Neutrino Excitation of the Giant Resonance in C^{12}

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The differential and total cross sections of the neutrino excitation of the giant resonance (gi. res.) in C^{12} are calculated using the Brown one-particle–one-hole theory. We find that the total cross section levels out at 1.5×10^{-39} cm^2 above 500 MeV. We also calculate the $N^{12}{}_{\text{gi. res.}} \rightarrow C^{11}+p$ spectrum using Boeker's method, because this decay can identify the neutrino reaction in a bubble chamber. To check our programs we recalculated muon-capture matrix elements for transitions to the $B^{12}{}_{\text{gi. res.}}$ state and found that some published numerical values should be changed.

I. INTRODUCTION

WHEN an accelerator is used as a source of high-energy neutrinos, it is very important to know the neutrino energy spectrum. Theoretical calculations of the spectrum are difficult, and the results are not extremely reliable. At present neutrino fluxes are thought to be known to better than 50% in the energy range from 1 to 10 GeV.[1] Thus it is desirable to check the neutrino spectrum further by experimentally observing the rate of neutrino reactions having a fairly well-known theoretical cross section.

Originally this kind of check was made[2] using Lee and Yang's[3] calculation of the elastic reaction. More recently, such checks have been made using calculations similar to those of Lee and Yang, but various models are employed to take into account the influence of nuclear structure on the reactions. In the same way a calculation of the cross section of the neutrino excitation of the giant resonance state in C^{12} can be used. Some aspects of this reaction make it especially appropriate as a check on the spectrum.[4] First, the total cross section is relatively large, ~5% of the direct reaction. Second, the energy of the lepton created is equal to that of the incoming neutrino minus that of the nuclear excited state (which is only ~0.02 GeV). Third, the cross section flattens out for energies above 0.4 GeV, so the observed lepton spectrum will have the same shape as the neutrino spectrum. Fourth, the characteristic decay of the giant-resonance (gi.res.) state is observable by the emission of one or more low-energy nucleons (especially protons) from the nucleus. In Ref. 4 the total and differential cross sections of the neutrino excitation of the giant resonance in C^{12} were calculated using a generalized Goldhaber-Teller model of the resonance.

The present work was done to see how sensitive this cross section is to the choice of nuclear model. We replace the Goldhaber-Teller giant resonance model with the more detailed one-particle–one-hole Brown model. Also using the Brown model, we find the approximate energy spectrum of the protons emitted in the decay of the giant-resonance state.

II. THEORY

Reference 4 gives the following expression for the cross section of the $\nu(\bar{\nu})+C^{12}\rightarrow B(N)^{12}{}_{\text{gi.res.}}+l(\bar{l})$ reaction:

$$d\sigma=\frac{2\pi}{2J+1}\sum_{MM'}\sum_{s_l s_\nu}(2\pi)^3\delta(\boldsymbol{\nu}-\mathbf{l}-\mathbf{q})\,|\mathcal{H}|^2\times\frac{d^3q}{(2\pi)^3}\,\frac{d^3l}{(2\pi)^3}\delta(E_l+E^{(i)}-\nu) \tag{1}$$

and

$$\begin{aligned}\sum|\mathcal{H}|^2=&\,G_V^2|\mathfrak{M}|^2(1+\hat{\nu}\cdot\mathbf{l}/E_l)-2G_VG_A[\mathrm{Re}\,\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot(\hat{\nu}+\mathbf{l}/E_l)\pm\mathrm{Im}\,\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot(\hat{\nu}\times\mathbf{l}/E_l)]\\&+G_A^2[2\mathrm{Re}\,\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\,\boldsymbol{\mathfrak{M}}\cdot(\mathbf{l}/E_l)+|\boldsymbol{\mathfrak{M}}|^2(1-\hat{\nu}\cdot(\mathbf{l}/E_l))\mp i(\hat{\nu}-\mathbf{l}/E_l)\cdot(\boldsymbol{\mathfrak{M}}^*\times\boldsymbol{\mathfrak{M}})]+2G_VG_P(\mu/E_l)\mathrm{Re}\,\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot(\mathbf{q}/2m)\\&-2G_AG_P(\mu/E_l)\mathrm{Re}\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\,\boldsymbol{\mathfrak{M}}\cdot(\mathbf{q}/2m)+G_P^2|(\mathbf{q}/2m)\cdot\boldsymbol{\mathfrak{M}}|^2(1-\hat{\nu}\cdot\mathbf{l}/E_l)-2G_VG_M[\pm(\hat{\nu}+\mathbf{l}/E_l)\cdot((\mathbf{q}/2m)\times\mathrm{Im}\,\mathfrak{M}^*\boldsymbol{\mathfrak{M}})\\&+((\mathbf{l}/E_l)\times\hat{\nu})\cdot((\mathbf{q}/2m)\times\mathrm{Re}\,\mathfrak{M}^*\boldsymbol{\mathfrak{M}})]-2G_AG_M\{\mathrm{Re}[\boldsymbol{\mathfrak{M}}^*\times(\hat{\nu}-\mathbf{l}/E_l)]\cdot(\boldsymbol{\mathfrak{M}}\times(\mathbf{q}/2m))\\&\pm\mathrm{Im}[\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\,\boldsymbol{\mathfrak{M}}\cdot((\mathbf{q}/2m)\times(\mathbf{l}/E_l))+\boldsymbol{\mathfrak{M}}^*\cdot(\mathbf{l}/E_l)\,\boldsymbol{\mathfrak{M}}\cdot((\mathbf{q}/2m)\times\hat{\nu})+(1-\hat{\nu}\cdot\mathbf{l}/E_l)(\mathbf{q}/2m)\cdot(\boldsymbol{\mathfrak{M}}^*\times\boldsymbol{\mathfrak{M}})]\}\,,\end{aligned} \tag{2}$$

* Supported in part by the National Science Foundation.

[1] R. Burns, K. Goulianos, E. Hyman, L. Lederman, W. Lee, N. Mistry, J. Rettberg, M. Schwartz, J. Steinberger, J. Sunderland, and G. Danby, presented at the CERN Informal Conference in Experimental Neutrino Physics, 1965, CERN Report No. 65–32, pp. 97 ff. (unpublished).

[2] H. Faissner, Acta Phys. Austriaca, Suppl. I, 189 (1964).

[3] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).

[4] H. Überall, Phys. Rev. 137, B502 (1965).

where $\hat{\nu}=\mathbf{v}/v$, $E^{(i)}$ is the excitation energy of the ith resonance state, and m is the nucleon mass. The upper sign refers to the neutrino reaction; the lower sign to the antineutrino reaction. We use the same values of the G_i's as in Ref. 4. The nuclear matrix elements are given by

$$\mathfrak{M}=(\Phi_{Z\pm1}{}^{\dagger}\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}^{(i)}}\tau_{\pm}{}^{(i)}\Phi_Z)\,, \tag{3}$$

$$\boldsymbol{\mathfrak{M}}=(\Phi_{Z\pm1}{}^{\dagger}\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}^{(i)}}\boldsymbol{\sigma}^{(i)}\tau_{\pm}{}^{(i)}\Phi_Z)\,, \tag{4}$$

where here the index i labels the ith nucleon, Φ_Z is the $J^P=0^+$ wave function of the C^{12} ground state, and $\Phi_{Z\pm1}$ is the $J^P=0^-$, 1^-, or 2^- excited one-particle–one-hole giant-resonance state of the B^{12} or N^{12} nucleus.

The nuclear matrix elements $\mathfrak{M}$ and $\boldsymbol{\mathfrak{M}}$ needed for the calculation of the cross section have previously been calculated in Ref. 4 using the Goldhaber-Teller model of the giant resonance generalized to take spin–isobaric-spin oscillations into account. In the present work these matrix elements are calculated from the Brown[5] theory of the giant resonance, using wave functions calculated by Gillet and Vinh Mau[6] and Lewis and Walecka[7] and deForest (LWD)[8]. In the Brown theory the matrix elements are given by

$$\mathfrak{M}_{JM}{}^{(i)}=\frac{-4\pi i^J M_T}{(2J+1)^{1/2}}Y_{JM}{}^*(\theta_q,\varphi_q)\sum_{n'l'j'\,nlj}\alpha_{(i)}{}^{(n'l'j')(nlj)^{-1}}(-1)^{l'+1/2+j+J}\times\left[\frac{(2J+1)(2l+1)(2j+1)(2j'+1)(2l'+1)}{4\pi}\right]^{1/2}\begin{Bmatrix}l' & j' & \frac{1}{2}\\ j & l & J\end{Bmatrix}\begin{pmatrix}l' & J & l\\ 0 & 0 & 0\end{pmatrix}(n'l'|j_J(qr)|nl)\,, \tag{5}$$

$$\boldsymbol{\mathfrak{M}}_{JM}{}^{(i)}=\frac{-4\pi M_T}{(2J+1)^{1/2}}\sum_{L=J-1}^{L=J+1}i^L\mathbf{Y}_{JLM}{}^*(\theta_q,\varphi_q)\sum_{n'l'j'nlj}\alpha_{(i)}{}^{(n'l'j')(nlj)^{-1}}\times\left[\frac{(2J+1)(2L+1)(2l+1)(2l'+1)(2j+1)(2j'+1)}{4\pi}\right]^{1/2}\begin{Bmatrix}l' & l & L\\ \frac{1}{2} & \frac{1}{2} & 1\\ j' & j & J\end{Bmatrix}\begin{pmatrix}l' & L & l\\ 0 & 0 & 0\end{pmatrix}(-1)^{l'}(\sqrt{6})(n'l'|j_L(qr)|nl)\,. \tag{6}$$

The $\alpha_{(i)}{}^{(n'l'j')(nlj)^{-1}}$ are the components which describe the mixing in the final-state wave function of components describing a hole in the (nlj) shell and a particle in the $(n'l'j')$ shell. M_T is the z component of isotopic spin of the final nuclear state. We use the definitions of Edmonds[9] for the above coefficients and spherical harmonics.

III. THE PROTON SPECTRUM

We estimate the spectrum of the protons from the decay

$$N^{12}{}_{\text{gi.res.}}\rightarrow C^{11}+p \tag{7}$$

because through this decay reaction (2) can most easily be recognized. We made an analysis of theoretical and experimental results to determine the energy levels and decay widths of the important states. We have concentrated on the following two cases:

(a) $E_\nu=500$ MeV, $\theta_l=15°$ LWD's wave functions,
(b) $E_\nu=500$ MeV, $\theta_l=30°$ LWD's wave functions.

For purposes of discussion we denote each of the states in the resonance by Ψ_i, and we number the states according to the following convention: states Ψ_1 through Ψ_4 are 1^- states, states Ψ_5 and Ψ_6 are 0^- states, and states Ψ_7 through Ψ_9 are 2^- states; the states are arranged in order of increasing energy within these classes. For example, Ψ_7 is the lowest-energy 2^- state and Ψ_9 is the highest-energy 2^- state. In case (a) states Ψ_2, Ψ_3, Ψ_4, and Ψ_8 have ~80% of the total differential cross section. In case (b) the states Ψ_2, Ψ_3, Ψ_4, Ψ_7, Ψ_8, and Ψ_9 have ~90% of the total differential cross section, so we neglect other states. Some authors have already identified certain of the Brown model states with experimental peaks in photonuclear and electron-scattering cross-section curves of C^{12}. Lewis and Walecka[7] identify Ψ_2 with an experimental state at 22.5 MeV and Ψ_3 with one at 24.5 MeV. They also mention that Ψ_4 may be identified with a possible

[5] G. E. Brown, L. Castillejo, and J. A. Evans, Nucl. Phys. 22, 1 (1961).
[6] V. Gillet and N. Vinh Mau, Nucl. Phys. 54, 321 (1964).
[7] F. H. Lewis and J. D. Walecka, Phys. Rev. 133, B849 (1964).
[8] T. deForest, Phys. Rev. 139, B1217 (1965).
[9] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1960) 2nd ed.

state at ~36 MeV. Überall[10] has identified Ψ_2 with a 22.5-MeV state and Ψ_3 with one at 25.5 MeV. deForest *et al.*,[11] identify Ψ_8 with a state seen in inelastic electron scattering at 19.2 MeV.

For the present calculation, we accept the identification of Refs. 7 and 10 that Ψ_2 occurs at 22.5 MeV, since a broad peak at this energy is seen in the recent experiments of Cook *et al.*,[12] Lochstet and Stephens,[13] and Fultz *et al.*[14] We take the energy of Ψ_3 to be at 25.5 MeV in accordance with Ref. 10 rather than at 24.5 MeV, as in Ref. 7, since 25.5 MeV seems to agree better with the recent experiments mentioned above. We accept the energy value of 19.2 MeV for Ψ_8. For Ψ_4 we take the average of the theoretical energies given by Refs. 6 and 8, which is $\frac{1}{2}(35.8+33.8)=34.8$ MeV. We notice that this assignment may be nearly correct, since there is a bump at ~35.2 MeV in the experimental curve of Fultz *et al.* Experimental evidence for the energies of Ψ_7 and Ψ_9 seems to be lacking, so for these states we take the average of the theoretical values given by Refs. 6 and 8. We obtain the energies of the resonance states in N^{12} by just adding 2.35 MeV Coulomb energy to the resonance-state energies of C^{12}. The energy assignments are summarized in Table I.

In the spirit of the one-particle–one-hole Brown theory, we assume that the excited states of N^{12} decay by emitting a proton through the nuclear surface. The residual C^{11} nucleus then has a hole in either its $1p_{3/2}$ or $1s_{1/2}$ shell. The $(1p_{3/2})^{-1}$-hole state is the ground state of C^{11}; so we may obtain the kinetic energy of this decay by subtracting the ground-state energy of the $C^{11}+p$ system from the energy of the excited states of $N^{12}{}_{\text{gi.res.}}$. If instead the residual nucleus has a hole in the $1s_{1/2}$ shell, then it still has approximately 16 MeV (following Ref. 6) more energy than the ground state of $C^{11}+p$. Only state Ψ_4 in $N^{12}{}_{\text{gi.res.}}$ has sufficient energy to decay to this excited state of $C^{11}+p$. Protons from the decay of Ψ_4 then will be emitted in two groups. One group has high energy and leaves the C^{11} nucleus in its ground state. The second group has lower energy and leaves the C^{11} nucleus in an excited $(1s_{1/2})^{-1}$ state. The kinetic energies of the proton decays of the $N^{12}{}_{\text{gi.res.}}$ are shown in Table I.

Other parameters which are important for predicting the shape of the proton spectrum are the particle-decay widths $\{\Gamma^{(i)}\}$ of the excited states. These were obtained from a mixed analysis of theory and experiment. First the decay widths were calculated using Boeker's[15] method. Boeker's expression can be put in the following convenient form:

$$\Gamma^{(i)}=(11.1/a_c)^2\sum_l P_l(\rho_c)(\gamma'_{jl}{}^{(i)})^2, \tag{8}$$

where $\Gamma^{(i)}$ is obtained in MeV, $P_l(\rho_c)$ is the penetrability of R-matrix theory, the channel radius a_c is measured in fermi, and the reduced widths are

$$\gamma'_{jl}{}^{(i)}=\left|\sum_n (-1)^{n-1}\alpha_{(i)}{}^{(nlj)(n'l'j')^{-1}}\right|. \tag{9}$$

The $\alpha_{(i)}{}^{(nlj)(n'l'j')^{-1}}$ of Refs. 6 and 8 were used for calculating the decay widths. The average values of the two calculations are shown in column four of Table I. In column three of Table I we show the approximate widths of the giant-resonance states found in the experiments discussed previously.

TABLE I. Proton energies and decay widths used in calculating the $N^{12}{}_{\text{gi.res.}}\rightarrow C^{11}+p$ decay spectrum.

i	$E_p{}^{(i)}$ (MeV) assumed	$\Gamma^{(i)}$ (MeV) approx. experimental	$\Gamma^{(i)}$ (MeV) average theoretical	$\Gamma^{(i)}$ (MeV) assumed
2	6.9	3.5	4.6	4.6
3	9.9	2.0	8.4	5.2
4	19.2 and 2.9	3.0	4.7	4.7
7	2.9		7.2	
8	3.6	2.0	2.0	2.0
9	7.9		6.1	6.1

The agreement with experiment is not too bad, which is encouraging because the theory is very rough and the decay of $N^{12}{}_{\text{gi.res.}}$ would naturally be different from that of $C^{12}{}_{\text{gi.res.}}$. For example $N^{12}{}_{\text{gi.res.}}$ should decay by proton emission, whereas $C^{12}{}_{\text{gi.res.}}$ decays by either proton or neutron emission. The largest discrepancy between theoretical and experimental widths occurs for Ψ_3 where the theoretical widths appears to be too great by 6 MeV. A similar discrepancy was found in Ref. 15 for the decay of Ψ_3 in the C^{12} nucleus. We arbitrarily reduce this discrepancy by one-half by using the average of the theoretical and experimental widths for our spectrum plot. The theoretical width of Ψ_7 is very great, and we have no experimental evidence of its value. It is doubtful whether the contribution of Ψ_7 could be resolved at all. For this reason we henceforth expel Ψ_7 from our set of important states to be used in obtaining the proton spectrum. The set of widths adopted for the present calculation of the proton spectra are given in column four of Table I.

IV. NUMERICAL CALCULATIONS

We repeated deForest's[8] calculation of the nuclear matrix elements $(M_V{}^2)_D{}^{(i)}$ and $(M_A{}^2)_{DJ}{}^{(i)}$, which are involved in the reaction $(\mu^-+C^{12}\rightarrow \nu_\mu+B^{12}{}_{\text{gi.res.}})$, to check the accuracy of our reduced matrix-element calculation, since both the muon-capture reaction and our

[10] H. Überall, Nuovo Cimento **38**, 669 (1965).
[11] T. deForest, Jr., J. D. Walecka, G. Vanpraet, and W. C. Barber, Phys. Letters **16**, 311 (1965).
[12] B. C. Cook, J. E. E. Baglin, J. N. Bradford, and J. E. Griffin, Phys. Rev. **143**, 724 (1966).
[13] W. A. Lochstet and W. E. Stephens, Phys. Rev. **141**, 1002 (1966).
[14] S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, Phys. Rev. **143**, 790 (1966).
[15] E. Boeker, thesis, University of Amsterdam, 1963 (unpublished).

TABLE II. Comparison of our numerical values for the $(M_A{}^2)_{DJ}{}^{(i)}$ muon-capture matrix elements with those of Ref. 8.

i	J	$E^{(i)}$ (MeV)	$(M_A{}^2)_{DJ}{}^{(i)}$ Ref. 8	$(M_A{}^2)_{DJ}{}^{(i)}$ Present work
1	1	19.57	0.013	0.0147
2		23.26	0.017	0.0318
3		25.01	0.197	0.1140
4		35.80	0.010	0.0506
5	0	25.66	0.091	0.0366
6		35.78	0.001	0.0351
7	2	18.91	0.011	0.0106
8		20.76	0.210	0.2089
9		23.94	0.059	0.0588

present neutrino reaction depend on the same reduced matrix elements. Our values of $(M_V{}^2)_D{}^{(i)}$ agree well with those of Ref. 8; however, we found that certain values of $(M_A{}^2)_{DJ}{}^{(i)}$ given in Ref. 8 are incorrect because a factor $(-1)^{l'}$ was omitted from the calculation.[16] In Table II we list the correct values of $(M_A{}^2)_{DJ}{}^{(i)}$ and those of Ref. 8 for comparison. It is interesting that the near equality $(M_V{}^2)_D \cong (M_A{}^2)_D$ discussed in Ref. 8 holds to within 1% when our correct values are used, and only within 8% when the values of Ref. 8 are used.

When we were certain that we could calculate the matrix elements correctly, we programmed the Hamiltonian [Eq. (2)] for the computer also. We combined these programs under a main program and were able to calculate the differential cross section of a transition to any final state having energy $E^{(i)}$ and quantum numbers J and M_J.

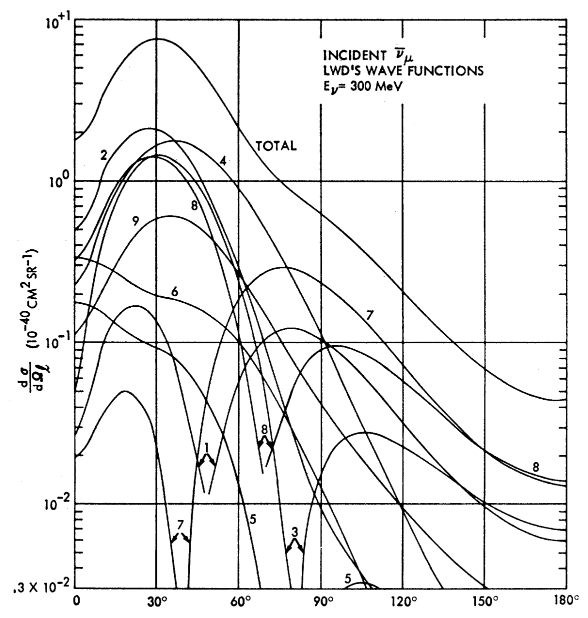

FIG. 1. Differential cross section for muon antineutrinos of 300 MeV calculated using the wave functions of Lewis, Walecka, and deForest (LWD).

[16] We are indebted to Dr. T. deForest for a helpful communication of this point.

We chose the z axis of the coordinate system of the spherical harmonics to be in the $\mathbf{q}$ direction because then the only spherical harmonics in the matrix elements are $Y_{JM}(0,0)$ and $\mathbf{Y}_{JLM}(0,0)$. Also, in Eq. (2) all terms containing products of $\mathfrak{M}_{JM}{}^{(i)}$ and $\mathfrak{M}_{JM}{}^{(i)}$ vanish, since $Y_{JM}(0,0)=0$ for $J=1$ and $M\neq 0$ and $\mathbf{Y}_{JLM}(0,0)=0$ for $J=1$, $L=1$ or 3 and $M=0$.

In addition we numerically evaluated the following quantities:

$$\frac{d\sigma}{d\Omega_l}=\sum_i \frac{d\sigma^{(i)}}{d\Omega_l},$$

$$\sigma^{(i)}=\int \frac{d\sigma^{(i)}}{d\Omega_l}d\Omega_l,$$

and $\sigma=\sum \sigma^{(i)}$. We changed the signs of the appropriate terms in the Hamiltonian to see what difference in cross section occurs between neutrino and antineutrino reactions. We found no difference at all providing that the threshold energies were not changed, because those terms in the Hamiltonian which change sign gave zero when all values of M_J in the final state were summed over. We calculated the widths and plotted the $\partial^2\sigma/\partial E_p\partial\Omega_l$ spectra by hand. We obtained the penetrabilities from standard tables,[17] and we took the $\partial\sigma^{(i)}/\partial\Omega_l$ data from our own computer output.

V. RESULTS

In Fig. 1 we show a graph of $\partial\sigma^{(i)}/\partial\Omega_l$ for $E_\nu=300$ MeV as a function of lepton angle θ_l. The shapes of the $\partial\sigma^{(i)}/\partial\Omega$ curves reflect the behavior of the matrix elements $\mathfrak{M}_{JM}{}^{(i)}$ and $\mathfrak{M}_{JM}{}^{(i)}$. The minima in the $\partial\sigma^{(i)}/\partial\Omega_l$ curves occur for small values of $\mathfrak{M}_{JM}{}^{(i)}$ and $\mathfrak{M}_{JM}{}^{(i)}$, and the tendency above $\theta_l=30°$ for $\partial\sigma^{(i)}/\partial\Omega_l$ to decrease

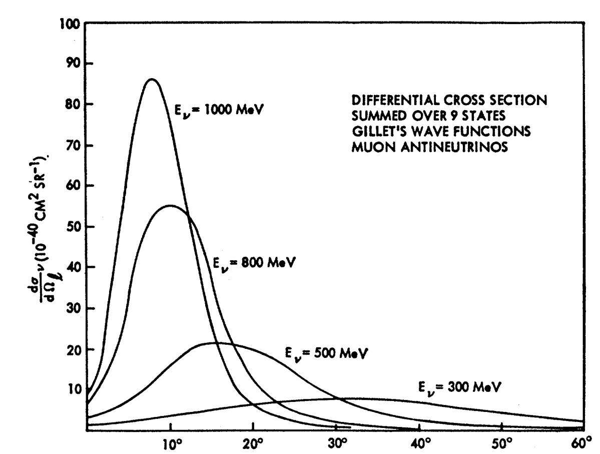

FIG. 2. Differential cross section of muon antineutrino reactions summed over nine final states using Gillet's wave functions.

[17] W. T. Sharp, H. E. Gove, and E. B. Paul, Atomic Energy Commission of Canada, Ltd., Chalk River Report No. AECL 268, 1965 (unpublished).

as θ_l increases is caused by the decrease of these matrix elements as q increases. The curve for $\partial\sigma_\nu/\partial\Omega_l$ does not have the structure seen in the curve of each state. Figure 2 shows graphs of $\partial\sigma/\partial\Omega_l$ for a range of neutrino energies. Here again $\partial\sigma/\partial\Omega_l$ peaks more sharply forward as E_ν increases. This behavior is seen in many kinds of high-energy processes and was also discussed in Ref. 4.

In Fig. 3 we give graphs of

$$\sigma_\nu^{(i)}=\int\frac{d\sigma^{(i)}}{d\Omega_l}d\Omega_l$$

as a function of E_ν. Figure 4 shows the total cross section $\sigma_\nu^{\text{total}}=\sum_i\sigma^{(i)}$ using both $\bar{\nu}_l$ and $\bar{\nu}_\mu$ and LWD's and Gillet's wave functions. It also shows the values of Ref. 4. The $\sigma_\nu^{\text{total}}$ curves using LWD's and Gillet's wave functions are very much alike for both $\bar{\nu}_l$ and $\bar{\nu}_\mu$. They differ quite considerably from the curve of Ref. 4 for $E_\nu<300$ MeV, but for $E_\nu>300$ MeV they agree within 30%.

Figures 5 and 6 show the graphs of

$$\frac{\partial^2\sigma^{(i)}}{\partial E_p\partial\Omega_l}$$

and

$$\frac{\partial^2\sigma^{\text{total}}}{\partial E_p\partial\Omega_l}=\sum_i\frac{\partial^2\sigma^{(i)}}{\partial E_p\partial\Omega_l},$$

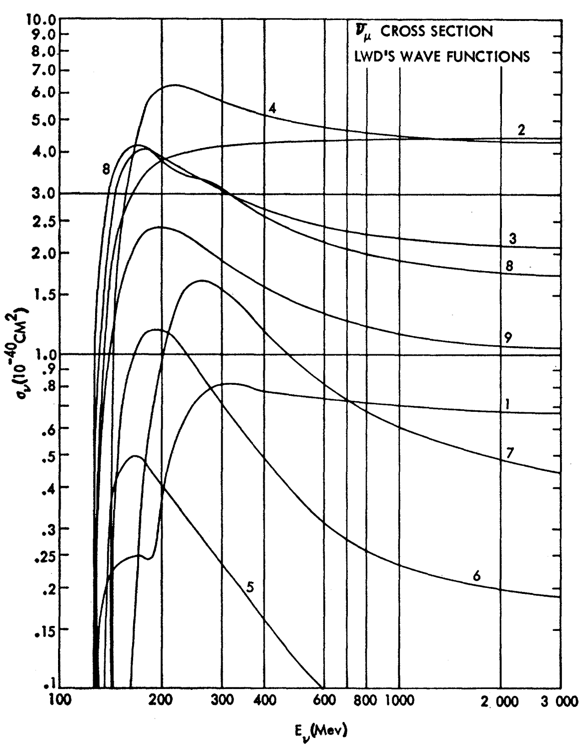

FIG. 3. Total cross sections of muon-antineutrino reactions calculated from LWD's wave functions.

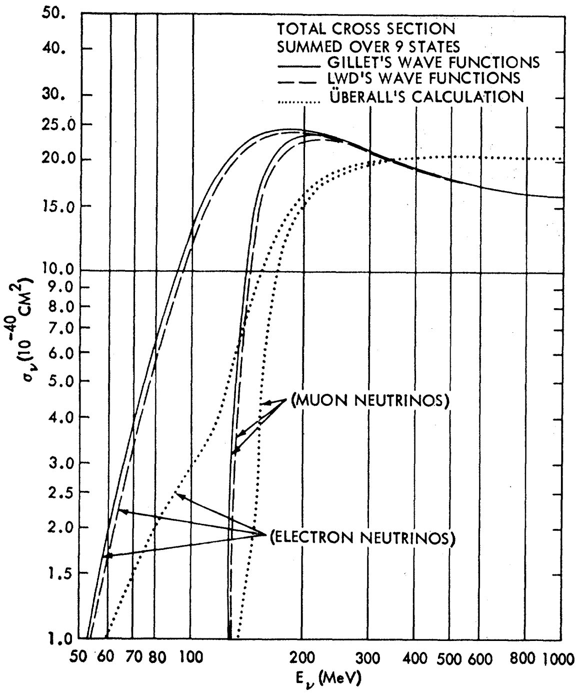

FIG. 4. Comparison of total cross sections calculated using Gillet's and LWD's wave functions with Überall's cross sections.

using wave functions of Ref. 8. In addition they show prominently the curves of $\partial^2\sigma^{(\text{total})}/\partial E_p\partial\Omega_l$ which one gets when the protons from the decay Ψ_4 are dropped from the sum. We are in doubt about the protons from Ψ_4 because the energy of the $(1s)^-$-hole state in C^{11} is not known with great accuracy. Also the low-energy

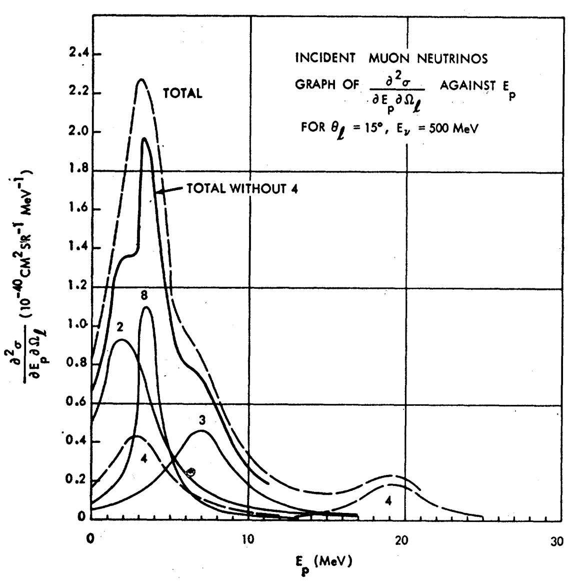

FIG. 5. Graphs of proton spectrum for an incident 500-MeV muon neutrino creating a lepton at 15° from the neutrino direction.

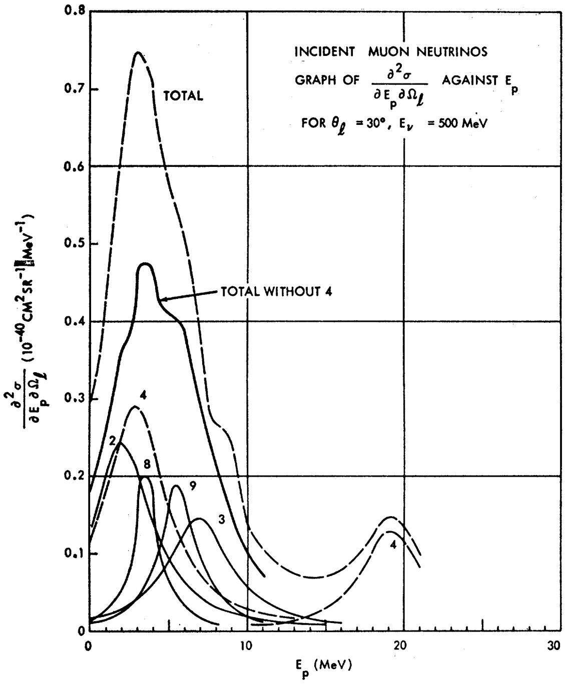

FIG. 6. Graphs of proton spectrum for an incident 500-MeV muon neutrino creating a lepton at 30° to the neutrino direction.

proton decay of Ψ_4 leaves the C^{11} nucleus in an excited state which may cause emission of another particle. If an experiment detects the second particle, it may be harder to interpret the event. In addition, the wave functions of Ref. 6 predict that $\partial\sigma^{(4)}/\partial\Omega_l$ is about 50% of the value that the wave functions of Ref. 8 predict, while the other states keep about the same transition strengths relative to one another. Finally, Gillet's photonuclear calculations predict that Ψ_4 should have more of the strength (19%) of the giant resonance than the small hump in the data of Fultz *et al.*, indicates. Whether we include the $\partial^2\sigma^{(4)}/\partial E_p\partial\Omega_l$ or not, the graphs show that as θ_l increases we see relatively more protons at higher energies, i.e., a change of the proton spectrum takes place. Such a shift of proton numbers with lepton angle can help in identifying the reaction mechanism in the bubble chamber.

VI. DISCUSSION

Although our total cross-section results agree fairly well with those of Ref. 4 for $E_\nu \geq 300$ MeV, we expect that our results overestimate the actual value of the cross section by about a factor of 2. We think that this occurs because many other calculations using the one-particle–one-hole Brown theory with harmonic-oscillator wave functions also predict rates which are approximately twice those seen in experiment. For example, Danos and Fuller[18] discuss this factor-of-2 discrepancy in calculations of photonuclear reactions. Lewis and Walecka[7] have shown that the factor-of-2 discrepancy originates from using approximate wave functions and including experimental energies for the diagonalization of the particle-hole Brown-model Hamiltonian. DeForest[8] states that the $(M_V{}^2)_{\mathrm{UD}}$ that he calculates is about twice the value which Foldy and Walecka[19] predict from an analysis of experimental photonuclear data. Since deForest shows that $(M_A{}^2)_{\mathrm{UD}} \approx (M_V{}^2)_{\mathrm{UD}} \approx (M_P{}^2)_{\mathrm{UD}}$ in the Brown model, he feels that all three matrix elements will be too large by about the same factor of 2. Since our matrix elements are essentially the same as those of deForest, we expect that our results are also probably too large by a factor of 2.

In addition we introduced a smaller source of error by dropping terms in the weak-interaction Hamiltonian of order $p/m = v/c$ in the nucleon's velocity. We can estimate that these terms could give a contribution of $\sim 25\%$ by noting that the highest momentum of a nucleon in a Fermi gas-nuclear model of C^{12} is ~ 250 MeV/c, so that the factor p/m is $\sim 25\%$. If we consider a harmonic-oscillator nucleus having an oscillator spacing $\hbar\omega = 15$ meV, the energy of a nucleon in the third level is 37.5 MeV which would have a classical maximum momentum of ~ 274 MeV/c and (by the virial theorem) an average momentum ~ 194 MeV/c. All of these estimates give a value of $p/m \lesssim 30\%$. Foldy and Walecka,[19] in computing muon-capture rates, estimate that these terms give a total nucleon-recoil contribution of about 10%, and they neglect these terms as was done in Ref. 4. In view of the over-all uncertainty inherent in the Brown model, we feel justified in likewise neglecting these terms.

VII. CONCLUSIONS

1. The wave functions of Refs. 6 and 8 give values of $\sigma(\nu + C^{12} \rightarrow N^{12}{}_{\mathrm{gi.res.}} + l)$ and $\sigma(\bar{\nu} + C^{12} \rightarrow B^{12}{}_{\mathrm{gi.res.}} + \bar{l})$, which agree very well with one another for all values of neutrino energy, and they agree with the values of Ref. 4 within 30% above $E_\nu = 300$ MeV. Also, the total cross sections from all three calculations remain approximately constant with increasing E_ν for $E_\nu \geq 500$ MeV. Thus it seems possible to use this reaction to help check on the spectrum of the machine neutrinos above 500 MeV.

2. For $E_\nu < 300$ MeV the curves of $\sigma(\nu + C^{12} \rightarrow N^{12}{}_{\mathrm{gi.res.}} + l)$ predicted by the wave functions of Refs. 6 and 8 agree very well with one another, but the curve calculated in Ref. 4 using a different nuclear model does not agree well with these two.

3. The wave functions of Refs. 6 and 8 give differential cross sections $(\partial\sigma^{(i)}/\partial\Omega_l)(\nu + C^{12} \rightarrow N^{12}{}_{\mathrm{gi.res.}} + l)$ (where $N^{12}{}_{\mathrm{gi.res.}}$ is in the ith excited state) which show a complex structure. However, when we sum over

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all final states, the $(\partial\sigma^{(\mathrm{total})}/\partial\Omega_l)(\nu+C^{12}\rightarrow N^{12}{}_{\mathrm{gi.res.}}+l)$ curve is very smooth.

4. The graphs of $\partial\sigma^{\mathrm{total}}/\partial\Omega_l$ are more peaked in the forward direction as neutrino energy increases.

5. Except for small effects caused by differences in threshold energies, all of the cross sections calculated for neutrinos are equal to those calculated for antineutrinos. This occurs because the interference terms in the Hamiltonian cancel to zero.

6. In the decay of $N^{12}{}_{\mathrm{gi.res.}}\rightarrow C^{11}+p$, the wave functions of Ref. 8 predict that for greater momentum transfer the proton-spectrum curves shift in strength to higher proton energies.

7. The numerical values for the $(M_A{}^2)_{\mathrm{UD}J}{}^{(i)}$ matrix elements calculated by Ref. 8 for the excitation of the giant-resonance state of C^{12} by muon capture have been corrected by not overlooking the factor $(-1)^{l'}$ in the expression for their value. With this correction the total dipole matrix elements $(M_V{}^2)_{\mathrm{D}}$ and $(M_A{}^2)_{\mathrm{D}}$ are equal within 1%.

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Self-Consistent Perturbation of Hartree-Fock-Bogoliubov Equations and Nuclear Rotational Spectra. II*

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The coefficient $\mathcal{B}$, occurring in the angular-momentum expansion $E_I=\mathcal{A}I(I+1)+\mathcal{B}I^2(I+1)^2+\cdots$ of the energy levels of the ground-state rotational band of even-even deformed nuclei, is numerically calculated on the basis of a microscopic model derived in a previous paper. Numerical estimates of the z factors, which measure the effect of band-mixing on γ-ray branching ratios from vibrational to ground-state bands, are also presented.

I. INTRODUCTION

THE energies of members of the ground-state rotational band of strongly deformed even-even nuclei can be represented by the expansion

$$E_I=\tfrac{1}{2}(\hbar^2/\mathcal{I})I(I+1)+\mathcal{B}I^2(I+1)^2+\cdots \qquad (1)$$

for not too large values of the angular momentum quantum number I. In a previous paper hereafter referred to as I,[1] the author derived expressions for the coefficients $\mathcal{I}$ and $\mathcal{B}$ within the framework of Hartree-Fock-Bogoliubov (HFB) theory. The model consists of a perturbation treatment of the Coriolis force $\mathbf{\Omega}\cdot\mathbf{J}$ in a reference frame rotating with angular velocity $\mathbf{\Omega}$ for stationary solutions of the HFB equations. Deviations from the $I(I+1)$ term in Eq. (1) result mainly from three effects in this model: centrifugal stretching of the deformed field (vibration-rotation interaction), attenuation of the Copper pair correlations (Mottelson-Valatin effect), and perturbation of the independent quasiparticle motion by the Coriolis force. The model is a kind of microscopic analog of the classical macroscopic centrifugal stretching model as formulated by Diamond, Stephens, and Swiatecki,[2] and Moszkowski,[3] but the latter model only includes the stretching, and, because it is entirely phenomenological, necessarily ascribes the entire deviation from the $I(I+1)$ term to this one effect, whereas the former model is capable of assessing the relative importance of all three effects, and, in fact, leads to the result that stretching is of relatively minor importance in most cases.

In the present paper, the microscopic expressions for the quantities $\mathcal{I}$ and $\mathcal{B}$ are numerically evaluated for the case of particles interacting through the quadrupole plus pairing force for several rare-earth and actinide nuclei. Preliminary calculations have been previously reported.[4] Since the appearance of I, several other closely related calculations have been published.[5–7] The relation of these to the present work will be mentioned

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