Baryonic Regge Recurrences and $U(6)_W \times O(2)_W$ Symmetry with Applications to Decays and Photoproduction*

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It is shown that if $N_{5/2^+}{}^*(1688)$ and $\Delta_{1/2^+}{}^*(1920)$ are the Regge recurrences of $N_{1/2^+}(940)$ and $\Delta_{3/2^+}(1238)$, respectively, they can be classified along with other particles in the $(56, 1; L=2)$ representation of the rest-symmetry group $U(6)\times U(6)\times O(3)$. The couplings of these baryons are then fixed by the collinear $U(6)_W \times O(2)_W$ group. This leads to the following predictions:

($R1$) $(D/F)_{B^*_{5/2+}\rightarrow B_{1/2+}+\text{pseudoscalar meson}}=\frac{3}{2}$;
($R2$) $\Gamma(\Delta^*(1920)\rightarrow N\pi)/\Gamma(N^*(1688)\rightarrow N\pi)=1.84$;
($R3$-$R4$) the photoformation (i.e., the process $\gamma+p\rightarrow$ resonance $\rightarrow N\pi$) of $\Delta^*(1920)$ and $N^*(1688)$ should occur with the multipole ratios $(E_{3+}/M_{3+})_{\Delta^*(1920)}=0$ and $(M_{3-}/E_{3-})_{N^*(1688)}\approx 0.1$, respectively.

These predictions are shown to be in agreement with experiment. Generalizations to higher Regge recurrences are presented.

EVER since complex-angular-momentum techniques were first applied to elementary-particle physics, it was suspected that the $N^*(1688)$ $J^P=\frac{5}{2}^+$ resonance is the first Regge recurrence of the nucleon $N(940)$.[1] Similarly, the $\Delta^*(1920)$ $J^P=\frac{7}{2}^+$ resonance was suggested as the first recurrence of the famous 33 resonance $\Delta(1240)$. In the course of time these ideas have received some support. Thus, e.g., it has been argued[2] that the $SU(3)$ octet assignment for $N^*(1688)$ is strongly favored over all other possible $SU(3)$ representations. Unfortunately, even if one accepts that $N^*(1688)$ belongs to an octet [along with (?) $Y_0^*(1815)$, $Y_1^*(1915)$, and $\Xi^*(1933)$] this does not yet imply that this octet lies on the same Regge trajectory as the fundamental baryon $J^P=\frac{1}{2}^+$ octet. In this paper we wish to present stronger evidence favoring the Regge-recurrence hypothesis for $N^*(1688)$ and $\Delta^*(1920)$. Based on this hypothesis, we shall also make some predictions that can easily be tested with available experimental techniques.

The idea of our approach is that along a Regge trajectory internal quantum numbers do not change. The $J^P=\frac{1}{2}^+$ baryon octet B and the $J^P=\frac{3}{2}^+$ baryon decimet D belong, as is commonly accepted, to a $(56, 1; L=0)$ representation of the group $U(6)\times U(6)\times O(3)$.[3] The Regge trajectory that accomodates this representation can be symbolically represented as $(56, 1; L=\tilde{\alpha}(t))$, where $\tilde{\alpha}(t)=\alpha_B(t)-\frac{1}{2}$, $\alpha_B(t)$ being the Regge trajectory of the baryon octet. The first recurrence of the basic $(56, 1; L=0)$ (taking into consideration the signature rule $\Delta L=2$) will therefore be a $(56, 1; L=2)$ multiplet.[4] This multiplet contains pieces corresponding to all total angular momenta that can be obtained by combining the $S=\frac{1}{2}$ octet and the $S=\frac{3}{2}$ decimet of (56, 1) with $L=2$ according to the usual rules of addition for angular momenta. $(56, 1; L=2)$ thus contains $J^P=\frac{5}{2}^+$ and $\frac{3}{2}^+$ octets and $J^P=\frac{7}{2}^+, \frac{5}{2}^+, \frac{3}{2}^+$, and $\frac{1}{2}^+$ decimets.[4] In particular, the $\frac{5}{2}^+$ octet and $\frac{7}{2}^+$ decimet are the first recurrences of the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decimet contained in $(56, 1; L=0)$, whereas the remaining multiplets of $(56, 1; L=2)$ lie on Regge trajectories that become "nonsense"[4] at $L=0$. Our arguments will explore the experimental consequences of this $(56, 1; L=2)$ assignment for $N^*(1688)$ and $\Delta^*(1920)$ and show that they are in accord with experiment. Specifically, we shall find that:

R1: The D/F ratio for the couplings of the $J^P=\frac{5}{2}^+$ baryon octet to the $J^P=\frac{1}{2}^+$ baryon octet and pseudoscalar mesons is $D/F=\frac{3}{2}$.

R2: The partial widths $\Gamma(\Delta^*(1920)\rightarrow N\pi)$ and $\Gamma(N^*(1688)\rightarrow N\pi)$ are related by

$$\frac{\Gamma(\Delta^*(1920)\rightarrow N\pi)}{\Gamma(N^*(1688)\rightarrow N\pi)}=\frac{72}{175}\left(\frac{q_{7/2}}{q_{5/2}}\right)^7\frac{M_{5/2}}{M_{7/2}}=1.84, \quad (1)$$

where $M_{7/2}$ ($M_{5/2}$) and $q_{7/2}$ ($q_{5/2}$) are, respectively, the mass of $\Delta^*(1920)$ ($N^*(1688)$) and the center-of-mass (c.m.) momentum of its decay products.

R3: Photoformation of $\Delta^*(1920)$ (i.e., the process $\gamma+p\rightarrow\Delta^*\rightarrow\pi N$) occurs purely in the magnetic octopole (M_{3+}) channel. The ratio

$$(E_{3+}/M_{3+})_{\Delta^*(1920)}=0.$$

R4: Photoformation of $N^*(1688)$ receives contributions from both the electric quadrupole (E_{3-}) and

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[1] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962); G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962).

[2] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963); C. A. Heusch, C. Y. Prescott, and R. F. Dashen, *ibid.* **17**, 1019 (1966); D. Brudnoy, *ibid.* **14**, 273 (1965).

[3] The group $U(6)\times O(3)$ has first been used in elementary particle physics by P. G. O. Freund and B. W. Lee [Phys. Rev. Letters **13**, 592 (1964)].

[4] A similar argument in the case of $U(6)\times O(3)$ symmetry has been advanced by M. Gell-Mann [Phys. Rev. Letters **14**, 77 (1965)].

magnetic octopole (M_{3-}) amplitudes[5] and their ratio is

$$(M_{3-}/E_{3-})_{N^*(1688)}\approx 0.1\,.$$

We now present the proof of statements R1–R4. The most convenient way to describe a (56, 1; $L=2$) representation is by means of the kinetic supermultiplet technique,[6] which is a generalization of the Rarita-Schwinger formalism to the $U(6)$ case. We introduce a kinetic supermultiplet $(\phi_{abc})_{\mu\nu}$ for the (56, 1; $L=2$) representation. Here a, b, and c are 12-valued pairs of Dirac and $SU(3)$ indices ($a=\alpha A$, $\alpha=1,\cdots,4$, $A=1,2,3$; $b=\beta B,\cdots$ etc.) and μ,ν are usual Lorentz tensor indices $\mu,\nu=0,1,2,3$. The fact that ϕ describes a (56, 1; $L=2$) representation is expressed by the following set of subsidiary conditions:

$$(\phi_{abc}(p))_{\mu\nu}=\text{totally symmetric in } a,b,c,$$
$$(\gamma\cdot p-M)_a{}^{a'}(\phi_{a'bc}(p))_{\mu\nu}=0 \quad \text{(Bargmann-Wigner equation)}, \tag{2a}$$

and

$$(\phi_{abc}(p))_{\mu\nu}=(\phi_{abc}(p))_{\nu\mu}\,,$$
$$p^\mu(\phi_{abc}(p))_{\mu\nu}=0\,,\quad g^{\mu\nu}(\phi_{abc}(p))_{\mu\nu}=0\,. \tag{2b}$$

The conditions (2a) ensure the requisite $U(6)\times U(6)$ transformation properties, whereas (2b) project out the $L=2$ part of the second-rank Lorentz tensor. Using the conditions (2), we can explicitly construct ϕ. We find

$$\begin{aligned}(\phi_{abc}(p))_{\mu\nu}=&(\gamma^\lambda C)_{\alpha\beta}[(D_{\lambda\mu\nu}{}^*(p))_\gamma]_{ABC}\\&+\frac{i}{2M}(\sigma^{\lambda\rho}C)_{\alpha\beta}[(p_\lambda D_{\rho\mu\nu}{}^*(p)-p_\rho D_{\lambda\mu\nu}{}^*(p))_\gamma]_{ABC}\\&+\frac{1}{3}\left\{\left[\left(1+\frac{\gamma\cdot p}{M}\right)\gamma_5 C\right]_{\alpha\beta}[(B_{\mu\nu}{}^*(p))_\gamma]_C{}^D\epsilon_{ABD}\right.\\&\left.+\text{cyclic permutations of } a,b,c\right\}+\cdots.\end{aligned} \tag{3}$$

Here C is the usual charge-conjugation matrix; $\alpha,\beta,\gamma=1,2,3,4$ are Dirac indices; $A,B,C=1,2,3$; the corresponding $SU(3)$ indices a, b, and c stand for the pairs of indices αA, βB, and γC, respectively, and $\lambda,\rho,\mu,\nu=0,1,2,3$ are Lorentz four-vector indices. D^* and B^* are the Rarita-Schwinger spin vectors that describe the $J^P=\frac{7}{2}^+$ decimet and the $J^P=\frac{5}{2}^+$ octet, respectively. The dots in Eq. (3) stand for the remaining spin-unitary spin multiplets contained in (56, 1; $L=2$) which we have chosen not to write out explicitly because in this paper we will not consider any processes that involve them.

[5] Throughout this paper we use the multipole amplitudes defined by G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu [Phys. Rev. **106**, 1345 (1957)].

[6] R. Gatto, L. Maiani, and G. Preparata, Nuovo Cimento **39**, 1192 (1965).

We will first construct the couplings of (56, 1; $L=2$) to the basic (56, 1; $L=0$) baryons and (6,$\bar{6}$) mesons. To this end we describe the latter by the familiar spinors ψ_{abc} and $M_a{}^b$, respectively.[7] To construct the couplings we will break the rest symmetry $U(6)\times U(6)\times O(3)$ down to the collinear $U(6)_W\times O(2)_W$ symmetry.[8] In a system of reference in which the collinear motion proceeds in the z direction, $O(z)_W$ is just the group of plane rotations generated by L_z. We will achieve this breakdown by standard kineton techniques.[9] In practice this means that the orbital indices $\mu\nu$ of ϕ should be contracted with momenta and never with γ matrices,[8] for the latter would mix $O(3)$ and $U(6)$ indices and thus cause the breakdown of $U(6)_W$. The most general $U(6)_W\times O(2)_W$-invariant $\phi\psi M$ vertex contains two independent coupling constants g and g' and has the explicit form

$$\begin{aligned}\Gamma_{\psi\phi M}=&g\bar{\psi}^{abc}(p')(\phi_{abd}(p))_{\mu\nu}q^\mu q^\nu M_c{}^d(q)\\&+g'\bar{\psi}^{abc}(p')(\phi_{abc}(p))_{\mu\nu}q^\mu q^\nu[\lambda_0\gamma\cdot(p+p')]_d{}^e M_e{}^d(q),\end{aligned} \tag{4}$$

with $q=p'-p$.

Observe that because of the λ_0 in the kineton factor $([\gamma\cdot(p+p')\lambda_0]_d{}^e)$ the second term only couples $SU(3)$-singlet vector mesons to ϕ and ψ. Since in this paper we are only interested in the couplings to the pseudoscalar and vector *octets*, all relevant couplings can be expressed in terms of only *one* coupling constant g. Inserting Eq. (3) into Eq. (4), we find in particular for the couplings of the pseudoscalar octet P to ϕ and ψ the form

$$\Gamma_P=\bar{g}q^\alpha q^\beta\left\{-2(q^\mu/m)(\bar{B}(p')B_{\mu\alpha\beta}(p)P(q))_{10}+\frac{(M+m)^2-\mu^2}{2mM}(\bar{B}PB)_{D+(2/3)F}+\cdots\right\}, \tag{5}$$

where M is the central mass of the (56, 1; $L=2$) supermultiplet $\approx M_{N^*(1688)}\approx M_{\Delta^*(1920)}$, m is the central mass of the (56, 1; $L=0$) supermultiplet ≈ 1100 MeV, μ is the central mass of the (6, $\bar{6}$; $L=0$) supermultiplet ≈ 750 MeV, $q=p'-p$. $(\;)_{10}$ means $SU(3)$-invariant $10\times 8\times 8$ coupling, and $(\;)_{D+(2/3)F}$ an $8\times 8\times 8$ invariant coupling with the strength of D and F equal to 1 and $\frac{2}{3}$, respectively. The dots now stand for the couplings of the pseudoscalar octet to other pairs of multiplets from ϕ and ψ, like $\Delta^*\Delta P$, etc., which we shall not discuss here. The constant $\bar{g}$ in (5a) is proportional to g of Eq. (4). R1 is already contained in Eq. (5). This equation also fixes the ratio of the $\Delta^*N\pi$ and $N^*N\pi$ couplings. Calculating the corresponding partial widths one readily obtains Eq. (1) of R2.

[7] See, e.g., B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).

[8] P. G. O. Freund, Phys. Rev. Letters **16**, 291 (1966).

[9] P. G. O. Freund, Phys. Rev. Letters **14**, 803 (1965); R. Oehme, *ibid.* **14**, 664 (1965).

We now consider the processes of photoformation ($\gamma p \to$ resonance $\to \pi N$) of $N^*(1688)$ and $\Delta^*(1920)$. The relevant matrix elements are

$$\begin{aligned}\langle N_{p'}^*(1688)|\,j_\mu^{\mathrm{elm}}(0)\,|p_p\rangle &= \bar{u}^{\alpha\beta}(p')\\ &\times\{q_\alpha g_{\beta\mu}(M-m)F^0(q^2)+q_\alpha q_\beta\\ &\times[\gamma_\mu F_1(q^2)+(i/(M+m))\sigma_{\mu\nu}q^\nu F_2(q^2)\\ &\quad-(q_\mu/(M+m))F_3(q^2)]\}u(p) \qquad (6a)\end{aligned}$$

and

$$\begin{aligned}\langle \Delta_{p'}^*(1920)|\,j_\mu^{\mathrm{elm}}(0)\,|p_p\rangle &= \bar{u}^{\alpha\beta\gamma}(p')\\ &\times\{(1+M/2m+m/2M)q_\alpha q_\beta g_{\gamma\mu}G_0(q^2)+q_\alpha q_\beta q_\gamma\\ &\times[((p+p')_\mu/2mM)G_1(q^2)-(\gamma_\mu/m)G_2(q^2)\\ &\quad-(q_\mu/m^2)G_3(q^2)]\}\gamma_5 u(p)\,, \qquad (6b)\end{aligned}$$

using the notation of Eq. (5). Gauge invariance requires

$$\begin{aligned}&F_0(q^2)+F_1(q^2)=q^2F_3(q^2)/(M^2-m^2)\,,\\ &[G_0(q^2)-G_2(q^2)]+(m/2M)[G_0(q^2)-G_1(q^2)]\\ &\quad+(M/2m)[G_0(q^2)+G_1(q^2)-2G_2(q^2)]\\ &\qquad=(q^2/m^2)G_3(q^2)\,. \qquad (7)\end{aligned}$$

The nonexistence of massless scalar hadrons requires the right-hand sides of these equations to vanish at $q^2=0$. Thus

$$\begin{aligned}&F_0(0)+F_1(0)=0\,,\\ &[G_0(0)-G_2(0)]+(m/2M)[G_0(0)-G_1(0)]\\ &\quad+(M/2m)[G_0(0)+G_1(0)-2G_2(0)]=0\,. \qquad (8)\end{aligned}$$

Assuming the electromagnetic (vector) form factors in Eqs. (6) to be dominated by vector-meson poles at low momentum transfers, we find using the $U(6)_W\times O(2)_W$ couplings [Eqs. (3), (4)][10]

$$\begin{aligned}&F_2(0)/F_1(0)=(M+m)/\mu-1\,,\\ &G_0(0)=G_1(0)=G_2(0)\,. \qquad (9)\end{aligned}$$

Furthermore, $G_1(0)$ and $F_1(0)$ can be related by $U(6)_W\times O(2)_W$ symmetry but we shall not explore this point here. In photoformation by real photons, the terms proportional to F_3 and G_3 in Eqs. (6) do not contribute. Therefore both matrix elements (6) are determined in terms of a single parameter. Using the formulas worked out by Brudnoy[11] that express the CGLN[5] electric and magnetic multipole amplitudes in terms of the values of the vertices (6) at $q^2=0$, we can translate our Eqs. (8) and (9) into relations for these multipole amplitudes. We find that for the photoformation of the $\Delta^*(1920)$ the electric 16-pole E_{3+} amplitude should vanish. This is precisely R3. For the $N^*(1688)$ we find in the same way

$$\left(\frac{M_{3-}}{E_{3-}}\right)_{N^*(1688)}=\frac{2}{3}\left[\frac{1}{3}+\frac{3M^2+m^2+4mM-2\mu M}{M^2-m^2}\right]^{-1}. \qquad (10)$$

Inserting $M=1688$ MeV, $m=940$ MeV, and $\mu=750$ MeV, we find $(M_{3-}/E_{3-})_{(1688)}=0.09$. If instead we had used $M\approx m+\mu$ and $2m/\mu=\mu_p=2.79$, so that $F_2(0)/F_1(0)\approx 2.79$ by Eq. (9), we would have obtained $=0.1$. Because of $(M-m)/\mu=O(1)$, the result is thus insensitive to small variations in the values of the central masses μ and m. $(M_{3-}/E_{3-})_{N^*(1688)}\approx 0.1$ is precisely our result R4.

We now wish to compare our results R1–R4 with experiment.

R1: The D/F ratio for the B^*BP couplings can be determined from a study of $B^*\to BP$ decay rates. Preliminary results[2] appear to indicate that $D/F\gtrsim 1$ which is compatible with our prediction. A more detailed comparison of R1 with experiment first requires more precise experimental determinations of as many $B^*\to BP$ decay rates as possible (at present only 6 are known and 3 of them very poorly) and/or a better theory of the breakdown of $SU(3)$ coupling-constant relations.

R2: Experimentally,[12]

$$\frac{\Gamma(\Delta^*(1920)\to N\pi)}{\Gamma(N^*(1688)\to N\pi)}=1.4\pm0.4\,, \qquad (11)$$

in agreement with our prediction R2.

It is interesting to note at this point that recently Sakita and Wali[13] have written down a superconvergent dispersion relation for a particular PB scattering amplitude. Saturating their sum rule with the nucleon N and the 33-resonance $\Delta(1238)$, they obtain precisely the $SU(6)$ relation between $\Gamma(\Delta(1238)\to N\pi)$ and the πN coupling constant. If in addition they include the $N^*(1688)$ and $\Delta^*(1920)$, they obtain the relation

$$\begin{aligned}\frac{\Gamma(\Delta^*\to\pi N)}{\Gamma(N^*\to\pi N)}&=\frac{8}{3}\frac{3-(D/F)^2}{(1+D/F)^2}\\ &\times\frac{24mM_{5/2}+3[(M_{5/2}-m)^2-\mu^2]}{24mM_{7/2}-4[(M_{7/2}-m)^2-\mu^2]}\,. \qquad (12)\end{aligned}$$

One may wonder whether this relation is connected with our result R2, or in other words, whether the sum rule of Ref. 13 is equivalent to $U(6)_W\times O(2)_W$ symmetry. To make a meaningful comparison we go to the case of exact symmetry ($M_{5/2}=M_{7/2}$, $q_{5/2}=q_{7/2}$). Equation (1) then becomes

$$\frac{175}{72}\frac{\Gamma(\Delta^*\to\pi N)}{\Gamma(N^*\to\pi N)}=1\,. \qquad (13)$$

[10] It is useful to observe that in as far as the $\frac{5}{2}^+$ octet and $\frac{7}{2}^+$ decimet are concerned only the combinations $q_\alpha q_\beta D^{*\mu\alpha\beta}$ and $q_\alpha q_\beta B^{*\alpha\beta}$ appear in Eq. (4). These moreover appear precisely in the same way as the $L=0$ $\frac{3}{2}^+$ and $\frac{1}{2}^+$ multiplets, so that one can readily obtain the required results from the $L=0$ work of Ref. 7. Note that this work includes $M_{\mathrm{final}}\neq M_{\mathrm{initial}}$, as is our case. In this sense the first Eq. (9) is a straightforward generalization of the well-known $SU(6)_W$ result $\mu_p^{am}=[(2m_p/\mu)-1]$.

[11] D. Brudnoy, Phys. Rev. **145**, 1229 (1966). We wish to thank Dr. J. Schechter for calling this very useful reference to our attention.

[12] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[13] B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967).

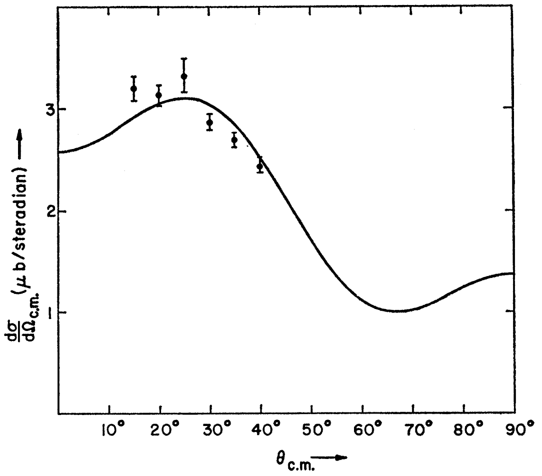

FIG. 1. Comparison of the differential cross section for photoproduction of π^+ at the $\Delta^*(1920)$ resonance [Eq. (16)] following from our prediction $(E_{3+}/M_{3+})_{\Delta^*(1920)}=0$ with the experimental results of the DESY group (Ref. 14) at this energy ($E_\gamma=1.48$ BeV). The over-all normalization of the theoretical curve is arbitrary.

Even in this limit, Eq. (12) is still dependent on the central masses M, m, and μ through the kinematical factor on the right-hand side. For the particular case $M=m+\mu$, this kinematical factor is unity, and for $D/F=\frac{3}{2}$ Eq. (12) becomes

$$\frac{175}{72}\frac{\Gamma(\Delta^*\to\pi N)}{\Gamma(N^*\to\pi N)}=\frac{7}{9}. \tag{14}$$

The relative factor 7/9 between Eqs. (13) and (14) is presumably to be accounted for by the fact that $M=m+\mu$ does not hold exactly even though it is a fair first approximation (for $m\approx 1100$ MeV $\mu\approx m_\rho=750$ MeV, the result M would be ≈ 1850 MeV).

R3: Ignoring in a rough first-approximation interference effects between the *resonant pure* M_{3+} amplitude in photoproduction at $E_{\text{c.m. total}}=1.92$ BeV [i.e., at the position of $\Delta^*(1920)$] and nonresonant background, we obtain the c.m. angular distribution:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}=\alpha(-175)(\cos\theta)^6 +255(\cos\theta)^4-65(\cos\theta)^2+17. \tag{15}$$

This function (in an arbitrary normalization) is compared with the experimental data of the DESY group[14] at $E_\gamma=1.48$ BeV ($E_{\text{c.m. total}}=1.92$ BeV corresponds to $E_\gamma=1.49$ BeV) in Fig. 1. Our prediction of pure M_{3+} photoformation of $\Delta^*(1920)$, as can be seen in Fig. 1, appears to be in agreement with experiment[14] even in the approximation of neglecting interference with the nonresonant background.

R4: In the case of photoproduction in the $N^*(1688)$ region, several analyses[15–17] have been made that include possible interference effects with a background estimated as reasonable at the time when these analyses were performed. We quote the results of these analyses: Peierls[15] argues that $N^*(1688)$ photoformation occurs predominantly via the E_{3-} multipole, Salin[16] finds $M_{3-}/E_{3-}\approx 1.0$, and Beder[17] quotes $M_{3-}/E_{3-}\approx 0.5$. Our prediction $M_{3-}/E_{3-}\approx 0.1$ thus gives the correct sign and agrees with the common finding of these authors that $M_{3-}/E_{3-}\lesssim 1$. It is noteworthy, however, that none of the above analyses includes the S_{11} and D_{15} N^*'s recently discovered very close to 1688 MeV.[18] We understand that a detailed phenomenological analysis is now under way by a CERN group[19] and this should throw more light on the validity of both our predictions R3 and R4.

We finally wish to add two more remarks.

(1) The fact that the mass splittings within the $\frac{5}{2}^+$ octet and the $\frac{7}{2}^+$ decimet are very nearly equal to those of their lower-spin counterparts (e.g., $m_{\Gamma_0^*(1815)}-m_{N^*(1688)}\approx m_\Lambda-m_N$) supports the Regge-recurrence hypothesis.

(2) Our results are readily generalized to higher Regge recurrences, and, specifically we find:

(R3′) The nth Regge recurrence of $\Delta(1238)$ having spin $J^P=(\frac{3}{2}+2n)^+$ can be photoformed only with a pure magnetic $M_{(J-\frac{1}{2})+}$ multipole transition; and

(R4′) the photoformation of the nth Regge recurrence of the nucleon of spin $J^P=(\frac{1}{2}+2n)^+$ and mass M_J proceeds with a ratio of magnetic to electric multipoles of

$$\xi_J=\frac{M_{(J+\frac{1}{2})-}}{E_{(J+\frac{1}{2})-}} =\frac{J-\frac{1}{2}}{J+\frac{1}{2}}\left[\frac{1}{J+\frac{1}{2}}+3\frac{[M_J+(4/9)m]^2+(1/9)m^2}{M_J^2-m^2}\right]^{-1}. \tag{16}$$

Equation (10) is a special case of this equation for $J=\frac{5}{2}$. Observe that for $J\to\infty$, provided that $M_J/m\to\infty$ or approaches a limit $\gg 1$, ξ_J will approach the limiting value of $\frac{1}{3}$. Photoformation should thus be a useful tool in investigating the Regge parentage of known πN resonances to the nucleon or to $\Delta(1238)$.

[14] G. Buschhorn, J. Carroll, R. D. Eaudi, P. Heide, R. Hübner, W. Kern, U. Kotz, P. Schrunser, and H. J. Skronn, Phys. Rev. Letters **17**, 1027 (1966). *Note added in proof.* Here recent results of this group (to be published) show in the near-forward direction a sizeable departure from Eq. (16). This could well be due to interference with a possibly diffractive background.

[15] R. F. Peierls, Phys. Rev. **118**, 325 (1960).

[16] Ph. Salin, Nuovo Cimento **28**, 1294 (1963). Salin's parameters N_1 and N_2 are related to E_{3-} and M_{3-} by $N_1=-(45/2)M_{3-}$ and $N_2=15(M_{3-}+E_{3-})$.

[17] D. Beder, Nuovo Cimento **33**, 94 (1964).

[18] According to R. Moorhouse [Phys. Rev. Letters **16**, 771 (1966); **16**, 968(E) (1966)] the photoformation of the S_{11} and D_{15} πN resonances is forbidden under certain assumptions about their $U(6)\times U(6)\times O(3)$ classification or equivalently of their quark structure.

[19] F. Berends, A. Donnachie, and D. Weaver, CERN Report No. 66, 1120/5-TH 703 (unpublished).