Nonresonant π-π Contributions to Electromagnetic Form Factors

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The effect of the exchange of two nonresonant pions on the electromagnetic structure of the pion and the nucleon is investigated. The results show that such an exchange is not negligible. By taking it into account, the value of the ρ mass needed to fit the nucleon isovector form factor may be changed from its previous anomalously low value to its experimental value. In addition, the coupling constant $g_{\rho NN}$, as deduced from the nucleon electromagnetic structure, is affected significantly.

I. INTRODUCTION

IN fitting the experimental values of the electromagnetic form factors of the neutron and proton, pole fits are frequently used. This is not only a convenient parametrization of the data but is also the form which results from the exchange of narrow resonances, e.g. the ϕ, ω, ρ, etc. For the isovector part of the nucleon form factors the symmetries of the system allow only the exchange of mesons with $T=1$, $P=-$ $(G=+1)$ and since the only meson possessing these properties is the ρ meson one should expect a one-pole approximation to fit the isovector form factors. Such a fitting has been made[1] but one finds that the mass of the exchanged particle which gives the best fitting is 560 MeV as compared to the experimental mass of the ρ which is 750 MeV.

This may be rectified by introducing more parameters in the form of a second pole at a higher mass value.[1] However, we investigate here an alternative which is that there is exchanged together with the ρ meson a system with substantially lower mass but the same quantum numbers. The only system which can fill this role is two pions in a nonresonant state. Our aim in this work is to investigate the effects of such a nonresonant contribution on electromagnetic form factors using dispersion relations. Thus the information to be fed in will be the $J=1$ pion-pion phase shift. Since little is known about this phase shift apart from the neighborhood where it passes through $\pi/2$, we assume several reasonable and different forms and investigate the consequences.

In Sec. II possible forms of the pion-pion phase shift are suggested and discussed. Section III contains the evaluation and presentation of the pion form factors resulting from these phase shifts and this leads in Sec. IV to the evaluation of the isovector form factors of the nucleon which are compared with experimental data. Finally the consequences of a nonresonant contribution on the determination of the ρ-nucleon-nucleon coupling constant $g_{\rho NN}$, for the electromagnetic form factors is discussed.

[1] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, 458 (1965).

II. PION-PION *P*-WAVE PHASE SHIFT

Experimentally, information about the π-π p wave phase δ_1 shift may be obtained indirectly by investigating the final-state interaction between the two pions in the process $\pi+N\rightarrow\pi+\pi+N$. The results show conclusively the existence of a resonance at an energy of 750 MeV (in the barycentric system of the two pions) and width about 100 MeV,[2–4] the ρ meson. However, away from the resonance where $\sin\delta_1$ becomes smaller it is difficult to extract a reliable value of δ_1. Also the method is based on a simple assumption, one-pion exchange, for the reaction mechanism and any lack of validity of this assumption would cause errors in the phase shift deduced. The phase shift resulting from such an analysis of the data by Wolf[5] is included in Fig. 1.

A cleaner experiment to analyze is the decay $K\rightarrow\pi+\pi+e+\nu$, but as yet few data are available.[6]

Attempts to evaluate theoretically the p-wave phase shift by means of dispersion relations have so far been notably unsuccessful in predicting the observed width of the resonance.[7–9] Thus, outside the region around the position of the ρ there are very few restrictions on the phase shift so long as it does not become too large. We have therefore adopted the procedure of choosing several reasonable phase shifts, requiring them to have the correct threshold behavior, to pass through $\pi/2$ at the experimental mass of the ρ, to give the experimental width of the ρ and at higher energies not to fall faster than allowed by Wigner's theorem.[10]

There are three energy intervals to be considered. Using t for the square of the total energy of the two pions in their barycentric system (and $m_\pi=1$), these are:

(*i*) $25<t<35$. The ρ meson corresponds to $t=30$ and

[2] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **29**, 515 (1963).
[3] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **25**, 365 (1962).
[4] J. Baton and J. Regnier, Nuovo Cimento **36**, 1149 (1965).
[5] G. Wolf, Phys. Letters **19**, 328 (1965).
[6] R. Birge *et al.*, Phys. Rev. **139**, B1600 (1965).
[7] G. F. Chew and S. Mandelstam, Nuovo Cimento **19**, 752 (1961).
[8] L. Balázs, Phys. Rev. **128**, 1939 (1962).
[9] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).
[10] E. Wigner, Phys. Rev. **98**, 145 (1955).

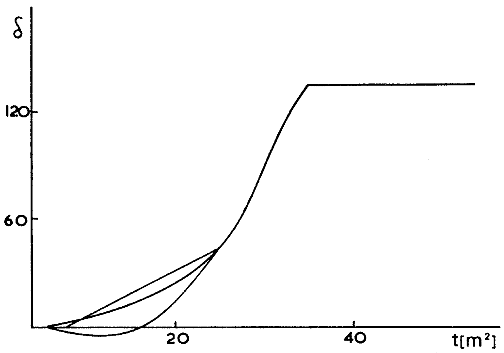

FIG. 1. Models for the p-wave phase shift.

in this region a Breit-Wigner form was used with a mass and width corresponding to those of the ρ meson.

(*ii*) $4<t<25$. In this interval several forms were used, each having the correct threshold behavior for a p wave [i.e. $\delta(t)=\alpha(t-4)^{3/2}$ near $t=4$] and each joining continuously to the Breit-Wigner form at $t=25$. We list here just four of the cases we considered.

(a) $\delta(t)=0.05q^3$ where $t=4(q+1^2)$. In this case the threshold behavor is continued up to the end of the interval, the constant being chosen so that this joins with the Breit-Wigner form.

(b) $\delta(t)=0.02q^3(q^3-3)$. Here we investigate the possibility of a negative low-energy phase shift as suggested recently by Jones.[11]

(c) $\delta=0$, $4<t<8$; $\delta=(\pi/4)(t-8/17)$, $8<t<25$. This corresponds to the phase shift resulting from the analysis of Wolf.

(d) $\delta(t)=0$. No nonresonant contribution at all.

(*iii*) $t>35$. In this region the phase shift was continued downwards from its value at $t=35$ by a straight line reaching the value $\delta(t)=0$ at some point $t=\Lambda$ and beyond this point taken to be zero:

$$\delta(t)=\delta(35)\left(\frac{t-\Lambda}{35-\Lambda}\right) \quad \text{for } 35<t<\Lambda,$$

$$\delta(t)=0 \quad \text{for } t>\Lambda.$$

Thus we have a one-parameter description of the high-energy phase shift and it seems reasonable that within dispersion integrals an appropriate choice of Λ should be able to reporduce the effect of the real phase shift.

There is one restriction we have placed on the value of Λ. This is that the rate at which the phase shift falls should be restricted by the Wigner theorem[10] according to which $d\delta(q)/dq\geqslant -R$, where R is the range of the interaction.

Hence

$$\frac{d\delta(q)}{dq}=\frac{d\delta(t)}{dt}\frac{dt}{dq}=\frac{\delta(35)}{35-\Lambda}, \quad \delta q\geqslant -R$$

or $\delta(35)\delta q\leqslant(\Lambda-35)R$.

Taking a value of q corresponding to $t=40$ and the range $R=2m_\pi$ gives $\Lambda\geqslant 150$. Choosing R to be given by the exchange of a ρ meson leads to $\Lambda\geqslant 350$. The phenomenological analysis of Wolf supports a slow rate of fall of the phase shift beyond the resonance. The high-energy phase shift used by Vick[12] violates the Wigner condition.

The phase shifts are illustrated in Fig. 1.

III. THE PION FORM FACTOR

The pion from factor $F_\pi(t)$, satisfies the dispersion relation

$$F_\pi(t)=1+\frac{t}{\pi}\int_4^\infty \frac{\operatorname{Im} F_\pi(t')}{t'(t'-t)}dt'$$

and in addition has the property that its phase is equal to the phase of the $T=1$, $J=1$ π-π elastic scattering amplitude for $4\leqslant t\leqslant 16$. Indeed we may write the Omnes[13] expression for $F_\pi(t)$

$$F_\pi(t)=\exp\left\{\frac{t}{\pi}\int_4^\infty \frac{\delta(t')dt'}{t'(t'-t)}\right\}, \tag{1}$$

where $\delta(t)$ is the phase of $F_\pi(t)$ on the cut and is equal to the phase shift for elastic π-π scattering in the $J=1$ state when inelastic processes are negligible. Since the $\rho\to 4\pi$ decay is small the inelastic processes are negligible at least up to $t\simeq 36$. Above this $\delta(t)$ will not be the elastic phase shift but simply the phase of $F_\pi(t)$. However, since neither the elastic phase shift nor the phase of $F_\pi(t)$ are known in this region the assumed $\delta(t)$ is simply a reasonable and convenient parametrization of

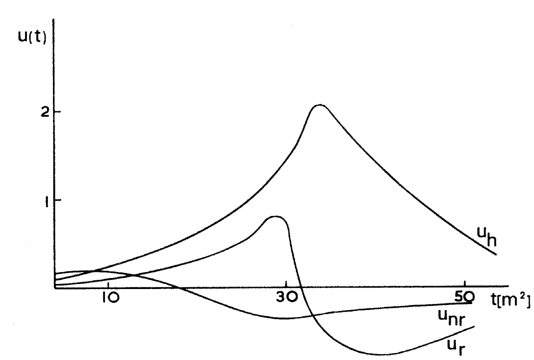

FIG. 2. The contributions to the pion's spectral function due to the low-energy region u_{NR}, to the resonance u_R, and to the high-energy phase shift u_H, respectively.

[11] L. W. Jones, D. O. Caldwell, B. Zacharov, D. Harting, E. Bleuler, W. C. Middlekoop, and B. Elsner, Phys. Letters **21**, 590 (1966).

[12] L. L. J. Vick, Nuovo Cimento **31**, 643 (1964).

[13] R. Omnes, Nuovo Cimento **8**, 316 (1958).

the phase for these values of t. Thus a knowledge of $\delta(t)$ for $t>4$ completely determines the pion form factor.

Breaking up the range of integration into three intervals as in Sec. II we have

$$\frac{t}{\pi}\int_4^{\infty}\frac{\delta(t')}{t'(t'-t)}dt'=\frac{t}{\pi}\int_4^{25}\frac{\delta(t')}{t'(t'-t)}dt'+\frac{t}{\pi}\int_{25}^{35}\frac{\delta(t')dt'}{t'(t'-t)}$$
$$+\frac{t}{\pi}\int_{35}^{\infty}\frac{\delta(t')}{t'(t'-t)}dt'=u_{NR}(t)+u_R(t)+u_H(t)\,.$$

The resonance contribution $u_R(t)$ which contains no adjustable parameters was evaluated by fitting the Breit-Wigner phase shift with a low-order polynomial and then the integration may be done analytically giving the curve shown in Fig. 2.

Also in Fig. 2 are shown the contributions from the nonresonant phase shifts we used and the high-energy contribution $u_H(t)$ for several values of the parameter Λ.

Using these values we have plotted in Figs 3 and 4 the spectral function Im $F_\pi(t)$ for the pion form factor. In Fig. 3 the nonresonant contribution is kept constant and only the high-energy contribution changed. The effect of decreasing the phase shift more rapidly at high energies is to decrease the spectral function in the neighborhood of the resonance. Increasing the nonresonant phase shift at low t also has this effect as shown in Fig. 4.

In terms of the spatial distribution of charge over the pion this means that any interaction away from the ρ resonance tends to shift charge either towards the center of the pion in the case of a high-energy interaction or outwards in the case of a low-energy interaction. Thus one of the effects of a nonresonant exchange would be the increase in radius of the charge distribution of the pion. There is only a little experimental information at present available on the charge structure of the pion. Nordberg and Kinsey[14] measuring the difference in π^+-α and π^--α elastic scattering arrive at a value of the rms charge radius of the pion r_π of 1.8 ±0.8 F.

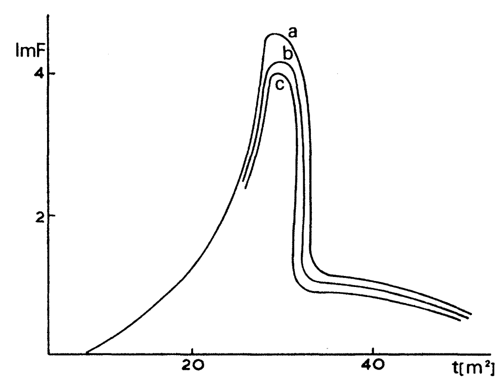

FIG. 3. The pion's spectral function for different values of the parameter Λ. The low-energy phase shift corresponds to Wolf's results. Curve (a) $\Lambda=\infty$; (b) $\Lambda=670$; (c) $\Lambda=350$.

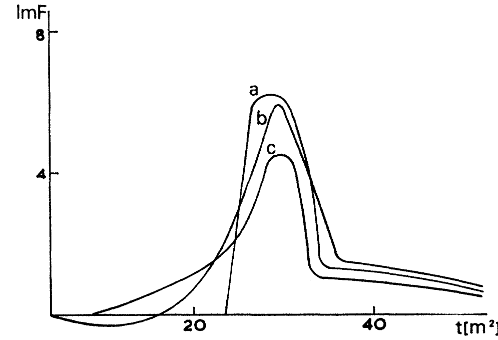

FIG. 4. The pion's spectral function for different choices of the low-energy phase shift. In all cases $\Lambda=\infty$. In curve (a) $\delta=0$; in (b) $\delta<0$; in (c) δ corresponds to Wolf's phase shift.

In order to compare this with the theoretical calculation we may write[15]

$$F_\pi(t)=1+(t/6)\langle r_\pi^2\rangle \text{ for } t\simeq 0$$

and find r_π from the slope of the form factor at $t=0$.

The corresponding value of the slope in this experiment is

$$\tfrac{1}{6}\langle r_\pi^2\rangle=(0.27\pm0.053)m_\pi^{-2}\,.$$

In a different experiment using pion electroproduction Ankerlof *et al.*[16] find a value

$$\tfrac{1}{6}\langle r_\pi^2\rangle=(0.041_{-0.020}^{+0.026})m_\pi^{-2}\,,$$

which agrees well with the value obtained using current-algebra techniques.[17] We may compare these results with the charge radius deduced from the spectral functions determined above.

In our analysis, the inclusion of a significant low-energy nonresonant phase shift like that of Wolf's and the use of a subtracted dispersion relation for the $F_\pi(t)$ leads to a value

$$\tfrac{1}{6}\langle r_\pi^2\rangle=0.029m_\pi^{-2}.$$

For a phase shift of a simple Breit-Wigner form with no nonresonant contribution and in the simple narrow resonance approximation, we would have

$$F_\pi^{(\rho)}(t)=1-g_\rho+\frac{g_\rho t_\rho}{t_\rho-t}.$$

The constant 1-g_ρ which determines the core charge of the pion may be determined from our spectral func-

[14] M. E. Nordberg, Jr., and K. F. Kinsey, Phys. Letters **20**, 692 (1966).

[15] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).

[16] C. W. Akerlof, W. W. Ash, K. Berkelman, and C. A. Lichtenstein, Phys. Rev. Letters **16**, 147 (1966).

[17] F. J. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966).

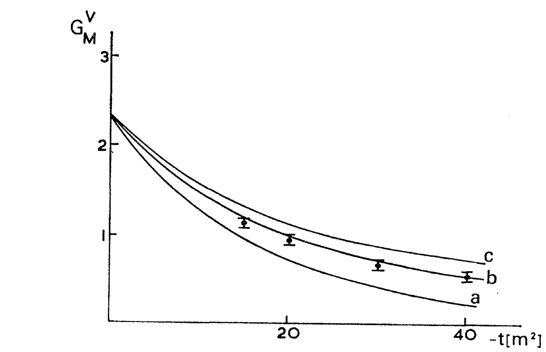

FIG. 5. Magnetic isovector form factor $G_M{}^V$ corresponding to the pion's spectral function of Fig. 3. The low-energy phase shift is that of Wolf. Curve (a) $\Lambda=\infty$; (b) $\Lambda=670$; (c) $\Lambda=350$. Shown also are the experimental data.

tions using the relation

$$1-g_\rho=\frac{1}{\pi}\int_4^\infty \frac{\operatorname{Im} F_\pi(t')}{t'}dt'.$$

In the case of Wolf's phase shift and $\Lambda\simeq 650$ we obtain $1\text{-}g_\rho=0.31$, and therefore in the absence of nonresonant background the slope becomes

$$\tfrac{1}{6}\langle r_\pi{}^2\rangle=g_\rho/t_\rho$$

or

$$\tfrac{1}{6}\langle r_\pi{}^2\rangle=0.023m_\pi{}^{-2}.$$

We therefore conclude that the inclusion of a nonresonant background results in an increase of the radius corresponding to simple pole exchange by 10–15%. Although the effect of a nonresonant background is not very large in the physical region of the pion's form factor it is important in the spectral region as illustrated in Figs. 3 and 4. We expect that the processes in which this region coincides with the physical energy range should be more sensitive to this background, as for example the processes in which an electromagnetic decay to a pion pair is involved. In addition in processes

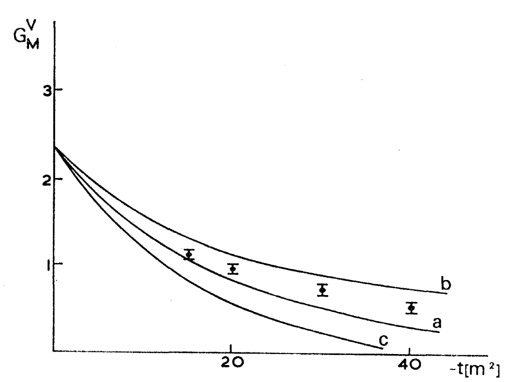

FIG. 6. Magnetic isovector form factor corresponding to $\Lambda=350$ and to various choices of the low-energy phase shift. Curve (a) $\delta=0.05q^3$; (b) $\delta=$Wolf's value (Ref. 5); (c) $\delta=0$. Shown also are the experimental data.

where the pion spectral function is a factor of another spectral function this difference may be important. This is demonstrated in the next section for the case of the nucleon form factors.

IV. THE NUCLEON FORM FACTORS

We shall be concerned only with the isovector nucleon form factors, $F_1{}^V$ and $F_2{}^V$, since it is these to which the $T=1$, $J=1$ π-π exchange contributes. They satisfy the dispersion relations[18,19]

$$\frac{2}{e}F_1{}^V(t)=1+\frac{t}{\pi}\int_4^\infty\frac{g_1(t')dt'}{t'(t'-t)},$$

$$\frac{2}{e}F_2{}^V(t)=\frac{g}{m}+\frac{t}{\pi}\int_4^\infty\frac{g_2(t')dt'}{t'(t'-t)},$$

where

$$g_i(t)=-2t^{-1/2}q^3F_\pi{}^*(t)\Gamma_i(t)\nu\,,\quad (i=1,2)$$

and the $\Gamma_i(t)$ are the helicity amplitudes for π-N scat-

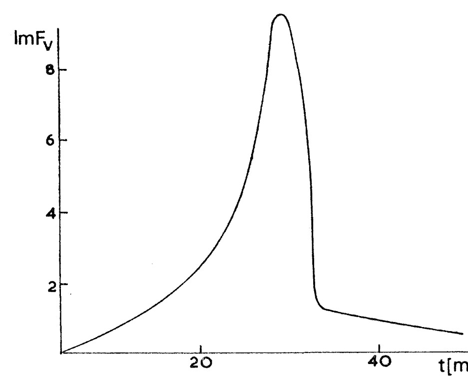

FIG. 7. Spectral function of the nucleon Im $F_v(t)$ for $\Lambda=670$ and corresponding to Wolf's phase shift.

tering as defined by Frazer and Fulco.[20] They satisfy a dispersion relation

$$\Gamma_i(t)=\frac{1}{\pi}\int_{-\infty}^{a}\frac{\operatorname{Im}\Gamma_i(t')dt'}{t'-t}+\frac{1}{\pi}\int_4^\infty\frac{\operatorname{Im}\Gamma_i(t')dt'}{t'-t}$$

$$\left(a=4\mu^2-\frac{\mu^4}{m^2}\right),$$

and also have the property that for $4<t<16$ the phase is equal to the π-π, $J=1$ scattering phase. Thus if we write $\Gamma_i(t)=J_i(t)/F_\pi(t)$ the $J_i(t)$'s satisfy

$$J_i(t)=\frac{1}{\pi}\int_{-\infty}^{a}\frac{\operatorname{Im}J_i(t')dt'}{t'-t}+\frac{1}{\pi}\int_{16}^\infty\frac{\operatorname{Im}J_i(t')}{t'-t}dt'.$$

[18] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. 110, 265 (1958).

[19] P. Federbush, M. L. Goldberger, and S. Treiman, Phys. Rev. 112, 642 (1958).

[20] W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).

These $J_i(t)$'s which depend on π-N dynamics through the left-hand cut have been evaluated by Vick[12] and we shall assume here that they remain unmodified by changes in the low-energy π-π p-wave interaction.

Thus,

$$g_i(t) = -2t^{-1/2}q^3|F_\pi(t)|^2 J_i(t),$$

and we may substitute the different forms of $F_\pi(t)$ which we calculated in Sec. III. The results are shown in Figs. 5 and 6 for the $G_M{}^V(t)$. We see that with zero low-energy π-π phase shift is it not possible to fit the experimental data, but that by including a phase shift of the same type as that deduced by Wolf a good fit may be made to the data with the physical mass and width of the ρ meson. The effect of the nonresonant interaction is strengthened here because the $J_i(t)$'s are large for small values of t and more weight is given to these low values through the nonresonant spectral function of the pion. This is shown clearly in Fig. 7.

Ball and Wong[21] found a decrease in the effective mass of the ρ in form-factor calculations which they attributed to the finite width of the ρ. However, as they fed in the π-π phase shift by means of an N/D solution for the partial wave it is not clear how much of their phase shift is what we have called "nonresonant." In our case the peaks of the spectral functions stay almost fixed at the position of the ρ and a smoothly varying nonresonant part builds up for lower t values.

We find in addition that the nucleon form factors also depend upon the high-energy behavior of the π-π phase shift (see Fig. 5). The best fit is obtained using a value of $\Lambda=650$. It seems therefore that in making a detailed fit, the dependence on both the low-energy and high-energy phase shifts should be taken into account simultaneously. However, our calculations show that, with the form of high-energy phase shifts used here, it is not possible to fit $G_M{}^V(t)$ without inserting some low-energy nonresonant phase shift and that conversely a reasonably small nonresonant phase shift enables a good fit to be made.

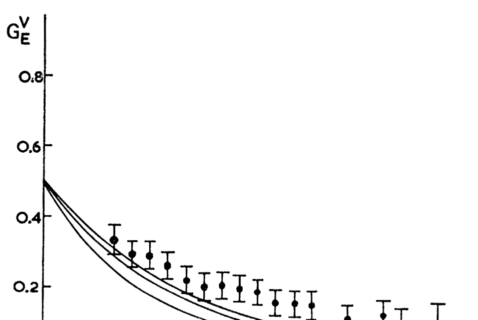

FIG. 8. Electric isovector form factor $G_E{}^V$, corresponding to the low-energy phase shift given by Wolf and various values of Λ: Curve (a) $\Lambda=\infty$; (b) $\Lambda=670$; (c) $\Lambda=350$. Shown also are the experimental data.

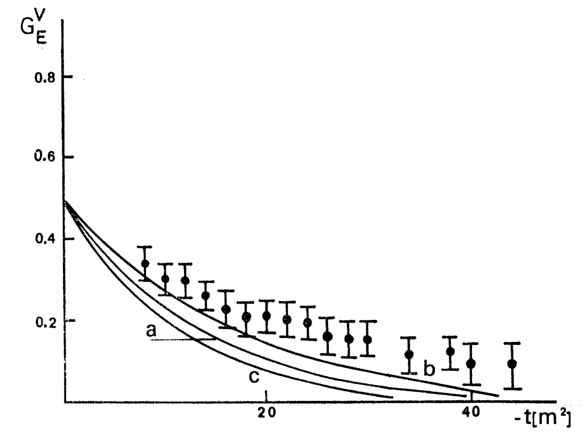

FIG. 9. Electric isovector form factor $G_E{}^V$, corresponding to $\Lambda=350$ and various choices of the low-energy phase shift. Curve (a) $\delta=0.05q^3$; (b) $\delta=$ Wolf's value (Ref. 5); (c) $\delta=0$.

The corresponding curves for $G_E{}^V(t)$ are shown in Figs. 8 and 9. Here the fit is not so good although the experimental errors are much larger than for $G_M{}^V(t)$. This is possibly due to the fact that we have neglected any indirect dependence of the $J_i(t)$ on the π-π phase shift and modified $\Gamma_i(t)$ only through the factor $F_\pi(t)$. Indeed the $J_1(t)$ and $J_2(t)$ quoted by Vick are almost identical, a fact which leads to $F_1{}^V(t)$ and $F_2{}^V(t)$ being almost identical. This is not borne out experimentally and leads to the lack of agreement for $G_E{}^V(t)$.

Inelastic effects for $t>30m_\pi{}^2$ could also also modify the spectral functions and improve the agreement.

V. COUPLING CONSTANTS

A final interesting point we want to discuss is the effect of the nonresonant contribution on the coupling of the ρ meson to the nucleon-antinucleon pair. It is known that knowledge of the electromagnetic structure of the nucleons and the pions provide a test of the coupling of the vector mesons to baryons and pseudoscalar particles in the simple vector-meson exchange models. In our case we expect that the inclusion of the uncorrelated spectrum of the two pions has an important effect on these coupling constants. If the exchange of a single ρ meson is assumed to dominate the isovector electromagnetic structure of the nucleon, the expressions for $G_{E,M}{}^V$ are

$$G_E{}^V(t) = C_E{}^V + G_{E\rho}{}^V g_{\rho\gamma}/1 - t/m_\rho{}^2,$$
$$G_M{}^V(t) = C_M{}^V + G_{M\rho}{}^V g_{\rho\gamma}/1 - t/m_\rho{}^2,$$

where the G_ρ's and $g_{\rho\gamma}$'s are the coupling constants of the ρ meson to the nucleons and electromagnetic field, respectively. As mentioned previously in order to fit experiment the ρ mass must be a parameter. These coupling constants may then be extracted by fitting

[21] J. S. Ball and D. Y. Wong Phys. Rev. **130**, 2112 (1963).

experiment and be compared with the predictions of some symmetry scheme.[22]

However, if we fix the ρ mass at its true value and include a nonresonant contribution, then the resulting forms of G are different from those above and we must re-extract the coupling constants. To do this we take as the effective coupling constant the area under the resonance in the spectral function from $t-\Gamma/2$ to $t+\Gamma/2$.

We then find $g_{\rho\gamma}G_{E\rho}{}^V=0.48$, $g_{\rho\gamma}G_{M\rho}{}^V=1.41$, and therefore $G_{M\rho}{}^V/G_{E\rho}{}^V=3$. This is to be compared with the narrow-resonance, variable mass values[1] of $g_{\rho\gamma}G_{E\rho}{}^V=0.525$, $g_{\rho\gamma}G_{M\rho}{}^V=2.47$, $G_{M\rho}{}^V/G_{E\rho}{}^V=4.7$. This indicates that in extracting coupling constnats from experiment the use of single-mesons-exchange formulas is not always justified and that inclusion of nonresonant effects may change significantly the results. Finally the coupling $g_{\rho\gamma}$ may be estimated by taking the area of the spectral function $\operatorname{Im} F_\pi(t)$ under the resonance. We find $g_{\rho\pi\pi}g_{\rho\pi}=0.35$, where $g_{\rho\pi\pi}$ is the coupling of the ρ to the π-π system given by the ρ resonance width ($\Gamma_\rho=100$ MeV)

$$\frac{g_{\rho\pi\pi}{}^2}{4\pi}=2\,.$$

Therefore, $g_{\rho\gamma}=0.07$ and

$$G_{E\rho}{}^V=6.83\,,\quad G_{M\rho}{}^V=20.11\,.$$

We want now to extract the vector and tensor couplings of the ρ meson to the nucleon-antinucleon pair corresponding to a vertex function of the form

$$F_\mu=f^V{}_{\rho NN}\gamma_\mu+f^T{}_{\rho NN}\sigma_{\mu\mu}q_\mu/2M\,.$$

The relations of $f_{\rho NN}{}^V$, $f_{\rho NN}{}^T$ to the $G_{E\rho}{}^V$, $G_{\mu\rho}{}^V$ are simply the equations defining the Sachs form factors from the Pauli-Dirac form factors and therefore we obtain the following system:

$$G_{E\rho}{}^V=f_{\rho NN}{}^V+\frac{m_\rho{}^2}{4M^2}f_{\rho NN}{}^T\,,$$

$$G_{M\rho}{}^V=f_{\rho NN}{}^V+f_{\rho NN}{}^T\,,$$

and

$$f_{\rho NN}{}^V=4.2,\quad f_{\rho NN}{}^{\mathrm{T}}=15.88\,.$$

According to a universal coupling of vector mesons to the isospin current proposed by Sakurai[23] we must have $2f_{\rho NN}{}^V/g_{\rho\pi\pi}=1$ whereas according to our results $2f_{\rho NN}{}^V/g_{\rho\pi\pi}=1.64$. Therefore, the inclusion of a nonresonant interaction in the p-wave π-π system seems also to affect the coupling constant $f_{\rho NN}{}^V$.

VI. CONCLUSIONS

We have shown that a reasonable nonresonant π-π p-wave phase shift can have quite important consequences. It can increase the charge radius of the pion and give a sizeable contribution to the isovector nucleon form factor.

It is therefore of interest to see if there is any theorectical evidence for the existence of such a phase shift. One way of investigating this is to consider forward dispersion for the $T=1$ π-π system and by substituting all known contributions try to evaluate the low-energy π-π phase shift by imposing unitarity. Such a program has been undertaken by one of us (N.G.A.) and preliminary results seem to indicate the necessity of a low-energy phase shift of the magnitude which we have used.

ACKNOWLEDGMENT

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[22] N. J. Papastamatiou, Nuovo Cimento, **41A**, 625 (1966).

[23] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).