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Resonance Saturation of Axial Charge Commutators, $SU(3)$ Symmetry, and Scalar Mesons*

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Resonance saturation of axial charge commutators taken between states of the pseudoscalar-meson octet is examined and found to be strongly suggestive of the existence of a 0^+ octet and a 0^+ singlet in the 500–1000-MeV region. The vector-meson contribution to the matrix element of the vector current taken between K and π states is shown to give only 50% or so of the experimental value, in contrast to the conclusions of other authors, but in agreement with general SU(3) considerations and the estimate of Adler of the ρ contribution to the sum rule for the π–π scattering. The approach to π_{e3} decay based on dispersion theory and charge-current commutators already used by Marshak *et al.* for K_{e3} decay is shown to give good agreement with experiment if vector-meson dominance is assumed. It is pointed out that in both cases the resulting expressions for the form factors at zero momentum transfer differ exactly by a factor of 2 from the corresponding ones obtained in the approach using only commutators of charges.

INTRODUCTION

In many of the recent calculations utilizing the ideas of current algebra[1] and PCAC (partially conserved axial-vector-current hypothesis) a "saturation" assumption is made: A sum over a complete set of intermediate states, sandwiched between the two factors of a commutator, is replaced by a sum over both stable and unstable one-particle states, the latter representing an approximation to the contribution of some of the multiparticle states. In order to use experimental information, it is also often assumed that off-mass-shell effects are either not very important, or can be simply accounted for by a kinematic correction. When the matrix element of a current (or charge) is more or less known, either from experiment, or because one is dealing with a conserved or partially conserved current, the evaluation of the same matrix element for a commutator which is proportional to this current (or charge) yields insight into the validity of these assumptions. In particular, one may ask: (a) To what extent do corresponding saturation assumptions made in different commutators yield correspondingly "good" or "bad" results? (b) To what extent are such assumptions consistent with approximate $SU(3)$ invariance? (c) To what extent are calculations using only the charge algebra in agreement with related ones making use of dispersion relations and charge-current commutators?

For example, we note that the assumption of vector-meson dominance in the saturation of the sum rule for π-π scattering obtained from the commutator

$$[A_{\pi^+},A_{\pi^-}]=2V_{\pi^0}, \qquad (1)$$

taken between $|\pi^+\rangle$ states, yields less than half of the value of the right-hand side,[2] whereas the same assumption in the evaluation of the commutator

$$[A_{\bar{K}^0},A_{\pi^-}]=V_{K^-}, \qquad (2)$$

taken between $|\pi^0\rangle$ and $|K^+\rangle$ states, apparently yields $\sim$100% of the value of the right-hand side,[3] known approximately from experiment on K_{e3} decay.

It is the purpose of this paper to point out that: (a) The vector-meson–dominance assumption in the evaluation of various charge commutators taken between appropriate states of the pseudoscalar meson octet P is consistent with approximate $SU(3)$ symmetry, yielding about one half of the expected value in all cases. (b) The assumption that the remaining 50% or so largely represents the contribution of 0^+ and 2^+ resonances in the commutators considered seems to require and is consistent with the existence of a 0^+ meson with $I=\frac{1}{2}$, $Y=1$ and of two 0^+ mesons with $I=0$, $Y=0$. (c) A calculation of the form factor at zero momentum transfer for π_{e3} decay using ρ dominance for the absorptive part, similar to the calculations of Pandit *et al.*[4] for the K_{e3} form factor, using K^* dominance, yields equally good results; however, both calculations yield a result twice as large as that obtained from the vector mesons in the computation making use only of the charge algebra.

We consider points (a), (b), and (c) in turn. Besides Eqs. (1) and (2), commutator relations of interest will include[5]

$$[A_{\pi^-},A_{\pi^0}]=V_{\pi^-}, \qquad (3)$$

$$[A_{K^-},A_{\pi^0}]=\tfrac{1}{2}V_{K^-}, \qquad (4)$$

and

$$[A_{K^-},A_{K^0}]=-V_{\pi^-}. \qquad (5)$$

The form factors $F_+(t)$ and $f_\pm(t)$ for π_{e3} and K_{e3} decay

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[1] M. Gell-Mann, Physics **1**, 63 (1964).

[2] S. Adler, Phys. Rev. **137**, B1022 (1965).

[3] V. S. Mathur, S. Okubo, and L. K. Pandit, Phys. Rev. Letters **16**, 370 (1966); **16**, 601(E) (1966).

[4] V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters **16**, 947 (1966); **16**, 1135(E) (1966).

[5] Vector and axial-vector currents are denoted by $V_\mu^{\pi^+}(x)$, $V_\mu^{K^+}(x)$, $\cdots$, and $A_\mu^{\pi^+}(x)$, $A_\mu^{K^+}(x)$, $\cdots$, respectively, normalized so that in a quark model we would have, e.g., $V_\mu^{\pi^+}(x)=i\bar{q}\gamma_\mu\times(\lambda_1+i\lambda_2)q/2$, $A_\mu^{K^+}(x)=i\bar{q}\gamma_\mu(\lambda_4+i\lambda_5)q/2$, etc.; the space integral of, say, $A_0^{\pi^+}(\mathbf{x},0)\equiv -iA_4^{\pi^+}(\mathbf{x},0)$ is denoted by A_{π^+}. The PCAC relations are used in the form $\partial_\mu A_\mu^{\pi^\pm}=C_\pi\phi^{\pi^\pm}$ and $\partial_\mu A_\mu^{K^\pm}=C_K\phi^{K^\pm}$, where, e.g., ϕ^{π^+} creates π^+ mesons.

are defined, respectively, by

$$(4q_0q_0')^{1/2}\langle\pi^0(q')|V_\mu^{\pi^-}(0)|\pi^+(q)\rangle=\sqrt{2}F_+(t)(q+q')_\mu$$

and

$$(4q_0q_0')^{1/2}\langle\pi^0(q')|V_\mu^{K^-}(0)|K^+(q)\rangle = 1/\sqrt{2}[f_+(t)(q+q')_\mu+f_-(t)(q-q')_\mu],$$

with $t=-(q-q')^2$. The factors of $\sqrt{2}$ have been chosen so that the conserved-vector-current hypothesis and $SU(2)$ symmetry implies $F_+(0)=1$ and in the limit of $SU(3)$ symmetry we also have $f_+(0)\to 1$.

VECTOR MESON CONTRIBUTIONS AND $SU(3)$

On taking the matrix element of Eq. (3) between $|\pi^0\rangle$ and $|\pi^+\rangle$, we get, following the standard technique[6] as applied in Ref. 3, i.e., letting $|\mathbf{q}|\to\infty$,

$$F_+(0)=F_+(0)|_\rho+\cdots,$$

where

$$F_+(0)|_\rho=(\tfrac{1}{2})\left(\frac{C_\pi^2}{m_\pi^4}\right)\frac{G_{\rho^+\pi^-\pi^0}{}^2}{m_\rho^2} \tag{6}$$

is the ρ-meson contribution and the dots represent the remainder.[7] Similarly, the matrix element of Eq. (4) taken between $|\pi^0\rangle$ and $|K^+\rangle$ states yields

$$f_+(0)=f_+(0)|_{K^*}+\cdots,$$

where

$$f_+(0)|_{K^*}=2\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{C_K}{m_K^2}\right)\frac{G_{K^{*+}K^-\pi^0}{}^2}{m_{K^*}{}^2} \tag{7}$$

is the K^*-meson contribution. Numerical evaluation of Eqs. (6) and (7) gives[8]

$$F_+(0)|_\rho=0.48, \tag{6'}$$

$$f_+(0)|_{K^*}=0.59. \tag{7'}$$

Since one expects, on the basis of approximate $SU(3)$ symmetry and the Ademollo-Gatto theorem,[9] that $f_+(0)\approx 1$, we see that the vector mesons make very similar contributions to the commutators (3) and (4). On the other hand, Mathur *et al.*,[3] using the commutator (2), to which both ρ and K^* contribute, found $f_+(0)|_{K^*,\rho}\approx 1$, i.e., *saturation* of this commutator by vector mesons. However, the matrix element of Eq. (2) taken between $|\pi^0\rangle$ and $|K^+\rangle$ yields

$$f_+(0)=f_+(0)|_{\rho,K^*}+\cdots,$$

where

$$f_+(0)|_{\rho,K^*}=\frac{C_\pi}{m_\pi^2}\frac{C_K}{m_K^2}\left(\frac{2G_{\rho^+\pi^-\pi^0}G_{\rho^-K^+K^0}}{m_\rho^2}-\frac{2G_{K^{*0}K^0\pi^0}{}^2}{M_{K^*}{}^2}\right), \tag{8}$$

which implies[10]

$$f_+(0)|_{\rho,K^*}=0.63. \tag{8'}$$

Thus, the vector-meson contribution to all three commutators in question is in fact much the same. That this should be so can be seen as follows: (i) In the limit of $SU(3)$ symmetry the right-hand sides of Eqs. (6), (7), and (8) all become equal, and (ii) the actual vector-meson masses and the VPP coupling constants do not show large departures from $SU(3)$ symmetry, while $(C_\pi/m_\pi^2)/(C_K/m_K^2)$ is fairly close to unity. The fact that the $SU(3)$ limit is manifest for the vector-meson contributions can be understood on general grounds; in contrast it should be noted that an octet of normal scalar mesons does *not* give equal contributions to the right-hand side of Eqs. (6)–(8), even in the $SU(3)$ limit.[11]

SATURATION AND SCALAR MESONS

On taking the matrix element of Eq. (1) between $|K^+\rangle$ states and the matrix element of Eq. (5) between $|\pi^0\rangle$ and $|\pi^+\rangle$ states, we obtain, noting that only $I=\frac{1}{2}$, $Y=1$ states can contribute in either case,

$$1=\left(\frac{C_\pi^2}{m_\pi^4}\right)\left[\frac{G_{K^{*0}K^-\pi^+}{}^2}{m_{K^*}{}^2}+\frac{1}{24}\frac{(m_{K^{**}}{}^2-m_K^2)^2}{m_{K^{**}}{}^4}\times G_{K^{**0}K^-\pi^+}{}^2+\frac{G_{K'^0K^-\pi^+}{}^2}{(m_{K'}{}^2-m_K^2)^2}\right]+\alpha \tag{9}$$

and

$$1=\left(\frac{C_K^2}{m_K^4}\right)\left[\frac{G_{K^{*0}K^-\pi^+}{}^2}{m_{K^*}{}^2}+\frac{1}{24}\frac{(m_{K^{**}}{}^2-m_K^2)^2}{m_{K^{**}}{}^4}\times G_{K^{**0}K^-\pi^+}{}^2+\frac{G_{K'^0K^-\pi^+}{}^2}{(m_{K'}{}^2-m_\pi^2)^2}\right]+\beta, \tag{10}$$

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[6] S. Fubini and G. Furlan, Physics 1, 229 (1965).

[7] The effective coupling constants for $S(0^+)$, $V(1^-)$, and $T(2^+)$ meson decays into pseudoscalar mesons P and P' are defined by writing the corresponding effective interaction Lagrangians in the form $G_{SPP'}SPP'$, $iG_{VPP'}V_\mu[P\cdot\partial_\mu P'-(\partial_\mu P)\cdot P']$, and $G_{TPP'}T_{\mu\nu}\times(\partial_\mu P)(\partial_\nu P')$.

[8] From the rate for $\pi\to\mu+\nu$ decay we get $(C_\pi/m_\pi^2)^2=1.65\times10^4$ $(\mathrm{MeV})^2/\cos^2\theta_A=1.73\times10^4$ $(\mathrm{MeV})^2$, using $\theta_A=\theta_V$ and $\cos\theta_V=0.978$. From the rate for $K^+\to\mu^++\nu$ decay, we get $(C_K/m_K^2)=1.22\times10^3$ $(\mathrm{MeV})^2/\sin^2\theta_A=2.82\times10^4$ $(\mathrm{MeV})^2$, with $\theta_A=\theta_V$. (Throughout this paper, all experimental numbers for meson masses and decay rates have been taken from the recent compilation of A. H. Rosenfeld *et al.*, Ref. 13.) See N. Brene *et al.* [Phys. Rev. **149**, 1288 (1966)] for a discussion of the possibility that $\theta_A\neq\theta_V$.

[9] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964); see also Ref. 18.

[10] Here we assume that ρ couples with the isotopic spin current [J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960)], which implies $G_{\rho^+\pi^-\pi^0}=\sqrt{2}G_{\rho^-K^+K^0}$, neglecting off-mass shell effects.

[11] The general form of a matrix element $\langle P_i|A_j^{\mathrm{op}}\cdot|V_k\rangle$ is, in the $SU(3)$ limit, $af_{ijk}+bd_{ijk}$ (suppressing space-time indices). But C invariance and the assumption that the axial currents $A_i^{\mathrm{op}}\cdot$ are currents of the first class, i.e., transform under C as they would in the quark model, imply that $b=0$. The vector-meson contribution to $\langle P_i|[A_j^{\mathrm{op}}\cdot,A_{j'}^{\mathrm{op}}\cdot]|P_{i'}\rangle$ is thus proportional to $[F_j,F_{j'}]_{ii'}=if_{jj'k}(F_k)_{ii'}$ and so is the same as obtained using $[A_j^{\mathrm{op}}\cdot,A_{j'}^{\mathrm{op}}\cdot]=if_{jj'k}V_k^{\mathrm{op}}\cdot$ and, in the SU(3) limit, $\langle P_i|V_k^{\mathrm{op}}\cdot|P_{i'}\rangle\propto(F_k)_{ii'}$. [In Eq. (6) of Ref. (3) there is a plus sign in front of $G_{K^*K\pi}{}^2$, which seems to account for the discrepancy. We thank Dr. V. Mathur for a conversation confirming this point.] For a normal scalar octet S (or a 2^+ octet), C invariance implies $\langle P_i|A_j^{\mathrm{op}}\cdot|S_k\rangle\propto d_{ijk}$ in the $SU(3)$ limit and hence a contribution proportional to $[D_j,D_{j'}]_{ii'}$ which is not equal to $if_{jj'k}(F_k)_{ii'}$.

where the first, second, and third terms in either Eqs. (9) or (10) represent, respectively, the contributions of 1^+, 2^+, and 0^+ states as approximated, respectively, by K^*, $K^{**}\equiv K^*(1415)$, and a hypothetical scalar resonant state called K'. The quantities α and β represent the effect of neglected higher angular momentum states, as well as the effect of the approximations made in arriving at the first three terms. If we assume that α and β are relatively small, Eqs. (9) and (10) impose rather stringet conditions on $m_{K'}$ and $G_{K'K\pi}$. In particular, if for the sake of definiteness we assume $|\alpha|<0.1$, $|\beta|<0.1$, we get[12]

$$500<m_{K'}<740\ \text{MeV}. \tag{11}$$

Thus, the assumption that the relevant matrix elements of the commutators (1) and (5) can be saturated, to within 10%, by single-particle states with $J\leq 2$, implies that there should exist at least one 0^+ state with $I=\frac{1}{2}$, $Y=1$; if there is only one such state, its mass should be in the range given by Eq. (11).

There is in fact a candidate for such a particle[13]: the so-called kappa meson (an $I=\frac{1}{2}$, $Y=1$ K-π resonance around 700 MeV) whose existence, however, continues to be in doubt. If we take $m_{K'}=m_\kappa\approx 725$ MeV, we get, from Eqs. (9) and (10), $\Gamma_K\sim 20$–30 MeV; reported values of Γ_κ range from 10–50 MeV. As a consistency check, we note that if we now reconsider Eq. (8), obtained from Eq. (2) taken between $|\pi^0\rangle$ and $|K^+\rangle$, we get, including the contribution of K^{**} and the conjectured K',

$$f_+(0)=f_+(0)|_{\rho,K^*}+\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{C_K}{m_K^2}\right)\times\left[\frac{G_{K^{**0}K^-\pi^+}{}^2(m_{K^{**}}{}^2-m_\pi{}^2)(m_{K^{**}}{}^2-m_K{}^2)}{24m_{K^{**}}{}^2}+\frac{G_{K'^0K^-\pi^+}{}^2}{(m_{K'}{}^2-m_\pi{}^2)(m_{K'}{}^2-m_K{}^2)}\right]+\gamma, \tag{12}$$

where γ is analogous to α and β in Eq. (9) and (10). If we identify K' with κ, Eq. (12) yields, with $m_{K'}=725$ MeV, $f_+(0)=0.95+\gamma$ for $\Gamma_K=20$ MeV and $f_+(0)=1.05+\gamma$ for $\Gamma_K=30$ MeV, gratifyingly consistent with $|\gamma|<0.1$ assumed for α and β. From the viewpoint of this calculation, the existence of the K meson is indeed an attractive possibility. On the other hand, we note that the range (11) includes the interesting alternative that $m_{K'}<m_\pi+m_K$ (=629–638 MeV), i.e., a K' which could be regarded as a $K\pi$ bound state, with the electromagnetic decays $K'\to K+2\gamma$, $K+e^++e^-$. That such a particle might exist does not seem to be ruled out by present experimental evidence.[14]

In a similar spirit we consider resonance saturation of the commutators (3) and (4) taken between $|\pi^0\rangle$, $|\pi^+\rangle$ and $|\pi^0\rangle$, $|K^+\rangle$, respectively. Apart from the 1^- states already considered, we also include the contributions of states with $j=2$ (K^{**} and f mesons) and with $j=0$. Allowing for two resonant 0^+ states with $I=0$, $Y=0$, as suggested by $SU(3)$ considerations, and calling these σ and η' [in the $SU(3)$ limit, we may regard σ as a singlet and η', K' as members of an octet] we get, in an obvious notation,

$$F_+(0)=\left(\frac{C_\pi{}^2}{m_\pi{}^4}\right)\left[\frac{G_{\rho^+\pi^-\pi^0}{}^2}{2m_\rho{}^2}+\frac{1}{12}\frac{(m_f{}^2-m_\pi{}^2)^2}{(m_f{}^4)}G_{f^0\pi^0\pi^0}{}^2+\frac{2G_{\eta'^0\pi^0\pi^0}{}^2}{(m_{\eta'}{}^2-m_\pi{}^2)^2}+\frac{2G_{\sigma^0\pi^0\pi^0}{}^2}{(m_\sigma{}^2-m_\pi{}^2)^2}\right]+\alpha', \tag{13}$$

and[15]

$$f_+(0)=\left(\frac{C_\pi}{m_\pi{}^2}\right)\left(\frac{C_K}{m_K{}^2}\right)\left[\frac{2G_{K^{*+}K^-\pi^0}{}^2}{m_{K^*}{}^2}-\frac{(m_{K^{**}}{}^2-m_K{}^2)(m_{K^{**}}{}^2-m_\pi{}^2)}{24m_{K^{**}}{}^2}G_{K^{**+}K^-\pi^0}{}^2-\frac{2G_{K'^+K^-\pi^0}{}^2}{(m_{K'}{}^2-m_K{}^2)(m_{K'}{}^2-m_\pi{}^2)}-\frac{4G_{\eta'^0\pi^0\pi^0}{}^2}{(m_{\eta'}{}^2-m_K{}^2)(m_{\eta'}{}^2-m_\pi{}^2)}+\frac{8G_{\sigma^0\pi^0\pi^0}{}^2}{(m_\sigma{}^2-m_K{}^2)(m_\sigma{}^2-m_\pi{}^2)}\right]+\beta'. \tag{14}$$

α' and β' are analogous to α and β in Eqs. (9) and (10), representing the effect of approximations for off-mass-shell effects, etc.[16] We have $F_+(0)=1$ and experiment on K_{e3}, π_{e3}, μ, and β decay, analysed within the framework of Cabbibo's theory of leptonic decays, is

[12] For every solution of Eqs. (9) and (10) with $m_{K'}>m_K$ there is a solution with $m_{K'}<m_K$ which we discard as being in contradiction with the metastability of the K meson.

[13] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[14] We have not made detailed estimates of production cross sections for this case. The associated dimensionless coupling constant for, e.g., $m_{K'}=590$ MeV is quite small: $(G/2m_{K'})^2/4\pi\sim 0.016$.

[15] In arriving at Eq. (14), $SU(3)$ values have been assumed for the ratios $G_{\sigma K^+K^-}/G_{\sigma\pi^+\pi^-}$ and $G_{\eta' K^+K^-}/G_{\eta'\pi^+\pi^-}$. Contributions from $f\to K\bar{K}$ and f' (1500) $\to\pi\pi$ appear to be negligible.

[16] It should be pointed out that sum rules derived by (a) treating resonances as stable particles and identifying vertex functions such as $G_{VPP'}(q^2=0)$ with effective coupling constants to be used in the computation of $\Gamma(V\to P+P')$ leads to expressions for the resonance contribution which are not quite the same as those obtained by (b) making the narrow-resonance approximation in the integrals over cross sections for 0-mass incident particles, related to physical cross sections by use of the kinematical correction of Adler (Ref. 2). The difference is a factor of k_0/k, where k is the final c.m. momentum in the two-particle decay and k_0 is the same quantity with the mass of one of the final particles set equal to zero. This factor is fairly close to unity in most of the cases of interest in this paper.

consistent with $f_+(0)=1.04$, to within a few percent.[17,18] To gain insight into the implications of Eqs. (13) and (14), we set $\alpha'=\beta'=0$, $f_+(0)=1$, and assume that $G_{\eta'\pi\pi}/G_{K'K\pi}$ has its $SU(3)$ value. For given $m_{K'}$, $\Gamma_{K'}$, and $m_{\eta'}$, Eqs. (13) and (14) then determine m_σ and Γ_σ. Considering only $m_{\eta'}>m_K\simeq 500$ MeV, and $m_{K'}$ in the range (11), one finds that $m_{\eta'}<600$ MeV if m_σ^2 is to be positive, and rather generally, that $m_\sigma>m_{\eta'}$. For example, if $m_{K'}=m_\kappa=725$ MeV, $\Gamma_\kappa=25$ MeV, then $m_\sigma=720$ MeV, $\Gamma_\sigma\simeq 155$ MeV for $m_{\eta'}=550$ MeV, whereas $m_\sigma\simeq 565$ MeV, $\Gamma_\sigma=60$ MeV for $m_{\eta'}=525$ MeV. Qualitatively similar results are obtained if K' is a bound state.[19]

In summary, under the indicated circumstances, with the SPP coupling constants not departing too violently from their $SU(3)$ values, Eqs. (9), (10), (13), and (14) taken together are strongly suggestive of the existence of a 0^+ octet with fairly narrow widths and a rather broader 0^+ singlet, with masses in the region 500–1000 MeV. If κ exists and K' is identified with it, we expect 500 MeV $<m_{\eta'}<600$ MeV and $m_\sigma>m_{\eta'}$ with $\Gamma_\sigma\sim 50$–200 MeV.[20] The existence of a broad σ resonance is consistent with conclusions drawn recently by Lovelace *et al.*[21] from an examination of backward π-p scattering.

[17] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[18] S. Oneda and J. Sucher, Phys. Rev. Letters **15**, 927 (1965); **15**, 1049(E) (1965).

[19] It is interesting to note that Eqs. (13) and (14), with $\alpha'=\beta'=0$, seem to require the existence of both a 0^+ octet and a 0^+ singlet. The assumption that $G_{\eta'\pi\pi}=G_{K'K\pi}=0$ (absence of a 0^+ octet) is inconsistent with the requirement that m_σ^2 be positive, unless the ratio $G_{\sigma\pi\pi}/G_{\sigma KK}$ is very different from its $SU(3)$ value of unity. If we set $G_{\sigma\pi\pi}=0$ (no 0^+ singlet) and use $SU(3)$ for $r=G_{\eta'\pi\pi}/G_{K'K\pi}$, Eqs. (13) and (14) are inconsistent for any value of $m_{\eta'}$. If r is regarded as a free parameter one finds $m_{\eta'}\simeq 300$ MeV and a value for $\Gamma_{\eta'}$ which is inconsistent with the rate for K_{e4} decay. Incidentally, the contribution of the 2^+ states to all the sum rules in this paper is never more than 10%.

[20] Our conclusions about the possible importance of a K' meson in the saturation of matrix elements of axial-charge commutators differ from those of V. S. Mathur and L. K. Pandit, Phys. Rev. **143**, 1216 (1966). This difference may in part be the result of a missing factor of π on the right-hand side of Eq. (9) of this reference (we thank Dr. V. S. Mathur for a conversation confirming this point), and in part the result of the somewhat different off-mass-shell corrections obtained from the method used by these same authors in Ref. 3, which we follow here (see Ref. 16). For related work see K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965); D. A. Geffen, University of Minnesota (unpublished); G. Segrè and J. D. Walecka, University of California, Berkeley (unpublished).

[21] C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966).

CHARGE-CURRENT COMMUTATOR AND VECTOR-MESON DOMINANCE

The approach to $f_+(0)$ based on an unsubtracted dispersion relation for $f_+(t)$, combined with K^* dominance for $\mathrm{Im}f_+(t)$ and use of the charge-current commutator

$$[A_{K^-},A_\mu^{\pi^0}(x)]=\tfrac{1}{2}V_\mu^{K^-}(x) \tag{15}$$

gives[4]

$$f_+(0)\simeq 4\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{C_K}{m_K^2}\right)\cdot\frac{G_{K^{*+}K^-\pi^0}{}^2}{m_{K^*}{}^2}. \tag{16}$$

A similar calculation of $F_+(0)$, using ρ dominance and the commutator

$$[A_{\pi^-},A_\mu^{\pi^0}(x)]=V_\mu^{\pi^-}(x) \tag{17}$$

yields

$$F_+(0)\simeq\left(\frac{C_\pi^2}{m_\pi^4}\right)\frac{G_{\rho^+\pi^-\pi^0}{}^2}{m_\rho^2}. \tag{18}$$

It does not appear to have been noticed that the right-hand side of Eq. (16) is just twice the right-hand side of Eq. (7), as is the right-hand side of Eq. (18) compared to the right-hand side of Eq. (6). Numerically we therefore obtain from Eqs. (16) and (18)

$$f_+(0)\simeq 2\times 0.59=1.18\,,$$
$$F_+(0)\simeq 2\times 0.48=0.96\,,$$

in very good agreement with the expected values $f_+(0)\simeq 1.04$ and $F_+(0)=1$.

The factor of 2 entering into the comparison of Eqs. (16) and (18) with (7) and (6) is reminiscent of other "factor of 2" problems encountered in the derivation of sum rules using current algebra.[22] We hope to discuss the relation between these two rather different types of calculation in a future paper.

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[22] F. Buccella *et al.*, Phys. Rev. **149**, 1268 (1966).