dependence is unchanged. The transformation (ii) mixes the x' and y' motions but leaves the Hamilton-Jacobi equation separable, and thus by the argument given previously establishes the degeneracy of ν_x''' and ν_y''', in the form $\nu_y'''=n\nu_x'''$ with integer n. It can easily be shown as in the paragraph following (9) that n must be unity. The Fourier series of O $(x''', p_x''', y''', p_y''')$ has the single fundamental frequency $\nu_x'''=\nu_x$ for the orbit considered. The transformation (iii) leaves x''' unchanged, i.e., $x''=x'''$, $p_x''=p_x'''$, but makes a multivalued transformation of y''' to y''. This means that the Fourier series for O expressed in the variables x'', p_x'', y'', p_y'' involves two distinct fundamental frequencies, ν_x from the x'', p_x'' dependence, and a multiple of this from the y'', p_y'' dependence. Only when the transformation $y'''\rightarrow y''$ is finitely multivalued, which means that a finite number m of cycles of y''' corresponds to a number n of cycles of y'', can these two frequencies be combined to yield a single fundamental frequency for the Fourier series for O. Thus, when $\alpha/\beta=m/n$ with integer m, n, the existence of the symmetry $SU(2)$ establishes the existence of a degeneracy in the two-dimensional harmonic oscillator, because this renders the transformation (31) finitely multivalued.

The method of argument can be applied to any system separable in some one-coordinate system. Step (i) transforms the system into a system having the symmetry of interest as a single-valued transformation group, in step (ii) a symmetry operation of the group is carried out; in (iii) the transformation inverse to (i) is made. Only when this transformation is finitely multivalued does the degeneracy of the symmetric system survive the inverse transformation. Our conclusion is, then, that the existence of a semisimple group G whose generators X_i have zero Poisson bracket with the Hamiltonian of a separable dynamical system implies the existence of a degeneracy of that system only when the group G can be realized in the phase space of the system by finitely multivalued transformations.

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Electromagnetic Interaction of the Bargmann-Wigner Field with Spin $\frac{1}{2}$*

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It is shown that the Bargmann-Wigner multispinor field for particles of spin $\frac{1}{2}$ gives the usual result for the magnetic moment, provided it is ensured that the auxiliary field appearing in the Lagrangian formulation remains vanishing in the presence of electromagnetic interaction.

ALTHOUGH the Bargmann-Wigner multispinor field equations for elementary particles are found to be particularly suitable for the treatment of the symmetries of strong interactions,[1] the electromagnetic interaction of such fields has not yet been fully explored. Indeed, recently it has been claimed[2] that the multispinor formulation leads to particles of spin $\frac{1}{2}$ without any intrinsic magnetic moment, which casts doubt on the validity of such a formalism for the description of charged particles. We shall, however, show that it is possible to establish complete equivalence between the multispinor field with spin $\frac{1}{2}$ and the usual Dirac field by following a more judicious treatment of the electromagnetic interaction.

The Lagrangian density for a Bargmann-Wigner field with spin $\frac{1}{2}$ can be expressed in terms of two third-rank multispinors $\psi_{\alpha\beta\gamma}$ and $\Omega_{\alpha\beta\gamma}$ as[3,4]

$$L=-\bar{\psi}[(\gamma\partial)_1+m]\psi-\bar{\Omega}[(\gamma\partial)_3-\tfrac{1}{2}m]\Omega+\tfrac{1}{2}[\bar{\psi}(\gamma\partial)_3\Omega+\bar{\Omega}(\gamma\partial)_3\psi], \quad (1)$$

with

$$\bar{\psi}^{\alpha\beta\gamma}=\psi^{*\alpha'\beta'\gamma'}(\gamma_4)_{\alpha'}{}^{\alpha}(\gamma_4)_{\beta'}{}^{\beta}(\gamma_4)_{\gamma'}{}^{\gamma},$$
$$[(\gamma\partial)_1\psi]_{\alpha\beta\gamma}=(\gamma\partial)_{\alpha}{}^{\alpha'}\psi_{\alpha'\beta\gamma}, \text{ etc.,} \quad (2)$$

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[4] We denote the space-time coordinates as $x_\mu=(x_i,ix_0)$, and take the γ_μ as Hermitian matrices with $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$.

where the field $\psi_{\alpha\beta\gamma}$ of mixed symmetry satisfies the relations

$$\psi_{\alpha\beta\gamma}=-\psi_{\beta\alpha\gamma},\quad \psi_{\alpha\beta\gamma}+\psi_{\beta\gamma\alpha}+\psi_{\gamma\alpha\beta}=0, \tag{3}$$

while the auxiliary field $\Omega_{\alpha\beta\gamma}$ is totally antisymmetrical. Upon variation it can be shown that $\Omega_{\alpha\beta\gamma}$ vanishes, and $\psi_{\alpha\beta\gamma}$ satisfies the field equations

$$(\gamma\partial)_1\psi=(\gamma\partial)_2\psi=(\gamma\partial)_3\psi=-m\psi. \tag{4}$$

The Lagrangian density (1) can be cast in a more familiar form by utilizing the transformation[5]

$$\begin{aligned}\psi_{\alpha\beta\gamma}=2^{-3/2}[&(C)_{\alpha\beta}(\eta)_\gamma\\&+i(\gamma_\mu\gamma_5C)_{\alpha\beta}(\psi_\mu)_\gamma+(\gamma_5C)_{\alpha\beta}(\psi)_\gamma],\\ \Omega_{\alpha\beta\gamma}=2^{-3/2}[&(C)_{\alpha\beta}(\omega)_\gamma\\&+i(\gamma_\mu\gamma_5C)_{\alpha\beta}(\Omega_\mu)_\gamma+(\gamma_5C)_{\alpha\beta}(\Omega)_\gamma],\end{aligned} \tag{5}$$

where the symmetry properties of $\psi_{\alpha\beta\gamma}$ and $\Omega_{\alpha\beta\gamma}$ imply

$$\begin{aligned}&\eta=\gamma_5(\psi-i\gamma_\nu\psi_\nu),\\&\omega=-\gamma_5\Omega,\quad \Omega_\mu=i\gamma_\mu\Omega,\end{aligned} \tag{6}$$

and thus (1) is transformed to

$$\begin{aligned}L=-\tfrac{1}{2}i(\bar\psi_\mu\partial_\mu\psi+\bar\psi\partial_\mu\psi_\mu)-\tfrac{1}{2}m(\bar\psi-i\bar\psi_\mu\gamma_\mu)(\psi-i\gamma_\nu\psi_\nu)\\+\tfrac{1}{2}m\bar\psi_\mu\psi_\mu-\tfrac{1}{2}m\bar\psi\psi+\bar\Omega(\gamma\partial+\tfrac{3}{2}m)\Omega\\+\tfrac{1}{2}(\bar\psi\gamma\partial\Omega+\bar\Omega\gamma\partial\psi)-\tfrac{1}{2}i(\bar\Omega\partial_\mu\psi_\mu+\bar\psi_\mu\partial_\mu\Omega).\end{aligned} \tag{7}$$

The field equations obtained from (7) are

$$\begin{aligned}-i\partial_\mu\psi+im\gamma_\mu(\psi-i\gamma_\nu\psi_\nu)+m\psi_\mu-i\partial_\mu\Omega=0,\\-i\partial_\mu\psi_\mu-m(\psi-i\gamma_\nu\psi_\nu)-m\psi+\gamma\partial\Omega=0,\\-2\gamma\partial\Omega-3m\Omega+i\partial_\mu\psi_\mu-\gamma\partial\psi=0,\end{aligned} \tag{8}$$

from which it follows that

$$(\gamma\partial+m)\psi=0,\quad \psi_\mu=(i/m)\partial_\mu\psi, \tag{9}$$

and

$$\Omega=0. \tag{10}$$

The equivalence of the above free-field equations with the Dirac equation is essentially a consequence of the vanishing of the auxiliary field Ω.

It is well known that the electromagnetic interaction of a charged field is not completely specified by gauge invariance. If we follow the usual rule of replacing ∂_μ by $D_\mu=\partial_\mu-ieA_\mu$ to introduce the electromagnetic interaction in (7), the resulting field equations yield

$$\Omega=(e/6m^2)\sigma_{\mu\nu}F_{\mu\nu}(\psi+\Omega), \tag{11}$$

$$(\gamma_\mu D_\mu+m)(\psi+\Omega)=-(e/4m)\sigma_{\mu\nu}F_{\mu\nu}(\psi+\Omega). \tag{12}$$

The relation (11) shows that Ω is nonvanishing in the presence of interaction, while (12) shows that $\psi+\Omega$ represents a charged field of spin $\frac{1}{2}$ without any intrinsic magnetic moment in agreement with Chang's result.[2] However, it seems to us more appropriate to choose the electromagnetic interaction in such a way that it is not only gauge invariant, but also ensures the vanishing of the auxiliary field. We therefore try a Lagrangian density of the form

$$\begin{aligned}L_{\text{total}}=L_0+L'+a\bar\psi\sigma_{\mu\nu}\psi F_{\mu\nu}\\+b(\bar\psi\sigma_{\mu\nu}\Omega+\bar\Omega\sigma_{\mu\nu}\psi)F_{\mu\nu}+c\bar\Omega\sigma_{\mu\nu}\Omega F_{\mu\nu},\end{aligned} \tag{13}$$

where L_0 is the Lagrangian density of the photon field, L' is obtained from (11) on replacing ∂_μ by D_μ, while a, b, and c are some constants. The resulting field equations then show that Ω vanishes if and only if

$$a=b=c=(e/4m), \tag{14}$$

and in that case we obtain for the ψ field

$$(\gamma D+m)\psi=0,\quad \psi_\mu=(i/m)D_\mu\psi, \tag{15}$$

together with the photon field equation

$$\square^2A_\mu=(e/2)(\bar\psi_\mu\psi+\bar\psi\psi_\mu)-(e/2m)\partial_\nu(\bar\psi\sigma_{\mu\nu}\psi)$$

or

$$\square^2A_\mu=-ie\bar\psi\gamma_\mu\psi. \tag{16}$$

Thus, the requirement of the vanishing of the auxiliary field in the presence of electromagnetic interaction ensures the equivalence of the Bargmann-Wigner and Dirac formulations for the field of spin $\frac{1}{2}$.

[5] With the present notation, C satisfies the relation $C^{-1}\gamma_\mu C=-\gamma_\mu^T$, where γ_μ^T is the transpose of γ_μ, and $C^{-1}=C^*$.