Possibility of Faster-Than-Light Particles*

G. FEINBERG†
The Rockefeller University, New York, New York
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We consider the possibility of describing, within the special theory of relativity, particles with spacelike four-momentum, which therefore have velocities greater than that of light in vacuum. The usual objections to such particles are discussed, and they are found to be unconvincing within the framework of relativistic quantum theory. A quantum field theory of noninteracting, spinless, faster-than-light particles is described. The field theory is Lorentz-invariant, but must be quantized with Fermi statistics. The associated particle theory has the property that the particle number is not Lorentz-invariant, and the no-particle state is not Lorentz-invariant either. Nevertheless, the principle of relativity is satisfied. The Lorentz invariance implies a relation between emission and absorption processes, in contradiction to the usual case. Some comments are made about the problem of introducing interactions into the field theory. The limiting velocity is c, but a limit has two sides.

I. INTRODUCTION

It is generally, although not universally,[1] believed that the validity of the special theory of relativity precludes the possibility of transmitting energy from point to point in space-time at velocities greater than c, the speed of light in a vacuum. The first statement to this effect appears in the first paper[2] of Einstein on the special theory. After pointing out that the relativistic formula for kinetic energy approaches infinity as $v \to c$ from below, Einstein concludes "... velocities greater than that of light ... have no possibility of existence." Other versions of this argument, as well as some others which will be considered later, are presented in the standard textbooks on relativity.[3]

It is the purpose of this paper to note that the standard arguments are not compelling in the context of relativistic quantum mechanics, with its characteristic discontinuous creation of particles. This has already been noted by other physicists.[1] The possibility of particles whose four-momenta are always spacelike, and whose velocities are therefore always greater than c is not in contradiction with special relativity, and such particles might be created in pairs without any necessity of accelerating ordinary particles through the "light barrier."

There are other problems with faster-than-light particles in relativistic quantum theory which arise from the fact that for a spacelike momentum vector, the sign of the energy can be changed by a Lorentz transformation, implying a more direct connection between the positive- and negative-energy solutions of the wave equation than for timelike momenta. This connection has been thought to imply that faster than light particles would necessarily involve the existence of negative-energy states with their resultant unphysical properties. This is, however, not the case, and we shall see that the negative-energy solutions for faster-than-light particles can be dealt with in a way quite similar to that used for ordinary particles, i.e., these solutions are associated with creation operators instead of annihilation operators in a quantum field theory.

It is perhaps worth noting that particles which travel faster than light do not involve logical inconsistencies. Indeed, no observations can be logically inconsistent.[4] To determine that a particle is moving faster than light it is only necessary to measure its position at two times and then calculate its velocity, by division, to be greater than c. None of these operations would seem to involve inconsistencies. Alternatively, one could measure the energy and momentum of the particle and note that

$$E^2 < p^2c^2.$$

Such a measurement could easily be done for example in a bubble chamber. Because of the problems associated with localization of faster-than-light particles, to be discussed below, the latter alternative seems more promising for demonstrating their existence.

In this paper we begin the program of describing the properties that faster-than-light particles would have if they exist within the context of the special theory of relativity. Our considerations will be mainly restricted here to noninteracting particles, although I hope to return to the description of interactions of such particles at another time.

In the absence of such a description of interactions, it cannot be regarded as demonstrated that it would be possible to detect faster-than-light particles even if they exist. It is perhaps worth mentioning here that the problems involved in describing the interactions of faster-than-light particles within quantum field theory

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† Permanent address: Department of Physics, Columbia University, New York, New York.

[1] For some dissents from the standard view, see O. M. P. Bilaniuk, V. K. Deshpande and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962); S. Tanaka, Progr. Theoret. Phys. (Kyoto) **24**, 171 (1960); G. Feinberg (unpublished).

[2] A. Einstein, Ann. Physik **17**, 891 (1905).

[3] See, e.g., D. Bohm, *Relativity* (W. A. Benjamin and Company, Inc., New York, 1965), p. 155ff.; P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1942), p. 43ff.; C. Moller, *The Theory of Relativity* (Oxford University Press, London, 1952), p. 52ff.

[4] I am indebted to Dr. M. Tausner for this remark.

are very similar to those in describing interactions of ordinary particles, i.e., in reconciling the singularities in local interaction with the requirements of Lorentz invariance and quantization. It is partly through long familiarity with ordinary particles that such problems are considered inessential for ordinary quantum fields by most physicists. In any case, it is as clear to the author as it will be to the readers that the theory presented here remains incomplete until the possibility of at least electromagnetic interactions of the particles is demonstrated. Some remarks in this direction will be found in Sec. VI and Appendix D.

While it is possible to discuss qualitatively the properties of faster-than-light particles and to answer the objections usually raised to their existence, within a relativistic theory, one cannot be sure about what new problems might arise without a detailed mathematical description of such particles. Such a description can be constructed, at least for spinless faster than light particles, within the formalism of relativistic quantum field theory. One description is presented in Sec. IV for noninteracting faster than light particles, which we call *tachyons*.[5] It is not clear whether there is a unique description of this type, as is the case for ordinary particles.

In the following, it will be found that a number of peculiar properties have been ascribed to tachyons. Some of these, such as the increase in velocity with decrease in energy follow directly from the kinematics of faster than light motion. Others, such as the necessity of the exclusion principle for spinless particles may be peculiar to our description, and come from trying to follow the conventional quantum-field-theory formalism as closely as possible.

II. OBJECTIONS TO FASTER-THAN-LIGHT PARTICLES

A number of different arguments have been advanced to demonstrate that the transmission of energy[6] at velocities greater than c is impossible if the special theory of relativity is true. We shall present several of these arguments followed by the reasons that we consider them insufficient to warrant the conclusion that faster-than-light objects can not exist. The counter-arguments will lead us to some of the properties of the particles.

1. From the usual expressions for the energy-momentum of a relativistic particle, we have

$$E=\frac{mc^2}{(1-v^2/c^2)^{1/2}}, \quad |p|=\frac{mv}{(1-v^2/c^2)^{1/2}}. \qquad (2.1)$$

Hence as $v\rightarrow c$ from below, $|p|$ and E become infinite, and would become imaginary if we take $v>c$.

[5] The name "tachyon" is suggested by the Greek work ταχις (tachys) meaning swift.

[6] I refer to the transmission of energy, rather than a signal, because the concept of a signal is somewhat more ambiguous. See the discussion in Appendix B.

Taken literally, the first part of this argument only shows that if a particle is at one time moving with $v<c$, it cannot be made to move with $v>c$. The argument does not rule out objects for which $v>c$ always. After all, we know of photons, for which $v=c$, and the argument taken literally would seem to rule them out as well.

To deal with the second part of the argument, we note that the quantity m, or the rest mass, that appears in Eq. (1) is not a directly measurable quantity, unless the particle can be brought to rest. We are therefore free to hypothesize particles for which $v>c$ always, and m is an imaginary quantity

$$m=i\mu \quad (\mu \text{ real}). \qquad (2.2)$$

In this case, the energy and momentum will remain real quantities, satisfying

$$E=\frac{\mu c^2}{(v^2/c^2-1)^{1/2}}, \quad |p|=\frac{\mu v}{(v^2/c^2-1)^{1/2}}, \qquad (2.3)$$

and $c^2p^2-E^2=\mu^2c^4$, so that the four-momentum is a spacelike vector. We note that $|pc|>E$, and that the velocity may still be defined by

$$\frac{v}{c}=\frac{|p|}{E}>1. \qquad (2.4)$$

Furthermore, the range of the energy and momentum are given by

$$0<E<\infty, \quad \mu c<|p|<\infty, \qquad (2.5)$$

and both the energy and momentum are monotonic decreasing functions of the velocity, so that tachyons, as we shall call these particles, speed up as they lose energy.

The value $v=\infty$ is allowed for tachyons, and at this velocity we have $E=0$ and $|p|=\mu c$, so tachyons at infinite speed carry momentum but no energy. Of course, infinite velocity is not invariant since the usual velocity transformation law remains valid:

$$\mathbf{v}'=\frac{\mathbf{v}+(\gamma-1)(u\cdot v\mathbf{u}/u^2)-\gamma\mathbf{u}}{(1-u\cdot v/c^2)\gamma}, \qquad (2.6)$$

where

$$\gamma=(1-u^2/c^2)^{-1/2}.$$

It is easy to see that

$$\frac{v'^2}{c^2}=1+\left(\frac{v^2}{c^2}-1\right)\frac{(1-u^2/c^2)}{(1-u\cdot v/c^2)^2}, \qquad (2.7)$$

so that $v'>c$ if $v>c$.

This formulation of the kinematics of faster-than-light particles leads directly to another objection to their existence which has been often raised.

2. When the momentum four-vector is spacelike, the sign of the fourth component, or energy, can be changed by an ordinary (orthochronous) Lorentz transforma-

tion. Hence, a particle which is seen by one observer to have positive energy will have negative energy to another observer. By the principle of relativity, any state which is possible for one observer must be possible for all observers, and hence faster-than-light particles can exist in negative-energy states for all observers.

This can be seen directly from the transformation equations for energy and momentum

$$E'=\gamma(E-p\cdot u),$$
$$\mathbf{p}'=\mathbf{p}+(\gamma-1)\frac{\mathbf{p}\cdot\mathbf{u}\mathbf{u}}{u^2}-\frac{\gamma E\mathbf{u}}{c^2}. \tag{2.8}$$

Since $|pc|>E$, one can always choose u so that for instance $E'=-E$. The occurrence of negative-energy states for particles has always been objected to on the grounds that no other system could be stable against the emission of these negative-energy particles, an entirely unphysical behavior.

Before analyzing this objection further, it is useful to cite another objection, as the two are really part of the same problem.

3. When the velocity of an object is greater than c, it is possible to change the sense of the propagation in time by an ordinary Lorentz transformation. That is, suppose that for one observer, a particle moves through two points x_1, x_2 at times t_1, t_2, with

$$\frac{\Delta x}{\Delta t}\equiv\frac{|x_2-x_1|}{t_2-t_1}>c \quad\text{and}\quad \Delta t=t_2-t_1>0. \tag{2.9}$$

For a second observer moving along the z axis with velocity u, we have

$$\Delta x'=(\Delta x-u\Delta t)\gamma,$$
$$\Delta t'=\left(\Delta t-\frac{u\Delta x}{c^2}\right)\gamma=\Delta t\left(1-\frac{uv}{c^2}\right)\gamma,$$

and clearly by choosing $uv>c^2$, we can make $\Delta t'$ have the opposite sign to Δt, i.e., change the time ordering of the points along the trajectory.

It can be seen that this change occurs under the same circumstances as the change in the sign of the energy. In fact from (2.8) and (2.9) we have

$$E'/E=\Delta t'/\Delta t=\gamma(1-uv/c^2). \tag{2.10}$$

This circumstance provides the key to the understanding of the negative energies.[7] Any observer will insist on a time ordering of events consistent with primitive ideas of causality, such as that emission occur before absorption. However, emission generally refers to the production of a positive-energy system and absorption to the destruction of a positive-energy system. It is clear that at a single point there is no distinction between absorption of a positive-energy particle and emission of a negative-energy particle. This distinction can only be made on the basis of whether the particle is again detected in the future, or has been detected in the past. But this is exactly the result that is altered by a Lorentz transformation which changes the sign of Δt. That is, suppose a process occurs which can be interpreted by one observer as emission of a positive-energy tachyon at one space-time point and absorption of the tachyon at a later spacetime point. For a different, Lorentz-transformed, observer the second point may be earlier in time than the first, and the energy of the tachyon may be transformed to a negative value by the Lorentz transformation. This observer will interpret the process as the emission of a positive-energy tachyon at point 2′ and its absorption at the later point 1′, and therefore need not introduce the concept of negative-energy particles at all. The consequence of Lorentz transformations is therefore to relate the rates of emission and absorption rather than to require the introduction of negative-energy states. While this is a novel situation in physics, it does not seem to be intrinsically unphysical. We shall see that such a relationship naturally occurs in a quantum field theory of tachyons. A more detailed analysis of some thought experiments involving the emission and absorption of tachyons as viewed by different Lorentz observers is given in Appendix A.

4. If faster-than-light particles existed, it might appear natural to use them to synchronize the clocks of observers in relative motion. Such observers would be related not by Lorentz transformations, but by a new group of transformations, and part of the justification for the requirement of Lorentz invariance would be lost.

A more detailed analysis shows that this remark is misleading. We know from general relativity that it is possible to relate the measurements made by any two observers, with clocks synchronized in an arbitrary way. However, in general the laws of physics will not be invariant under the transformations obtained in this way. In particular, it is clear that if infinite speed tachyons are used to synchronize clocks of different observers, the velocity of light would not then be the same for the different observers. Of course, within one particular coordinate system, the laws of physics will still be those of special relativity. This does not depend on the method of clock synchronization of distinct observers.

It might be further objected that the transformations resulting from the new type of synchronizations could leave some other laws invariant, such as the velocity of propagation of tachyons. It would seem that this cannot be the case. While the velocity of tachyons of a single energy might be the same for two relatively moving observers, this cannot be the case in general. The invariance of the velocity of light from observer to observer depends not only on the use of light rays to

[7] This interpretation is suggested in the work of Bilaniuk, Deshpande and Sudarshan (Ref. 1) who have also described much of the material in Sec. II of this paper.

synchronize clocks, but also on the empirical fact that for any observer, the velocity of light is independent of its energy, i.e., of the velocity of the source of light. Since this cannot be made the case for tachyons, their velocity will also vary from observer to observer.

In view of the close connection between the form of the laws of physics as seen by one observer, and the requirement of the invariance of certain quantities under transformations between observers, it is not surprising that a particular set of transformations and hence of clock synchronizations should be more natural than another, once the laws of physics have been determined by one observer. In our world, these are the Lorentz transformations, and any other synchronization will only produce complications.

5. Because of the possibility of changing the time ordering of events along the path of a tachyon by a Lorentz transformation, it seems possible to transmit signals into the past of a single observer.[8] This is in apparent conflict with the natural view that one is free to decide whether or not to carry out an experiment up until the time that one actually does so.

Although this argument seems to be the most serious qualitative objection to faster-than-light particles, its force seems somewhat weaker than is generally stated. A conclusion warranted by this argument is that tachyons cannot be used to send reliable signals, either forward or backward in time, in the sense that one cannot completely control the outcome of an experiment to produce or absorb them. Indeed, this also follows from the relation between emission and absorption that is contained in the theory to be presented. It does not seem to me to follow from this argument that one could never detect a faster-than-light particle, or to devise an apparatus which may produce them. Although this difficulty of experimentally handling tachyons may seem strange on the basis of our experience with ordinary particles, it cannot by itself be used to conclude that they do not exist.

A more detailed analysis of a particular example of acausal behavior, from the standpoint of the interpretation given previously, will be presented in Appendix B.

I conclude that the usual objections to the existence of faster-than-light particles need not be valid, and that it may be possible to describe them consistently within the special theory of relativity. It is to the construction of such a description that we now turn.

III. IMAGINARY-MASS SCALAR FIELDS

In this section, we study the solutions to the Klein-Gordon equation for a scalar c-number field $\phi(x)$, with an imaginary mass $m=i\mu$. This is a necessary preliminary to quantizing the field. We restrict the discussion to scalar fields because it is known that there are no finite-dimensional representations of the Lorentz group corresponding to imaginary mass, other than the one-dimensional representation.

We consider the equation

$$(\Box^2+\mu^2)\phi=\left(\nabla^2-\frac{\partial^2}{\partial t^2}+\mu^2\right)\phi=0. \tag{3.1}$$

A set of elementary solutions to this are clearly

$$\phi_{+,k}=\frac{1}{(2\pi)^{3/2}}\exp i(\mathbf{k}\cdot\mathbf{x}-\omega t)\equiv\frac{1}{(2\pi)^{3/2}}\exp(ikx) \tag{3.2}$$

and

$$\phi_{-,k}=\frac{1}{(2\pi)^{3/2}}\exp[-i(\mathbf{k}\cdot\mathbf{x}-\omega t)]\equiv\frac{1}{(2\pi)^{3/2}}\times\exp(-ikx).$$

Here $\omega=+(k^2-\mu^2)^{1/2}$ always. We restrict $\mathbf{k}$ by the condition

$$|k|\geq\mu. \tag{3.3}$$

This is done in order that ω as defined should be real, i.e., that the solutions should be oscillatory in time. This would appear to be necessary if we are to interpret a superposition of such solutions as the wave function of a particle with real energy, which we require.

Because of the restriction of the wave numbers given by (3.3), the set of functions $\phi_{+,k}^*(\mathbf{x},t=0)=\phi_{-,k}(\mathbf{x},t=0)$ does not form a complete set. Instead of the usual completeness relation,

$$\sum_{\text{all }k}\phi_{+,k}^*(\mathbf{x},t=0)\phi_{+,k}(\mathbf{x}',t=0)=\delta^3(x-x'),$$

we have

$$\sum_{|k|\geq\mu}\phi_{+,k}^*(\mathbf{x},t=0)\phi_{+,k}(\mathbf{x}',t=0)\equiv\bar{\delta}^3(x-x') \tag{3.4}$$

with

$$\begin{aligned}\bar{\delta}^3(x-x')&=\int d^3k\,\theta(|k|-\mu)\exp i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')\\&=\delta^3(x-x')-\int_0^\mu k^2d\,k\int d\Omega\frac{\exp i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}{(2\pi)^3}\\&=\delta^3(x-x')+\frac{(\lambda\cos\lambda-\sin\lambda)}{|x-x'|^3 2\pi^2}\end{aligned} \tag{3.5}$$

with $\lambda=\mu|x-x'|$.

The incompleteness of the allowed set of solutions has several consequences.

1. Tachyons cannot be localized in space, i.e., a superposition of solutions of the form

$$\psi(x)=\int\phi_{+,k}(x)f(k)d^3k,\quad(|k|\geq\mu),$$

which could be a tachyon wave function, cannot be made into $\delta^3(x)$. In fact, such a superposition cannot be made to vanish outside a sphere of finite radius, but

[8] Essentially this argument is given by D. Bohm (Ref. 3). It was pointed out to me by Dr. P. B. Kantor.

rather necessarily has a finite tail. The tail can, however, be made to decrease with an arbitrary power of x for large x, by choosing the weight function $f(k)$ to have a zero of suitable order at $|k|=\mu$.

2. The Cauchy initial-wave problem must be restricted somewhat for the ϕ field. In order to determine $\phi(\mathbf{x},t)$, it is still possible to prescribe $\phi(\mathbf{x},0)$ and $(\partial\phi/\partial t)(\mathbf{x},0)$. However, these functions cannot be prescribed arbitrarily. Instead, they must both be restricted to functions whose Fourier transforms vanish when $|k|<\mu$, i.e.,

$$\phi(\mathbf{x},\,t=0)=\int g(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}d^3x \tag{3.6}$$

with

$$g(\mathbf{k})=0 \quad \text{for} \quad |k|<\mu\,,$$

and similarly for $\partial\phi/\partial t$. These conditions insure that no increasing or decreasing exponentials will occur in $\phi(\mathbf{x},t)$. When they are imposed on the initial data, the Cauchy problem may be solved as usual by

$$\phi(\mathbf{x},t)=\int d^3x'\,G_1(\mathbf{x}-\mathbf{x}',\,t)\phi(\mathbf{x}',0)+\int d^3x'\,G_2(\mathbf{x}-\mathbf{x}',\,t)\frac{\partial\phi}{\partial t}(\mathbf{x}',0)\,, \tag{3.7}$$

$$\begin{aligned} G_2&=-\int\frac{d^3k}{(2\pi)^3}\frac{\exp i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}{\omega}\sin\omega t\,,\\ G_1&=-\frac{\partial G_2}{\partial t}\,. \end{aligned} \tag{3.8}$$

The function G_2 is the analog for this field of the function Δ for the ordinary Klein-Gordon field. Unlike that function, it does not vanish for space like separations. This may be seen for instance by noting that $G_1=-\partial G_2/\partial t$ reduces at $t=0$ to $\bar\delta^3(x-x')$ which is not zero for $x\neq x'$. The fact that G_2 does not vanish for spacelike separations is not in contradiction with the fact that $G_2(x-x',\,t=0)$ vanishes. This is because G_2 is not an invariant function. To see this, we can rewrite G_2 as a four-dimensional integral

$$G_2(x)=\int e^{ik\cdot x}\delta(k^2-\mu^2)\,\epsilon(k_0)\frac{d^4k}{(2\pi)^3}\,. \tag{3.9}$$

The noninvariance of this integral follows from the fact that the step function $\epsilon(k_0)$ is not an invariant function for spacelike four-momenta.

3. Because of the impossibility of localizing $\phi(x)$, the discussion of propagation for the solutions of (3.1) is more complex than for the real mass Klein-Gordon equation. In the usual discussion, one chooses $\phi(x,\,t=0)$ to be a pulse, confined to some region about $x=x_0$, and then the propagation of the pulse is determined by the form of the Green's function G for large x. In the present case, such a pulse cannot be constructed from the allowed solutions, and the values of $\phi(x,t)$ at large x come not only from the fact that G_2 does not vanish at spacelike separation, but also from the fact that $\phi(x,0)$ will not vanish at large x. It is not hard to see that $G_1(x,t)$ is the response to a disturbance whose value at $t=0$ is given by $\bar\delta^3(x)$, which as we have indicated, does not vanish identically for large x.

In order to discuss the propagation of solutions of (3.1), it appears necessary to consider initial data with some definite shape near $x=0$, and examine how this shape propagates itself. This problem will not be considered further in this article. Because of this, the "velocity" of the tachyons is a somewhat loosely defined concept operationally. I shall continue to write as if $v/c=p/E$ is the measurable velocity of a tachyon, but a more detailed discussion of this point is clearly required. Some of the mathematical literature on the propagation character of partial differential equations is probably relevant here.[9]

This completes the discussion I shall give here of the classical field theory. I turn now to the construction of the quantized field theory.

IV. QUANTUM FIELD THEORY OF NONINTERACTING TACHYONS

In order to describe many-tachyon systems, and to prepare for the introduction of interactions, we consider the field ϕ satisfying (3.1) to be an operator, with commutation relations to be determined, and examine the quantum field theory of such operators. We expand ϕ by

$$\begin{aligned}\phi(x,t)&=\int\frac{d^3k}{(2\omega_k)^{1/2}}\phi_{+,k}a(k)+\int\frac{d^3k}{(2\omega_k)^{1/2}}\phi_{-,k}a^\dagger(k)\\ &=\int\frac{d^3k}{(2\omega_k)^{1/2}(2\pi)^{3/2}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}a(k)\\ &\quad+\int\frac{d^3k}{(2\omega_k)^{1/2}(2\pi)^{3/2}}e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)}a^\dagger(k)\,.\end{aligned} \tag{4.1}$$

We have taken a Hermitian field to describe neutral particles. The integral is as before over the allowed $|k|\geq\mu$. The $a(k)$, $a^\dagger(k)$ are operators with undetermined commutation relations. The argument of $a(k)$ is the three-vector momentum.

It is not possible to requre ϕ, $\partial\phi/\partial t$ to satisfy canonical commutation relations. This follows from the same reason that the classical ϕ were not localizable, i.e., the incompleteness of the functions $\phi_{\pm,k}$. There is however another restriction on the commutation relations following from the Lorentz invariance of the theory,

[9] See, for example, L. Hormander, *Linear Partial Differential Operators* (Academic Press Inc., New York, 1963), Ch. 5. I would like to thank Dr. E. Lubkin for bringing this work to my attention.

which is not related to the incompleteness of the ϕ_k. To see this, we consider the condition of Lorentz invariance of the *field* theory, which is the existence of unitary operators $L(l,a)$, associated with a Lorentz transformation $x'=lx+a$, and satisfying

$$L(l,a)\phi(x)L^{-1}(l,a)=\phi(lx+a)\,. \tag{4.2}$$

The space-time translations are generated by the energy and momentum operators in the usual way, and Eqs. (4.2) are equivalent to the differential equations

$$\partial_\mu\phi(x)=-i[P_\mu,\phi(x)] \tag{4.3}$$

with P_μ the 4-vector momentum operators. Upon substitution of (4.1), this implies

$$[P_\mu,a(k)]=-a(k)k_\mu\,,\quad [k_0=+(k^2-\mu^2)^{1/2}]\,, \tag{4.4}$$

which has the natural solution, although perhaps not the unique solution

$$P_\mu=\int_{k\geq\mu} k_\mu a^\dagger(k)a(k)d^3k\,, \tag{4.5}$$

where the $a(k)$ can be of one of the usual types of annihilation operators, i.e., fermions, bosons, or parafermions, parabosons.[10] Note that P_0 has no negative eigenvalues.

It is remarkable that by choosing the Lorentz transformation in (4.2) to be a homogeneous Lorentz transformation (boost), we can rule out the boson and paraboson solutions. In order to see this, we compute, for a "boost" with velocity $\mathbf{u}$, the exponent in $\phi(lx)$

$$k(lx)\equiv k_\mu l_{\mu\nu}x_\nu\equiv\bar{k}_\nu x_\nu\,,$$

where

$$\bar{k}_i=k_i+(\gamma+1)\frac{(\mathbf{k}\cdot\mathbf{u})u_i}{u^2}+\gamma\omega u_i\,,\quad i=1,2,3 \tag{4.6}$$
$$\bar{k}_0=\gamma(k_0+\mathbf{k}\cdot\mathbf{u})\,.$$

(Note that $\bar{k}_0$ can be negative, although k_0 is taken positive.) For any $\mathbf{u}$ there will be a set of $\mathbf{k}$ such that $\bar{k}_0$ will be negative. These are just those $\mathbf{k}$ whose energies are made negative by the inverse Lorentz transform. The corresponding terms in the positive-frequency part of $\phi(x)$ must therefore be transformed by L into terms in the negative-frequency part of $\phi(x)$, that is, the operators $a(k)$ multiplying these terms must be transformed by L into the $a^\dagger(k)$ operators. It is not hard to show that the condition (4.2) implies that

$$\begin{aligned} La(k)L^{-1}&=\left(\frac{k_0'}{k_0}\right)^{1/2}a(k') &&\text{if}\quad k_0'/k_0>0 \\ & &&\text{and}\quad k_0,\,k_0'\neq0\,, \\ La(k)L^{-1}&=\left(\left|\frac{k_0'}{k_0}\right|\right)\;a^\dagger(-k') &&\text{if}\quad k_0'/k_0<0 \\ & &&\text{and}\quad k_0,\,k_0'\neq0\,. \end{aligned} \tag{4.7}$$

Here $k_\mu'=l_{\mu\nu}k_\nu$.

[10] O. W. Greenberg and A. M. L. Messiah, Phys. Rev. **138**, B1155 (1965).

It is clear that these relations and the assumption that L is unitary are inconsistent with the canonical commutation relations

$$\begin{aligned}&[a(k),a^\dagger(k')]=\delta^3(k-k')\,,\\&[a(k),a(k')]=0\,,\end{aligned} \tag{4.8}$$

since a Lorentz transform changing $a(k)$ into $a^\dagger(k)$ will change the sign of the left-hand side of (4.8) without changing the right-hand side. On the other hand, if we quantize with anticommutators, no such trouble will arise. Therefore, we shall take the a, $a^\dagger$ to satisfy

$$\begin{aligned}&a(k)a^\dagger(k')+a^\dagger(k')a(k)=\delta^3(k-k')\,,\\&a(k)a(k')+a(k')a(k)=0\,,\quad (k_0,\,k_0'\neq0)\end{aligned} \tag{4.9}$$

and these are consistent with (4.7) and a unitary L. Therefore the tachyons are fermions, even though they have spin-zero. Such a violation of the connection between spin and statistics is not in contradiction with the known theorems on this connection,[11] since we do not assume "microscopic causality."

A special remark must be made about the momenta satisfying $k_0=0$, since a change of sign of k_0 is not defined for them. The distinction between creation and annihilation of particles with such momenta is arbitrary, since the emission of a zero-energy particle with momentum $\mathbf{k}$ is indistinguishable from the absorption of a zero-energy particle of momentum $-\mathbf{k}$. We can introduce the linear combination

$$\begin{aligned}&\alpha(k)\equiv a(k)+a^\dagger(-k)\,,\quad [\omega_k=0]\\&\alpha(k)=\alpha^\dagger(-k)\,,\end{aligned}$$

for which we require the commutation relations

$$\{\alpha(k),\alpha^\dagger(k')\}=\delta(k-k')\,,$$

and the $\alpha(k')$ commute with all $a(k)$ for $k_0\neq0$. We then obtain for the Lorentz transformation of operators corresponding to zero energy

$$L\alpha(k)L^{-1}=\left(\left|\frac{k_0'}{k_0}\right|\right)^{1/2}a(k')\qquad\text{if}\quad k_0'>0\,,$$

$$L\alpha(k)L^{-1}=\left(\left|\frac{k_0'}{k_0}\right|\right)^{1/2}a^\dagger(-k')\quad\text{if}\quad k_0'<0\,.$$

On the other hand, if a given nonzero energy is transformed to zero by a Lorentz transformation, we have

$$La(k)L^{-1}=\left(\frac{k_0'}{k_0}\right)^{1/2}\alpha(k')\,.$$

Finally, if both k_0 and k_0' vanish, we have

$$L\alpha(k)L^{-1}=\alpha(k')\,.$$

[11] See R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (W. A. Benjamin, Inc., New York, 1964), p. 146ff.

Because of the singular factor $1/\sqrt{k_0}$ appearing in the expansion of ϕ in terms of $a(k)$ or $\alpha(k)$ and in the Lorentz transformations, the zero-energy states should only appear as part of a range of momentum states. In this case the singularity will be integrated over and is unlikely to cause trouble.

We are now free to interpret $a^\dagger(k)$ as a creation operator, and $a(k)$ as an annihilation operator, for particles of momentum k. This interpretation is justified by the form of the energy-momentum operators, (4.5), and the anticommutation relations (4.9). When this is done, it can be seen that we have treated the negative-energy solutions of (3.1), just as indicated qualitatively in Sec. II. Furthermore, the transformation of a into $a^\dagger$ by a Lorentz transformation which changes the sign of k_0 is again consistent with the remark that these transformations should interchange the roles of emission and absorption. We defer for the moment a discussion of how the states transform under Lorentz transformations.

In order to satisfy the conditions (4.2), (4.3), the energy-momentum operators (4.5) must transform as a four-vector under Lorentz transformations, up to a possible additive constant. It is easily verified that under a Lorentz transformation, we have

$$LP_\mu L^{-1}=l_{\mu\nu}P_\nu+\textstyle\sum_{\bar{k}} k_\mu\,, \tag{4.10}$$

where $\sum_{\bar{k}}$ means a sum over all momenta whose fourth component changes sign under the inverse Lorentz transformation, i.e., those momenta satisfying

$$\mathbf{k}\cdot\mathbf{u}+\omega<0. \tag{4.11}$$

Note that this is a finite region of momentum space, so that the additive constants that appear in (4.10) are finite, at least if the volume of space is taken finite. The constants can be removed by a redefinition of P_μ, taking instead

$$P_\mu'=\int_{k\geq\mu} k_\mu[a^\dagger(k),a(k)]d^3k. \tag{4.12}$$

However, we prefer the other definition, since the energy is not positive with (4.12). There is no physical difference, except perhaps for gravitation, between the two expressions.

It is of some interest to express H $(=P_0)$ in terms of the field ϕ. A simple calculation, using the lemma of Appendix C, yields the result

$$H=\frac{1}{2}\int d^3x\{\dot\phi^2(x)+\phi(x)[-\nabla^2-\mu^2]\phi(x) +i[(-\nabla^2-\mu^2)^{1/2}\phi(x),\dot\phi(x)]\}. \tag{4.13}$$

To simplify this expression, we evaluate the anticommutator at equal times, of the fields

$$\{\phi(x),\phi(y)\}=\frac{1}{(2\pi)^3}\int_{k\geq\mu}\frac{d^3k}{2\omega}[\exp ik\cdot(x-y)+\exp ik\cdot(y-x)]=\frac{1}{(2\pi)^3}\int\frac{d^3k}{\omega}\exp ik\cdot(x-y) \quad \text{for } x_0=y_0 \tag{4.14}$$

$$\{\phi(x),\,(-\nabla_y^2-\mu^2)^{1/2}\phi(y)\}=\frac{1}{(2\pi)^3}\int d^3k\,\exp ik\cdot(x-y)=\bar\delta^3(x-y) \quad \text{for } x_0=y_0 \tag{4.15}$$

$$\left\{\phi(x),\frac{\partial\phi}{\partial t}(y)\right\}=\frac{-i}{(2\pi)^3}\int \tfrac{1}{2}d^3k[\exp ik\cdot(x-y)-\exp ik\cdot(y-x)]=0 \quad \text{for } x_0=y_0 \tag{4.16}$$

$$\left\{\frac{\partial\phi}{\partial t}(x),\frac{\partial\phi}{\partial t}(y)\right\}=\frac{-1}{(2\pi)^3}\int \tfrac{1}{2}d^3k\,\omega[\exp ik\cdot(x-y)+\exp ik\cdot(y-x)]. \tag{4.17}$$

These evidently differ exceedingly from the canonical quantization of a fermion field.

The first two terms in H are c numbers and can be dropped. The remaining term is not the integral of a quantity local in space, since the operator $(-\nabla^2-\mu^2)^{1/2}$ is really an integral operator. Nevertheless, we can check that the Hamiltonian in this form also gives the Heisenberg equations of motion, i.e.,

$$[\mathcal{H},\phi(x)]=\frac{i}{2}\int d^3x[[(-\nabla^2-\mu^2)^{1/2}\phi(x),\,\dot\phi(x)],\,\phi(y)]$$

$$=-i\int d^3x\,\dot\phi(x)\{(-\nabla^2-\mu^2)^{1/2}\phi(x),\,\phi(y)\}$$

$$=-i\dot\phi(y) \text{ by (4.15) and the lemma of Appendix C.} \tag{4.18}$$

$$[\mathcal{H},\dot\phi(y)]=\frac{i}{2}\int d^3x[[(-\nabla^2-\mu^2)^{1/2}\phi(x),\,\dot\phi(x)],\,\dot\phi(y)]$$

$$=i\int d^3x(-\nabla^2-\mu^2)^{1/2}\phi(-\nabla^2-\mu^2)^{1/2}\bar\delta^3(x-y)$$

$$=i(\nabla^2-\mu^2)\phi(y). \tag{4.19}$$

It is also possible to write the other generators of the Poincaré group as explicit functions of the fields ϕ, and to check that the structure relations of the group are satisfied, apart from a constant that we have added to H to make it positive-definite.

It does not appear possible to derive the field equation from an invariant Lagrangian, in the presence of the anticommutation relations (4.14) and (4.17). A direct use of the conventional form

$$L=-H+\phi\frac{\partial H}{\partial\phi}$$

leads to the result $L=0$. This can be traced to the fact that any invariant formed of ϕ and its derivatives of finite order, other than ϕ itself, reduces to a c number by the anticommutation relations.[12] This does not seem to be a real problem for the free fields, although it makes the introduction of self interactions difficult. As we shall see, there is no corresponding problem for complex fields.

It is of interest to evaluate the anticommutator at arbitrary space-time points, which is also a c number function.

$$\{\phi(x,t),\phi(x',t')\}$$

$$=\int\frac{d^3k}{\omega}\frac{\exp i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}{(2\pi)^3}\cos\omega t \tag{4.20}$$

$$=I(x-x')=\int d^4k\frac{[\exp ik\cdot(x-x')]\delta(k^2-\mu^2)}{(2\pi)^3}. \tag{4.21}$$

Unlike the Green's function G_2, this is clearly an invariant function. It can be evaluated explicitly, using the reduction formulas for Fourier transforms of invariant functions.[13]

$$I(y^2)=\frac{-\mu}{4\pi\sqrt{(y^2)}}Y_1(\mu\sqrt{(y^2)})\theta(y^2)-\frac{\mu}{2\pi^2\sqrt{(-y^2)}}\times K_1(\mu\sqrt{(-y^2)})\theta(-y^2). \tag{4.22}$$

The anticommutator does not vanish outside the light cone, and in fact falls off only as $(x-x')^{-2}$ at large spacelike separations.

One other point worth noting about the field theory is the existence of a conserved current $\mathcal{J}_\mu$. The current is given by

$$\mathcal{J}_\mu=\phi\partial_\mu\phi-\partial_\mu\phi\phi, \tag{4.23}$$

and clearly the field equation implies

$$\partial_\mu\mathcal{J}_\mu=\phi\Box^2\phi-\Box^2\phi\phi=0. \tag{4.24}$$

The conserved charge associated with $\mathcal{J}_\mu$ can be written

$$\phi=-\int\mathcal{J}_4d^3x=\int d^3k\,a^\dagger(k)a(k)+\text{infinite constant}, \tag{4.25}$$

so that ϕ is essentially the tachyon number. This quantity is not however Lorentz invariant.

It is interesting that in this theory, the tachyon number can be written as the integral of a local charge density, while the Hamiltonian cannot be written as the integral of a local energy density. This is precisely the reverse of the situation in the field theory of ordinary neutral particles.

The current $\mathcal{J}_\mu$ does not generate a local transformation of the field ϕ. Instead, it generates opposite phase transformations on the fields ϕ_+, ϕ_-, defined as the positive- and negative-frequency parts of ϕ. Since ϕ_+, ϕ_- are not scalar fields, it is not feasible to write invariant interactions using them, and the conservation of $\mathcal{J}_\mu$ would not be valid in the presence of interactions.

This completes the field theory of the noninteracting real tachyon field. The generalization of this field theory to noninteracting complex fields is straightforward and is done in Appendix D. The main additional feature that occurs in that case is that for Lorentz transformations that would geometrically change the sign of the energy, the creation operators for particles $a^\dagger(k)$ are changed into annihilation operators for antiparticles $b(k')$ and vice versa. The necessity for this is clear in view of the fact that emitting a particle induces the same change of charge at a vertex as absorbing an antiparticle.

The particle aspects of the field theory we have constructed involves several new features, however, which will now be considered.

V. PARTICLE ASPECTS OF TACHYON FIELDS

The particle interpretation of a conventional field theory is obtained by assuming the existence of a no-particle, or vacuum, state defined by

$$a(k)|0\rangle=0, \tag{5.1}$$

and forming the particle states by acting on the vacuum with various creation operators. The vacuum will be a null eigenstate of four-momentum, and the eigenstate of lowest energy.

These conditions will also be satisfied in the tachyon theory, and the particle states can be constructed in the same way, i.e., a state containing particles of momentum $k_1,\cdots,k_n$ is

$$|k_1,k_2,\cdots,k_n\rangle=a^\dagger(k_1)a^\dagger(k_2)\cdots a^\dagger(k_n)|0\rangle. \tag{5.2}$$

The state so constructed are eigenstates of four-momentum in the usual way, i.e.,

$$P_\mu|k_1,\cdots,k_n\rangle=(k_{1_\mu}+k_{2_\mu}+\cdots k_{n\mu})|k_1,\cdots k_n\rangle. \tag{5.3}$$

Of course, since the spinless tachyons are fermions, the momenta $k_1,\cdots k_n$ must be distinct. Also, there are other states of zero energy than $|0\rangle$.

A novel feature of the tachyon-particle theory is the behavior of the particle states under Lorentz transformations. In conventional theories, the vacuum state is Lorentz invariant. This follows from the fact that it is

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[12] There does exist a nonvanishing pseudoscalar which is quartic in the field ϕ. This is the quantity $\epsilon_{\mu\nu\alpha\beta}\partial_\mu\phi\partial_\nu\phi\partial_\alpha\phi\partial_\beta\phi\equiv\partial_1\phi\partial_2\phi\partial_3\phi\partial_4\phi$.

[13] See G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963), Appendix B.

the unique null eigenstate of four-momentum, and the behavior of the energy-momentum operators under Lorentz transformations. Since the number of particles in a state is also Lorentz-invariant, it follows that in conventional theories a Lorentz transformation can only change the momenta and spins of the particles in a state, without affecting the number.

The situation is quite different for tachyons. It follows directly from Eq. (4.7) that the vacuum state is not Lorentz invariant. To see this, we consider the state

$$|k\rangle = a^\dagger(k)|0\rangle. \tag{5.4}$$

By (4.7), there exists a Lorentz transformation L that changes $a^\dagger(k)$ into some annihilation operator $a(\bar{k})$. If the vacuum were invariant under L, then we would have

$$L|k\rangle = La^\dagger(k)|0\rangle = a(\bar{k})|0\rangle = 0 \tag{5.5}$$

which is impossible for a unitary operator. We can calculate what the Lorentz transform of the vacuum is by using (4.7). Let us denote the transform of the vacuum corresponding to the Lorentz transformation L by

$$|\Omega_L\rangle \equiv L|0\rangle.$$

Clearly $|\Omega_L\rangle$ will be different for different Lorentz transformations. If k is a momentum such that $\bar{k}_0$ is positive when k_0 is positive, then

$$\begin{aligned} a(k)|\Omega_L\rangle = a(k)L|0\rangle &= \left(\left|\frac{\bar{k}_0}{k_0}\right|\right)^{1/2} La(\bar{k})|0\rangle, \\ &= 0 \end{aligned} \tag{5.6}$$

so that these momenta are not present in $|\Omega_L\rangle$. If however k is a momentum such that $\bar{k}_0$ is negative when k_0 is positive, then

$$\begin{aligned} a^\dagger(k)|\Omega_L\rangle = a^\dagger(k)L|0\rangle &= \left(\left|\frac{\bar{k}_0}{k_0}\right|\right)^{1/2} La(-\bar{k})|0\rangle \\ &= 0. \end{aligned} \tag{5.7}$$

Therefore $|\Omega_L\rangle$ contains one particle with each momentum whose energy changes sign under the inverse Lorentz transformation. As we have seen, these are the momenta k^* satisfying the condition

$$\omega^* + \mathbf{k}^*\cdot\mathbf{u} < 0, \tag{5.8}$$

where u is the velocity corresponding to the "boost."

It is of interest to compute the number of particles in the state $|\Omega_L\rangle$. As this is proportional to the quantization volume, we consider the number per unit volume $\rho|\Omega_L\rangle = (N/V)|\Omega_L\rangle$:

$$\frac{N}{V} = \frac{1}{V}\sum_k a_k{}^\dagger a_k \to \int d^3k\, a^\dagger(k)a(k), \tag{5.9}$$

$$\rho|\Omega_L\rangle = \int d^3k\, a^\dagger(k)a(k)L|0\rangle \tag{5.10}$$

$$= \left(\int_{k^*} d^3k\right)|\Omega_L\rangle, \tag{5.11}$$

where the integral is over the k^* satisfying (5.8). This integral is easily done, and gives $\frac{2}{3}\pi\mu^3(\gamma-1)$, which is clearly finite. Therefore, a state which contains no tachyons according to one observer will be seen by another observer to contain a large number of particles. This can occur since the four-momentum does not transform geometrically in this theory, but instead there are additional constant terms appearing in Eq. (4.10). As a result, the state $|\Omega_L\rangle$ does not have the same four momentum as the vacuum.

Similarly, we can calculate the transformation properties of states containing some number of particles.

The following rules describe what happens to a general state under Lorentz transformation. We suppose that in the initial coordinate system there are particles of momentum $k_1, \cdots k_n$, whose fourth components do not change sign under the given Lorentz transformation, and particles of momentum $k_{n+1}, \cdots, k_m$ whose fourth components do change sign under the Lorentz transformation. The Lorentz-transformed state then will contain all those particles satisfying (5.8) with the exception of those whose momenta are the negative of the Lorentz transforms of $k_{n+1}, \cdots, k_m$. In addition, it will contain the Lorentz transforms of $k_1, \cdots, k_n$.

For example, a state containing one particle of momenta k_a, whose energy sign changes under a given Lorentz transformation will, by the new observer, be seen as a state with all the particles whose momenta k' satisfy (5.8), omitting the momentum $(-k_a{}', -\omega_a{}')$ which is in that set.

This is clearly a very strange result, and upon first sight might be taken to imply that the particle theory was in fact not Lorentz invariant. I do not think that this is the case, however. Every observer will have a vacuum state and the particle states constructed from it with a common set of creation operators. A Lorentz transformation induces a transformation between these states which is unitary (in a finite space). The fact that this transformation links states with different numbers of particles is a new feature of tachyons, but does not imply noninvariance, since all the equations of the theory are invariant under the transformation.

A more subtle question involves the usual use of the vacuum as an invariant reference state which is taken as the natural state of the world. That is, the states that occur in theoretical physics are the vacuum state for most kinds of particles and contain only a small number of the other particles, which have been produced through known definite processes. This cannot be done invariantly for tachyons, and it can therefore be asked what would be a reasonable tachyon state to occur in some situation for a particular observer. I do not have an answer to suggest to this, but it is worthwhile to remark that even for particles such as electrons, the actual state of the world is not approximately the vacuum, but rather contains some 10^{80} particles, and is not at all Lorentz invariant. We do not regard this as any indication of a lack of Lorentz invariance in nature, but rather as a part of the boundary conditions for any physical situation. A similar approach may be useful in dealing with tachyons if they indeed exist.

We note that the change in the number of particles present from observer to observer is required in order to bring about the relation between emission and absorption that was described qualitatively in Sec. II. 3. This relation requires that a situation described by one observer as the emission at a point of a tachyon, which then escapes to infinity, may be described by another observer as the absorption of a tachyon which enters from infinity. Clearly, in order for this to work, it is necessary that the initial state, which contains no tachyon for the first observer, contain at least one tachyon for the second observer. In fact, from the rule for transforming states, we see that a process described by one observer as emission of a single tachyon, would by another observer, for whom the energy of the initial tachyon would have changed sign geometrically, be described as a transition from the state $|\Omega_L\rangle$ to a state with one less tachyon. In general, suppose we have a transition of the form:

$$|k_1, \cdots k_n; k_{n+1}, \cdots k_m\rangle \rightarrow |p_1, \cdots p_r; p_{r+1} \cdots p_s\rangle,$$

as viewed in one coordinate system, and the quantities $k_{1,4}, \cdots k_{n,4}$; $p_{1,4} \cdots p_{r,4}$ do not change sign under some Lorentz transformation, while $k_{n+1,4} \cdots k_{m,4}$ and $p_{r+1,4} \cdots p_{s,4}$ do change sign. Then in the transformed coordinate system, this will be seen as a transition between an initial state containing the momenta k_1', $\cdots k_n'$, and the momenta in $|\Omega_L\rangle$, missing $-k_{n+1}'$, $\cdots$, $-k_m$, to a final state containing the momenta in $|\Omega_L\rangle$, missing the momenta $-p_{r+1}'$, $\cdots -p_s'$, and including in addition the momenta p_1', $\cdots p_r'$.

Note that most of the large number of momenta present will not have changed in the transition, and indeed would occur as disconnected lines in a Feynman graph. For that reason, the high density of tachyons implied by (5.11) may not be in obvious disagreement with experiment.

These results for the relations between the transitions seen by Lorentz-transformed observers are valid under the assumption that it is possible to define a Lorentz-invariant S matrix in the theory of interacting tachyons. This would in turn be the case if the S matrix is a scalar function of the field operators $\phi(x)$, just as for ordinary field theories. However, in order to tell if this is so, it is necessary to produce a theory of interacting tachyons. This will not be done in this paper, although some comments on interactions are to be found in the next section.

VI. INTERACTIONS OF TACHYONS

In order to understand better the interactions possible for tachyons, we shall study qualitatively what production and scattering processes tachyons can take part in. We consider first restrictions coming from the conservation of energy and linear momentum. We shall define both initial and final states so that all tachyons appearing therein have positive energy. In accordance with our previous discussion this implies that a given tachyon may be transferred from initial to final state by a Lorentz transformation. Therefore, the decision as to whether a given process is allowed by four-momentum conservation must be considered separately in each coordinate system, and the rules we obtain will depend on the energies of the particles involved.

We must consider five possibilities, in which either the initial or final states contain the following particles.

Case A. State contains only normal particles, with timelike total momentum.

Case B. State contains normal particles and tachyons, with spacelike total momentum.

Case C. State contains normal particles and tachyons, with timelike or null total momentum.

Case D. State contains only tachyons, with spacelike total momentum.

Case E. State contains only tachyons, with timelike total momentum.

In considering selection rules, it is sufficient to consider that part of the total state which undergoes the reactions, and I shall take the cases A–E to refer to that. The detailed selection rules are given in Table I. It is clear that transitions in which the total momentum changes from spacelike to timelike, or vice versa are forbidden.

One consequence of the selection rules is that any system of normal particles, including a single particle at rest is energetically unstable against emission of tachyons. This occurs because a system of two tachyons can be found with any value of total mass whatsoever, by varying the energies and directions of the two tachyons. Because of this, strong restrictions must exist on the interactions of tachyons in order to be consistent with the observed behavior of the proton, electron, neutrino, and photon.

It also follows from Table I that tachyons can emit massless particles such as photons or neutrinos without changing their own mass, that is, the decay

$$T \rightarrow T + \gamma$$

is allowed, where T represents a tachyon with a fixed value of μ^2 and any energy. This is just a restatement of the long known fact that Čerenkov radiation can be emitted in free space by a charged particle that moves faster than light. It has been well known for many years that this was possible.[14] I shall call the emission of a particle or particles without a change of mass of the tachyon, elastic decay. The elastic decay of a tachyon of mass μ^2 with emission of a particle of mass m is energetically possible when the tachyon has an energy

[14] This seems to have been first recognized by A. Sommerfeld, K. Akad. Wet. Amsterdam Proc. 8, 346 (1904). See also G. A. Schott, *Electromagnetic Radiation* (Cambridge University Press, Cambridge, England, 1912).

TABLE I. Kinematic restrictions on tachyon transitions. In this table are given the minimum initial-state energies E for which a transition is allowed. Forbidden transitions are marked x. Transitions which can occur as elastic scattering are marked 0. I have taken all tachyons to have mass μ, and all normal particles to have mass m. The generalization to other cases is straightforward. The labels A, B, C, D, E are defined in paragraph 2 of Sec. VI. The initial and final states contain at most two particles.

Content of initial state \ Content of final state	A	B	C	D	E
A	0	x	$E\geq(m^2+\frac{1}{4}\mu^2)^{1/2}$	x	Possible for all values of E
B	x	0	x	$E>(m^2+m^4/4\mu^2)^{1/2}$	x
C	$\begin{cases} E>\frac{1}{2}m+\mu^2/m & \text{if} \quad m^2>2\mu^2 \\ E>(\mu^2+m^4/4\mu^2)^{1/2} & \text{if} \quad m^2>2\mu^2 \end{cases}$	x	0	x	Possible for all values of E
D	x	$E>m+\mu^2/2m$	x	0	x
E	$E>m$	x	Possible for all values of E	x	0

at least equal to

$$E_T^2=m^2+\frac{m^4}{4\mu^2}. \tag{6.1}$$

Hence, if the mass of tachyons is much smaller in absolute value than that of the normal massive particles, elastic decays involving emission of these particles will be possible only for very energetic tachyons.

Finally, we note that it is always energetically possible for a single tachyon to decay into several tachyons with the same value of μ^2. This implies that unless the tachyon self-interaction is very weak, there will be a rapid degradation of the energy spectrum of tachyons produced in any way, as the tachyons decay into numbers of less energetic tachyons.

It is of interest to apply these considerations to the particles present in the state $|\Omega_L\rangle$. That is, we assume that for some given system, one observer sees no tachyon present, and another, related by a Lorentz transformation L, sees the state $|\Omega_L\rangle$. The particles in this state have energies E_i and momenta $\mathbf{p}_i$ satisfying

$$E_i<\mathbf{p}_i\cdot\mathbf{u}, \tag{6.2}$$

where u is the velocity of the "boost." If we take any subset of these particles, their total energy and momentum will therefore also satisfy

$$E_{\text{tot}}=\sum E_i<\mathbf{u}\cdot\sum \mathbf{p}_i=\mathbf{u}\cdot\mathbf{p}_{\text{tot}}. \tag{6.3}$$

Hence the total momentum is also a spacelike vector, and the tachyons in $|\Omega_L\rangle$ cannot make a spontaneous transition into normal particles alone. This, of course, also follows from the fact that for the original observer there were no particles present at all, and so no such transitions could occur.

If other particles are present, either tachyons or normal, in addition to those in $|\Omega_L\rangle$, then transitions could in general occur. But these transitions must be the Lorentz transform of a transition which occurs, according to the first observer, without the presence of the particles in $|\Omega_L\rangle$. Thus the large number of particles in $|\Omega_L\rangle$ is rather irrelevant insofar as their physical effects are concerned, since these effects can be determined by examining those processes that can occur in the tachyon vacuum state.

It is interesting to examine whether a tachyon in $|\Omega_L\rangle$ could emit an ordinary particle or particles. According to the comment of the last section this is impossible, since in the Lorentz-transformed state with no tachyons originally present, this is forbidden by energy conservation. To see why this cannot occur in any coordinate sytem, we note that if a tachyon emits a particle, the four-momentum conservation implies that

$$p'=p-k \tag{6.4}$$

with k a timelike or null momentum and p, p' spacelike momenta, with p satisfying

$$\omega_p+\mathbf{p}\cdot\mathbf{u}\leq 0, \tag{6.5}$$

where $\mathbf{u}$ is the velocity associated with the Lorentz transformation L. Now

$$\omega_{p'}+\mathbf{p}'\cdot\mathbf{u}=\omega_p+\mathbf{p}\cdot\mathbf{u}-(\omega_k+\mathbf{k}\cdot\mathbf{u}). \tag{6.6}$$

However

$$\omega_k+\mathbf{k}\cdot\mathbf{u}=\omega_k+ku\cos\theta>0, \tag{6.7}$$

since $\omega_k\geq k$, and $u<1$. Therefore

$$\omega_{p'}+\mathbf{p}'\cdot\mathbf{u}<0,$$

and therefore p' is also in $|\Omega_L\rangle$. However, by the exclusion principle, this is impossible unless $\mathbf{p}'=\mathbf{p}$, in which case $k=\omega_k=0$ which is no transition. Therefore, the quantization by Fermi statistics is required here to ensure that a transition forbidden in one coordinate system is not the Lorentz transform of an allowed process. This suggests, but hardly proves, that fermion quantization is necessary for any particle of spacelike four-momentum.

In addition to the energy and momentum restrictions on tachyon processes, there are restrictions following from the conservation of statistics. I shall simply assume here that if we assign a number $+1$ to bosons and -1 to fermions and multiply these numbers for a multiparticle system that these products are conserved in any transition.[15] Since the tachyons are fermions of zero spin, while all other fermions have half integral spin, it immediately follows that the number of tachyons is conserved modulo 2, that is, the following selection rule holds:

$$N_T(\text{initial})-N_T(\text{final})=\text{even integer}\,, \qquad (6.8)$$

where N_T means the number of tachyons present in the state.

As a result of this selection rule, it is impossible to produce a single tachyon in a collision between ordinary particles.

In order to describe quantitatively the interactions of tachyons, either among themselves or with ordinary particles, it is necessary to introduce interactions into the equations satisfied by the tachyon fields. This is often done via a Lagrangian formalism, which does not seem to be available here. It is of course possible to introduce the interactions directly into the field equations, for example by introducing a source term into Eq. (3.1)

$$(\Box^2+m^2)\phi=J_\phi\,, \qquad (6.9)$$

where J_ϕ depends on the tachyon field and other particle fields. Because of the "conservation of statistics," the source J_ϕ should contain an odd number of factors of ϕ and its derivatives.

If the field equation (6.9) is still second order in time derivatives of ϕ, we could still assume that the equal-time anticommutation relations (4.14)–(4.17) are satisfied. There are then two problems to be faced. One is the demonstration that the resulting *field* theory is still Lorentz invariant, a matter for which no really complete proof exists even in conventional field theories. The problem is accentuated in the present case by the breakdown of the usual "causality" condition on the commutator of the fields at spacelike separations. Under these circumstances, I have not been able to settle the question of whether the theory defined by (4.14)–(4.17) and (6.9) is a Lorentz-invariant field theory.

The other problem is to extract from the field equations an expression for the S matrix describing tachyon transitions. Here also the differences between the free-tachyon theory and the conventional theory are such as to make the standard formalisms, such as the Dyson interaction picture, or the LSZ (Lehmann-Symanzik-Zimmermann) formulation, difficult to apply. The Yang-Feldman method, based on the Heisenberg equations of motion, seems a more promising approach to this problem. However, no conclusive results are available yet here either.

Note that in order to obtain a Lorentz invariant S matrix, it is as far as we know sufficient to have a Lorentz-invariant field theory. Whether this is also necessary is not known, even in conventional theories. If the S matrix as an operator is Lorentz invariant, then because of the peculiar transformation property of the particle states described in Sec. V, the S-matrix elements for transitions between different numbers of particles will be related by Lorentz transformations. It does not seem possible to avoid this property in any Lorentz-invariant theory of faster-than-light particles.

[15] See for example, Greenberg and Messiah (Ref. 10) where this is proven, however, under assumptions that may be invalid for tachyon theories.

VII. CONCLUSION

We have investigated the possibility of describing particles that travel faster than light within the special theory of relativity. We find that the objections generally raised to the existence of such particles are not valid in a relativistic quantum theory. A description of such particles, called tachyons, by the formalism of relativistic quantum field theory is possible, at least for the case of spinless, noninteracting particles. The field theory constructed is explicitly Lorentz invariant. The particles described by this formalism have several peculiar properties. Among these are the following:

1. The spinless particle cannot be quantized by Bose statistics but can be quantized by Fermi statistics.
2. The vacuum state is not invariant under Lorentz transformations but rather changes into a state containing many tachyons.

These properties, although quite different from those of ordinary particles, would not appear to involve any fundamental contradictions with accepted physical principles.

In my opinion, there are several major problems remaining to be solved before the theory is complete. These include the following:

1. What is the propagation in space-time of the particles described here like? In particular, how does one define propagation for the unlocalizable states?
2. How can one extend the theory described here to the case of interacting particles?

As these two problems are solved, we can look forward to the solution of the more fundamental question, i.e.,

3. Do faster-than-light particles exist in nature, and can they be detected?

It is the hope of the author that he has convinced the reader that an affirmative answer to this question would not necessarily be in contradiction to Einstein's theory of relativity.

ACKNOWLEDGMENTS

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APPENDIX A. EMISSION AND ABSORPTION OF TACHYONS AS VIEWED IN DIFFERENT LORENTZ FRAMES

In this Appendix I shall illustrate the interpretation of the change of sign of energy under Lorentz transformation as an exchange of the role of emission and absorption. The main point to be made is that this interpretation is consistent with at least one form of the principle of relativity, which requires that any event that can occur for one observer be a possible event for any other, Lorentz-transformed, observer. The most striking new kinematical feature of this interpretation is that the stability of a system against emission of tachyons is a function of its velocity relative to the observer.

As our first illustration, imagine a situation in which an observer sees two atoms both originally at rest at points x_1, x_2. Atom 1 is assumed to be in its ground state at time t_0, while atom 2 is in an excited state at t_0, with an energy δE above its ground state. We suppose that at a time $t_1 > t_0$ for this observer, atom 2 emits a tachyon in the direction of atom 1, dropping to its ground state, and recoiling. I disregard here the fact that tachyons cannot be emitted or absorbed singly, which is irrelevant for the purpose of this discussion. This tachyon is absorbed by atom 1 at a later time t_2 (i.e. $t_2 > t_1$), and atom 1 makes a transition to one of its excited states, and moves in the direction of the tachyon.

Since the spacetime points (x_1,t_1) and (x_2,t_2) are connected by the trajectory of a tachyon, they are separated by a spacelike interval. It is therefore possible to make a Lorentz transformation to an observer for which $t_2' < t_1'$. We are then entitled to ask how this observer will describe the above process. For this second observer, atom 1 will be moving along the line joining the atoms at time t_0' and will be in its ground state at that time. Atom 2 will also be moving in this direction and will be in an excited state.

As described in Sec. II. 3, the change in energy ΔE_1 of atom 1 during the interaction with the tachyon will have opposite sign for the two observers, as will the change in energy ΔE_2 of atom 2, which for each observer is equal to the negative of the energy change of atom 1. Therefore, the second observer will interpret the event at t_1' as absorbtion by atom 2 of a particle of positive energy which was emitted by atom 1 at t_2'. However, since atom 1 was in its ground state before this occurred, it is necessary to conclude that a moving atom in its ground state can emit particles, simultaneously changing part of its kinetic energy of motion into the internal energy difference δE. This is of course kinematically impossible when the emitted particle has timelike four-momentum, since it cannot happen for an atom of rest. However, it is indeed kinematically possible when the emitted particle has spacelike four-momentum, because in this case the process will be seen as absorption in the frame in which the atom is at rest. The different interpretations of the event by the two observers are indicated schematically in Fig. 1.

Consider next a free tachyon moving through space. From the results of Table I, we see that it is energetically possible for this to emit photons, no matter how small the energy of the tachyon. Suppose that such emission occurs in one coordinate system, the energy of the final tachyon being positive in that system. There are other coordinate systems in which the energy of the final tachyon, as computed by Lorentz transformation, is negative, while that of the initial tachyon is still positive. According to our interpretation, the process will be viewed in this frame as the annihilation of a particle and antiparticle into a single photon, which is energetically possible for these objects.

If we look at this in more detail, we can describe the

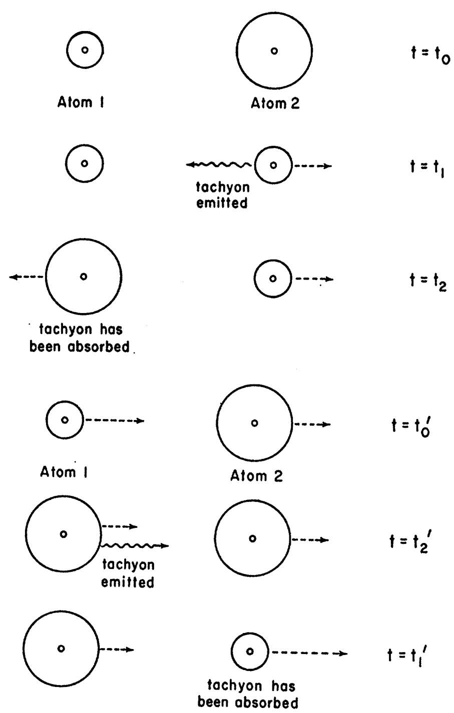

FIG. 1. (a) Emission and absorption of a tachyon as viewed by one observer. Atom 1 is at rest in its ground state at t_0. Atom 2 is at rest in excited state. At t_1 atom 2 emits a tachyon, dropping to its ground state and recoiling. At t_2, this tachyon is absorbed by atom 1, which jumps to an excited state and also recoils. In this situation we have $t_0 < t_1 < t_2$; (b) the same process viewed by another observer, for whom emission and absorption have been exchanged. Atom 1 is now moving at $t = t_0'$, but still in its ground state. This atom emits a tachyon at t_2' and jumps to an excited state, losing some of its translational energy. Atom 2, which for this observer is moving, and in an excited state at t_0', absorbs the tachyon at t_1', dropping to the ground state and gaining translational energy. For this observer, we have $t_0' < t_2' < t_1'$.

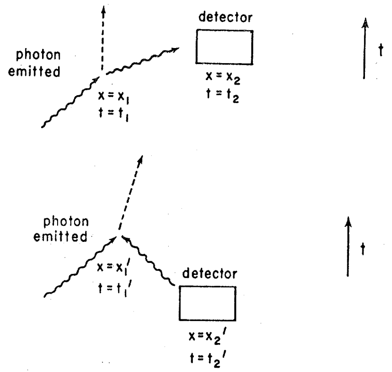

FIG. 2. (a) Spacetime diagram of Čerenkov radiation by a tachyon. A photon is emitted at time t_1 by a tachyon, whose energy remains positive to this observer. The tachyon is detected at a later time t_2. There is always 1 tachyon present for this observer, as well as a photon after t_1. Time is taken to run upward in the diagram; (b) spacetime diagram of the same process as viewed by an observer for whom the energy of the final tachyon would be negative if geometrically Lorentz transformed. According to this observer, a tachyon and antitachyon are present until $t=t_1'$, after which only a photon is present. Time is taken to run upward in the diagram.

situation as follows. Suppose that for observer 1, the emission of the photon occurs at the spacetime point (x_1,t_1). We must then imagine that this observer has a detector, which registers the existence of the recoil tachyon at another point (x_2,t_2), with $t_2>t_1$, and $|x_2-x_1|>c(t_2-t_1)$. For simplicity, I assume that the detection of the initial tachyon will be similar for the two observers, and also assume that the recoil tachyon escapes to infinity after passing through the detector.

In the transformed system, the photon is emitted at a point (x_1',t_1'), and an antitachyon is detected at the point (x_1',t_2'), with $t_2'<t_1'$. Therefore according to this observer, there are two particles present up until t_1', and none thereafter. The "extra" particle is one of those appearing under the Lorentz transform of the vacuum, described in Sec. V. If the situation occurs as I have described it, the relativistic invariance of the theory then implies a relationship between the rates of two processes that we are not used to thinking of as the same, i.e., Čerenkov radiation in one case, and pair annihilation in the other. However, if we extend the usual notion that two processes that are related by Lorentz transformation are really the same, then we may say that for tachyons, Čerenkov radiation and pair annihilation are the same process. The only really novel aspect of the situation is the fact that Lorentz transformations can, in this case, alter the number of particles that are present in a state at a given time. I have again indicated the different interpretations of the event by two observers in Fig. 2.

The above examples are meant to illustrate the assertion that the change in the sign of energy- and time-ordering under some Lorentz transformations can be systematically reinterpreted as an exchange between emission and absorption. A further logical problem arises when emission and absorption by sources in relative motion are considered. An example of this situation will be described in Appendix B.

APPENDIX B. ANALYSIS OF CAUSAL ANOMALIES PRODUCED WITH TACHYONS

In order to examine some of the causal anomalies that may arise if tachyons are exchanged between observers in relative motion, we consider a situation with two such observers. We take observer 1 with unprimed coordinates at rest at the origin, and observer 2 with primed coordinates at the point $(x_0,0,0)$ at time $t=0$, and moving with velocity u in the x direction. I suppose that each observer can emit and absorb tachyons and disregard the selection rule from the statistics.

The Lorentz transformation relating the two observers is (suppressing y and z) given by

$$x'=\gamma(x-x_0-ut)\,,\quad x=x_0+\gamma(x'+ut)\,,$$
$$t'=\gamma(t-ux+ux_0)\,,\quad t=\gamma(t'+ux')\,, \tag{B1}$$
$$(c=1 \text{ here})\,.$$

Let observer 1 emit a tachyon with velocity v_1 towards observer 2 at time $t=0$. This will be absorbed by 2 at $t=x_0/(v_1-u)$. These events will occur at the times $t'=\gamma ux_0$ and $t'=x_0/\gamma(v_1-u)$ respectively for observer 2, for whom the tachyon will have traveled with velocity $(v_1-u)/(1-uv_1)$. Therefore, if we take $uv_1<1$, the tachyon will have traveled forward in time and have positive energy for each observer.

Now suppose that by agreement, immediately upon absorbing the first tachyon, observer 2 emits a tachyon towards 1, travelling in his system with velocity $-v_2$. This will reach 1 at the time

$$t'=\frac{x_0}{\gamma(v_1-u)}+\frac{x_0v_1}{\gamma(v_1-u)(v_2-u)}\,,$$

and when 1 is at the point

$$x'=\frac{-x_0v_1v_2}{\gamma(v_1-u)(v_2-u)}\,.$$

Hence it will be absorbed at the time

$$t_F=\frac{x_0(v_1+v_2-u-uv_1v_2)}{(v_1-u)(v_2-u)}\,,$$

having traveled with velocity $-(v_2-u)/(1-uv_2)$ for observer 1. By rewriting t_F as

$$t_F=\frac{x_0}{(v_1-u)(v_2-u)}[v_1-u+v_2(1-uv_1)]\,, \tag{B2}$$

we see that a condition that t_F be >0 is just that $1-uv_1>0$, which we have assumed to be satisfied. In other words, there can be no causal anomaly if the outgoing tachyon propagates normally for both observers, i.e., if the things we have called emission and absorption really are such. Since t_F is symmetric in v_1 and v_2, it also follows that a causal anomaly ($t_F<0$) cannot occur if the final tachyon propagates normally for both observers.

Hence to obtain such an anomaly we consider a case where the observers disagree on the order of emission and absorption for each of the two tachyons. For simplicity, imagine that each tachyon is emitted, in the rest system of the emitter, with infinite velocity, i.e., $v_1=v_2=\infty$ in the previous formulas. It follows from (B2) that

$$t_F=-ux_0. \tag{B3}$$

That is, the tachyon (2) emitted by observer 2 will reach observer 1 before the first one (1) is emitted. One could then imagine that observer 1 after absorbing this tachyon would be free to refuse to generate the tachyon (1) that stimulated observer 2 to emit (2), thus producing a causal anomaly.

In order to see how this anomaly appears to each observer, we must use the reinterpretation of the absorption of a negative energy tachyon as an emission. In the previous discussion, we have seen that tachyon (1) will appear to have negative energy to observer (2). Therefore, he will not regard it as having been emitted by observer 1 to him, but rather emitted by himself toward observer 1. Similarly, tachyon (2) will appear to have negative energy to observer 1, and he therefore will regard his apparatus as having emitted it, rather than as receiving it as a signal from observer 2. It follows that if the two observers have entered into an agreement depending on the receipt of a signal emitted by the other, that the sequence of events we have described will not be regarded as having triggered the agreement.

Let us look at this situation more closely by seeing how each observer does the detection of the signal from the other. Suppose the detector is taken to be an atom at rest in its ground state, and we assume that upon absorption of a tachyon, the atom will start moving, and perhaps make a transition to an excited state. By choosing the atom at rest in its ground state we rule out the possibility that it can emit a tachyon, as seen by the observer that is using it for a detector. It is easy to see that under these circumstances, the detector cannot absorb a tachyon emitted with velocity v by the other observer unless the conditions

$$1-uv>0 \tag{B4}$$

is satisfied. That is, this type of detector cannot absorb a tachyon whose energy will change sign under the Lorentz transformation relating the two observers, and hence cannot be used to induce a causal anomaly.

Suppose, however, that another type of detector is used, which can absorb such tachyons. This could for example be an atom in an excited state, in which case the condition (B4) need not be satisfied for absorption. However, in this case, there is the possibility of spontaneous emission of tachyons, and an observer using this kind of detector cannot, simply by determining whether the detector has made a transition, decide whether such spontaneous emission has occurred, or whether a tachyon from outside has been absorbed. Indeed, in the case described above, observer 1 would naturally describe the second stage as such a spontaneous emission, independent of any activity by the second observer.

Therefore, while it does appear possible to construct kinematic closed cycles using tachyons in which signals are sent back to the past, a careful examine of the methods of detection, with due regard to the interpretation of absorption of negative-energy tachyons as emission of positive-energy tachyons, leads to the conclusion that such closed cycles will not be interpreted as reciprocal signaling, but rather as uncorrelated spontaneous emission. It therefore does not appear that causal anomalies can be used as an argument against the existence of tachyons.

APPENDIX C. PROPERTIES OF TRUNCATED δ FUNCTIONS

We prove here a simple lemma concerning what may be called truncated δ functions. We define a truncated δ function $\delta_S(x)$ relative to a set S of momenta $\mathbf{k}$ by the relation

$$\delta_S(\mathbf{x})=\frac{1}{(2\pi)^3}\sum_{k\subset S} e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{C1}$$

If S contains all possible momenta, then $\delta_S(\mathbf{x})=\delta^3(\mathbf{x})$, the usual delta function. When S is the set of momenta satisfying $k^2\geq\mu^2$, then $\delta_S(\mathbf{x})=\bar{\delta}(\mathbf{x})$. The point of the lemma is then just that $\delta_S(\mathbf{x})$ acts like $\delta^3(\mathbf{x})$ with respect to those functions whose Fourier transforms vanish outside the set S. That is, if

$$f(\mathbf{x})=\sum_{k\subset S} f(k)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{C2}$$

then

$$\int f(\mathbf{x}')\delta_S(\mathbf{x}-\mathbf{x}')d^3x'=f(\mathbf{x}). \tag{C3}$$

To see this, we substitute the Fourier decomposition of $f(\mathbf{x})$ and $\delta_S(\mathbf{x}-\mathbf{x}')$ into (C3)

$$\int d^3x' f(\mathbf{x}')\delta_S(\mathbf{x}-\mathbf{x}')$$

$$=\int d^3x' \sum_k \sum_{k'} \frac{1}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}'} e^{ik\cdot(\mathbf{x}-\mathbf{x}')} f(k) \tag{C4}$$

$$=\sum_{k\subset S}\sum_{k'\subset S} e^{+i\mathbf{k}'\cdot\mathbf{x}} f(k)\delta^3(k-k') \tag{C5}$$

$$=\sum_{k\subset S} f(k)e^{i\mathbf{k}\cdot\mathbf{x}}=f(\mathbf{x}), \tag{C6}$$

since the point $\mathbf{k}'=\mathbf{k}$ occurs somewhere in the set S over which $\mathbf{k}'$ ranges.

Similar generalizations can be made to derivatives of the function $\bar{\delta}_S(\mathbf{x})$.

In order to apply the lemma, it is useful to note the following property. If $f(\mathbf{x})$ satisfies (C2), then so does any finite derivative of $f(\mathbf{x})$. Also, the function $g(\mathbf{x})$ defined by

$$g(\mathbf{x})=(-\nabla^2-\mu^2 f)^{1/2}(\mathbf{x}) \tag{C7}$$

satisfies (C2), since we can write

$$g(\mathbf{k})=(k^2-\mu^2)^{1/2}f(\mathbf{k}). \tag{C8}$$

Using these results we can easily demonstrate Eq. (4.13) for the Hamiltonian.

$$H=\int_{k\geq\mu} d^3k\,\omega(k)a^\dagger(k)a(k) \tag{C9}$$

$$a(k)=\left(\frac{\omega}{2(2\pi)^3}\right)^{1/2}\int e^{-i\mathbf{k}\cdot\mathbf{x}}\left[\phi(x)+\frac{i\dot{\phi}(x)}{\omega}\right]d^3x \tag{C10}$$

$$H=\frac{1}{2(2\pi)^3}\int d^3k\omega^2(k)\int d^3xd^3x'e^{i(\mathbf{k}\cdot\mathbf{x}'-\mathbf{k}\cdot\mathbf{x})}$$
$$\times\left[\phi(x)-\frac{i\dot{\phi}(x)}{\omega}\right]\left[\phi(x')+\frac{i\dot{\phi}(x')}{\omega}\right] \tag{C11}$$

$$=\frac{1}{2}\int d^3xd^3x'[\phi(x)(-\nabla^2-\mu^2)\phi(x')\bar{\delta}(\mathbf{x}-\mathbf{x}')$$
$$+\dot{\phi}(x)\dot{\phi}(x')\bar{\delta}(\mathbf{x}-\mathbf{x}')+i\phi(x)(-\nabla^2-\mu^2)^{1/2}\dot{\phi}(x')$$
$$\times\bar{\delta}(\mathbf{x}-\mathbf{x}')-i(-\nabla^2-\mu^2)^{1/2}\dot{\phi}(x)\phi(x')\bar{\delta}(\mathbf{x}-\mathbf{x}')] \tag{C12}$$

$$=\frac{1}{2}\int d^3x\{\phi(x)(-\nabla^2-\mu^2)\phi(x)+\dot{\phi}(x)\dot{\phi}(x)$$
$$+i[(-\nabla^2-\mu^2)^{1/2}\phi(x),\,\dot{\phi}(x)]\}\,,$$

which is (4.13).

APPENDIX D. COMPLEX TACHYON FIELDS

In this section I shall describe noninteracting complex tachyon fields by a method analogous to the description of real fields in Sec. IV. We begin with a non-Hermitian scalar field $\phi(x)$, and its adjoint $\phi^\dagger(x)$, satisfying

$$(\Box^2+\mu^2)\phi(x)=0. \tag{D1}$$

Again we expand these in plane waves:

$$\phi(x)=\int[\phi_{+,k}a(k)+\phi_{-,k}b^\dagger(k)]d^3k\frac{1}{(2\omega_k)^{1/2}}, \tag{D2}$$

$$\phi^\dagger(x)=\int[\phi_{-,k}a^\dagger(k)+\phi_{+,k}b(k)]d^3k\frac{1}{(2\omega_k)^{1/2}}. \tag{D3}$$

Here $a^\dagger$, $b^\dagger$ will be the independent creation operators for particles and antiparticles, which we will require to satisfy

$$\{a(k),a^\dagger(k')\}=\{b(k),b^\dagger(k')\}=\delta(k-k')\,, \tag{D4}$$

$$\{a(k),a(k')\}=\{a(k),b(k')\}=\{a(k),b^\dagger(k')\}$$
$$=\{b(k),b(k')\}=0. \tag{D5}$$

The logic of doing this is again dictated by the requirement that ϕ transform as a Lorentz scalar, and the circumstances that there are Lorentz transformations which can mix the positive and negative frequencies in ϕ.

We will again take the Hamiltonian to be a sum of positive-energy particle and antiparticle terms

$$H=\int d^3k\,\omega(k)a^\dagger(k)a(k)+\int\omega(k)b^\dagger(k)b(k)d^3k. \tag{D6}$$

We also define the total number and charge operators

$$N=\int(a^\dagger(k)a(k)+b^\dagger(k)b(k))d^3k\,, \tag{D7}$$

$$Q=\int(a^\dagger(k)a(k)-b^\dagger(k)b(k))d^3k. \tag{D8}$$

We can express these in terms of the fields ϕ, $\phi^\dagger$ by using the results of Appendix C. We find

$$H=\tfrac{1}{2}i\int d^3x\left[(-\nabla^2-\mu^2)^{1/2}\phi^\dagger,\frac{\partial\phi}{\partial t}\right]-\tfrac{1}{2}i\int d^3x$$
$$\times\left[(-\nabla^2-\mu^2)^{1/2}\phi,\frac{\partial\phi^\dagger}{\partial t}\right]+c\text{ number}, \tag{D9}$$

$$N=-i\int d^3x\left(\frac{\partial\phi^\dagger}{\partial t}\phi-\phi^\dagger\frac{\partial\phi}{\partial t}\right)+c\text{ number}, \tag{D10}$$

$$Q=\int d^3x\frac{\partial\phi^\dagger}{\partial t}\frac{1}{(-\nabla^2-\mu^2)^{1/2}}\frac{\partial\phi}{\partial t}+\int\phi^\dagger(-\nabla^2-\mu^2)^{1/2}\phi d^3x$$
$$+c\text{ number}. \tag{D11}$$

Note again that the Hamiltonian density is nonlocal, as is the charge density $\rho(x)$ defined by $Q=\int\rho(x)dx$. However, the number density is again a local operator, given by the fourth component of the conserved current

$$2J_\mu=[\phi^\dagger,\partial_\mu\phi]-[\partial_\mu\phi^\dagger,\phi]. \tag{D12}$$

The Lorentz transformations are represented by the following unitary transformations on the a, b.

$$
\begin{aligned}
La(k)L^{-1}&=\left(\frac{k_0'}{k_0}\right)^{1/2} a(k') &&\text{if } \frac{k_0'}{k_0}>0\,,\\
La(k)L^{-1}&=\left(\left|\frac{k_0'}{k_0}\right|\right)^{1/2} b^\dagger(-k') &&\text{if } \frac{k_0'}{k_0}<0\,,\\
Lb(k)L^{-1}&=\left(\frac{k_0'}{k_0}\right)^{1/2} b(k') &&\text{if } \frac{k_0}{k_0}>0\,,\\
Lb(k)L^{-1}&=\left(\left|\frac{k_0'}{k_0}\right|\right)^{1/2} a^\dagger(-k') &&\text{if } \frac{k_0'}{k_0}<0\,.
\end{aligned}
\tag{D13}
$$

We note that while the total number of particles is not a Lorentz-invariant quantity, the total charge is. Evidently, this occurs because an equal number of particles and antiparticles appear from the vacuum after a Lorentz transformation, and the charge of these adds to zero. In particular, a state containing a single positively charged particle of momentum p in one Lorentz frame, appears in another frame in which the sign of the energy of this particle would geometrically reverse, to contain all the positively charged particles in the set $|\Omega_L\rangle$, and all the antiparticles in $|\Omega_L\rangle$ except for the momentum $-\bar{p}$, $-\bar{p}_0$.

It is possible to define a "charge-conjugation" operation on the fields ϕ, by

$$C\phi(x)C^{-1}=\phi^\dagger(x) \quad\text{and}\quad CC^\dagger=1\,, \tag{D14}$$

or

$$
\begin{aligned}
Ca(k)C^{-1}&=b(k)\,,\\
Cb(k)C^{-1}&=a(k)\,.
\end{aligned}
\tag{D15}
$$

The transformation commutes with H and N, while it anticommutes with Q. It is interesting that the number current J_μ, which has a similar functional form to the electric current of a normal particle, commutes rather than anticommutes with C, because of the fermion commutation relations of the fields. This would indicate that if we wish to preserve C invariance in the electromagnetic interaction of tachyons, it is necessary to use a nonlocal current, rather than the local current J_μ as a source for the electromagnetic field.

The electromagnetic current $\mathcal{J}_\mu(x)$ is therefore given by an expression whose fourth component is the charge $\rho(x)$. This expression can be determined by requiring that a conservation law

$$\partial_\mu \mathcal{J}_\mu=0 \tag{D16}$$

be satisfied. We find that

$$
\begin{aligned}
\mathcal{J}_i(x)&=-\phi^\dagger\frac{1}{(-\nabla^2-\mu^2)^{1/2}}\nabla_i\frac{\partial\phi}{\partial t}+\nabla_i\phi^\dagger\frac{1}{(-\nabla^2-\mu^2)^{1/2}}\frac{\partial\phi}{\partial t}+\text{H.c.}\,,\\
\mathcal{J}_0(x)&=\frac{\partial\phi^\dagger}{\partial t}\frac{1}{(-\nabla^2-\mu^2)^{1/2}}\frac{\partial\phi}{\partial t}+\phi^\dagger(-\nabla^2-\mu^2)^{1/2}\phi\,.
\end{aligned}
\tag{D17}
$$

This current is clearly nonlocal in coordinate space. It can be shown, however, that the current $\mathcal{J}_\mu(x)$ transforms as a four-vector under the transformations defined by Eq. (D13).

Since the complex field carries two degrees of freedom, it is possible to write local bilinear scalar functions of the field. One might therefore imagine it possible to write a Lagrangian which generates the field equation (D1) through a variational principle. This is indeed possible in contradiction to the case of the real field. We can write

$$\mathcal{L}(x)=-\partial_\mu\phi^\dagger\partial_\mu\phi+\mu^2\phi^\dagger\phi\,, \tag{D18}$$

which does not reduce to a c number. The variation of this Lagrangian considered as a classical operator clearly generates (D1).

If one now attempts to go over to a quantum-field theory by using a canonical procedure to obtain the Hamiltonian from the Lagrangian (D18), one finds that this canonical Hamiltonian will act as the generator of time displacements in the field $\phi(x)$ only if the canonical commutation relations are used to quantize the field. This possibility is ruled out by the requirement that ϕ transform as a Lorentz scalar, and therefore the Lagrangian formalism does not seem to be of much value in formulating the theory of the complex tachyon field either. However, the existence of q-number local scalar functions of the field would make the problem of writing interactions easier in this case.