The mean lives of the 677-keV level obtained from measurements with various targets are summarized in Table I. The average of all these measurements is

$$\tau(677 \text{ keV}) = (1.55 \pm 0.30) \times 10^{-13} \text{ sec}.$$

About half of the error arises from uncertainties in the stopping power and in the thicknesses of different target layers.

For the 709-keV level only a lower limit for the lifetime could be established. Assuming an experimental error equal to two standard deviations, this lower limit is

$$\tau(709 \text{ keV}) \geqslant 3.0 \times 10^{-12} \text{ sec}.$$

The energies of the first two levels in ^{30}P adopted from our measurements with various reactions are 677.0±1.0 and 709.0±1.0 keV.

DISCUSSION

The strength of the 677-keV $M1$ transition is related to the transition probability for the β decay $^{30}S \rightarrow {}^{30}$P by the formula given by Kurath.[1,2]

$$\tau = (4.0 \pm 0.4 \times 10^{-13}) \left[1 + 0.2125 \frac{\langle J_f T_f \| l\tau \| J_i T_i \rangle}{\langle J_f T_f \| \sigma\tau \| J_i T_i \rangle} \right]^{-2} \text{ sec}$$

for[12] $\log ft = 4.39 \pm 0.03$. As this is a rather slow transition, the lifetime obtained by ignoring the orbital interaction is much too slow. The ratio of the reduced matrix elements representing the orbital- and spin-dependent interaction, obtained from the above expression by substituting the measured lifetime for τ, might have either of two values: $+2.8 \pm 0.8$ or -12.2 ± 0.8.

Wiechers and Brussaard[13] have calculated the $M1$ transition probabilities for the first two states in ^{30}P using the shell-model wave functions of Glaudemans *et al.*[14] The lifetimes derived from their results are 1.6×10^{-13} sec for the lower state, in excellent agreement with our measurement, and 6.5×10^{-12} sec for the upper state, consistent with our experimental limit.

[12] G. Frick, A. Gallmann, D. E. Alburger, D. H. Wilkinson, and J. P. Coffin, Phys. Rev. **132**, 2169 (1963).
[13] G. Wiechers and P. J. Brussaard, Nucl. Phys. **73**, 604 (1965).
[14] P. W. M. Glaudemans, G. Wiechers, and P. J. Brussaard, Nucl. Phys. **56**, 529 (1964); P. W. M. Glaudemans, G. Wiechers, and P. J. Brussaard, *ibid.* **56**, 548 (1964).

PHYSICAL REVIEW VOLUME 158, NUMBER 4 20 JUNE 1967

Two-Nucleon Emission Process in π^--Meson Absorption

IL-TONG CHEON
Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan
(Received 19 December 1966)

The purpose of the paper is to investigate the effects of the short-range N-N correlations on the branching ratio $W(pn \rightarrow nn)/W(pp \rightarrow np)$, on the distribution of the absorption rate with respect to the projection of the openintg angle between the emitted nucleons, and on the total absorption rate in negative-pion absorption by C^{12} nuclei.

1. INTRODUCTION

As a useful method of getting some interesting information on nuclear structure, the absorption of stopped negative π mesons has attracted the attention of many theorists and experimentalists.[1–3] The information obtained from a study of the pion-capture process is as follows:

(1) The negative-pion absorption at rest occurs preferentially on a proton in a strongly correlated nucleon pair, and this ejects two nucleons out of the nucleus, leaving a two-hole excitation behind. Since the transition rate to a shell-model state of two holes has a sensitive state dependence, appreciable information about the two-hole states, in particular their spins and orbital angular momenta, is obtainable from an investigation of this transition process.

(2) Since the pion is absorbed at rest, the momentum distribution of the nucleons in the nucleus can be obtained by measuring the momenta of the ejected nucleons.

(3) From the energy and momentum conservation law, in the case of the π^- absorption by a single nucleon in the nucleus, the elementary processes $\pi^- + p \rightarrow n + \gamma$ and $\pi^- + p \rightarrow n + \pi^0$ are possible. However, in the case of nucleonic absorption (nonradiative and nonmesonic capture) the π^- mesons are dominantly absorbed by a strongly correlated nucleon pair in the nucleus, i.e., $\pi^- + p + n \rightarrow n + n$ or $\pi^- + p + p \rightarrow n + p$. This is con-

[1] T. Ericson, Phys. Letters **2**, 278 (1962); Il-T. Cheon, C. Nguyen-Trung, and Y. Sakamoto, *ibid.* **19**, 232 (1965); R. M. Spector, Phys. Rev. **134**, B101 (1964); S. G. Eckstein, *ibid.* **129**, 413 (1963); H. Byfield, J. Kessler, and L. M. Lederman, *ibid.* **86**, 17 (1952); M. S. Kozadayev, M. M. Kulyukin, T. M. Sylyayev, A. A. Fillippov, and Yu. A. Shcherbakov, Zh. Eksperim. i Teor. Fiz. **38**, 409 (1960) [English transl.: Soviet Phys.—JETP **11**, 300 (1960)]; A. T. Varfolomeev, Zh. Eksperim. i Teor. Fiz. **42**, 725 (1962) [English transl.: Soviet Phys.—JETP **15**, 496 (1962)].
[2] Il-T. Cheon, Nucl. Phys. **79**, 657 (1966).
[3] Il-T. Cheon, Phys. Rev. **145**, 794 (1966).

firmed by experimental evidence.[4] Since the two nucleons share between themselves the available energy (139.6 MeV, the pion rest mass) released in the pion capture, they acquire a relative momentum of about 730 MeV/c. From this it is concluded that the capture process occurs in relative distances of the order of 0.5 F. On the other hand, the range of the repulsive core of the nuclear force is of the same order. Therefore, the short-range *N-N* correlations due to the repulsive core of the nuclear force give to the pion capture the important effects shown in Ref. 1. When the nuclear correlations are not considered, the relative-momentum distributions of the ejected nucleons are shown by simple curves. However, a large dip appears in the distributions because of the effects of nuclear correlations. As the strength of the nuclear correlations increases, the depth of the dip becomes larger and its position shifts toward the low-momentum region. This means that the high-momentum parts of the absorption rate increase with the effects of the nuclear correlations. Furthermore, the relative angular distributions of the π^--meson absorption rates vary in large angles under the effects of nuclear correlations. This fact will be shown in Sec. 3. On the other hand, the pion capture gives us important information on the short-range *N-N* correlations.

(4) Information is obtained on the mechanism of the reaction, such as the ratio of the radiative to the nucleonic capture, the one- and two-nucleon ejection probability, the branching ratio $W(pn\rightarrow nn)/W(pp\rightarrow np)$, and so on. The ratio of the probabilities of π^--meson absorption by np and pp pairs depends on the effective nucleon-nucleon forces inside the nucleus. Since our understanding of nuclear structure depends on a knowledge of these forces and there is as yet no basis to assert *a priori* that they are different from the analogous forces for free nucleons, the determination of the ratio of the probabilities of π^--meson absorption by np and pp pairs presents an extremely important problem.

The purpose of this paper is to investigate the effects of the short-range *N-N* correlations on the branching ratio $W(pn\rightarrow nn)/W(pp\rightarrow np)$, on the distribution of the absorption rate with respect to the projection of the opening angle between the emitted nucleons, and on the total absorption rate in negative-pion absorption by C^{12} nuclei.

2. DERIVATION OF THE ABSORPTION RATES

The π-*N* interaction is taken to be the ordinary pseudovector interaction

$$\frac{f\hbar}{\mu c}\bar{\psi}_N\gamma_5\gamma_\mu\tau\psi_N\partial_\mu\phi\,, \tag{1}$$

where the coupling constant $f^2/4\pi\hbar c=0.08$, and the pion field is

$$\phi=\frac{c\hbar}{(2EL^3)^{1/2}}\exp\left[\frac{i}{\hbar}(\mathbf{p}_\pi\cdot\mathbf{r}-Et)\right].$$

In the nonrelativistic approximation this is[5]

$$\frac{f\hbar}{\mu c}\left[\bar{\psi}_N\sigma_k\tau\psi_N\partial_k\phi+\frac{i}{2Mc}\bar{\psi}_N\boldsymbol{\sigma}\cdot(\mathbf{p}+\mathbf{p}')\tau\psi_N\partial_4\phi\right]. \tag{2}$$

Since the process under consideration is the absorption of a *stopped* pion, $\mathbf{p}_\pi=0$, we can rewrite the π-*N* interaction as follows:

$$G\left[\bar{\psi}_N\tau\psi_N\boldsymbol{\sigma}\cdot\nabla_\pi\phi-\frac{\mu}{M}\bar{\psi}_N\tau\boldsymbol{\sigma}\cdot\nabla_N\psi_N\phi\right], \tag{3}$$

where

$$G=\frac{f\hbar}{\mu c}\frac{\hbar c}{(2\mu c^2)^{1/2}}.$$

In the interaction (3) the first term corresponds to pion capture from the $2P$ orbit and the second one to that from the $1S$ orbit. We assume that the pion is captured from the $2P$ orbit by C^{12} nuclei.

The Hamiltonian describing the π^--meson absorption by the strongly correlated nucleon pair is given in the form

$$\begin{aligned}\mathcal{H}^{NN\pi}&=G\sum_{i=1}^{2}\tau^-(i)\boldsymbol{\sigma}(i)\cdot\nabla(i)\phi^-(i)\\&=G[\tfrac{1}{2}(T^-\mathbf{S}+\tau^-\boldsymbol{\sigma})\cdot(\nabla_R\Phi^-+\nabla_r\phi^-)\\&\qquad+(T^-\boldsymbol{\sigma}+\tau^-\mathbf{S})\cdot(\tfrac{1}{4}\nabla_R\phi^-+\nabla_r\Phi^-)],\end{aligned} \tag{4}$$

by separating the center-of-mass and relative motions of the nucleon pair in the nucleus, where

$$\begin{aligned}&\nabla_R=\nabla(1)+\nabla(2)\,, &&\nabla_r=[\nabla(1)-\nabla(2)]/2\,,\\&\mathbf{S}=[\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)]/\sqrt{2}\,, &&\boldsymbol{\sigma}=[\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)]/\sqrt{2}\,,\\&T=[\tau(1)+\tau(2)]/\sqrt{2}\,, &&\tau=[\tau(1)-\tau(2)]/\sqrt{2}\,,\\&\Phi=[\phi(1)+\phi(2)]/2\,, &&\phi=\phi(1)-\phi(2)\,,\end{aligned}$$

and

$$T^-=(T_x-iT_y)/\sqrt{2}\,.$$

The wave function of the initial nucleus is given on the basis of the *L-S* coupling scheme

$$\begin{aligned}|i\rangle=\sum&(L_cM_c\lambda\mu|L_0M_0)(S\nu S'\nu'|S^0M_{S^0})\\&\times(T_1\Lambda_1T'\Lambda'|T\Lambda)\sum\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle\\&\times(lmLM|\lambda\mu)\Phi_c(L_cS'T')\\&\qquad\times\psi_{NLM}(\mathbf{R})\varphi_{nlm}(\mathbf{r})X_{S\nu}\chi_{T_1\Lambda_1}\,,\end{aligned} \tag{5}$$

where (|) and ⟨ | ⟩ are Clebsch-Gordan coefficients and transformation brackets; $\Phi_c(L_cS'T')$ is the wave

[4] S. Ozaki, R. Weinstein, G. Glass, E. Loh, L. Neimala, and A. Wattenberg, Phys. Rev. Letters 4, 533 (1960).

[5] For example, see J. J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1964), p. 27.

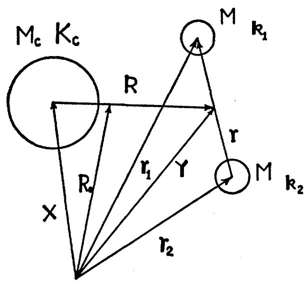

FIG. 1. The scheme of the initial or final states.

TABLE I. Spin and parity of target nucleus C^{12}.

				J_{pair}^P	λ
C^{12}	Be^{10} $J^P=0^+$	+	(pp) $S=0$	0^+	0
$J^P=0^+$	B^{10} $J^P=3^+$	+	(pn) $S=1$	3^+	2

function of the core nucleus; $\psi_{NLM}(\mathbf{R})$ and $\varphi_{nlm}(\mathbf{r})$ are the harmonic-oscillator wave functions of a nucleon pair for the motion of their center of mass and their relative motion; and X and χ represent the spin and isotopic spin wave functions of the pair. For the radial wave function of the relative motion of a nucleon pair in the nucleus, we can take account of the effect of correlations due to a repulsive core in the nucleon-nucleon interaction.[2,6] From the selection rules included in transformation brackets, $2n_1+2n_2+l_1+l_2=2n+2N+l+L$ and $\mathbf{l}_1+\mathbf{l}_2=\mathbf{l}+\mathbf{L}=\boldsymbol{\lambda}$, and spin-parity conservation, the quantum numbers of the pair coupled to the core nucleus are selected. The allowed quantum numbers are given in Tables I and II.

The ground state of C^{12} is constructed of $1s$-state nucleons and $1p$-state nucleons in the shell-model scheme. There are three ways to form the pair from the nucleons in these states.

Case 1: The pair is formed by two nucleons in the $1s$-state. From the selection rules, $2n_1+2n_2+l_1+l_2=2n+2N+l+L$ and $\mathbf{l}_1+\mathbf{l}_2=\mathbf{l}+\mathbf{L}=\boldsymbol{\lambda}$, the quantum numbers which the pair can take are $n=N=l=L=\lambda=0$. Therefore, it is concluded from Table I that this case contributes to the pion absorption by the relative S-state (pp) pair.

Case 2: The pair is formed by a nucleon in the $1s$ state and one in the $1p$ state. We get $\lambda=1$ by the selection rules. In the model under consideration such a pair does not exist in the ground state of C^{12}.

Case 3: The pair is formed by two nucleons in the $1p$ state. Under the same consideration of the quantum numbers for the pair, the pions can be captured by the relative S- and D-state (pn) pair and the relative S-state (pp) pair. In a similar way the final state is given as follows:

$$|F\rangle=V^{-1}\exp[i(\mathbf{K}_c\cdot\mathbf{X}+\mathbf{K}\cdot\mathbf{Y}+\mathbf{k}\cdot\mathbf{r})]\times\Phi_x(L_cS'T')\mathrm{X}_{S''\nu''}\chi_{T''\Lambda''},$$

where

$$\mathbf{k}=(\mathbf{k}_1-\mathbf{k}_2)/2,\qquad \mathbf{K}=\mathbf{k}_1+\mathbf{k}_2,$$
$$\mathbf{Y}=(\mathbf{r}_1+\mathbf{r}_2)/2,\qquad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,$$

and $\mathbf{K}_c$ and $\mathbf{X}$ are the recoil momentum and position vector of the residual nucleus. Using the coordinates given in Fig. 1 and considering momentum conservation, $\mathbf{K}+\mathbf{K}_c=0$, we can rewrite the wave function of the final state in the form

$$|F\rangle=V^{-1}\exp\left[i(\mathbf{K}_c+\mathbf{K})\cdot\mathbf{R}_0+i\left(\frac{M_c\mathbf{K}}{M_c+2M}-\frac{2M\mathbf{K}_c}{M_c+2M}\right)\mathbf{R}+i\mathbf{k}\cdot\mathbf{r}\right]\times\Phi_x(L_cS'T')\mathrm{X}_{S''\nu''}\chi_{T''\Lambda''}$$
$$=V^{-1}\exp(i\mathbf{K}\cdot\mathbf{R}+i\mathbf{k}\cdot\mathbf{r})\Phi_x(L_cS'T')\mathrm{X}_{S''\nu''}\chi_{T''\Lambda''},\quad(6)$$

where $\mathbf{R}_0=(M_c\mathbf{X}+2M\mathbf{Y})/(M_c+2M)$ and $\mathbf{R}=\mathbf{Y}-\mathbf{X}$. Here we assume the zero-range absorption, that a pion is absorbed by a nucleon pair at the center of mass, i.e., $\phi(\mathbf{r}_1)=\phi(\mathbf{r}_2)=\phi(\mathbf{R})$. Since the π-meson wave function for the $2P$ state, given in the well-known form[3])

$$\phi(\mathbf{R})=\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Z}{\sqrt{3}a_0}R\exp\left(-\frac{Z}{2a_0}R\right)Y_{1\nu}(\hat{R})\simeq\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Z}{\sqrt{3}a_0}RY_{1\nu}(\hat{R}),\quad(7)$$

varies slowly in the region of the nucleus, this assumption is reasonable. In (7) the approximation to the second line is made under the condition $ZR_0/a_0\ll1$ (with R_0 standing for the nuclear radius) which is fulfilled for light nuclei.

Thus we can calculate the matrix elements which describe π^--meson absorption by the nucleon pair using the method of Refs. 2 and 3. Using a correlation function of the type

$$f(r)=1-\exp(-\xi r^2),$$

we get the following results:

TABLE II. The allowed quantum numbers of the nucleon pair.

n	N	l	L	λ
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	0	2	2
0	0	2	0	2

[6] G. M. Shklyarevskii, Zh. Eksperim. i Teor. Fiz. **45**, 698 (1963) [English transl.: Soviet Phys.—JETP **18**, 480 (1964)]; R. I. Jibuti and T. I. Kopaleishvili, Nucl. Phys. **55**, 337 (1964).

(a) For pion capture by the relative D-state (pn) pair,

$$\langle F|\mathcal{H}^{NN\pi}|I;\,pn^D\rangle=\frac{G}{V}\frac{4\sqrt{3}\pi}{2\sqrt{20}}\left(\frac{Z}{a_0}\right)^{5/2}\sum_m(-)^m Y_{2m}(\hat{k})B_0\left(\frac{\pi}{30}\right)^{1/2}\frac{1}{\gamma}\left(\frac{1}{\gamma\alpha}\right)^{3/4}\times k^2\exp\left(-\frac{K^2}{4\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{7/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]. \quad (8)$$

(b) For pion capture by the relative S-state (pn) pair,

$$\langle F|\mathcal{H}^{NN\pi}|I;\,pn^S\rangle=\frac{G}{V}\frac{4\sqrt{3}\pi}{2\sqrt{20}}\left(\frac{Z}{a_0}\right)^{5/2}\sum_M(-)^M Y_{2M}(\hat{K})A_0\left(\frac{\pi}{30}\right)^{1/2}\frac{1}{\alpha}\left(\frac{1}{\gamma\alpha}\right)^{3/4}\times K^2\exp\left(-\frac{K^2}{4\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]. \quad (9)$$

(c) For pion capture by the relative S-state (PP) pair formed by $1p$-state protons,

$$\langle F|\mathcal{H}^{NN\pi}|I;\,pp_P\rangle=\frac{G}{V}\sum_{\Lambda'}(11T'\Lambda'|T\Lambda)\left(\frac{Z}{a_0}\right)^{5/2}\frac{\pi}{2\sqrt{2}}\left(\frac{1}{\gamma\alpha}\right)^{3/4}(\delta_{\nu''-1}+\delta_{\nu''0}+\delta_{\nu''1})$$
$$\times\exp\left(-\frac{K^2}{4\alpha}\right)\left\{A_0\left(\frac{3}{2}\right)^{1/2}\left(1-\frac{K^2}{3\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]\right.$$
$$\left.-A_1\left[\left(1-\frac{k^2}{3\gamma}\right)\exp\left(-\frac{k^2}{4\gamma}\right)+\left(\left(\frac{\gamma}{\eta}\right)^{3/2}-2\left(\frac{\gamma}{\eta}\right)^{5/2}+\left(\frac{\gamma}{\eta}\right)^{7/2}\frac{k^2}{3\gamma}\right)\right]\exp\left(-\frac{k^2}{4\eta}\right)\right\}. \quad (10)$$

(d) For pion capture by the relative S-state (pp) pair formed by $1s$-state protons,

$$\langle F|\mathcal{H}^{NN\pi}|I;\,pp_S\rangle=\frac{G}{V}\sum_{\Lambda'}(11T'\Lambda'|T\Lambda)\left(\frac{Z}{a_0}\right)^{5/2}\frac{\pi}{2}\left(\frac{1}{\gamma\alpha}\right)^{3/4}A_0(\delta_{\nu''-1}+\delta_{\nu''0}+\delta_{\nu''1})\times\exp\left(-\frac{K^2}{4\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]. \quad (11)$$

In (8), (9), (10), and (11), $\eta=\gamma+\xi$,

$$\frac{1}{A_0^2}=1-2^{5/2}\left(2+\frac{\xi}{\gamma}\right)^{-3/2}+\left(1+\frac{\xi}{\gamma}\right)^{-3/2},$$

$$\frac{1}{A_1^2}=\left[1-2^{5/2}\left(2+\frac{\xi}{\gamma}\right)^{-3/2}+\left(1+\frac{\xi}{\gamma}\right)^{-3/2}\right]-2\left[1-2^{7/2}\left(2+\frac{\xi}{\gamma}\right)^{-5/2}+\left(1+\frac{\xi}{\gamma}\right)^{-5/2}\right]+\frac{5}{3}\left[1-2^{9/2}\left(2+\frac{\xi}{\gamma}\right)^{-7/2}+\left(1+\frac{\xi}{\gamma}\right)^{-7/2}\right],$$

and

$$\frac{1}{B_0^2}=1-2^{9/2}\left(2+\frac{\xi}{\gamma}\right)^{-7/2}+\left(1+\frac{\xi}{\gamma}\right)^{-7/2},$$

which denote the shifts of the normalization constants due to introduction of the correlation function $f(r)$ into the harmonic-oscillator wave functions describing the relative motions of the nucleon pairs. If the nuclear correlations are neglected, i.e., $\xi\rightarrow\infty$, then $A_0=A_1=B_0=1$. The parameters γ and α are the constants of the harmonic-oscillator wave functions for the relative and center-of-mass motions of the pair. They are determined from the electron-scattering data, $\alpha=4\gamma=0.374$ F^{-2}.[7]

[7] R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957); Rev. Mod. Phys. 28, 214 (1956).

Thus the absorption rates are given by the Golden rule

$$W=\frac{2\pi}{\hbar}\int \sum_{\mathrm{av}} |\langle F|\mathcal{H}^{NN\pi}|I\rangle|^2 \rho_f\,,$$

where $\sum_{\mathrm{av}}$ denotes the average on the final state,

$$\rho_f dE_f=\frac{V^2}{(2\pi)^6}d\mathbf{k}d\mathbf{K}\,,$$

and

$$E_f=\frac{\hbar^2k^2}{M}+\frac{\hbar^2K^2}{4M}+\frac{\hbar^2K^2}{2M_c}\,.$$

In order to get the distribution of the opening angle between the emitted nucleons, it is convenient to use the coordinate system given in Fig. 2 in integration with respect to $\mathbf{k}$ and $\mathbf{K}$.

Under the assumption that the energy is shared equally between two emitted nucleons in the pion absorption, we have $|\mathbf{k}_1|=|\mathbf{k}_2|$, in other words $\beta=\pi/2$ in Fig. 2. When $\beta=\pi/2$,

$$\cos\Theta=\left[\left(\frac{1}{4}+\frac{M}{2m}\right)K^2-D\right]\Big/\left[\left(\frac{1}{4}-\frac{M}{2m}\right)K^2+D\right],$$

where

$$\frac{1}{m}=\frac{1}{2M}+\frac{1}{M_c}\,,\quad D=(M/\hbar^2)(\mu c^2-B)\,,$$

with B the separation energy.

The distributions of the absorption rates with respect to the projection of the opening angle Θ between the emitted nucleons are, then, given as follows:

(a′) For pion capture by the relative D-state (pn) pair,

$$\frac{dW(pn^D)}{d\cos\Theta}=\frac{\Gamma}{20}C(pn^D)\frac{B_0^2}{\gamma^2}\exp\left(-\frac{K^2}{2\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{7/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]^2\frac{k^6}{D}\left[\left(\frac{1}{4}-\frac{M}{2m}\right)K^2+D\right]^2. \quad (12)$$

(b′) For pion capture by the relative S-state (pn) pair,

$$\frac{dW(pn^S)}{d\cos\Theta}=\frac{\Gamma}{20}C(pn^S)\frac{A_0^2}{\alpha^2}\exp\left(-\frac{K^2}{2\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]^2\frac{K^4k^2}{D}\left[\left(\frac{1}{4}-\frac{M}{2m}\right)K^2+D\right]^2. \quad (13)$$

(c′) For pion capture by the relative S-state (pp) pair formed by $1p$-state protons,

$$\frac{dW(pp_P{}^S)}{d\cos\Theta}=\frac{\Gamma}{4}C(pp_P{}^S)\exp\left(-\frac{K^2}{2\alpha}\right)\left\{A_0\left(\frac{3}{2}\right)^{1/2}\left(1-\frac{K^2}{3\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]\right.$$
$$\left.-A_1\left(1-\frac{k^2}{3\gamma}\right)\exp\left(-\frac{k^2}{4\gamma}\right)-A_1\left[\left(\frac{\gamma}{\eta}\right)^{3/2}-2\left(\frac{\gamma}{\eta}\right)^{5/2}+\frac{k^2}{3\gamma}\left(\frac{\gamma}{\eta}\right)^{7/2}\right]\exp\left(-\frac{k^2}{4\eta}\right)\right\}^2\frac{k^2}{D}\left[\left(\frac{1}{4}-\frac{M}{2m}\right)K^2+D\right]^2. \quad (14)$$

(d′) For pion capture by the relative S-state (pp) pair formed by $1s$-state protons,

$$\frac{dW(pp_S{}^S)}{d\cos\Theta}=\tfrac{1}{2}\Gamma C(pp_S{}^S)A_0^2\exp\left(-\frac{K^2}{2\alpha}\right)\left[\exp\left(-\frac{k^2}{4\gamma}\right)-\left(\frac{\gamma}{\eta}\right)^{3/2}\exp\left(-\frac{k^2}{4\eta}\right)\right]^2\frac{k^2}{D}\left[\left(\frac{1}{4}-\frac{M}{2m}\right)K^2+D\right]^2. \quad (15)$$

The definition of Γ is

$$\Gamma=\frac{G^2M}{16\pi\hbar^3}\left(\frac{Z}{a_0}\right)^5\left(\frac{1}{\gamma\alpha}\right)^{3/2},$$

and C denotes the weights;

$$C(pn^D)=C(pn^S)=3NZ/4=12\,,$$
$$C(pp_P{}^S)=Z(Z-1)/4=3\,,$$
$$C(pp_S{}^S)=Z(Z-1)/4=\tfrac{1}{2}\,.$$

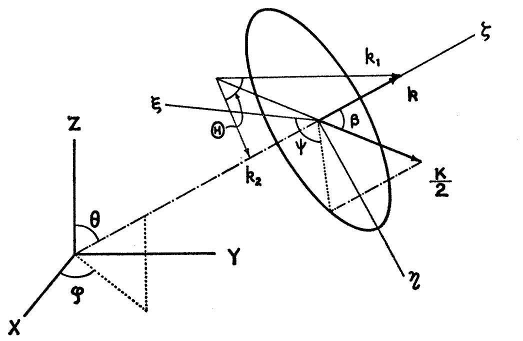

FIG. 2. Coordinates of the momenta, **k** and **K**, of the two ejected nucleons.

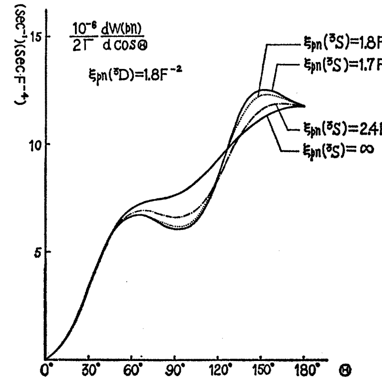

FIG. 3. Distributions of the absorption rate with respect to the projection of the opening angle between the emitted nucleons for various values of the correlation parameter $\xi_{pn}(^3S)$ when $\xi_{pn}(^3D)$ is fixed to be 1.8 F^{-2}.

3. NUMERICAL RESULTS AND DISCUSSION

In this section it is shown numerically how the short-range N-N correlations influence the branching ratio $W(pn\rightarrow nn)/W(pp\rightarrow np)$, the distributions of the absorption rate with respect to the projection of the opening angle between the emitted nucleons, and the total absorption rate in negative-pion absorption by C^{12} nuclei; the results obtained by our theory are compared with the experimental values.

A. The Angular Distribution of the Absorption Rates

The angular distributions of the absorption rates for (pn) pair are shown in Figs. 3 and 4. They are sensitive at large angles to the correlation parameters. When the correlation parameter for the triplet D-state (pn) pair is fixed, $\xi_{pn}(^3D)=1.8\ F^{-2}$; as the value of the correlation parameter for the triplet S-state (pn) pair $\xi_{pn}(^3S)$ becomes larger, the peak existing near 150° shifts to a larger angle, its height goes lower, and the depth of the dip at 90° becomes shallower. Increase of the value of the correlation parameter $\xi_{pn}(^3D)$ makes not only the position of the peak existing near 150° shift to larger angles but makes its height go higher for the fixed value of the correlation parameter $\xi_{pn}(^3S)=1.7\ F^{-2}$. It is known from Figs. 3 and 4 that the angular distributions are scarcely influenced by the short-range N-N correlations in angles smaller than 50°. The experimental values shown in Fig. 4 were obtained by Demidov *et al.*[8] The theoretical values are somewhat larger compared with the experimental values between 50° and 90° and there is, further, a discrepancy at 170°. The ratios of the capture rates at 180° and 90° are given in Table III. Our theoretical results are obtained by using the values $\xi_{pn}(^3S)=1.7\ F^{-2}$, $\xi_{pn}(^3D)=1.8\ F^{-2}$, and $\xi_{pp}(^1S)=1.6\ F^{-2}$. The angular distributions for pion absorption by the pp apir are shown in Fig. 5 for each value of the correlation parameter $\xi_{pp}(^1S)$. The values of separation energy of the nucleons in the single-particle energy state are taken from the report of Pugh and Riley.[9] Finally, it is worthwhile to note that the experimental errors are fairly large.

TABLE III. The ratio of the capture rate at 180° to that at 90° $W(NN;180°)/W(NN;90°)$. Our results are obtained for $\xi_{pn}(^3S)=1.7F^{-2}$, $\xi_{pn}(^3D)=1.8\ F^{-2}$, and $\xi_{pp}(^1S)=1.6\ F^{-2}$.

$\dfrac{W(pn\rightarrow nn;180°)}{W(pn\rightarrow nn;90°)}$	$\dfrac{W(pp\rightarrow np;180°)}{W(pp\rightarrow np;90°)}$
4.0[a]	...
6[b]	12.5
1.9[c]	5.5

[a] Reference 8. [b] Reference 4. [c] Present work.

B. The Branching Ratio $W(pn\rightarrow nn)/W(pp\rightarrow np)$

As was mentioned previously, since the π^- mesons are absorbed by the strongly correlated nucleon pair, the ratio of the probabilities of pion capture by pn and pp pairs depends on the effective nuclear forces in the nucleus. Therefore, the determination of the ratio presents an extremely important problem in the investigation of the property of the effective nuclear forces and of the nucleon wave function in the nucleus.

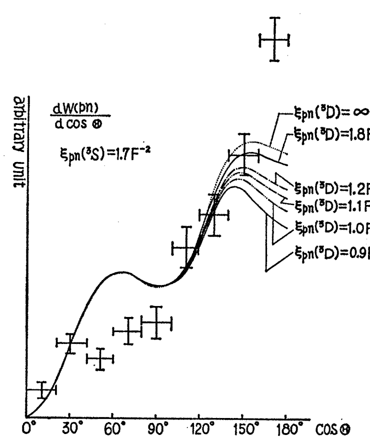

FIG. 4. Distributions of the absorption rate with respect to the projection of the opening angle between the emitted nucleons for various values of the correlation parameter $\xi_{pn}(^3D)$ when $\xi_{pn}(^3S)$ is fixed to be 1.7 F^{-2}.

[8] V. S. Demidov, V. G. Kirillov-Ugryumov, A. K. Ponosov, V. P. Protasov, and F. M. Sergeev, Zh. Eksperim. i Teor. Fiz. 44, 1144 (1963) [English transl.: Soviet Phys.—JETP 17, 773 (1963)].

[9] H. G. Pugh and K. F. Riley, in *Proceedings of the Rutherford Jubilee International Conference, 1961*, edited by J. B. Berks (Academic Press Inc., New York, 1961), p. 165.

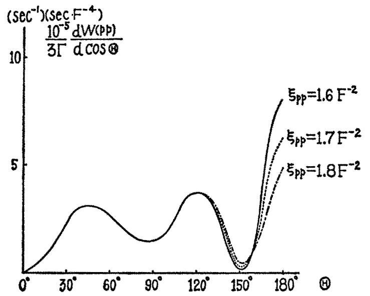

FIG. 5. Distributions of the absorption rate with respect to the projection of the opening angle between the emitted nucleons for various values of the correlation parameter $\xi_{pp}(^1S)$.

Using scintillation counters, Ozaki *et al.* evaluated this ratio for π^- capture by carbon nuclei.[4] Measurements were made with the counters at 180° to each other. The ratio obtained was

$$R=W(pn)/W(pp)=5.0\pm1.5.$$

It is, however, pointed but by Fedotov[10] that this work is subject to valid criticism from a methodological point of view. In view of the low threshold for nucleons ($\sim$9 MeV), coincidences could be recorded which were due to neutrons from the decay of the residual nucleus. Fedotov determined the ratio R on the basis of the consideration that practically all nucleons of the pair with energies above 30 MeV are primary, i.e., nucleons of the pair which captured the π^- meson. His result is

$$R=4\pm1.3.$$

The theoretical results can be obtained by putting $\Theta=180°$ in Eqs. (12), (13), (14), and (15). It is immediately known from Eq. (13) that the relative S-state pn pair does not contribute to the probabilities of the back-to-back emissions. The results are shown in Fig. 6. The ratio depends sensitively on the nuclear correlations. The sensitivity of the ratio is stronger for the nuclear correlations in the triplet state than singlet state. From Fig. 4, $\xi_{pn}(^3D)=1.8$ F^{-2}. Then $\xi_{pp}(^1S)$ $=1.63$ F^{-2} when $R=4$, and $\xi_{pp}(^1S)=1.72$ F^{-2} when $R=5$. Since the phase shift of the triplet S state is larger than that of the singlet S state—as is shown by using the semiphenomenological two-nucleon potential—the wave function of the triplet S state extends farther inward than that of the singlet S state. This means in the present term that the value of the correlation parameter for the singlet S state is smaller than that for the triplet S state, i.e., $\xi_{pp}(^1S)<\xi_{pn}(^3S)$. Taking into account all arguments mentioned above, we can conclude that $\xi_{pp}(^1S)=1.6$ F^{-2}, $\xi_{pn}(^3S)=1.7$ F^{-2}, and $\xi_{pn}(^3D)=1.8$ F^{-2} for $R=4$, while $\xi_{pp}(^1S)=1.7$ F^{-2}, $\xi_{pn}(^3S)=1.8$ F^{-2}, and $\xi_{pn}(^3D)=1.8$ F^{-2} for $R=5$. Conversely, using these values of the correlation parameters, we can explain the experimental data of the ratio of the pion-capture rates by the pn and pp pairs and of the angular distribution.

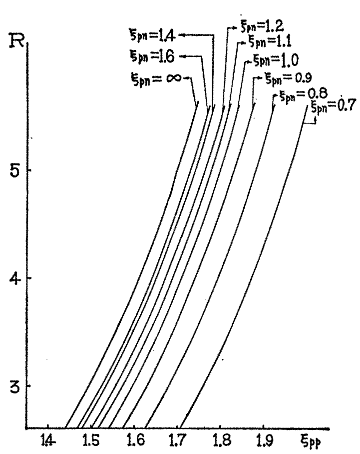

FIG. 6. The ratios of the pion capture rates by (pn) and (pp) pairs for the various values of the correlation parameters $\xi_{pn}(^3D)$ and $\xi_{pp}(^1S)$. The parameters are given in units of F^{-2}.

TABLE IV. Total absorption rates in units of 10^{15} sec^{-1}.

ξ	$W(ppp^S)$	$W(pps^S)$	$W(pn^D)$	$W(pn^S)$
0.7	0.83	0.40	12.1	4.4
0.8	0.69	0.39	12.2	4.0
0.9	0.58	0.38	12.3	3.7
1.0	0.49	0.38	12.3	3.5
1.1	0.43	0.38	12.3	3.3
1.2	0.39	0.38	12.4	3.2
1.3	0.35	0.38	12.4	3.1
1.4	0.33	0.38	12.4	3.1
1.5	0.31	0.39	12.4	3.0
1.6	0.30	0.39	12.4	3.0
1.7	0.29	0.39	12.4	3.0
1.8	0.28	0.39	12.4	3.0
1.9	0.28	0.40	12.4	3.0
2.0	0.27	0.40	12.4	3.0
2.2	0.27	0.40	12.5	2.9

C. Total Absorption Rate

Total absorption rates can be obtained by integrating Eqs. (12), (13), (14), and (15) with respect to $\cos\Theta$. The results are shown in Table IV. The total absorption rates are functions of the nuclear correlations, varying smoothly between $\xi=0.7$ F^{-2} and $\xi=0.2$ F^{-2}.

Finally it is shown that the one-nucleon absorption is 15%.[10,11]

ACKNOWLEDGMENTS

This work was partly financed by Gakujutsu Shinkokai in Japan. The author would like to express his gratitude to Professor H. Yukawa and the members of the Research Institute for Fundamental Physics for the hospitality extended to him, and to Professor M. Kobayasi for constant encouragement. Thanks are due to Miss S. Hirobe for her help in the numerical computation.

[10] P. I. Fedotov, Yadern. Fiz. **2**, 466 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 335 (1966)].

[11] V. S. Demidov, V. S. Verebryusov, V. G. Kirillov-Ugryumov, A. K. Ponosov, and F. M. Sergeev, Zh. Eksperim. i Teor. Fiz. **46**, 1220 (1964) [English transl.: Soviet Phys.—JETP **19**, 826 (1964)].