Sigma^-p\rangle$ stands for the sum of the two amplitudes, $\langle\Lambda N|\Sigma^-p\rangle$ and $\langle N\Lambda|\Sigma^-p\rangle$.

$$\langle pp|pp\rangle=\langle np|np\rangle+\langle pn|np\rangle, \quad (4.1)$$

$$\langle\Sigma^-p|\Sigma^-p\rangle=\langle\Xi^0p|\Xi^0p\rangle, \quad (4.2)$$

$$\begin{aligned}\langle\Sigma^0n+n\Sigma^0|\Sigma^-p\rangle&=3^{1/2}\langle\Lambda n+n\Lambda|\Sigma^-p\rangle\\&=-3^{1/2}\langle\Sigma^+n+n\Sigma^+|\Lambda p\rangle\\&=-6^{1/2}\langle\Sigma^0p+p\Sigma^0|\Lambda p\rangle\\&=\langle\Sigma^+n+n\Sigma^+|\Sigma^0p\rangle, \quad (4.3)\end{aligned}$$

$$\langle pp|pp\rangle+\langle\Sigma^-p|\Sigma^-p\rangle=2\langle\Sigma^0p+p\Sigma^0|\Sigma^0p\rangle, \quad (4.4)$$

$$\langle pp|pp\rangle-\langle\Sigma^-p|\Sigma^-p\rangle=2^{1/2}\langle\Sigma^0n+n\Sigma^0|\Sigma^-p\rangle, \quad (4.5)$$

$$\langle np+pn|np\rangle=\langle\Sigma^+p+p\Sigma^+|\Sigma^+p\rangle, \quad (4.6)$$

$$\langle\Sigma^+p|\Sigma^+p\rangle+\langle\Sigma^-p|\Sigma^-p\rangle=2\langle\Sigma^0p|\Sigma^0p\rangle. \quad (4.7)$$

From these, the relations between the total cross sections at high energies can be deduced via the optical theorem applied to the NSF relations for elastic scattering. Thus one has

$$\sigma^t(\Sigma^-p)=\sigma^t(\Xi^0p), \quad (4.8)$$

$$2\sigma^t(\Sigma^0p)=\sigma^t(\Sigma^+p)+\sigma^t(\Sigma^-p), \quad (4.9)$$

$$\tfrac{3}{2}[\sigma^t(\Lambda p)+\sigma^t(\Sigma^0p)]=\sigma^t(\Xi^-p)+\sigma^t(np)+\sigma^t(\Sigma^+p). \quad (4.10)$$

For inelastic processes, we have certain relations between the differential cross sections by considering

$$d\sigma/d\Omega=|A^{\mathrm{NSF}}|^2+|A^{\mathrm{SF}}|^2. \quad (4.11)$$

The latter are given as

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^0n)&=3\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Lambda n)\\&=\frac{d\sigma}{d\Omega}(\Sigma^0p\to\Sigma^+n), \quad (4.12)\end{aligned}$$

$$\frac{d\sigma}{d\Omega}(\Sigma^+p\to\Sigma^+p)=\frac{d\sigma}{d\Omega}(np\to np), \quad (4.13)$$

$$\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^-p)=\frac{d\sigma}{d\Omega}(\Xi^0p\to\Xi^0p), \quad (4.14)$$

$$\begin{aligned}4\frac{d\sigma}{d\Omega}(\Sigma^0p\to\Sigma^0p)+2\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^0p)&\\=\frac{d\sigma}{d\Omega}(pp\to pp)+\frac{d\sigma}{d\Omega}(\Sigma^-p\to\Sigma^-p). \quad (4.15)\end{aligned}$$

The relations (4.2) and (4.6) are merely $SU(3)$ predictions while a prediction analogous to (4.1) for charge-exchange np scattering from the Regge-pole model is, in the c.m. system,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta=0}(np\to pn)=\frac{k^2}{4\pi^2}[\sigma^t(pp)-\sigma^t(np)]^2. \quad (4.16)$$

This has been experimentally compared by Manning *et al.*,[19] but without much success. In our model, however, we obtain the following inequality for the differential cross section for charge exchange np scattering:

$$\frac{d\sigma}{d\Omega}(np\to pn)>\frac{k^2}{4\pi^2}[\sigma^t(pp)-\sigma^t(np)]^2, \quad (4.17)$$

where we have used (4.1) for the NSF amplitude and the result that, in (4.11), the right-hand side is reduced by merely dropping the term $|A^{\mathrm{SF}}|^2$ for the process $np\to pn$. The inequality (4.17) is in agreement with the experimental observations at $\theta=0$ of Manning *et al.*[19] As for hyperon-proton scattering, Eq. (4.12) predicts the following ratio for the differential cross sections of the processes $\Sigma^-p\to\Lambda n$ and $\Sigma^-p\to\Sigma^0n$, leading to the same relation for the total cross sections, viz.,

$$\sigma(\Sigma^-p\to\Sigma^0n)/[\sigma(\Sigma^-p\to\Sigma^0n)+\sigma(\Sigma^-p\to\Lambda n)]=\tfrac{3}{4}. \quad (4.18)$$

While a comparison of this result is meaningful for sufficiently high energy, this is not possible for lack of appropriate data. Some experimental data are available[20] only for $p_{\Sigma^-}(\mathrm{lab})\approx150$ MeV/c. According to the measurement of Engelmann *et al.*, the experimental

[18] By virtue of the appearance of the same spatial function $\psi_s^{(1)}, \psi_s^{(2)}$ in (3.9) as in (3.5).

[19] G. Manning *et al.*, Nuovo Cimento **41A**, 167 (1966).

[20] R. Engelmann *et al.*, Phys. Letters **21**, 587 (1966).

TABLE I. Negative-parity wave function under parastatistics. The coefficients of the appropriate unitary spin functions are listed under the respective $SU(3)$ multiplets. For the octet, only the coefficient of the state ϕ' is shown. For explanation of symbols, see Ref. 1 or 3.

L	S	J	[10]	[8]	[1]
1	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_\mu'\chi_\mu'+\psi_\mu''\chi_\mu''$	$\psi_\mu'\chi_\mu''+\psi_\mu''\chi_\mu'$	$\psi_\mu'\chi_\mu''-\psi_\mu''\chi_\mu'$
1	$\frac{3}{2}$	$\frac{1}{2}$	0	$\psi_\mu'\chi_\mu^s$	0
1	$\frac{1}{2}$	$\frac{3}{2}$	$(\psi_\mu'\sigma_\mu'+\psi_\mu''\sigma_\mu'')\chi_{3/2}^s$	$(\psi_\mu'\sigma_\mu''+\psi_\mu''\sigma_\mu')\chi_{3/2}^s$	$(\psi_\mu'\sigma_\mu''-\psi_\mu''\sigma_\mu')\chi_{3/2}^s$
1	$\frac{3}{2}$	$\frac{3}{2}$	0	$\psi_\mu'\sigma_\mu^s\chi_{3/2}^s$	0
1	$\frac{3}{2}$	$\frac{5}{2}$	0	$\psi_{1,1}'\chi_{3/2}^s$	0

ratio for the left-hand side of (4.18) is 0.47,[20] which, understandably, is not in good agreement with (4.18). For the B-B processes with decuplet production, the NSF amplitudes give the following sum rules, which are mostly predicted by $SU(3)$:

$$\begin{aligned}\langle\Delta^0p+p\Delta^0|np\rangle&=\langle\Delta^+p+p\Delta^+|pp\rangle\\&=-\langle\Delta^+n+n\Delta^+|np\rangle\\&=-3^{-1/2}\langle\Delta^{++}n+n\Delta^{++}|pp\rangle,\end{aligned}\quad(4.19)$$

$$\begin{aligned}\langle\Sigma^{*+}p+p\Sigma^{*+}|\Sigma^+p\rangle&=\langle\Delta^+\Sigma^++\Sigma^+\Delta^+|\Sigma^+p\rangle\\&=2^{1/2}\langle\Delta^0\Sigma^0+\Sigma^0\Delta^0|\Sigma^-p\rangle,\end{aligned}\quad(4.20)$$

$$\langle\Sigma^{*-}p|\Sigma^-p\rangle=-\langle\Xi^{*-}p|\Xi^-p\rangle\quad(4.21)$$

$$=4(6)^{-1/2}\langle\Delta^0\Lambda+\Lambda\Delta^0|\Sigma^-p\rangle,\quad(4.22)$$

$$\begin{aligned}\langle\Sigma^{*0}n+n\Sigma^{*0}|\Sigma^-p\rangle&=3(2)^{-1/2}\langle\Delta^0p+p\Delta^0|np\rangle\\&\quad-2(3)^{-1/2}\langle\Sigma^{*+}n+n\Sigma^{*+}|\Lambda p\rangle\\&=\langle\Sigma^{*+}n+n\Sigma^{*+}|\Sigma^0p\rangle.\end{aligned}\quad(4.23)$$

Experimental comparisons of these cannot be made at present. Finally, the relations that relate the NSF amplitudes for B-B scattering and decuplet production, respectively, are found to be

$$\begin{aligned}&\langle pp|pp\rangle+\langle\Xi^-p|\Xi^-p\rangle\\&\quad-2\langle\Lambda p+p\Lambda|\Lambda p\rangle+2^{1/2}\langle n\Lambda+\Lambda n|\Sigma^-p\rangle\\&=\langle\Delta^{++}n+n\Delta^{++}|pp\rangle-2^{1/2}\langle\Sigma^{*0}n+n\Sigma^{*0}|\Sigma^-p\rangle\\&=2^{1/2}\langle n\Sigma^0+\Sigma^0n|\Sigma^-p\rangle-4\langle\Lambda\Lambda|\Xi^-p\rangle,\end{aligned}$$

which again cannot be tested as yet in the absence of data on Ξ^--particle beams against proton targets.

5. PRODUCTION OF NEGATIVE-PARITY BARYONS

The cases of physical interest in such processes are those in which one of the final baryons is a negative-parity one, while the other continues to be an ordinary octet. The negative-parity baryons, according to Refs. 1 and 3, are members of the **(70**,3) representation of $SU(6)\times O(3)$, and the spatial parts of their wave functions are of the form (ψ_μ',ψ_μ''), μ being a three-dimensional tensor index. For convenience, we reproduce in Table I the complete wave functions of the various $SU(3)$ multiplets of (**70**,3). As there are two octets each of $J^P=\frac{1}{2}^-$ and $J^P=\frac{3}{2}^-$, we have chosen a basis in which these respective octets are classified according to their spin structures, (viz., spin-doublet and spin-quartet states). The relevant matrix elements are now of the form

$$\langle([\psi^\alpha\chi^\beta]_J\phi^\gamma)^{(1)}\Psi^{(2)}|\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}|\Psi^{(1)}\Psi^{(2)}\rangle,\quad(5.1)$$

and a second one with the roles of the indices (1), (2) interchanged in the final state. According to Table I, the index α takes only the prime and double-prime labels, while the indices β and γ take on the additional label s. The functions $\Psi^{(1,2)}$ are of course the same as used in Sec. 3.

If we now evaluate these matrix elements in successive steps as in the case of scattering (in Sec. 3), we note that the $SU(3)$ matrix elements have exactly the same structure as before. Thus, for the choice (5.1), we can arrange the matrix elements as suitable linear combinations of 24 (3×3) matrices (the elements correspond to nine possible ij values), there being now *three* possibilities (ϕ',ϕ'',ϕ^s) for the $SU(3)$ function associated with the negative-parity baryon. The evaluation of the spin matrix elements is also on identical lines. The only important difference lies in the structure of the radial integrals. Since there will now be a large number of them, and we are interested only in their qualitative structures, we shall merely indicate their possible types, instead of giving a complete list.

For each pair (ij) there is a multiplying spatial function $a_{ij}^{\mu\nu}$, which can be expressed in terms of three distinct types of spherical harmonics, viz., those of $l=0, 1, 2$. An inspection of the amplitude (2.5) shows that there are *four* distinct functions of *each* type. Each of these spherical harmonics can be associated with the final-state negative-parity function (ψ',ψ'') of $L^P=1^-$ to give a resultant orbital momentum 0, $(1)^3$, $(2)^2$, 3, the multiplicities of the repeated angular momenta being indicated by superscripts. Thus $7\times4=28$ radial integrals are obtained for each of the functions ψ' and ψ'', giving a total of 56 possible radial integrals.[21] It should therefore be surprising that any simple relation

[21] Note that since a preferred direction is defined through the incident beam, the problem is not just one of construction of possible invariants with these terms, rather one of all possible harmonics (cf. Ref. 1).

should exist at all with so many radial integrals to be eliminated. Since, on the other hand, there would not be much point in writing sum rules between hypothetical amplitudes, we have first explored the possibility of obtaining relations between production amplitudes corresponding to nonstrange particles due to an initial p-p state. Perhaps the only reason why one can hopefully look for amplitude relations involving only nonstrange particles is the appearance of a large number of $SU(3)$ states with the $(\mathbf{70},3)$ representation of $SU(6)\times O(3)$, each $SU(3)$ state having its own nonstrange components, all with the same spatial wave function. We have succeeded in finding the following relation[22] between the production amplitudes $\langle N^*p|pp\rangle_\lambda{}^{(SJ)}$ where the subscript λ denotes $SU(3)$ multiplicity of the negative-parity baryon and the superscripts SJ denote its spin (S) and J values as in Table I.

$$\langle N^*p|pp\rangle_8{}^{(\frac{1}{2}\frac{1}{2})}+\langle N^*p|pp\rangle_8{}^{(\frac{3}{2}\frac{1}{2})} -3^{1/2}\langle N^{*++}n|pp\rangle_{10}{}^{(\frac{1}{2}\frac{1}{2})}+(\tfrac{3}{2})^{1/2}\langle N^{*++}n|pp\rangle_{10}{}^{(\frac{1}{2}\frac{3}{2})} =(\tfrac{1}{2})^{1/2}\langle N^{*+}p|pp\rangle_8{}^{(\frac{1}{2}\frac{3}{2})}+(\tfrac{1}{2})^{1/2}\langle N^{*+}p|pp\rangle_8{}^{(\frac{3}{2}\frac{3}{2})}. \quad (5.2)$$

This is a rather interesting relation which offers a relatively better chance of detection than others involving strange particles. However, it is still too early to confront such a relation with experiment.

6. DISCUSSION

Most of our results, being in the form of sum rules, are independent of any detailed dynamics based on the quark structure of the baryons. The only dynamical information used for the model is of a qualitative nature, designed to provide a formal justification of the model proposed, not to predict any *absolute* magnitudes for the cross sections, total or differential. On the other hand, the sum rules obtained depend specifically on the group structures $(\mathbf{56},1)$ and $(\mathbf{70},3)$ of $SU(6)\times O(3)$ for the positive- and negative-parity states, respectively, and to that extent could provide a test of this particular group, as compared with the predictions of groups like $SU(6)$, $\tilde{U}(12)$, $SU(6,6)$, etc. In this respect, this quark model goes somewhat beyond the additivity model for Q-Q amplitudes,[5] where no such explicit group structures are assumed. Of course, one common feature it shares with the latter is the so-called impulse approximation which restrict its validity to the high-energy region, characterized by exchanges of not more than *one unit* of charge or strangeness (ΔQ or ΔS).

The model has a very simple prediction for the production of states like the Roper resonance. According to the interpretation given in Ref. 3, $3Q$ states like the Roper resonance are characterized by $L^P=1^+$ and antisymmetric (A) spatial wave functions. The calculation of amplitudes for the production of such baryons in the final state proceeds on lines identical with Sec. 5. The particular baryon characterized by an A function has a spin-unitary spin function which is also antisymmetric under parastatistics, in contrast with the symmetrical spin-unitary spin function in its initial state. The evaluation of these matrix elements then leads to *zero* value for such amplitudes. Qualitatively this result may be interpreted as an impossibility within our model to produce a final baryon state involving, so to speak, "two units of symmetry change, from an S function to an A function," if we suppose that a change from S to M or from M to A each signifies one unit of symmetry change. This is in the same language that we use to speak of one unit of charge or hypercharge transfer. Thus the most that this model can do is to produce the negative-parity baryons involving only one unit of symmetry change (from S to M), as it does with respect to ΔS or ΔQ. Apparently it is incapable of producing a change of two units of symmetry, just as it gives zero amplitudes for $\Delta S=2$ or $\Delta Q=2$.

Physically, such limitations, which are the result of the impulse approximation to the Q-Q amplitude, can be interpreted as due to the exchange of meson singlets or octets (whether pseudoscalar, vector, or tensor types). To produce changes of two units in any of these attributes, one must go beyond the impulse approximation characterized by double meson exchanges between two quarks, simultaneous scattering of two pairs of quarks, or similar mechanisms. Such amplitudes, which are necessarily several orders of magnitude smaller in the high-energy limit, might well be much less negligible at lower energies, where, however, symmetry-breaking effects like the mass differences would play a more important role. Inclusion of such effects within our model is in principle possible only through a more elaborate formulation of the Q-Q amplitude so as to take account of scattering of more than one quark pair, etc. Any estimate of their relative magnitudes compared with those of the impulse approximation cannot be made without more detailed dynamical assumptions on the nature of the Q-Q forces which generate the Q-Q scattering amplitudes as well as the $3Q$ bound states.

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[22] It may be noted that the sum rule is a result of cancellation of certain unwanted terms when the possible ij values are added up, according to the additivity assumption. This may be contrasted with the case of octet scattering, where the same amplitude relation was obtained for each (ij) pair.