A-Dependence of Nuclear Charge and Mass Distributions

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A discussion is given of the A dependence of the root-mean-square radii of the nuclear charge and mass distributions both along and at right angles to the valley of maximum stability. In the second case, i.e., for isotopes and isotones, it is pointed out that the charge radii never obey an $A^{1/3}$ dependence even approximately, but that for mass radii the experimental evidence is not in conflict with such a dependence. Attention is also drawn to the differences between the distribution of proton centers and the charge distribution.

I. INTRODUCTION

FOR many years now it has been an article of faith among nuclear physicists that "nuclear radii increase as $A^{1/3}$," and it is the purpose of this article to show the sort of restrictions that ought to be placed on this law. We shall show that for certain size parameters an approximate $A^{1/3}$ law is indeed to be expected and is found experimentally, but that for others no such dependence is to be expected. A mathematically exact $A^{1/3}$ law is of course never expected, and to that extent parametrizations of nuclear radii in terms of an exact $A^{1/3}$ proportionality place an artificial constraint on the choice of parameters. Further it is important to define exactly the size parameter one is discussing. We shall be concerned with root-mean-square radii of the nuclear mass and charge distributions, which will be denoted by R_M and R_C, and with the half-way radius R of an optical or shell-model potential, due to the mass distribution of radius R_M.

II. CHARGE DISTRIBUTION

The most accurately known nuclear size parameter is undoubtedly the root-mean-square radius R_C of the charge distribution along the valley of maximum stability. It has been known for a long time from elastic electron scattering[1] that this increases rather less fast than $A^{1/3}$, and this result has recently been confirmed from experiments on muonic x rays.[2] A semiempirical formula, which assumes that neutrons and protons are equally distributed with a constant central density, but which takes into account the finite thickness of the nuclear surface, fits the data on elastic electron scattering very well, the surface effect leading to exactly the observed amount of departure from the strict $A^{1/3}$ law.[3] Another procedure by which to derive the charge distribution is to build it up from single-particle wave functions in a spherical potential.[4,5] Reference 4 uses an energy-dependent Woods-Saxon potential of half-way radius R, and in order to fit the data it is found that $RA^{-1/3}$ must be allowed to decrease from 1.42 F for Li^6 to 1.30 F for Ca^{40}. If we may assume that R_M and R_C do not differ significantly from each other, a point to which we shall return in the next section, then such a decrease is a natural consequence of the assumption[6] that the potential is related to the mass radius R_M. In contrast, Ref. 5 uses an energy-independent Woods-Saxon well with $R=1.25A^{1/3}$ which gives at best a moderate fit to the same data. The energy dependence, which is of crucial importance when calculating separation energies,[7] is actually not important here, but the assumption of a strict $A^{1/3}$ proportionality leads to lack of agreement with experiment. The point to notice here is that the parameterization of an optical or shell-model potential in terms of a constant $r_0=RA^{-1/3}$ is never strictly justified, although it is valuable to parametrize in terms of an approximately constant r_0 and not allow too wide variations, which would be unphysical. In this way, the analysis of elastic electron scattering[4] shows that r_0 is a slowly decreasing function of A. In other cases, for instance in optical-model analyses of low-energy scattering, the depth of the potential V_0 and its half-way radius R are connected through the well-known V_0R^n ambiguity, and it is then not possible to make any precise statement about R by itself.

While then R_C for nuclei along the valley of maximum stability obeys an $A^{1/3}$ law at least approximately, there is every reason to expect serious departures from the law for different isotopes or isotones, i.e., at right angles to the mass valley. The effect of the Coulomb repulsion here is to make R_C increase less rapidly than $A^{1/3}$ for isotopes and more rapidly for isotones. Quantitatively, this effect has been explained both macroscopically in terms of the simple theory of nuclear compressibility[8] and microscopically in terms of single-particle wave functions with the correct separation energy of the least bound particle.[4,5] The departure from the $A^{1/3}$ law is usually given in terms of

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the quantity

$$\gamma = \frac{3A}{R_C}\frac{dR_C}{dA},$$

where $\gamma=1$ for strict $A^{1/3}$ proportionality. Experimentally it is found that $\gamma \simeq 0.65$ for isotopes of spherical nuclei,[9] while $\gamma \simeq 1.5$ for isotones.[10] These results, which have been obtained from isotope shift and muonic x-ray data, are confirmed[11] by electron scattering data on $Fe^{54,56}$ and $Ni^{58,60}$. The fact that we quite generally obtain $\gamma < 1$ for isotopes and $\gamma > 1$ for isotones was first noticed by Bodmer[11a] and verified[11] from the analysis of the electron scattering data. Additional effects near closed shells have been observed in the value of γ as a function of A for isotones[10], and these lead also to the exceptionally small values of γ for the Ca isotopes, which are observed experimentally.

III. MATTER DISTRIBUTION

For large nuclei, the saturation properties of nuclear forces result in a constant density of nuclear matter in the central region of the nucleus, although the imbalance between neutrons and protons in heavy nuclei lead to exclusion principle and Coulomb repulsion effects, which tend to reduce the central density. So even if we neglect surface effects, a strict $A^{1/3}$ law is not to be expected for R_M, although an approximate one should be valid. There is little experimental evidence to support this latter conclusion, since most experiments have determined the nuclear charge distribution. As these can be fitted on the assumption of a constant central-charge density, the effects due to the exclusion principle and the Coulomb repulsion are likely to be small.

The fact that our conclusion about the $A^{1/3}$ dependence of R_M is a result of the saturation properties of nuclear forces implies that the conclusion should be correct even when we move away from the valley of maximum stability. We therefore investigate the Ca isotopes for which R_C quite definitely does not obey an $A^{1/3}$ law.

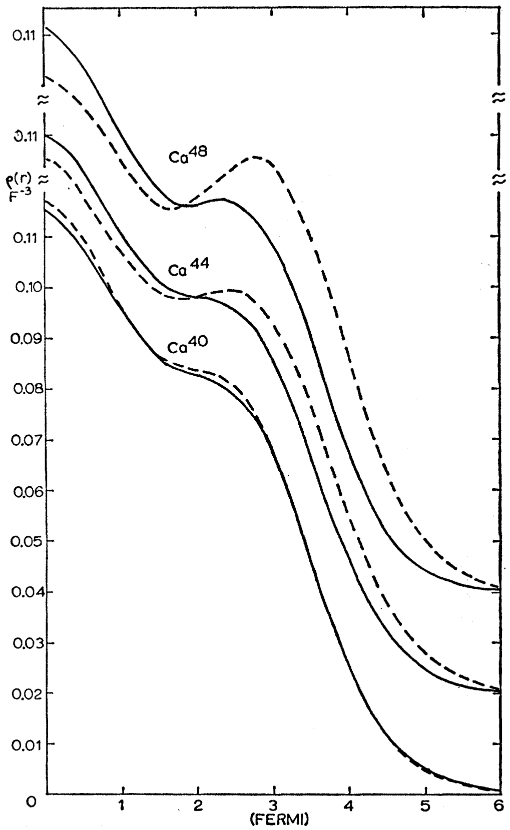

FIG. 1. Proton (solid line) and neutron (dashed line) point densities for $Ca^{40,44,48}$.

We use the procedure in which the charge distribution is built up from single-particle proton wave functions in a spherical potential, so as to fit the electron scattering data.[4,12] We then, with the help of the $(N-Z)/A$ symmetry term in the potential, which has been used for optical potentials by Perey,[13] determine the corresponding neutron potentials. The resulting proton and neutron density distributions are shown in Fig. 1 and the root-mean-square radii of the proton, neutron, and nucleon distributions are given in Table I. It will be seen that the $1f_{7/2}$ neutrons in Ca^{44} and Ca^{48} produce a neutron-rich surface, and that the nuclear root-mean-square radius increases almost exactly as $A^{1/3}$. This confirms earlier results[5] for $Ca^{40,48}$ which were based solely on the energy of the $2p_{3/2}$ neutron state. The accuracy of our results is limited by uncertainties in the application of the symmetry term to finite nuclei. We have assumed it to affect merely the depth of the potential and if this assumption is valid, then reasonable variations in the strength of this term lead to very small changes in the root-mean-square

TABLE I. Root-mean-square radii of the proton, neutron, and nucleon distributions for $Ca^{40,44,48}$. The values of the radius parameter $\langle(5/3)r^2\rangle^{1/2}A^{-1/3}$ for the equivalent homogeneous distribution are likely to be accurate to about ± 0.010.

Nucleus		Ca^{40}	Ca^{44}	Ca^{48}
$\langle r^2\rangle^{1/2}$ (F)	Protons	3.314	3.340	3.293
	Neutrons	3.274	3.525	3.630
	Nucleons	3.294	3.442	3.493
$\langle(5/3)r^2\rangle^{1/2}A^{-1/3}$ (F)	Protons	1.249	1.222	1.168
	Neutrons	1.236	1.290	1.290
	Nucleons	1.243	1.258	1.241

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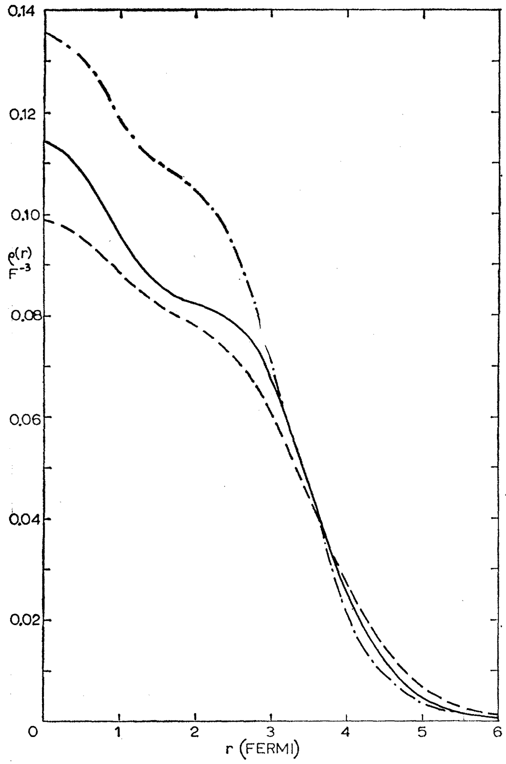

FIG. 2. Proton point density (solid line) and charge density (dashed line) for Ca^{40} compared with a Hartree-Fock calculation (Ref. 28) for the nuclear point density drawn to half scale (dot-dash line).

radii, of the order of 1%. If the symmetry term should be peaked in the surface, as has been found in optical-model analyses of the (p,n) reaction,[14,15] then this would directly affect the nuclear radius and lead to additional uncertainties in its value. It should be stressed, however, that the assumption that the symmetry term in the shell-model potential merely affects the depth of the potential has led to excellent agreement with experiment[4] for the single-particle levels of both protons and neutrons in the $2s$, $1d$, and $1f$ levels in $Ca^{40,48}$. Still, at this stage it is not possible to say more than that the measurements on the charge distributions in the Ca isotopes are not in contradiction with an $A^{1/3}$ dependence of the corresponding mass distributions.

The above conclusions could be confirmed by experiments on nuclear scattering and reactions on the Ca isotopes. Scattering by protons, deuterons, and α particles[16–18] has been analyzed in terms of optical potentials, but in view of the number of parameters involved and the well-known ambiguities such as the above-mentioned one, V_0R^n=constant, it is difficult to obtain definite information in this way. An interpretation of α-particle scattering from $Ca^{40,44}$, based on the diffraction model[19] is at least not in contradiction with the $A^{1/3}$ law.

IV. PROTON AND NEUTRON DISTRIBUTIONS

We now turn to the relative distributions of protons and neutrons in nuclei along the valley of maximum stability. It is generally assumed that these are very similar, except perhaps for the heaviest nuclei, and the evidence from $\pi^{\pm}$ scattering,[20,1] from π^- scattering,[21] from π^0 photoproduction,[22] and from neutron total and reaction cross sections[23] tends to support this. On the other hand, an analysis[24] of the recent experiments[25] on proton scattering at 20 BeV indicates systematically larger nuclear radii than are found in electron scattering, in agreement with theoretical considerations[6] which require a surface neutron skin for nuclei as light as $A\simeq60$. There is also evidence from K^- capture in heavy nuclei[26] that in the extreme tail of the distribution, to which none of the above experiments are sensitive, neutrons predominate. Clearly there is need here for more accurate experiments. What cannot be stressed too strongly is that our results in Sec. III, which indicate a neutron-rich skin for the Ca isotopes, i.e., for nuclei at right angles to the valley of maximum stability, are irrelevant to a discussion of the proton and neutron distributions in nuclei along the valley of maximum stability.

V. POINT AND CHARGE DISTRIBUTIONS

Because of the evidence that neutrons and protons have closely similar distributions in nuclei, it is common to identify the nuclear matter distribution with the nuclear charge distribution, as obtained, say, from electron scattering. What tends to get forgotten is that the latter is not a distribution of nucleon centers, but has the finite proton size folded in. In Fig. 2 we plot the proton point and charge distributions for Ca^{40}, and also one of the two very similar nucleon distributions obtained recently from Hartree-Fock calculations.[27,28]

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This must of course be compared with the nucleon point distribution, which for Ca^{40} is almost exactly twice the proton point distribution. (It is for that reason that in this case we are able to compare the proton distribution directly with the Hartree-Fock nucleon distribution plotted to half-scale.) It will be seen that the agreement is not unsatisfactory, and that both distributions depart very significantly from the so-called Fermi distribution which is flat in the central region. The Fermi distribution is of course a purely phenomenological one and has no deeper justification, in the way that one based on the Hartree-Fock approach may be considered to have. It will also be seen that the surface thickness of the point distribution is some 20% narrower than the surface thickness of the charge distribution. This effect is likely to be independent of A and indicates that the value of the surface thickness obtained from electron scattering should not be taken over uncritically into nuclear-structure calculations.

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He^3+p Elastic Scattering from 12.6 to 15.4 MeV*

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The elastic scattering of protons from He^3 has been studied in a search for a narrow level in Li^4 reported to lie about 10.6 MeV above He^3+p. Data were obtained at laboratory angles of 120° and 150° at proton energies from 12.6–15.4 MeV in 100-keV steps and from 13.84 to 14.74 MeV in overlapping 10-keV steps. Measurements were made relative to the elastic scattering of protons from He^4 by using a mixture of He^3 and He^4 in the gas target. The ratio of the $He^3(p,p)$ yield to the $He^4(p,p)$ yield was smooth to ±0.75% over the entire energy range. In this region an experimental upper limit of 10^{-5} times the Wigner limit was determined for the reduced width of any narrow singlet s-wave resonance.

I. INTRODUCTION

The observation of nine events forming a narrow peak in the energy spectrum of π^- mesons from the decay of $_\Lambda He^4$ was interpreted by Beniston *et al.*[1] as evidence for the possible existence of a narrow level in Li^4 located 10.62±0.20 MeV above He^3+p with a width of 0.23±0.20 MeV. Because of the narrowness of this level, an assignment of $T=2$ was suggested although the events were observed to break up via the $T=1$ He^3+p channel rather than the available $T=2$ channel (see Fig. 1). No confirmation of the existence of such a state was obtained by studies of the $He^3(p,p)$-He^3 excitation function in this region by Dangle *et al.*[2] and by Igo and Leland.[3] However, both of these experiments used energy steps that were larger than their respective target thicknesses and could, therefore, have missed a very narrow resonance. Estimates described in the Appendix indicate that if the resonance were $T=2$, then because of the small kinetic energy available

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