$$f_-(0)=\frac{2\sqrt{2}\mu}{m_K^2-m_\pi^2}-\frac{(m_K^2-m_\pi^2)}{2\sqrt{2}}\times\langle\pi^0|J''{}_3{}^1(0)|K^+\rangle_{p=\infty}. \quad (27b)$$

The static value of $f_+(q^2)$ agrees with the value predicted by exact $SU(3)$. Inspection of Eqs. (23a)–(23d) shows that, if the charge radii, and transition radius of the pseudoscalar mesons were measured, $f_+(q^2)$ would be completely determined. If further $f_-(q^1)$ had no pole at $q^2=-(m_K^2-m_\pi^2)$, it would be given in terms of three independent parameters.

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# Baryon Magnetic Moments and Assignment Mixing in $SU(6)$*

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An extension of the $SU(6)$ static model is used to formulate a phenomenological analysis of the magnetic moments of the baryons and of the $J^P=(\frac{3}{2})^+$ baryonic resonance transition moments. The results are dependent on a parameter, characteristic of the model, the assignment mixing angle. The calculations of the magnetic moments are divided into two parts. In the first only the contribution of the vector-meson resonances is taken into account, while in the second this pole model is combined with part of the nonresonant contribution to the magnetic moments. The strength of the electric and magnetic couplings of the vector mesons to the nucleons is also calculated within the framework of the pole model.

## I. INTRODUCTION

THE static model of Chew and Low[1] has been the basis of several calculations of nucleon magnetic moments. In fitting the pion-nucleon coupling constant, calculated in this model, to the experimental value, it was found that the most probable number of pions circulating about the nucleon was one, and this fact led to the idea that the anomalous moments were due to the interaction of the magnetic fields with the orbital current of the circulating charged pion.[2] There were difficulties attending this idea, however, for, while the isovector part of the anomalous moment could be made to approximately correspond to the observed value, the isoscalar part, determined by using the same cutoff energy (in the integrals arising in the course of the calculation) was not at all in agreement with experiment.[3] It was then suggested that the $K$ mesons, if they were pseudoscalar particles, would contribute to the isoscalar moment in a way which would make agreement with experiment more likely.[4] However, ensuing dispersion theoretic calculations using these ideas met with almost as little success as those based on the static model.[5]

At about the time of the advent of the Chew-Low model it was conjectured by Nambu[6] that meson resonances in the momentum-transfer channel were necessary for explaining the electromagnetic properties of the nucleons. Later, resonances, resulting from a strong pion-pion interaction in their $l=1$ partial wave ($J^P=1^-$, $I=0$, 1), were found (albeit at an uncomfortably high mass).

This idea of explaining phenomenologically the electromagnetic form factors of the nucleons by means of vector-meson poles has been refined to the point where these form factors can be quite well described even up to fairly high momentum transfers.[7]

Thus it became clear that one of the main reasons for the failure of both the static model and the dispersion theoretic calculations was the neglect of vector resonances in the pseudoscalar meson systems; that is, the neglect of a reasonably strong $p$-wave pseudoscalar-meson–pseudoscalar-meson interaction. Of course, such vector resonances can be included in the Chew-Low model by introducing them as elementary particles in their own right. This is done, however, at the expense of including two extra parameters, namely, the electric and magnetic couplings of the vector mesons to the nucleons.

## II. EXTENSION OF THE STATIC MODEL

All of these effects lying outside the scope of the original static model (e.g., vector and strange pseudoscalar mesons) are conveniently described by an exten-

* Work supported by the U. S. Atomic Energy Commission.

[1] G. F. Chew, Phys. Rev. **95**, 1669 (1954); F. E. Low, *ibid.* **97**, 1392 (1955); G. F. Chew and F. E. Low, *ibid.* **101**, 1570 (1956).

[2] H. Miyazawa, Phys. Rev. **101**, 1564 (1956); G. Salzman, *ibid.* **105**, 1076 (1957).

[3] H. Miyazawa, Phys. Rev., **101**, 1564 (1956).

[4] G. Sandri, Phys. Rev. Letters, **101**, 1617 (1956).

[5] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958); H. D. I. Abarbanel, C. G. Callen, Jr., D. H. Sharp, *ibid.* **143**, 1225 (1966).

[6] Y. Nambu, Phys. Rev. **106**, 1366 (1957).

[7] T. Massam and A. Zichichi, Nuovo Cimento **43**, 1137 (1966).

sion of the static version[8] of $SU(6)$ (CBC) made by Belinfante and one of the present authors (GHR).[9] This extended static model, $SU(6)_M$, contains as parameters the angles describing the mixing of vector ($V$) and pseudoscalar ($P$) mesons in the total angular momentum $J=1$ submodules of the 35-dimensional representation of $SU(6)$. These angles arise because of an ambiguity in the case of the above mesons, both having $J=1$ with respect to the (static) nucleons. The field operators representing the (3,8) and (3,1) submodules are, in general, the linear combinations

$$\mathbf{a}_8=\cos\theta_8\mathbf{a}_P{}^{\alpha}+\sin\theta_8\mathbf{a}_V{}^{\alpha}\,, \tag{1}$$
$$\mathbf{a}_1=\cos\theta_1\mathbf{a}_P+\sin\theta_1\mathbf{a}_V\,,$$

where $\alpha$ is an $SU(3)$ octet index. The octet describes the $P$ and $V$ mesons, and the singlet describes the $X_0$ and $\omega_1$ mesons. In the present case the $SU(3)$ singlet is excluded since we are considering only electromagnetic interactions.

The mixing of operators in Eq. (1) is called *assignment mixing*, and hence the name $SU(6)_M$. The special cases of CBC[8] and of the collinear (or $W$-spin)[10] $SU(6)_W$ versions of $SU(6)$ are particular cases of $SU(6)_M$, with the assignments

$$\cos\theta_1=\cos\theta_8=1,\quad \theta_1=\theta_8=0\ (\text{CBC})\,, \tag{2}$$

and

$$\cos\theta_1=\cos\theta_8=(\tfrac{1}{3})^{1/2},\quad \theta_1=\theta_8\cong 55°\ (W\text{-spin})\,. \tag{3}$$

The usual static version, $SU(6)_S$,[11] is not recovered, however.

The extension of the Chew-Low model to take into account the resonating pseudoscalar system can be thought of in the following way. The old calculations of the anomalous moments were based on the contribution of the process shown in Fig. 1 to the spatial part of the electromagnetic current. This process was then related to the total cross sections in the $(2I+1,\ 2J+1)$ channels, with isospin $I$ and total angular momentum $J$ zero or one, so that, in principle, this contribution could

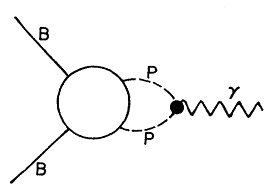


FIG. 1. The contribution of the virtual pseudoscalar meson to the baryon electromagnetic form factor in the static model.

[8] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965). In this paper, Belinfante and Cutkosky found by using only the $SU(3)$ baryon and peusdoscalar meson octets that the ratio $-\mu_p/\mu_n$ is relatively insensitive to the $F/D$ ratio and has a value close to the experimental one. This calculation supports the idea that the kaons are very important and cannot be neglected.

[9] J. G. Belinfante and G. H. Renninger, Phys. Rev. **148**, 1573 (1966).

[10] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965); Phys. Rev. **143**, 1269 (1966).

[11] F. Gürsey and L. A. Radicati, Phys. Rev. Letters, **13**, 173 (1964); A. Pais, *ibid.* 13, 175 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

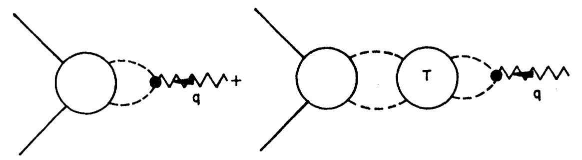


FIG. 2. The contribution of the virtual pseudoscalar meson to the baryon form factor, allowing for a pseudoscalar-meson–pseudoscalar-meson interaction.

be computed using experimental cross sections.[12] In this scheme, however, no allowance was made for the resonating pseudoscalar meson system. In other words, the possibility that the mesons will interact prior to their absorption at the photon vertex should be considered. For, even though the diagram of Fig. 1 is important, the rescattering of the strongly interacting $p$-wave pseudoscalar mesons is even more important. In order to include this interaction we should write, instead of the process picutred in Fig. 1, the two terms shown in Fig. 2. The new term contains the off-mass-shell meson-meson scattering amplitude $T$. The on-mass-shell amplitude resonates in the $J=1$, $I=0$, 1 channels, and if we approximate $T$ in the $t$ channel ($t=q^2$) by a sum of poles corresponding to these resonances, the magnetic-moment equation becomes what is schematically represented in Fig. 3. The second term shown there is now the *nonresonant* or *continuum* contribution and is a correction to the first term, which is, of course, the usual *pole model.*

The two new couplings of the vector mesons to the nucleons, $g_V{}^E$ (the electric coupling constant) and $g_V{}^M$ (the magnetic coupling constant), are hereby introduced, along with another parameter characteristic of the pole model, $g_{\gamma V}=em_V{}^2/2\gamma_V$.[13] A similar equation holds for the time component of the electromagnetic current (which contains the electric charge). This equation is used together with the one for the spatial components, described above, to normalize the over-all $SU(6)_M$ strength $g$, which appears in both equations.

Because of its success, and because it is able to give immediate information about the assignment angle $\theta_8$,[14] we shall examine the *pole contribution* to the electromagnetic current first [Sec. III].

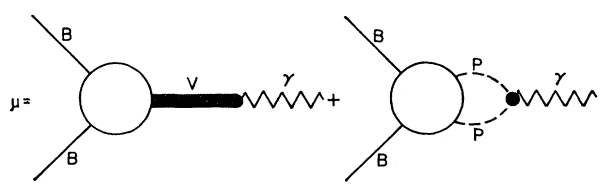


FIG. 3. Separation of the resonant and nonresonant parts of Fig. 2.

[12] H. Miazawa, Phys. Rev. **101**, 1564 (1956).

[13] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961); M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[14] R. E. Cutkosky and M. Jacobs have found the value $\sin\theta_8=\sin\theta_W=(\frac{2}{3})^{1/2}$ by using a Fermi-Yang bootstrap model of the mesons [Phys. Rev. (to be published)].

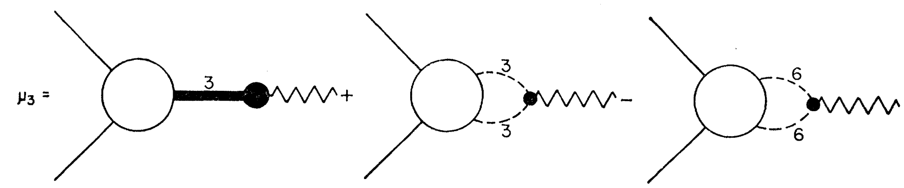


FIG. 4. The equations for the magnetic moments when the symmetry is reduced from $SU(6)$ to $SU(3)$. The numbers indicate where $SU(3)$ average masses and $SU(6)$ average masses are to be inserted.

We start by relating $g_V{}^E$ and $g_V{}^M$ to the pion-nucleon coupling constant $f_\pi$. Next we calculate the proton and neutron magnetic moments [Table I]. In a similar manner, both the hyperon magnetic moments and the transition moments entering the $J^P=(\frac{3}{2})^+$ baryonic resonance decay, $B^*\to B+\gamma$, can also be calculated, using the value of $\theta_8$ determined from the proton and neutron magnetic moments. Finally, by using the actual masses of the $\rho$, the $\omega$, and the $\varphi$, we can introduce some $SU(3)$ symmetry breaking into the results.

The numerical values of such magnetic-moment calculations can be somewhat refined by splitting the moments into isovector and isoscalar moments. The advantage of doing this is, of course, that the isovector moments involve the $\rho$ meson exclusively, and therefore depend only on its mass. The isoscalar moments, on the other hand, depend on the mass of the $\omega_8$, which is a linear combination of the physical mesons $\omega$ and $\varphi$.

We now return to the problem of including some of the continuum corrections. As far as the original equations involving contributions from both the resonant and nonresonant processes are concerned, we have thought of them as being $SU(6)$-invariant. If we wish, however, to take into account the fact that $SU(6)_M$ is a broken symmetry, while assuming that $SU(3)$ is still good, we need to know how the breaking affects the equation shown in Fig. 3. Denoting by $\mu_3$ the $SU(3)$-symmetric magnetic moment obtained from the $SU(6)_M$ symmetric moment $\mu_6$ by reducing the symmetry from $SU(6)_M$ to $SU(3)$, we may write

$$\mu_3=\mu_6+\delta\mu. \tag{4}$$

Upon separating the pole contribution from the continuum contribution, Eq. (4) may be written as

$$\mu_3=\mu_3{}^{\text{pole}}+\delta\mu^{\text{continuum}}, \tag{5}$$

since $\mu_6$, when account is taken of the $SU(6)_M$ symmetric charge equation, is essentially just the pole term (see the Appendix for a more complete discussion of this point). Pictorially, Eq. (5) is shown in Fig. 4.

TABLE I. Magnetic moments (in proton magnetons) resulting from the pole model.

| | 0° | 30° | 45° | $\theta_W$ | 60° | 75° | 90° |
|---|---|---|---|---|---|---|---|
| $\mu_p{}^a$ | 0 | 1.365 | 1.930 | 2.229 | 2.364 | 2.637 | 2.729 |
| $\mu_n{}^a$ | 0 | −0.910 | −1.287 | −1.486 | −1.576 | −1.758 | −1.820 |
| $\mu_p{}^b$ | 0 | 1.455 | 2.057 | 2.376 | 2.520 | 2.811 | 2.910 |
| $\mu_n{}^b$ | 0 | −1.048 | −1.482 | −1.711 | −1.815 | −2.025 | −2.096 |

[a] $SU(3)$ symmetric.
[b] $SU(3)$ broken symmetric values.

The results of these considerations together with some important modifications, are presented in Sec. IV.

## III. THE VECTOR-MESON POLE CONTRIBUTION

In order to discuss the electromagnetic properties of the nucleons we examine the matrix element of the electromagnetic current between physical baryon states:

$$(E_1E_2/M_1M_2)^{1/2}\langle p_2|j_\mu{}^{\text{em}}(0)|p_1\rangle. \tag{6}$$

The time component $j_0{}^{\text{em}}$ of this four-vector is related to the charge form factor, while the spatial components $\mathbf{j}^{\text{em}}$ are related to the magnetic form factor, both of them depending on the square of the momentum transfer $q^2=(p_2-p_1)^2$. In the limit of static nucleons (or in the Breit, or brickwall, frame) the current four-vector separates into

$$((G_E(q^2),\quad i\boldsymbol{\sigma}\times\mathbf{q}G_M(q^2)),$$

where the electric and magnetic form factors $G_{E,M}(q^2)$ are normalized to yield the observed static values:

$$G_{Ep}(0)=e,\quad G_{En}(0)=0,$$
$$G_{Mp}(0)=\mu_p=2.793,\quad G_{Mn}(0)=\mu_n=-1.913,$$

and where the magnetic moments are in units of $e/2M$.

In the pole model (first term in Fig. 3) the nucleon is pictured as emitting a vector meson, which in turn couples directly to the photon. In this model, then, the matrix element of Eq. (6) is replaced by

$$\sum_V (E_1E_2/M_1M_2)^{1/2}\frac{\langle p_2|j_\mu{}^V(0)|p_1\rangle}{-q^2+m_V{}^2}\frac{em_V{}^2}{2\gamma_V}. \tag{7}$$

The factor $(-q^2+m_V{}^2)^{-1}$ arises from the propagator of the virtual vector meson; $\langle p_2|j_\mu{}^V(0)|p_1\rangle$ is the matrix element of the vector-meson current between the same physical nucleon states as before; $\gamma_V$ is a constant related to the strength of the vector-meson couplings to the photon.[13]

The new matrix element in Eq. (7) can be evaluated with the help of $SU(6)_M$. In the static limit, then,

$$(E_1E_2/M_1M_2)^{1/2}\langle p_2|j_\mu{}^V(0)|p_1\rangle$$
$$=\chi_2{}^\dagger\Big\{(4\pi)^{1/2}g_V{}^EG_E(q^2)2(3)^{1/2}F_\alpha,\ (4\pi)^{1/2}g_V{}^MG_M(q^2)$$
$$\times(4(3)^{1/2}/5)[F_\alpha+((5)^{1/2}/2)D_\alpha]\frac{i}{m_V}\boldsymbol{\sigma}\times\mathbf{q}\Big\}\chi_1. \tag{8}$$

The constants $F_\alpha$ and $D_\alpha$ are $SU(3)$ Clebsch-Gordan coefficients for the couplings $8\otimes 8\to 8_a$ and $8\otimes 8\to 8_s$, respectively. The Pauli spinors $\chi$ also transform as members of the 56-dimensional representation of $SU(6)$. The time component is assumed to transform as the (1,8) part of the **35**, while the spatial components as the (3,8) part. The additional coefficients have been chosen so that if the meson is the $\rho$, then

$$(E_1E_2/M_1M_2)^{1/2}\langle p_2|j_0^\rho|p_1\rangle = \chi_2^\dagger\{(4\pi)^{1/2}g_\rho{}^E G_E(q^2)\boldsymbol{\tau}\}\chi_1, \quad (9)$$

$$(E_1E_2/M_1M_2)^{1/2}\langle p_2|\mathbf{j}^\rho|p_1\rangle = \chi_2^\dagger\left\{(4\pi)^{1/2}g_\rho{}^M G_M(q^2)\frac{i}{m_V}\boldsymbol{\tau\sigma}\times\mathbf{q}\right\}\chi_1. \quad (10)$$

In Eqs. (9) and (10) above we have allowed for the possibility that $m_V$, the average octet vector-meson mass, is not equal to the measured mass $m_\rho$ by simply replacing $m_v$ by $m_\rho$.

We are now able to connect $g_\rho{}^E$, $g_\rho{}^M$, and $f_\pi$. We start by writing down the general matrix element of the pseudoscalar mesons in the static limit:

$$(E_1E_2/M_1M_2)^{1/2}\langle p_2|j^P(0)|p_1\rangle=(4\pi)^{1/2}\frac{f_p}{m_p}\chi_2^\dagger \times(i\boldsymbol{\sigma}\cdot\mathbf{q})\frac{4}{5}(3)^{1/2}\left(F_\alpha+\frac{(5)^{1/2}}{2}D_\alpha\right)\chi_1, \quad (11)$$

where $f_p$ stands for an $SU(3)$ average of the pseudoscalar meson-nucleon coupling constants. This expression, in the case of isovector pions, reduces to

$$(4\pi)^{1/2}\frac{f_\pi}{m_\pi}\chi_2^\dagger(i\boldsymbol{\sigma}\cdot\mathbf{q})\boldsymbol{\tau}\chi_1.$$

The relation between the pseudoscalar coupling constant $f_p$ and the over-all $SU(6)_M$ strength $g$ appearing in the Lagrangian of Ref. 9 is

$$(4\pi)^{1/2}\frac{f_p}{m_p}=g\left(\frac{10}{3}\right)^{1/2}\frac{\pi}{3}\cos\theta_8. \quad (12)$$

Then, by calculating in a similar way the matrix elements of Eqs. (9) and (10), we can relate $g_\rho{}^E$ and $g_\rho{}^M$ to $g$. Finally, by using Eq. (12), we obtain

$$g_\rho{}^M/g_\rho{}^E=\tfrac{5}{2}(\tfrac{2}{3})^{1/2}\sin\theta_8, \quad (13)$$

with $g_\rho{}^E$ given by $g_\rho{}^E=[(3)^{1/2}/5](f_\pi/\cos\theta_8)(m_\rho/m_\pi)$. With the introduction of $G_\pi=(2M/m_\pi)f_\pi$, where $M$ is the nucleon mass, this last relation becomes

$$g_\rho{}^E=\frac{(3)^{1/2}}{5}\frac{G_\pi}{\cos\theta_8}\frac{m_\rho}{2M}. \quad (14)$$

Upon setting $\theta_8$ equal to the "$W$-spin value," $\theta_8=\theta_W$, given by Eq. (3), we find from Eq. (13)

$$g_\rho{}^M/g_\rho{}^E=5/3. \quad (15)$$

The vector current can also be written in the form

$$(4\pi)^{1/2}g^\rho\gamma_\mu\boldsymbol{\tau}+(4\pi)^{1/2}f^\rho i\sigma_{\mu\nu}\boldsymbol{\tau}q^\nu/m_\rho,$$

the relation of the new constants $g^\rho$ and $f^\rho$ to the ones used above being

$$g^\rho=g_\rho{}^E,\quad f^\rho=\frac{m_\rho}{2M}g_\rho{}^E+g_\rho{}^M.$$

Therefore, for $\theta_8=\theta_W$,

$$f^\rho/g^\rho=m_\rho/2M+5/3\cong 2.1, \quad (16)$$

which agrees with several different estimates.[15] For the same value of the mixing angle we obtain, from Eq. (14),

$$(g_\rho{}^E)^{1/2}=\frac{9}{50}G_\pi{}^2\frac{m_\rho{}^2}{2M^2}. \quad (17)$$

This last expression is to be compared with

$$f_\rho{}^2=(g_V/g_A)^2G_\pi{}^2(m_\rho{}^2/2M^2), \quad (18)$$

obtained from current algebra applied to $\rho$ decay into two pions,[16] or as found by Sakurai.[17] Then from Eqs. (15) and (16) we obtain the ratio $g_A/g_V=(25/18)^{1/2}=1.18$, an amusing result in itself, to which we shall direct our attention in a future work.

In the limit of zero momentum transfer,

$$[(E_1E_2/M_1M_2)^{1/2}\langle p_2|j_\mu{}^{\mathrm{em}}|p_1\rangle]_{q^2=0} =[\textstyle\sum_V (E_1E_2/M^2)^{1/2}\langle p_2|j_\mu{}^V|p_1\rangle]_{q^2=0}(1/2\gamma_V),$$

from which emerge the charge and magnetic-moment equations:

$$Q=\sum_V (4\pi)^{1/2}g_V{}^E 2(3)^{1/2}F_Q\frac{1}{2\gamma_V}, \quad (19)$$

$$\mu=\sum_V (4\pi)^{1/2}g_V{}^M\frac{4(3)^{1/2}}{5}[F_Q+\tfrac{1}{2}(5)^{1/2}D_Q]\frac{1}{2\gamma_V}2M_p, \quad (20)$$

where the index $Q$ stands for the sum of $SU(3)$ Clebsch-Gordan coefficients which transform as the charge operator. From the above equations we obtain

$$Q_p=1=(4\pi)^{1/2}\left(\frac{g_\rho{}^E}{2\gamma_\rho}+(3)^{1/2}\frac{g_{\omega_8}{}^E}{2\gamma_{\omega_8}}\right), \quad (21)$$

$$\mu_p=(4\pi)^{1/2}\left(\frac{g_\rho{}^M}{2\gamma_\rho}\frac{1}{m_\rho}+\frac{(3)^{1/2}}{5}\frac{g_{\omega_8}{}^M}{2\gamma_{\omega_8}}\frac{1}{m_{\omega_8}}\right)2M_p \quad (21')$$

[15] G. Köpp and P. Söding, Phys. Letters **23**, 494 (1966) and references therein.

[16] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966).

[17] J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966). The constant $g_\rho{}^E$ in the text differs from Sakurai's $f_\rho$ by a factor of $\frac{1}{2}$.

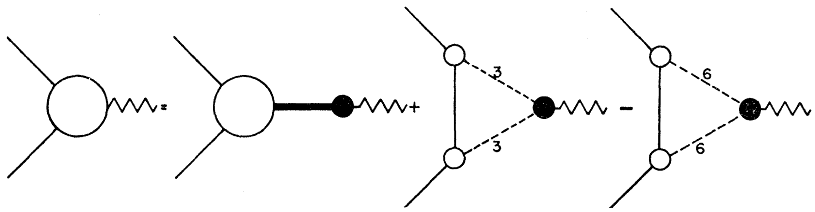

FIG. 5. The nonresonant part is approximated further by keeping the baryon and baryon-resonance Born terms in the nonresonant baryon–pseudoscalar-meson scattering amplitude.

for the proton, and

$$Q_n = 0 = (4\pi)^{1/2}\left(-\frac{g_\rho{}^E}{2\gamma_\rho} + (3)^{1/2}\frac{g_{\omega_8}{}^M}{2\gamma_{\omega_8}}\right), \tag{22}$$

$$\mu_n = (4\pi)^{1/2}\left(-\frac{g_\rho{}^M}{2\gamma_\rho}\frac{1}{m_\rho} + \frac{(3)^{1/2}}{5}\frac{g_{\omega_8}{}^M}{2\gamma_{\omega_8}}\frac{1}{m_{\omega_8}}\right)2M_p \tag{22'}$$

for the neutron. From Eqs. (21) and (22) it follows that $g_\rho{}^E/2\gamma_\rho = \sqrt{3}(g_{\omega_8}{}^E/2\gamma_{\omega_8})$ and

$$1 = (4\pi)^{1/2} g_\rho{}^E/\gamma_\rho. \tag{23}$$

Using these last two expressions, together with relation (13), we arrive at

$$(4\pi)^{1/2} g_\rho{}^M/2\gamma_\rho = \frac{1}{2}\left[(4\pi)^{1/2}\frac{2g_\rho{}^E}{2\gamma_\rho}\right]\frac{5}{2}\left(\frac{2}{3}\right)^{1/2} \times \sin\theta_8 = \frac{5}{4}\left(\frac{2}{3}\right)^{1/2}\sin\theta_8, \tag{24}$$

and similarly,

$$(4\pi)^{1/2} g_{\omega_8}{}^M/2\gamma_{\omega_8} = (\tfrac{1}{3})^{1/2}(\tfrac{2}{3})^{1/2}\sin\theta_8. \tag{25}$$

Expressions (24) and (25) permit us to write

$$\mu_p = \left(\frac{5}{m_\rho} + \frac{1}{m_{\omega_8}}\right)\frac{1}{4}\left(\frac{2}{3}\right)^{1/2}(\sin\theta_8)2M_p, \tag{26}$$

$$\mu_n = \left(-\frac{5}{m_\rho} + \frac{1}{m_{\omega_8}}\right)\frac{1}{4}\left(\frac{2}{3}\right)^{1/2}(\sin\theta_8)2M_p, \tag{27}$$

from which follows the well-known $SU(6)$ ratio[18] $\mu_p/\mu_n = -\frac{3}{2}$ in the limit of degenerate masses. In this limit, and for $\theta_8 = \theta_W$ we also obtain

$$\mu_p = 2M_p/m_V, \tag{28}$$

$$\mu_n = -\tfrac{2}{3}2M_p/m_V, \tag{29}$$

which are the results obtained by using current commutators and pole dominance.[19] By extending the calculation to the hyperons the well-known $SU(3)$ relations are regained.[20]

[18] B. Sakita, Phys. Rev. Letters **13**, 643 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1965).

[19] M. Ademollo, R. Gatto, G. Longhi, and G. Veneziano, Phys. Letters **22**, 521 (1966).

[20] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

If we introduce some symmetry breaking by using $m_\rho = 765$ MeV and $m_{\omega_8} = 941$ MeV (determined by using an $\omega-\varphi$ mixing angle of approximately 35°), we can determine $\sin\theta_8$ from the experimental value for $\mu_p$. Equation (26) then yields the value $\sin\theta_8 \cong 0.93$ corresponding to an angle $\theta_8 \cong 68°$. Insertion of this value of the assignment mixing angle into Eq. (27) gives $\mu_n = -2.01$. Table I shows the variation of $\mu_p$ and $\mu_n$ with $\theta_8$ in both the degenerate mass case and in the case of broken symmetry. By extending the use of this angle to the equations for the other baryon moments it is found, for example, that

$$\mu_\Lambda = -0.76 \quad \text{and} \quad \mu_{\Sigma^+} = 2.62.$$

These values are in good agreement both with the experimental determinations[21] and with the mass-corrected values obtained by Bég and Pais[22] in $SU(6)_S$.

Upon rearranging the results of Eqs. (26) and (27) we obtain the isovector and isoscalar moments as given by the pole term:

$$\mu_V \equiv \tfrac{1}{2}(\mu_p - \mu_n) = (\tfrac{3}{2})^{1/2}\tfrac{5}{6}\sin\theta_8\frac{2M_p}{m_\rho}, \tag{30}$$

$$\mu_S \equiv \tfrac{1}{2}(\mu_p + \mu_n) = (\tfrac{3}{2})^{1/2}\tfrac{1}{6}\sin\theta_8\frac{2M_p}{m_\rho}. \tag{31}$$

If the values given in Table II are compared with the experimental values $\mu_V{}^{\text{expt}} = 2.452$ and $\mu_S{}^{\text{expt}} = 0.440$, it will be seen that the pole term, as usual, describes the isovector part better than the isoscalar part.

We look next at the magnetic dipole decuplet-octet transitions. For $N_{33}{}^{*+} \to p + \gamma$, for example, we find

$$\mu^* \equiv \mu(N_{33}{}^{*+} \to p + \gamma) = \frac{2}{(3)^{1/2}}(\sin\theta_8)\frac{2M_p}{m_V},$$

TABLE II. Isovector and isoscalar magnetic moments in proton magnetons.

| $\mu$ \ $\theta_8$ | 0° | 30° | 45° | $\theta_W$ | 60° | 75° | 90° |
|---|---|---|---|---|---|---|---|
| $\mu_V$ | 0 | 1.251 | 1.770 | 2.043 | 2.167 | 2.418 | 2.503 |
| $\mu_S$ | 0 | 0.203 | 0.288 | 0.332 | 0.352 | 0.393 | 0.407 |

[21] The present experimental weighted averages are $\mu_\Lambda = -0.73 \pm 0.16$ and $\mu_{\Sigma^+} = 2.3 \pm 0.6$. [Arthur H. Rosenfeld, Angela Barbaro-Galtieri, William J. Podolsky, Leroy R. Price, Paul Soding, Charles G. Wohl, Matts Roos, and William J. Willis, Rev. Mod. Phys. **39**, 1 (1967)].

[22] M. A. Bég and A. Pais [Phys. Rev. **137**, B1514 (1965)], find $\mu_\Lambda = -0.78$ and $\mu_{\Sigma^+} = 2.20$.

which can be written in terms of $\mu_p$:

$$\mu^*=\tfrac{2}{3}\sqrt{2}\mu_p. \tag{32}$$

Since only the $\rho_0$ plays a role here, that is, only the (3,8) part of the **35** enters into the calculation of $\mu^*$, the prediction of $SU(6)_M$ in the pole model must agree with that of $SU(6)_S$.[23] The experimental value obtained by Dalitz and Sutherland[24] from an analysis of photoproduction data in the vicinity of the 3-3 resonance is $(1.28\pm0.02)[\tfrac{2}{3}\sqrt{2}\mu_p]$.

## IV. THE NONRESONANT CONTRIBUTION

We shall now modify the equations for the electromagnetic current by including part of the nonresonant (continuum) contribution. This modification will also take into account the fact that the meson masses are not degenerate in the $SU(6)$ 35-dimensional representation and so it represents, in addition, an attempt to reduce the symmetry from $SU(6)$ to $SU(3)$. We shall consider here the effect of separating only the meson masses but not the baryon masses. This can be thought of as reflecting the importance of the mesons in the actual value of the magnetic moments.

The electromagnetic vertex functions, then, have the form shown in Fig. 4. In order to discuss the continuum correction, we shall replace the nonresonant $B\bar{B}$ amplitude by the Born terms due the baryons and baryon isobars in the crossed channel, so that the equations which we shall actually treat are those represented in Fig. 5. The vector-meson pole term is treated here in exactly the same way as it was treated in Sec. III.[25]

In order to discover what the model of the nonresonant term is, we first consider the diagram of Fig. 5 relativistically and then take the limit in which the baryons become static. Only the pseudoscalar mesons are included in this contribution, the electromagnetic vertices for which are obtained from elementary considerations. The current coupling these mesons to the photons is

$$j_\mu{}^P=\frac{1}{2i}\phi_\alpha{}^\dagger\overleftrightarrow{\partial}_\mu\phi_\beta Q^{\alpha\beta}, \tag{33}$$

where $Q^{\alpha\beta}$ is the charge operator in $SU(3)$, $Q=I_Z+Y/2$, and $\phi_\alpha$ is a relativistic operator describing the pseudoscalar meson with $SU(3)$ index $\alpha$. When the above current is placed in the diagram of Fig. 5 and the static limit taken, the meson electromagnetic vertex transforms as an $SU(2)$ triplet times an $SU(3)$ octet. Thus, because of the $p$-wave nature of pseudoscalar mesons in the static limit, this vertex is forced to have a part transforming as the (3,8) of the **35** and a part transforming as the (3,8) of the **405**. The other two vertices are essentially proportional to the $SU(6)$ Clebsch-Gordan coefficients for $\mathbf{56}\otimes\mathbf{35}\rightarrow\mathbf{56}$ and are obtained from $SU(6)_M$. The group-theoretical part can then be readily calculated, with the continuum correction containing contributions from both the **35** and the **405**. The ratio $\mu_p/\mu_n$ is no longer fixed at $-\tfrac{3}{2}$, but depends now on two parameters, the assignment mixing angle $\theta_8$ and a parameter $\lambda$ describing to what degree the magnetic moment is accounted for by the pole term.[26]

Besides the Clebsch-Gordan coefficients there now appear terms resulting from the static limits of integrals of propagators and vertex functions over internal momenta (see Fig. 5):

$$I_M(m)=\int_0^\infty\frac{k^4dk\ v^2(k)}{\omega^4} \tag{34}$$

and

$$I_E(m)=\int_0^\infty\frac{k^4dk\ v^2(k)}{\omega^3}, \tag{35}$$

where $\omega^2=k^2+m^2$ and $v(k)$ is the usual function appearing in the static model to represent the spatial distribution of the nucleon. Actually, what occurs in the equations for the magnetic moments is, according to Fig. 5, the difference

$$I_M(m_3)-I_M(m_6)=\Delta I_M,$$

$m_3$ being the $SU(3)$ average pseudoscalar mass and $m_6$, the $SU(6)_M$ average meson mass given in the present case by[27]

$$m_6{}^2=24m_3{}^2\cos^2\theta_8+8m_V{}^2(1+3\sin^2\theta_8)+3m_{X^0}{}^2. \tag{36}$$

Similarly, the difference

$$I_E(m_3)-I_E(m_6)=\Delta I_E$$

appears in the proton charge equation.

Because of the form of the integral $I_M$ [Eq. (34)] it is clear that $\Delta I_M$ will not depend strongly on the momentum cutoff $k_\Lambda$ introduced by setting the square of the baryon density, $v^2(k)=\theta(k-k_\Lambda)$. We shall thus set $k_\Lambda$ equal to infinity in $\Delta I_M$. However, the charge part of the electromagnetic current, which involves the difference $\Delta I_E$, does depend strongly on the cutoff $k_\Lambda$.

For each baryon we again obtain two equations, each containing the over-all strength $g$ as a parameter. In order to eliminate this $g$ from both equations, which is equivalent to the normalization performed in the pole model [Eq. (23)], we write now $g=g_0\lambda$, where $g_0$ is the result of using the charge equation to normalize the

[23] M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

[24] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

[25] P. Signell and J. W. Durso [Phys. Rev. Letters **18**, 185 (1967)] have recently examined a model of the nonresonant contribution to the nucleon form factors.

[26] J. G. Belinfante has considered a simplified bootstrap model of the nucleon magnetic moments in which the ratio $\mu_p/\mu_n$ can vary between $-11/8$ and $-\tfrac{3}{2}$, depending on the magnetic properties assumed for the mesons (unpublished). A. Kanazawa and M. Saito have investigated a similar bootstrap model in a recent report; Hokkaido University, Sapporo, Japan (unpublished).

[27] J. G. Belinfante (private communication).

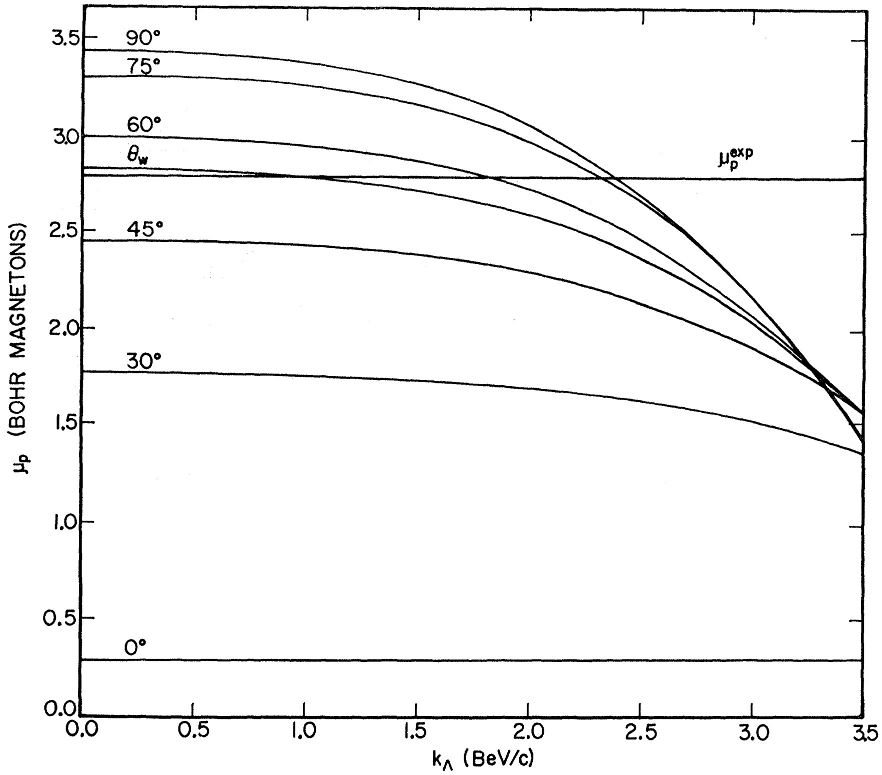


FIG. 6. The variation of $\mu_p$ with the momentum cutoff for various values of $\theta_8$.

$SU(6)_M$ coupling constant in Eq. (23). By making use of the relations between $g_V{}^E$ and $g$, derived above, we find that $\lambda^2=(g_V{}^E)^2/(\gamma_V{}^2/4\pi)$. As a function of $\lambda$ the equations for the magnetic moments of the nucleons are now

$$\mu_p=\left\{(\tfrac{3}{2})^{1/2}\sin\theta_8\lambda+15\frac{\gamma_V{}^2}{4\pi}\cos^2\theta_8\frac{\Delta m}{m_V}\lambda^2\right\}\frac{2M_p}{m_V},\qquad(37)$$

$$\mu_n=-\left\{(\tfrac{2}{3})^{1/2}\sin\theta_8\lambda+12\frac{\gamma_V{}^2}{4\pi}\cos^2\theta_8\frac{\Delta m}{m_V}\lambda^2\right\}\frac{2M_p}{m_V},\qquad(38)$$

with $\Delta m=m_6-m_3$, while the equation for the proton charge is

$$Q_p=1=\lambda+\frac{9}{\pi}\frac{\gamma_V{}^2}{4\pi}\cos^2\theta_8\frac{\Delta I_E}{m_V{}^2}\lambda^2.\qquad(39)$$

In order to make use of these equations we must decide how to evaluate $\Delta I_E$ and what to use for $g^2$ or, alternatively, for $\gamma_V{}^2$. The simplest way of evaluating $\Delta I_E$ is to put $k^4v^2(k)=k_\Lambda{}^5\delta(k-k\ )/5$. This choice of $v^2(k)$ in the charge equation [Eq. (39)] will be referred to as the "$\delta$-function model." Then, since the combination $g^2k_\Lambda{}^5/20\omega^3$, $\omega^2=k_\Lambda{}^2+m_6{}^2$ occurs in the continuum contribution, we can use the results of the Appendix of Ref. 9 to set this equal to $\frac{2}{3}$. Or we can simply replace $\gamma_V$ by $\gamma_\rho$ and use the experimental value $\gamma_\rho{}^2/4\pi=3/5$, determined by measuring the decay rate of $\rho$ into lepton pairs.[28] In both cases we vary $\lambda$ and $\theta_8$ to fit both the proton and the neutron moments. These two methods give similar results: When $\mu_p$ and $\mu_n$ are reasonably close to the observed values, we find that $k_\Lambda\cong250$ MeV/$c$, $\lambda=0.6$–$0.7$, and $\theta_8=45°$–$70°$. Thus, both of these treatments give a value of $\lambda$ smaller than desired (the pole contribution being then less important than expected) and only a fairly wide range of $\theta_8$ is determined.

The $\delta$-function model might be thought a bit extreme because of its strange dependence on $k_\Lambda$, so that the more usual model obtained by setting $v^2(k)=\theta(k-k_\Lambda)$ might be expected to improve the results somewhat. This "pure cutoff" model was also used, in conjunction with $\gamma_V{}^2/4\pi=\frac{3}{5}$. The only significant change is that now $k_\Lambda\cong1$-$1.5$ BeV/$c$, the parameters $\lambda$ and $\theta_8$ falling within the same range as in the $\delta$-function model.

An alternative procedure was suggested to us by Cutkosky: The factor $(\gamma_V{}^2/4\pi)\cos^2\theta_8\lambda^2$ appearing in Eqs. (37) and (38) can be replaced by $(3/25)(f_p{}^2/m_3{}^2)$ $\times m_V{}^2$ [obtained from Eq. (14) and the relation

[28] J. K. de Pagter, J. I. Friedman, G. Glass, R. C. Chase, M. Gettner, E. von Goeler, R. Weinstein, and A. M. Boyarski, Phys. Rev. Letters **16**, 35 (1966). These authors give $(\rho\to\mu^-\mu^+)/(\rho\to\pi^-\pi^+)=(0.33\pm0.04)\times10^{-4}$ and using

$$\Gamma(\rho\to2\pi)=124\text{ MeV},\quad m_\rho=765\text{ MeV}\qquad(40)$$

we get $\gamma_\rho{}^2/4\pi=4/5$. However, the result of de Pagter *et al.* should be corrected by a factor of $\frac{4}{3}$ [R. Weinstein, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967); see also J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966)].

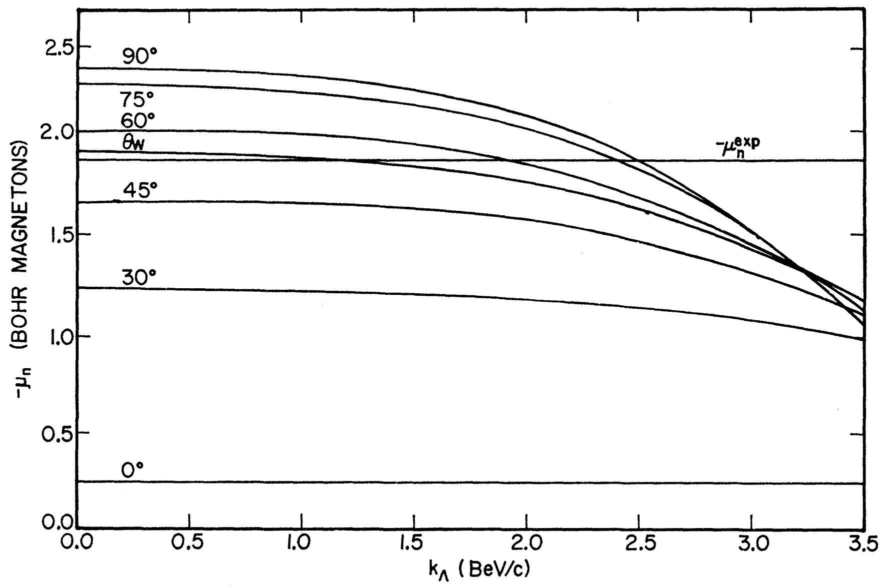


FIG. 7. The variation of $-\mu_n$ with the momentum cutoff for various values of $\theta_8$.

$\lambda^2(\gamma_V^2/4\pi)=(g_V^E)^2]$. Since the kaon-nucleon couplings, expressed in terms of $f$, are not significantly smaller than the corresponding pion ones, we will simply replace the $SU(3)$ average coupling $f_p^2$ introduced in Eq. (11), by the experimental number $f_\pi^2=0.08$. Instead of Eqs. (37) and (38), then, we have

$$\mu_p=\left[\left(\tfrac{3}{2}\right)^{1/2}\sin\theta_8\lambda+\frac{9}{5}f_\pi^2\frac{\Delta m}{m_V}\left(\frac{m_V}{m_8}\right)^2\right]\frac{2M_p}{m_V} \tag{41}$$

and

$$\mu_n=-\left[\left(\tfrac{2}{3}\right)^{1/2}\sin\theta_8\lambda+\frac{36}{25}f_\pi^2\frac{\Delta m}{m_V}\left(\frac{m_V}{m_8}\right)^2\right]\frac{2M_p}{m_V}, \tag{42}$$

while the charge equation, Eq. (39), becomes

$$1=\lambda+\frac{27}{25\pi}f_\pi^2\frac{\Delta I_E}{m_8^2}. \tag{43}$$

Here again we are faced with the problem of evaluating $\Delta I_E$. By using the pure cutoff model, mentioned above, in this difference, the results for $\mu_p$ and $\mu_n$ shown in Figs. 6 and 7 are obtained. The experimental values of these moments are now attained for a value of $\lambda$ quite close to unity, which is in accord with the concept of pole dominance. The cutoff momentum is now about 1 BeV/$c$. More significant than the new values of $\lambda$ and $k_\Lambda$ is the fact that this method permits a much better determination of the assignment mixing angle. Indeed, if we accept $k_\Lambda\cong 1$ BeV/$c$ as a reasonable value of the momentum cutoff, we see from Figs. 6 and 7 that $\theta_8\cong\theta_W$ turns out to be the angle which is definitely preferred (we recall that the pole term alone gave $\theta_8\cong 68°$). For values of the angle smaller than $\theta_W$, in fact, either the experimental values are never reached, or they are attained only for cutoff momenta very near zero. For larger values, the opposite is true: $\mu_p$ and $\mu_n$ take on reasonable values only at excessively high cut-off momenta (at 60° $k_\Lambda$ is already about 2 BeV/$c$. In Table III we list the values obtained for $k_\Lambda=1$ BeV/$c$. Furthermore, it is apparent from these two figures that only for angles quite close to $\theta_W$ is the fit to the magnetic moments fairly insensitive to $k_\Lambda$.

The ratio $R=-\mu_p/\mu_n$ obtained from Eqs. (26) and (27) decreases very slowly with $k_\Lambda$ and is also quite insensitive to the angle; for example, $R(45°)=R(\theta_W)=R(60°)=1.44$ for values of $k_\Lambda$ in the range from 0.5 to 1.5 BeV/$c$. This value of the ratio is very close to the experimental value, $R_{\text{expt}}=1.46$.

The calculations and considerations made above for the proton and neutron can be extended to the remaining members of the baryon octet and also to the $M1$ decuplet-octet transitions.

## V. SUMMARY OF RESULTS

The simple pole model of the baryon magnetic moments in conjunction with $SU(6)_M$ not only gives an estimate of the assignment mixing angle of $\theta_8=68°$, but also determines the $M1$ transition moments involved in the electromagnetic decays of the baryon resonances rather well. There are arguments, however, that $\theta_8$ should be about 55°, the $W$-spin value. By including some of the nonresonant effects of the baryon-pseudoscalar meson scattering amplitude, we have been able

TABLE III. Magnetic moments (in proton magnetons) from Eqs. (41) and (42), using $k_\Lambda=1$ BeV/$c$.

| $\mu$ \ $\theta_8$ | 0° | 30° | 45° | $\theta_W$ | 60° | 75° | 90° |
|---|---|---|---|---|---|---|---|
| $\mu_p$ | 0.296 | 1.769 | 2.429 | 2.785 | 2.946 | 3.276 | 3.388 |
| $\mu_n$ | −0.237 | −1.235 | −1.688 | −1.935 | −2.046 | −2.275 | −2.352 |
| $\lambda$ | 0.998 | 0.994 | 0.990 | 0.988 | 0.987 | 0.985 | 0.984 |

to construct a phenomenological model of the magnetic moments which does, in fact, determine $\theta_8$ to be about 55°. In this correction to the pole model, the angle $\theta_8$ now depends on a parameter $\lambda$ describing the relative amounts of resonant and nonresonant effects, while $\lambda$ depends on a momentum cutoff $k_\Lambda$ arising in the charge form factor. The ratio $-\mu_p/\mu_n$ turns out to be almost constant and $=1.44$ for a wide range of $k_\Lambda$, while the angle $\theta_8$ has a preferred value approximately equal to $\theta_W$, the $W$-spin angle, for values of $\lambda$ quite close to unity and $k_\Lambda \cong 1$ BeV/$c$.

## ACKNOWLEDGMENTS

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## APPENDIX

Here we will discuss in greater detail the steps connecting Eqs. (4) and (5). Both the $SU(6)$ symmetric $\mu_6$ and the broken moment $\mu_3$ consist of two terms, arising from the pole contribution and the nonresonant correction. Thus, Eq. (4) may be rewritten as

$$\mu_3=\mu_6+\delta\mu^{\text{pole}}+\delta\mu^{\text{continuum}}.$$

If we can show that $\mu_6$ (when the charge equation is taken into account) is essentially just the pole term, then the equation

$$\mu_3=\mu_6{}^{\text{pole}}+\delta\mu^{\text{pole}}+\delta\mu^{\text{continuum}}$$

holds. By making use of the definition $\delta\mu^{\text{pole}}=\mu_3{}^{\text{pole}}-\mu_6{}^{\text{pole}}$ in this last equation, we are led immediately to Eq. (5).

That $\mu_6$ should be the simple pole term is suggested by group theoretic and dimensional arguments. To show that, in fact, this form emerges from the symmetric moment and charge equations is the purpose of the ensuing discussion. The photon is now considered as belonging solely to the 35-dimensional representation of $SU(6)$, so that the proton moment in this limit is

$$\mu_p=(\tfrac{3}{2})^{1/2}\sin\theta_8\frac{\lambda}{m_6}+\frac{9\gamma_V{}^2}{2\pi^2}\cos^2\theta_8\frac{\lambda^2}{m_6{}^2}I_M,$$

where $I_M$ is the integral given by Eq. (34). The corresponding charge equation is

$$\lambda=1-\frac{1}{2}\frac{9\gamma_V{}^2}{2\pi^2}\cos^2\theta_8\frac{\lambda^2}{m_6{}^2}I_E,$$

where $I_E$ is the integral given by Eq. (35). We can now use the charge equation to eliminate the coupling constant from the magnetic-moment equation and so obtain

$$\mu_p=(\tfrac{3}{2})^{1/2}\sin\theta_8\frac{\lambda}{m_6}+2(1-\lambda)\frac{I_M}{I_E}.$$

The ratio $I_E/I_M$ represents some sort of average virtual meson energy in the cloud surrounding the nucleon, so that this ratio can be denoted by $\langle\omega\rangle$, whose dependence on the meson mass is governed by the specific model used. The proton moment then becomes

$$\mu_p=(\tfrac{3}{2})^{1/2}\sin\theta_8\frac{\lambda}{m_6}+\frac{2(1-\lambda)}{\langle\omega\rangle},$$

which is of the form

$$\mu_p=(\tfrac{3}{2})^{1/2}\sin\theta_8\frac{1}{m}.$$

This is a definition of the mass $m$. If $\lambda=1$ or if $\langle\omega\rangle^{-1}=(\tfrac{3}{2})^{1/2}\sin\theta_8/2m_6$, then $m=m_6$, and the moment resumes the form of the pole contribution. The case $\lambda=1$ is just the pole term with no continuum correction, while the case $m=m_6=(\tfrac{3}{2})^{1/2}\sin\theta_8\langle\omega\rangle/2$ is not trivial. In the $\delta$-function model of Ref. 9, the average meson energy is

$$\langle\omega\rangle=\omega_\Lambda=(k_\Lambda{}^2+m^2)^{1/2},$$

where $k_\Lambda$ is the cutoff in momentum, and the charge equation [after using Eq. (12)] becomes

$$\lambda=1-\frac{27}{125\pi}\frac{f_P{}^2}{m_3{}^2}\frac{k_\Lambda{}^5}{\omega_\Lambda{}^3}.$$

By using $f_p{}^2\cong f_\pi{}^2=0.08$, we find that

$$m/m_3\cong 3[10(1-\lambda)]^{1/2}.$$

From this relation we find, for example, that when $\lambda=0.90$, $m=1260$ MeV, and $k_\Lambda=2180$ MeV/$c$, while when $\lambda=0.98$, $m=570$ MeV and $k_\Lambda=990$ MeV/$c$. For values of $k_\Lambda$ in the vicinity of 1 BeV/$c$, then, the parameters $\lambda$ and $m=m_6$ assume reasonable values and, as a consequence, the assumption that the exact $SU(6)$ limit of the model we have used should be equivalent to the pole model is made quite plausible.