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$He^3/(He^3+He^4)$ Ratios in Cosmic Rays, Path Lengths in Space, and Energy Spectrum of Helium Nuclei in Local Interstellar Space

S. BISWAS, S. RAMADURAI, AND N. SREENIVASAN
Tata Institute of Fundamental Research, Bombay, India
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Recent data on the ratios of $He^3/(He^3+He^4)$ in cosmic rays in the energy interval 45–300 MeV/nucleon in 1963–65 have been used to determine the path length of helium nuclei in space. In these calculations we have taken into account (i) the energy dependence of the cross sections for the production of He^3 and H^3 from $p+He^4$ reactions and that of the total inelastic cross section as obtained from all available data, (ii) the ionization loss of He^4 and He^3 nuclei in space, and (iii) the solar modulation of the fluxes of He^3 and He^4 nuclei. It is shown that using the data of the measured $He^3/(He^3+He^4)$ ratios, the accurately measured differential spectrum of He nuclei in the energy interval 30–1500 MeV/nucleon in 1965, and the $R\beta$ (i.e., rigidity×velocity) dependence of the solar modulation, we can determine both the path lengths in space and the residual solar modulation, and hence the energy spectrum of the He nuclei in local interstellar space, assuming a simple form of the source spectrum. The path lengths determined from $He^3/(He^3+He^4)$ ratios are in agreement with those from Li/M ratios (M-nuclei: $6\leqslant Z\leqslant 9$). The path length of multiply charged nuclei in space is found to have a maximum value of about 7–9 g cm^{-2} of hydrogen at 200–300 MeV/nucleon and to drop to about 2–3 g cm^{-2} at 50 MeV/nucleon and to 3.5 g cm^{-2} at 1000 MeV/nucleon. The source spectrum is found to be consistent with a power spectrum in rigidity, but not with a power spectrum of total energy per nucleon or power spectrum of kinetic energy per nucleon with an exponent of 2.5.

I. INTRODUCTION

THE relative abundance of He^3 nuclei in the primary cosmic rays can be used to determine the amount of matter traversed by the cosmic-ray helium nuclei in space, since He^3 nuclei are presumably absent in the source regions and are mainly produced in fragmentation of He^4 nuclei in collision with hydrogen in the interstellar space (and in the source region). The data available at present indicate that for helium nuclei of kinetic energy of about 150–350 MeV/nucleon, the path length is about 3–12 g cm^{-2} of hydrogen.[1–10] Recently Fan *et al.*[11] determined the ratios $\Gamma(E)=He^3/(He^3+He^4)$ in the low-energy interval, $E=40$–115 MeV/nucleon, using satellite-borne detectors and also obtained the flux of He^3 nuclei in the above energy interval. By combining these results with those obtained near the solar minimum (in 1963–65) by other investigators,[6,8,9] the energy dependence of $\Gamma(E)$ is obtained in the energy interval 40–350 MeV/nucleon as shown in Fig. 1. Using these results, we have determined the amount of matter traversed in space by the low-energy helium nuclei as a function of energy, taking into account the energy dependence of the cross sections for the production of He^3 and H^3, the energy dependence of the total inelastic cross sections of the reaction

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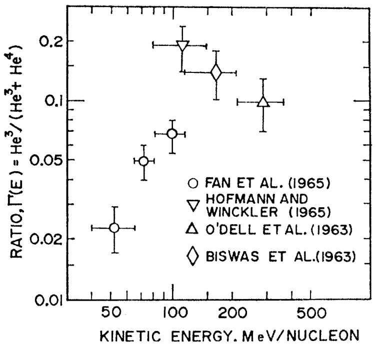

FIG. 1. The ratio $\Gamma(E)$ of $He^3/(He^3+He^4)$ as a function of kinetic energy per nucleon as measured during 1963–1965.

$p+He^4$, the ionization loss of He^4 and He^3 nuclei in space, and the solar modulation of the fluxes of He^4 and He^3 nuclei. The last effect is important as pointed out by Fan *et al.*[11] since He^3 and He^4 have different mass-to-charge ratios and since solar modulation is found by Gloeckler and Jokipii[12] to be dependent on $R\beta$ (i.e., rigidity×velocity) of the nuclei. It is shown that using available data and assuming a simple form of source spectrum, both the path lengths and the residual solar modulation in 1965, and hence the differential spectrum of He nuclei in the local interstellar space, can be determined.

II. CROSS SECTIONS FOR THE PRODUCTION OF He^3 AND H^3

We have shown in Table I the nuclear reactions producing He^3 and H^3 nuclei by the bombardment of He^4 targets by protons and neutrons and all available data of cross sections of these reactions, together with the total inelastic cross sections. The σ for H^3 is also taken into account since all H^3 nuclei, having a half-life of 12.3 years, would decay to He^3 in cosmic time scale. In certain cases, for energies >90 MeV, we have also used the cross sections of equivalent reactions produced by neutrons, shown in Table I, as representing proton-induced cross sections. Experimental data seem to justify this assumption. For instance, the σ for the reactions $n(He^4,d)H^3$ for 90-MeV neutrons agrees very well with σ for the reactions $p(He^4,d)He^3$ for 93-MeV protons. In Fig. 2, we have plotted these cross sections and have shown their energy dependence.

In the energy interval 300–1000 MeV, the total σ for He^3+H^3 is assumed to be constant. This is inferred from the results of interactions of helium nuclei of energy 225 MeV/nucleon[13] and >6 BeV/nucleon[14] with nuclear emulsions which indicate that the fraction of inelastic interactions leading to He^3 fragments remains almost constant ($\sim$0.07) at these two energies.

The cross sections for the production of H^3 from the bombardment of heavy-element targets by protons and their energy dependence was given by Badhwar and Daniel.[10] From these and the relative abundance of He^4 and nuclei of $Z\geqslant 6$ in the primary cosmic rays, it is found that for energy <1 BeV/nucleon, the contribution of heavy nuclei to the production of He^3 and H^3 is $\lesssim 5\%$ of that of He^4 nuclei. Therefore the contribution of He^3 by heavy nuclei has been neglected in the calculation of the path length of He nuclei in space.

The other source of He^3 in interstellar space could be high-energy cosmic-ray protons interacting with interstellar He^4, whose abundance is estimated as $\sim 7\%$. In these collisions, He^3 nuclei would be produced preferentially with very low energies. For example, laboratory measurements in equivalent reactions, $n(He^4,pn)H^3$ with 90-MeV[15] and 300-MeV[16] neutrons, indicate that about 90% of H^3 are of energy 0–25 MeV. Therefore He^3 nuclei from this source are expected to be of energy much smaller than those considered here and can be neglected.

III. DIFFUSION EQUATIONS

Having obtained the energy dependence of relevant cross sections, we next consider the diffusion equations involving ionization loss and fragmentations[17] for calculating the fluxes of He^3 and He^4 nuclei. We assume that acceleration in interstellar space is negligible and the interstellar medium is composed of neutral hydrogen. Let $J_i(E_1,x)dE_1$ be the flux of the ith group of nuclei with energies between E_1 and E_1+dE_1 at a distance x g cm^{-2} from the source. If this group of nuclei pass through Δx g cm^{-2} of hydrogen, they will emerge with energies between E_2 and E_2+dE_2. Thus the differential spectrum is given as

$$J_i'(E_2,x+\Delta x)dE_2=J_i(E_1,x)\left[1-\frac{\Delta x}{\Lambda_i(E_1)}\right]dE_1 +\sum_{j\neq i} J_j(E_2,x)dE_2\frac{\Delta x}{m}\sigma_{ji}(E_2)\,,$$

where $\Lambda_i(E)$ is the absorption mean free path of the ith type of nuclei. In case of He^3 and He^4 nuclei, $\Lambda_i=\lambda_i$, the interaction mean free path in g cm^{-2} and $\lambda_i(E)=m/\sigma_t(E)$, $\sigma_t(E)$ being the total inelastic cross section at energy E, and m the proton mass in g cm^{-2}; $\sigma_{ji}(E)$ is the cross section for the production of ith type of nuclei from the fragmentation of jth type of nuclei, at energy E. The range-energy relation is used in the form of $E=cx^n$, where E is the kinetic energy per nucleon, x is g cm^{-2} of neutral hydrogen, c and n are constants for a particular group of nuclei and particular energy interval. In the present case, we derived two

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TABLE I. Nuclear reactions and the cross sections for the production of He^3 and H^3 by protons and neutrons on He^4 targets.

Reactions	Q values (MeV)	Label
$p(He^4,d)He^3$	−18.3	(A)
$p(He^4,pn)He^3$	−20.6	(B)
$p(He^4,pp)H^3$	−19.8	(C)
$n(He^4,d)H^3$	−17.6	(A')
$n(He^4,nn)He^3$	−20.5	(B')
$n(He^4,pn)H^3$	−19.8	(C')

Proton energy (MeV)	Cross section σ (in mb) for reaction (A)	(B)	(C)	$(B+C)$	Total inelastic	Reference
28	38.5±13	4.8±1.3	8.9±1.0	13.7±3.4	52.2±1.4	a
31.5	35.5±3.5	···	···	···	···	b
53	28.2±2.2	47.6±2.8	31.9±2.5[c]	79.5±8[c]	105	d
55	40 ±5.9	···	···	50−80	···	e
93	11 ±2	···	···	···	···	f
630	···	(47.3±5)[g]	29 ±3	(76.3±6)[g]	(126 ±14)[g]	h
970	···	···	···	···	93 ±13	i
Neutron energy	(A')	(B')	(C')	$(B'+C')$		
90	13 ±2.5	16	42 ±6	58	94 ±17	j
200	···	···	···	···	62	k
300	1.1±0.6	2.4±1.0	38.5±3.8	41 ±4	70 ±10	l

[a] A. F. Wickersham, Phys. Rev. **107**, 1050 (1957).
[b] J. Benveniste and B. Cork, Phys. Rev. **98**, 422 (1953).
[c] Includes all three-pronged events.
[d] D. J. Cavins, T. C. Griffith, G. J. Lush, A. J. Methringham, and R. H. Thomas, Nucl. Phys. **60**, 369 (1964).
[e] Reference 18.
[f] Reference 19.
[g] These values have not been included in Fig. 2 because these seem to be inconsistent with those expected from the results of other investigators. The possibility of some systematic error in these data cannot be excluded as was pointed out earlier by Badhwar and Daniel (Ref. 10).
[h] M. S. Kozodaev, M. M. Kulyukin, R. M. Sulyaev, A. I. Filippov, and Yu. A. Shcherbakov, Zh. Eksperim. i Teor. Fiz. **38**, 300 (1960) [English transl.: Soviet Phys.—JETP **11**, 511 (1960)].
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[j] Reference 15.
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[l] Reference 16.

equations for He^4 and He^3 nuclei. The contribution of $Z\geqslant 6$ nuclei to the production of He^4 and He^3 nuclei is found to be small ($<5\%$) as discussed before and hence has been neglected.

We have assumed that He^3 is absent in the source and that the source spectrum has one of the following two forms: (1) $J^s(\epsilon,A,Z)d\epsilon=K(A,Z)/\epsilon^{2.5}d\epsilon$ and (2) $J^s(R,A,Z)dR=C(A,Z)/R^{2.5}dR$, where ϵ and R are total energy per nucleon and rigidity of the particle, respectively, and K and C are constants depending on A and Z. As discussed later, the rigidity spectrum (2) is found to be more appropriate and this has been used. In this case the differential rigidity flux is transformed into differential energy flux.

The effect of elastic scattering of He^4 nuclei with protons has been evaluated using experimental data of differential cross sections of elastic scattering as a function of the scattering angle at 55 MeV [18] and at 95 MeV.[19] It is found that the mean change in the energy of He^4 nuclei, $\langle\Delta E\rangle_{av}/E$ is only 7.6% at 55 MeV/nucleon and 4.1% at 95 MeV/nucleon. Hence the effect of elastic scattering has been neglected. The second effect, that in inelastic collisions the energy per nucleon of the He^3 fragment may be different from that of the primary He^4 nucleus, has been calculated for the reaction (C') of Table I for 90-MeV neutrons using the kinematics of the collision process and the measured differential cross section of H^3 as a function of the angle of scattering.[15] It is found that mean energy per nucleon of H^3 fragment is only $\sim$6% less than the primary energy of 90 MeV/nucleon. The He^4 nuclei of lowest energy considered here, namely, 50 MeV/nucleon at the earth

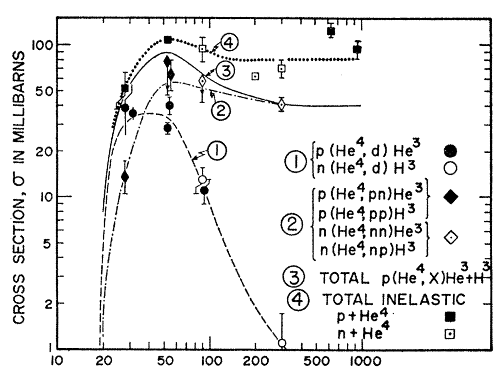

FIG. 2. Experimental values of cross sections for the production of He^3 and H^3 nuclei from He^4 targets bombarded with protons and neutrons. The energy dependence of these cross sections and also that of the total inelastic cross section are shown.

[18] S. Hayakawa, N. Horikawa, R. Kajikawa, K. Kikuchi, H. Kobayakawa, K. Matsuda, S. Nagata, and Y. Sumi, Phys. Letters **8**, 330 (1964).
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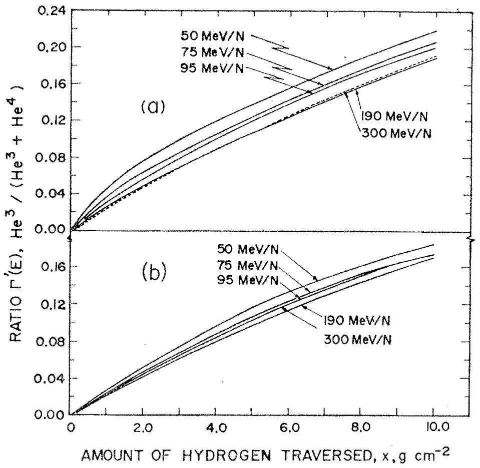

FIG. 3. The growth curves of the ratios $\Gamma'(E)$ of $He^3/(He^3+He^4)$ as a function of x, the mean amount of hydrogen traversed in space for different energies at the earth assuming that the particles of a given energy (a) tranverse same amount of matter and (b) have a distribution of path length about the mean value as given by the distribution function suggested by Balasubrahmanyan *et al.* (Ref. 21).

correspond to particles of energy ~90 MeV/nucleon at source before traversing about 2 g cm^{-2} of hydrogen. Therefore the effective change in the energy of He^3 as compared with the primary energy would be about 10% for particles in the lowest energy interval, i.e., 50 MeV/nucleon, and would be still smaller for higher-energy particles. Hence this effect is small and has been neglected in the present calculations. Similar calculations were done by Badhwar and Kaplon.[20]

The calculations were done with a CDC-3600 computer and iterations were made at every 0.1-g cm^{-2} interval for the amount of matter traversed from 0 to 15 g cm^{-2}. Thus the differential fluxes of He^3 and He^4 nuclei were obtained at every 0.5 g cm^{-2} and at every 10-MeV/nucleon interval from 30–100 MeV/nucleon and at every 50-MeV/nucleon interval from 100–1000 MeV/nucleon. From these data the variation of the ratio $\Gamma'(E)=He^3/(He^3+He^4)$ outside the solar system has been calculated as a function of x, the amount of matter traversed according to two models. In model I, we assume that all particles of a given energy E at the earth have traversed the same amount of matter in space. On this assumption the growth curves of the ratio $\Gamma'(E)$ versus x are calculated and these are shown in Fig. 3(a) for energies E of 50, 75, 95, 190, and 300 MeV/nucleon. In model II, we assume that particles of a given energy E at the earth could arrive at the earth after traversing different path lengths and the probability that the path length lies between x and $x+dx$ is given by the normalized distribution function[21]

$$p(x)dx=(5/\pi)^{1/2}/(1.886\bar{x})\exp[-(5/4)(x-\bar{x})^2/\bar{x}^2]dx,$$

where $\bar{x}$ is the mean path length. In this case we computed the ratio $\Gamma'(E)$ for a particular value of $\bar{x}$ from the relation

$$\Gamma'(E)=\frac{\int J_{He^3}(E,x)p(x)dx}{\int[J_{He^3}(E,x)p(x)dx+J_{He^4}(E,x)p(x)dx]},$$

where $J_i(E,x)$ is flux of the ith component of energy E (MeV/nucleon) after traversal of a path length x g cm^{-2}. The growth curves of $\Gamma'(E)$ versus $\bar{x}$, the mean amount of matter traversed, calculated in this manner are shown in Fig. 3(b) for the same values of energies. It is seen that the differences between the growth curves of different energies are narrowed down because of the distribution of the path lengths.

Before using the curves in Figs. 3(a) and 3(b) for determining the path lengths, it is necessary to correct the measured ratios $\Gamma(E)$ for the residual solar modulation so as to obtain the values of $\Gamma'(E)$ outside the solar system. This is discussed in the next section.

IV. SOLAR MODULATION AND PATH LENGTHS FROM $He^3/(He^3+He^4)$ RATIOS

Recently Gloeckler and Jokipii[12] have shown that the measured interplanetary magnetic-field irregularities require that the diffusion coefficient K in Parker's theory of solar modulation[22] be proportional to $R\beta$ of the particle, and the measured changes in the fluxes of both protons and helium nuclei in 1963–65 can be explained by $R\beta$ dependence of the solar modulation as given by the relation

$$J_i^e(E)=J_i^0(E)\exp[-\eta(t)/R\beta], \quad (1)$$

where $J_i^e(E)$ is the differential flux of the ith type of nuclei at earth at kinetic energy E MeV/nucleon, $J_i^0(E)$ the corresponding flux outside the solar system, and $\eta(t)$ a constant which is a function of time only and is independent of R and β. Therefore we assume that the relation (1) gives the correct representation of the solar modulation during 1963–65 in the energy region under consideration here. The studies of solar modulation of protons and helium nuclei by Dorman and Dorman,[23] Nagashima *et al.*,[24] and Silberberg[25] have also shown that experimental results are roughly consistent with $R\beta$ dependence of solar modulation. The results of Webber[26] are also consistent with this as pointed out by Gloeckler and Jopikii.[12] Other models of

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interplanetary and interstellar modulations have been discussed by Fichtel and Reames.[27]

The observed changes in the fluxes of protons and helium nuclei only give the change of η, i.e., $\Delta\eta$, and it has not been possible so far to derive the value of η of Eq. (1) without any additional assumptions. It is shown in this work that using the data of $He^3/(He^3+He^4)$ ratios and the spectral shape of He nuclei it is possible to determine both the path length of He nuclei in space and the value of η, and hence the spectral shape outside the solar system with a simple assumption of the source spectrum.

It is well known that in the energy interval $E>1.5$ BeV/nucleon cosmic-ray helium and proton spectra can be represented by either of the two forms (i) $N(>\epsilon)=K/\epsilon^{1.5}$, (ii) $N(>R)=C/R^{1.5}$ up to about 1000 BeV/nucleon or more. Since modulation and energy-loss effects are negligible here the source spectrum must be given by either of the two forms in this energy interval. Therefore we assume that the source spectrum of He nuclei in the energy interval 50–1500MeV/nucleon is given by one of these forms, i.e., by a differential spectrum having an exponent of 2.5 in total energy per nucleon or in rigidity. An alternative form of the source spectrum, $J(E)dE\propto E^{-2.5}dE$ is also discussed later.

In Fig. 4 we have shown the differential spectrum of He nuclei in mid-1965 measured precisely by a number of investigators.[11,8,28,29] It is seen that the results are in good agreement with one another and give an accurate differential spectrum of He nuclei in the interval 40–1500 MeV/nucleon considered here. This experimental spectral shape shows that in the energy interval 500–1500 MeV/nucleon, total energy per nucleon spectrum is proportional to $\epsilon^{-3.2}$. Even without correction for solar modulation, this observed index 3.2 is much larger than the index 2.5 at high energies; hence the form of source spectrum as $\epsilon^{-2.5}$ in the entire energy interval $E>50$ MeV/N can be ruled out, unless one assumes a radical change in the spectral shape in the interval <1500 MeV/nucleon. Therefore we assume the source spectrum as power spectrum in rigidity with an index 2.5, same as in the high-energy region.

The observed differential energy spectrum of He^3 nuclei in mid-1965 is also shown in Fig. 4. Here the experimental points in the energy interval of 115–250 MeV/nucleon measured in 1963 have been corrected for relative modulation between 1963 and 1965 using the value of $\Delta\eta=0.25$ obtained[9] from the spectral shapes of He nuclei measured in 1963 and 1965.

The next problem is to determine the appropriate value of η, corresponding to the residual modulation in 1965, and to calculate the path length of He nuclei from $\Gamma'(E)=He^3/(He^3+He^4)$ ratios. This is done by trial by

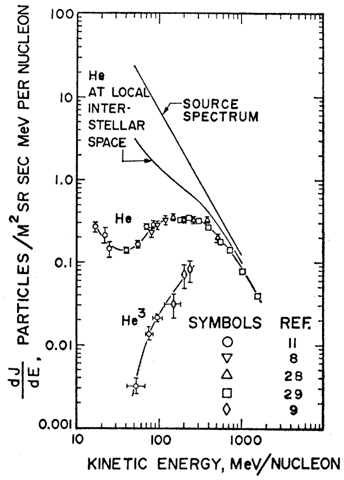

FIG. 4. The differential energy spectra of He and He^3 nuclei in 1965. The spectrum of He nuclei at local interstellar space is obtained by demodulating the spectrum observed in 1965 for $\eta=0.60$. The source spectrum $J(R)dR\propto R^{-2.5}dR$ is also shown.

choosing different values of η and by demodulating the measured ratios of $He^3/(He^3+He^4)$. (It is assumed that the measured He flux corresponds to that of He^4 nuclei as the contamination of He^3 nuclei is $\lesssim 10\%$.) For each assumed value of η, the path length and its variation with energy can be obtained by two separate methods as follows. (1) By demodulating the measured ratios of $He^3/(He^3+He^4)$ nuclei with a particular η value we get the ratio $\Gamma'(E)=He^3/(He^3+He^4)$ at local interstellar space. Using these values of $\Gamma'(E)$ and the growth curves of the ratio as given in Fig. 3 path lengths of He nuclei in space are obtained. (2) For the particular assumed value of η, the demodulated He spectrum and the source spectrum are related to each other by the path length traversed in space. The fluxes calculated from the diffusion equation are normalized to the demodulated fluxes at 1000 MeV/nucleon assuming that the amount of matter traversed by He nuclei of energy 1 BeV/nucleon is 3.5 g cm^{-2} as determined by the Li/M ratio (M nuclei: $6\leqslant Z\leqslant 9$). Thus using the calculations of the diffusion equation, and the demodulated experimental fluxes, the path length can be evaluated for helium nuclei in the energy interval 50–1000 MeV/nucleon. Various values of η ranging from 0.4 to 1.0 were tried. For values of $\eta>0.65$, the path lengths derived from method (1) yield values higher than those obtained from method (2), whereas for values of $\eta<0.55$ the reverse is true. These differences exist even if the dispersion in path lengths is taken into account. Thus the best fit is obtained for the value of $\eta=0.60$.

In Fig. 5 we have shown, for $\eta=0.60$, the path lengths determined from the ratios $\Gamma(E)=He^3/(He^3+He^4)$ and those from the demodulated He^4 spectrum and the source spectrum. It is seen that the path lengths determined from the two methods are in good agreement within experimental errors. Hence we regard this value of $\eta=0.60$ as close to the correct factor for the residual modulation in mid-1965 and the path lengths derived for this value of η as the appropriate path lengths. It may be pointed out that ratios $\Gamma'(E)$ and path lengths

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[29] J. F. Ormes and W. R. Webber in *Proceedings of the International Conference on Cosmic Rays* (The Institute of Physics and The Physical Society, London, 1966), Vol. 1, p. 349.

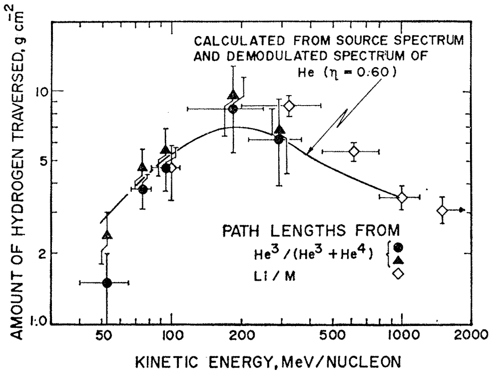

FIG. 5. The path lengths of He and heavy ($Z \geqslant 6$) nuclei as a function of kinetic energy per nucleon. The path lengths determined from demodulated $He^3/(He^3+He^4)$ ratios are shown by closed circles for model I and by closed triangles for model II, as given in Table II, for $\eta = 0.60$. The open diamonds denote the path lengths of $Z \geqslant 6$ nuclei derived from demodulated Li/M ratios for model I using $\eta = 0.73$.

at about 200 and 300 MeV/nucleon are not very sensitive to small changes of η; whereas the $\Gamma'(E)$ ratios and path lengths for $E < 100$ MeV/nucleon are very sensitive to the choice of η. Hence the recent measurements of $\Gamma(E)$ ratios in the low-energy interval allow us to determine the η value within rather narrow limits, $\eta = 0.55$–0.65, and hence the path lengths. The values of $\Gamma(E)$, $\Gamma'(E)$, and path lengths are shown in Table II.

We wish to point out that this value of η is not sensitive to the value of the path length at 1 BeV/nucleon at which normalization is done. For example, if we take this path length as 2.5 g cm^{-2} instead of 3.5 cm^{-2}, the value of η remains essentially unchanged. We have also examined the dependence of the value of η on the assumed spectral index of the source spectrum. If the index is taken as 2.7 instead of 2.5, it is found that the value of η which gives the best fit is 0.65.

The errors in the path length of He nuclei shown in Fig. 5 and Table II have been derived from the experimental errors in determining the ratios $\Gamma(E)$. As the statistical errors in the cross sections are small ($\sim 10\%$) compared to those of the $He^3/(He^3+He^4)$ ratios (~ 20–30%), the former errors have been neglected in determining the uncertainties in the path lengths.

In Fig. 5 we have also shown the amount of matter traversed in space by heavy nuclei of $Z \geqslant 6$ as determined by Biswas *et al.*[30] from Li/M ratios.[31] We have made corrections to the measured Li/M ratios for the residual solar modulation and hence derived the path lengths accordingly. These are shown in Table II. The correction for solar modulation is made in the following manner. Since experimental Li/M ratios are the average of measurements made in 1963–65, we have used $\eta = 0.73$ ($\eta = 0.60$ for 1965 and $\Delta\eta = 0.25$ for 1963 to 1965). The mass-to-charge ratio of Li is taken as 2.2 since cross sections for the production of Li^6 and Li^7 are about the same. This results in the modulation corrected values of Li/M ratios being lower than the measured values as opposed to $He^3/(He^3+He^4)$ ratios. Similar effects on $(Li+Be+B)/M$ ratios have been discussed by Hildebrand and Silberberg.[32] As seen in Fig. 5 and Table II, the path lengths of He nuclei derived from $He^3/(He^3+He^4)$ ratios are in agreement with those of heavy nuclei ($Z \geqslant 6$) determined from Li/M ratios. The general nature of variation of path length with kinetic energy is such that it has a broad maximum of about 7–9 g cm^{-2} at 200–300 MeV/nucleon and it drops to a value of about 3.5 g cm^{-2} at 1000 MeV/nucleon and to about 2–3 g cm^{-2} at 50 MeV/nucleon. These results are in disagreement with the conclusions of Badhwar and Kaplon[20] on the energy dependence of the path length. They have concluded that the experimental ratios $\Gamma(E)$ are not inconsistent with a path length of 2.5 g cm^{-2} independent of the particle energy, although the agreement between the calculated and experimental values seems to be poor. In addition their conclusions are based on the assumptions of the velocity-dependent solar modulation and of arbitrary value of η for the residual solar modulation.

In Fig. 4 we have shown the differential energy spectrum of the helium nuclei at local interstellar space in the energy interval 50–1000 MeV/nucleon. This is derived by demodulating the measured spectrum in 1965 with relation (1) with $\eta = 0.60$ as determined in this work. Source spectrum, $J(R)dR \propto R^{-2.5}dR$, translated into kinetic energy per nucleon scale is also shown in the same figure. This has been obtained by converting the flux in rigidity interval $J(R)dR$ into that in kinetic energy per nucleon interval $J(E)dE$ by the relation $J(E)dE = (E+m_0c^2)(1/R)(A/Z)^2 J(R)dR$, where m_0c^2 is the nucleon rest mass. This spectrum has been normalized to the measured flux at 1 BeV/nucleon corrected for residual solar modulation and traversal of 3.5 g cm^{-2} of hydrogen in space. This source spectrum in kinetic-energy plot turns out to be very close to the form $J(E)dE \propto E^{-\gamma}dE$ in a particular kinetic-energy interval, where $\gamma = 1.77$ in the energy interval 50–1000 MeV/nucleon. The values of γ would be 1.75 and 2.5 in the extreme nonrelativistic and extreme relativistic energy regions, respectively.

We have also examined the possibility of source spectrum being of the form $J(E)dE \propto E^{-2.5}dE$ in the entire energy interval, since for $E \geqslant m_0c^2$, the source spectra considered above reduce to this form. Using identical procedure as described above, we find that it is not possible, for any value of η, to get an agreement between the path lengths derived from the two separate methods outlined earlier, for the entire energy interval of 50–1000 MeV/nucleon. For example, if we choose the value of $\eta = 0.9$ to get agreement between the path

[30] S. Biswas, S. Ramadurai, and N. Sreenivasan, Phys. Rev. **149**, 1037 (1966).

[31] For references see Ref. 30 and also G. M. Comstock, C. Y. Fan, and J. A. Simpson, Astrophys. J. **146**, 51 (1966).

[32] B. Hildebrand and R. Silberberg, Phys. Rev. **141**, 1248 (1966).

TABLE II. Path lengths in space of He and heavy nuclei.

Method	Kinetic energy (MeV/nucleon)	Experimental ratios[a]	Ratios corrected for residual modulation[b]	Path length (g cm^{-2} of hydrogen) Model I[c]	Model II[d]
$He^3/(He^3+He^4)$	39–67	0.025±0.006[e]	0.062±0.015	1.5±0.5	2.4±0.6
	67–82	0.050±0.010[e]	0.103±0.015	3.8±0.7	4.7±0.9
	82–108	0.068±0.012[e]	0.115±0.020	4.7±1.0	5.6±1.3
	117–250	0.120±0.030[f]	0.166±0.048	8.4±3.0	9.6±3.2
	215–368	0.100±0.030[g]	0.132±0.039	6.2±2.3	6.8±2.4
Li/M	80–120	0.092±0.027[k]	0.077±0.023	4.6±1.2	5.5±1.5
	200–450	0.164±0.018[h]	0.155±0.017	8.7±0.9	10.0±1.1
	450–800	0.125±0.011[h]	0.121±0.010	5.5±0.5	6.0±0.5
	800–1200	0.110±0.013[h]	0.108±0.013	3.5±0.4	3.6±0.5
	>1500	0.099±0.012[h]	0.099±0.012	3.1±0.4	3.2±0.4

[a] All these data refer to measurements made in 1963–1965. The data of Hofmann and Winckler (Ref. 8) are not included for reasons as discussed by Fan *et al.* (Ref. 11) and Biswas *et al.* (Ref. 9).
[b] See text for the correction for the residual solar modulation.
[c] Assuming all nuclei of a given energy per nucleon traverse the same path length.
[d] Assuming the nuclei of a given energy per nucleon have a distribution of path lengths as given by Balasubrahmanyan *et al.* (Ref. 21).
[e] Reference 11.
[f] Reference 9. The ratio given here is slightly different because this is obtained from the measured flux of He^3 in the interval 117–250 MeV/nucleon Ref. 9 and the best estimate of He flux in the same energy interval during the same period from the data of several investigators.
[g] Reference 6.
[h] Average of the measurements in 1963–65 by a number of investigators; see Ref. 31.

lengths derived from the two methods in the energy interval 50–100 MeV/nucleon, the path lengths for the energy interval 200–1000 MeV/nucleon from the two methods differ widely. On the other hand, if we choose the value of $\eta=1.5$ to match the path lengths at higher energies, this results in a very large disparity at low energies. Hence this form of source spectrum is not consistent with observations.

In the investigations made so far, it has not been possible to determine the value of η for the residual solar modulation from the measured spectral shapes without other assumptions besides the form of the source spectrum. For example, Silberberg[25] calculated the value of $\eta=2.9$ for 1963 in the energy interval 1–10 BeV/nucleon assuming that above 1 BeV/nucleon, the ratio of helium/proton is constant at local interstellar space and the differential spectra of protons and helium at the earth can be obtained from the integral proton spectrum and helium/proton ratios. This value of η leads to difficulties in the energy region below 1 BeV/nucleon as discussed by Silberberg.[25] Balasubrahmanyan *et al.*[21] assumed the value of $\eta=1$ for 1963 and calculated the proton spectrum at local interstellar space on the velocity-dependent solar modulation. Nagashima *et al.*[24] have obtained only the lower limit of the value of η as $\sim$0.8 for 1958.

V. CONCLUSION

The results obtained in the present work can be briefly summarized as follows:

1. From the available data on cross sections of He^4+p (or n) reactions, the energy dependence of the cross section for the production of He^3 and H^3 as well as of total inelastic cross section have been obtained in the interval 30–1000 MeV/nucleon. The diffusion equations involving ionization loss, energy-dependent fragmentation process, and energy-dependent total inelastic cross section have been used to calculate the growth curve of the ratio $\Gamma'(E)=He^3/(He^3+He^4)$ as a function of amount of neutral hydrogen traversed in space.

2. The assumed source spectrum in the form of a power law in rigidity, $J(R)dR\propto R^{-2.5}dR$ for all rigidities $R>0.6$ BV (kinetic energy >45 MeV/nucleon), is found to be consistent with observations, whereas a source spectrum in the form of a power law in total energy per nucleon, $J(\epsilon)d\epsilon\propto\epsilon^{-\gamma}d\epsilon$ with $\gamma=2.5$, is not consistent with observations unless we assume that γ is significantly higher, $\geqslant3.2$, in the energy interval $E\leqslant1000$ MeV/nucleon. It is also found that a source spectrum in the form of a power law in kinetic energy, $J(E)dE\propto E^{-2.5}dE$, is not consistent with observations.

3. Solar modulation of the $He^3/(He^3+He^4)$ ratio has been analyzed using the $R\beta$ dependence of the solar modulation as suggested recently.[12] It is shown that the path length of He nuclei as a function of energy can be evaluated in two separate manners and only for a value of $\eta=0.60$, which determines the residual solar modulation in mid-1965; the path lengths and their variation with energy give consistent results from the two methods. Hence this determines the appropriate values of the path lengths from $He^3/(He^3+He^4)$ ratios, the residual solar modulation, and hence the spectrum of helium nuclei at local interstellar space.

4. The path lengths of helium nuclei in space are in agreement with those derived for heavy nuclei from Li/M ratios. The path length is found to have a maximum of $\sim$7-9 g cm^{-2} at 200–300 MeV/nucleon and it drops to a value of $\sim$2–3 g cm^{-2} at about 50 MeV/nucleon and to 3.5 g cm^{-2} at 1000 MeV/nucleon.

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