Hop-Conduction Magnetoresistance in p-Type Germanium*

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Measurements of the resistivity ρ of p-type, Ga-doped germanium samples having room-temperature carrier concentrations between 2.2×10^{15} and 1.4×10^{16} cm^{-3} have been made at liquid-helium temperatures and for comparison at 77°K employing magnetic inductions B up to 25 kG. At liquid-helium temperatures $\rho/\rho_0\approx\exp[\gamma B^2]$ with $\gamma\sim N_A^{-0.7}$, where N_A is the acceptor concentration, ρ/ρ_0 is almost independent of temperature and, in a transverse magnetic field, ρ/ρ_0 exhibits a different type of anisotropy than is characteristic of valence-band conduction. The dependence of ρ/ρ_0 on the strength and orientation of the magnetic field is explained qualitatively in terms of the influence of the magnetic field on the acceptor wave functions and thereby on the phonon-assisted hopping of holes between acceptor sites, which is the process responsible for conduction at low temperatures. The ratio of the transverse to the longitudinal magnetoresistance at 4.2 and at 3.5°K is found to be in qualitative agreement with that predicted by theory for hop-conduction magnetoresistance in weak fields for a semiconductor with a simple spherical band at $k=0$.

I. INTRODUCTION

PREVIOUS experiments[1,2] on the resistivity ρ of phonon-assisted hop conduction[3] employing magnetic field strengths up to 25 kG have been confined to n-type Ge. They revealed effects which could be attributed to the influence of the magnetic field on the wave functions of the donor impurities and thereby on the conduction caused by preferential hopping of electrons from full to empty donor sites in the presence of an applied dc electric field. Phonons are needed to assist the hopping because the presence of minority acceptor impurities makes the corresponding electronic states on adjacent donors nonequivalent in energy.

In view of such results on n-type Ge, it was felt that generally similar effects should be observable in p-type Ge but that some new features would be found in view of the differences between acceptor and donor impurity states.[4]

In this paper we report magnetoresistance results on p-type germanium samples lightly doped with gallium which generally fulfill these expectations. However, a detailed explanation of the anisotropy of the transverse magnetoresistance in terms of how the acceptor wave functions are affected by the magnetic field has not been worked out because of the complexity of these functions.[4,5]

The temperature dependence of ρ/ρ_0 and the ratio of transverse to longitudinal magnetoresistance $\Delta\rho_T/\Delta\rho_L$ were also measured. Various features of our experimental results are discussed using a theory[6] developed for the weak-field magnetoresistance of hop conduction in a semiconductor having a simple spherical band at $k=0$.

II. EXPERIMENTAL DETAILS

Preparation of the single-crystal samples was similar to that in the preceding paper. The samples employed and some of their characteristics are listed in Table I. Each sample is designated by an abbreviation for the total hole concentration deduced from the Hall coefficient measured at room temperature with a field of 25 kG. By this field, R_H had become independent of field strength so that the hole concentration p is given by[7] $p=1/R_H|e|$, where e is the electronic charge. It is assumed that the excess of acceptor concentration over the donor concentration is equal to the value of p at 298°K.

The Dewar system, magnet, and electrical circuitry and instrumentation were the same as those used in the preceding paper.

III. RESULTS AND DISCUSSION

A. Temperature Dependence of ρ_0 at Low Temperatures

The resistivity of each of our samples in zero magnetic field is plotted as a function of reciprocal temperature between about 5° and 2°K in Fig. 1. The curves are similar to those obtained for p-type, indium-doped Ge by Blakemore,[8] who fitted his data with the Mott[9,10]-Price[11,12] model, which predicts that the number of

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TABLE I. Some characteristics of the p-type, Ga-doped germanium samples. ϵ_3 is the activation energy derived from the resistivity at lowest temperatures, and N_D/N_A is the ratio of the concentration of donors N_D to the concentrations of acceptors N_A.

Sample	ρ_0(298°K) (Ω cm)	ϵ_3 (meV)	N_D/N_A	N_A (10^{15} cm^{-3})
2.2–15	1.41	1.60	0.015	2.2
5.0–15	0.68	1.09	0.11	5.6
6.4–15	0.61	1.72	0.02	6.5
8.0–15	0.44	1.19	0.11	9.0
1.4–16	0.25	1.36	0.16	16.7

majority sites which take part in impurity conduction varies with temperature. Applying instead the more complete theory developed by Miller and Abrahams[3] for n-type Ge to our results, we expect an exponential temperature dependence for ρ_0 due to phonons being needed to assist holes in jumping between acceptor ions. From Fig. 1 we see that ρ_0 is an exponential function of temperature for each of our samples at least at the lower temperatures. This allows an activation energy ϵ_3 to be obtained for each sample. Then, from each value of ϵ_3 we deduced a value for the compensation N_D/N_A (where N_D is the donor concentration and N_A the acceptor concentration) by using the theory of Miller and Abrahams.[3] Finally, N_A is deduced from the values of N_A-N_D and N_D/N_A. The values we obtained for ϵ_3, N_D/N_A, and N_A are given in Table I.

B. Magnetic-Field Dependence of ρ at Low Temperatures

Figures 2 and 3 show that for each of our samples $\ln(\rho/\rho_0)$ is proportional to B^2 up to 25 kG, in agreement with the model of Sladek and Keyes[1] (SK) developed for n-type Ge. (Actually the lower field data for sample 5.0–15 deviate from such a simple dependence. We attribute this to unknown experimental difficulties since the deviations are in opposite directions at 4.1 and 3.3°K). Since their model explained hop conduction magnetoresistance in terms of the shrinkage of the wave functions of the majority impurities by the magnetic field, we infer that the magnetoresistance we observe is due to the influence of the magnetic field on the wave functions of acceptor impurities. Our smaller values of ρ/ρ_0 have a field dependence which is also in accord with the predictions of theory for weak-field, phonon-assisted, hop-conduction magnetoresistance in simple band semiconductors by Mikoshiba[6] (M). This theory includes the fact that the magnetic field causes a difference in phase of the wave functions on adjacent impurities as well as causing a shrinkage of the wave function on each impurity.

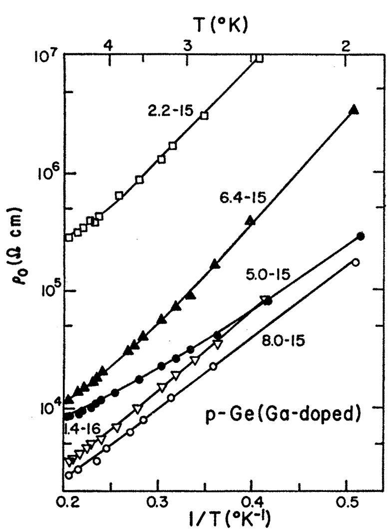

FIG. 1. Resistivity in zero magnetic field versus reciprocal temperature.

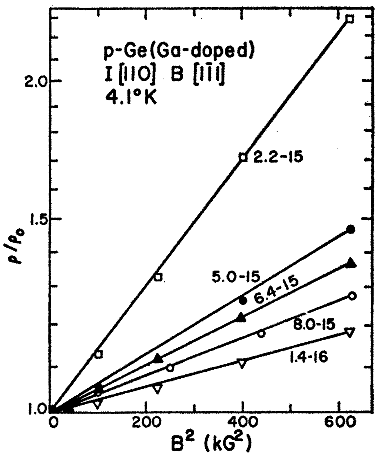

FIG. 2. Magnetoresistance ratio ρ/ρ_0 versus the square of the magnetic induction. A logarithmic scale is used for ρ/ρ_0.

To display the concentration dependence of ρ/ρ_0 we plot the slope γ of the $\ln(\rho/\rho_0)$-versus-B^2 curve for each sample at 3.3°K versus acceptor concentration in Fig. 4. From Fig. 4 it can be seen that all but one of the values of γ fall along a straight line of slope≈ -0.72. Thus, the dependence of γ on impurity concentration which we

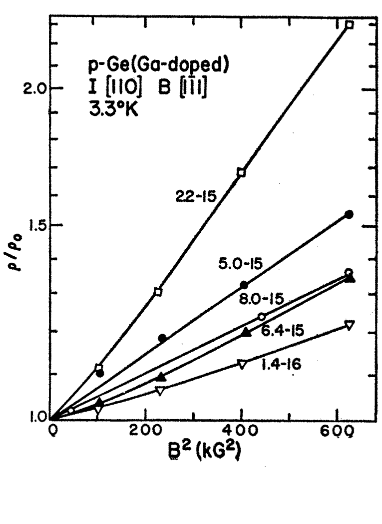

FIG. 3. Magnetoresistance ratio ρ/ρ_0 versus the square of the magnetic induction. A logarithmic scale is used for ρ/ρ_0.

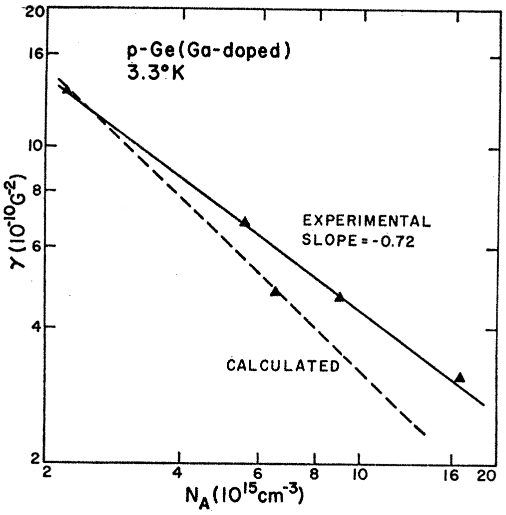

FIG. 4. Dependence of $\gamma \equiv [\ln(\rho/\rho_0)]/B^2$ on the concentration of acceptors. The solid line is drawn through the data points. The dashed line is calculated assuming the applicability of results of weak-field theory (see text).

find is less strong than the $1/N_A$ dependence predicted by the model of SK.

The theory of M for weak fields contains a factor which could, in principle, account for the dependence of γ on N_A which we observe while still preserving the B^2 dependence of $\ln(\rho/\rho_0)$. Specifically, for weak fields M obtained the formula

$$\Delta\rho_T/\rho_0 \approx \lambda a^3[1.6(R/a)^3+5.36(R/a)^2], \qquad (1)$$

in which $\lambda = KB^2/24\, m^*c^2$, a is the effective Bohr radius measuring the size of the impurity wave function, and R is the average distance between impurities. In the expression for λ, K is the dielectric constant and m^* is the effective electron mass in the semiconductor, B is the applied magnetic induction, and c is the velocity of light.

If we assume that the right-hand side of Eq. (1) divided by B^2 should be used in computing γ for our case, we obtain the curve in Fig. 4 by taking $R = (N_A)^{-1/3}$ and $a \approx 41$ Å (deduced from the ionization energy of Ga in Ge). The curve is very close to the data point at smallest N_A but drops more and more below the data for larger N_A. In fact the curve is very close to a straight line with a slope only very slightly less than -1.0. In view of the uncertainties in choosing values for R and a, the calculated curve cannot be viewed as having more than a semiquantitative significance. We have not tried to choose values of R and a to force a fit for two reasons. First, M's factor is strictly applicable only for values of $\Delta\rho_T/\rho_0 \ll 1$. Second, a curve calculated using values of R and a which fitted the experimental concentration dependence of γ better would be displaced far from the data because the magnitudes of the computed γ's would not be close to experimental ones.

C. Anisotropy of ρ/ρ_0 in a Transverse Magnetic Field

Figures 5 and 6 show the transverse magnetoresistance as a function of the crystallographic orientation of the magnetic field for each of our samples at 4.1 and 3.3°K, respectively. The anisotropy exhibited is different than that characteristic of valence-band conduction[13,14] which we checked for our samples by making measurements at 77°K and is also somewhat different than that found in low concentration n-type Ge.[1,2,15] We attribute the anisotropy of ρ/ρ_0 of our p-type samples at low temperatures to the anisotropy of the acceptor wave functions[5] whose overlap controls the phonon-assisted hopping process responsible for conduction at low temperatures.

A specific formula which would account for the anisotropy of ρ/ρ_0 in our p-type samples is not available because of the complexity of the acceptor wave functions.[4,5] However, from Figs. 5 and 6 we see that the anisotropy of ρ/ρ_0 is almost independent of temperature and impurity concentration. This implies that the symmetry of the wave function does not change with distance from the impurity nucleus.

D. Temperature Dependence of ρ/ρ_0

For all of our samples, ρ/ρ_0 is almost independent of temperature. This can be seen for four of our samples in Fig. 7. Data for sample 8.0–15 have been omitted to avoid clutter. They lie generally between the curves for samples 6.4–15 and 1.4–16 and have a temperature dependence similar to sample 5.0–15. These results are in general agreement with theory for carriers making direct, phonon-assisted transitions between singlet ground states on adjacent impurities.[3] Specifically such theory[6] predicts that the magnetic field affects the energy of carriers resonating between impurities but

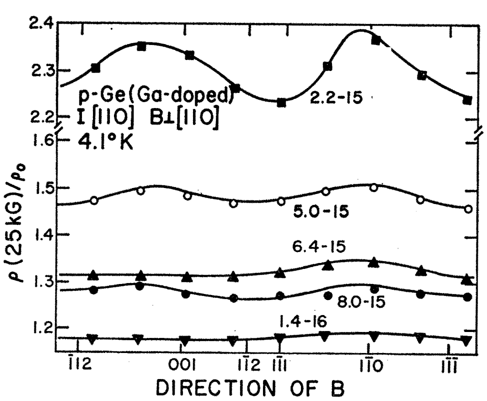

FIG. 5. Dependence of ρ/ρ_0 on the direction of the transverse magnetic field.

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does not affect the temperature dependence of the resistivity, so that ρ/ρ_0 should be completely independent of temperature. The temperature dependence of ρ/ρ_0 due to transitions involving spin reversals[16] is also not expected, because the fields we used are not strong enough even if the nature of the acceptor states would be suitable for the occurrence of such transitions.

The occurrence of some temperature dependence of ρ/ρ_0 may simply indicate that the approximations involved in the theory prevent such small effects from being predicted. Some effect might arise due to the magnetic field splitting the degeneracy of the acceptor ground state in an appropriate fashion. This could occur, for example, due to the magnetic field causing a quantum shift of the light-hole band relative to the heavy-hole band. We do not have a specific model for how this would affect the magnetoresistance.

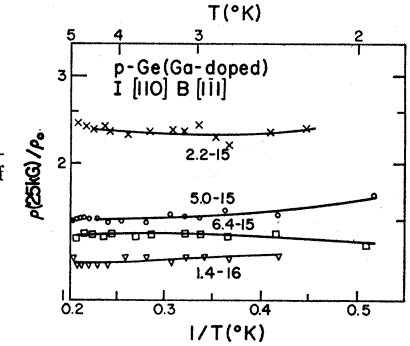

FIG. 7. Temperature dependence of ρ/ρ_0.

E. Ratio of Transverse and Longitudinal Magnetoresistances

Figure 8 shows the ratio of the change in resistivity due to a transverse magnetic field $\Delta\rho_T$, to the change in resistivity due to a longitudinal magnetic field $\Delta\rho_L$, as a function of field strength for one of our samples at 4.1 and 3.5°K. From Fig. 8 it can be seen that $\Delta\rho_T/\Delta\rho_L$ is greater than unity and falls off somewhat with increasing field strength. Also shown in Fig. 8 are values calculated using an expression from M's theory for weak fields:

$$\Delta\rho_T/\Delta\rho_L = (1.6t^3+5.36t^2)/(0.8t^3+3.68t^2), \qquad (2)$$

where $t=R/a$ as before. One value is calculated using $R=(4\pi N_A/3)^{-1/3}$ and the other using $R=(N_A)^{-1/3}$, with $a=41$ Å in both cases.

Before comparing the calculated values of $\Delta\rho_T/\Delta\rho_L$ with our data we mention two caveats: (1) The weak-field approximation is not applicable to most of our data given in Fig. 8 since the field strengths we used were too high. (Measurements attempted at lower fields were not accurate enough to determine $\Delta\rho_T/\Delta\rho_L$ where the weak field approximation would be most applicable.) (2) The simple spherical band semiconductor model employed by M is not appropriate for p-type Ge since the valence band is comprised of warped heavy and light hole bands degenerate at $\mathbf{k}=0$.

In view of items (1) and (2), no quantitative agreement with theory is to be expected. However, it is satisfying that our data are in qualitative accord with the theory in that $\Delta\rho_T/\Delta\rho_L$ is appreciably greater than unity. The slight dependence of $\Delta\rho_T/\Delta\rho_L$ on magnetic-field strength might arise because $\Delta\rho_T$ is more dependent on the phase effect[6] (see Sec. III B) than is $\Delta\rho_L$, and the importance of the phase effect relative to the size effect (see Sec. III B) begins to decrease at higher fields.[6] The temperature dependence of $\Delta\rho_T/\Delta\rho_L$ which we find is not explainable in terms of theory. Perhaps it is connected with the randomness of the impurity distribu-

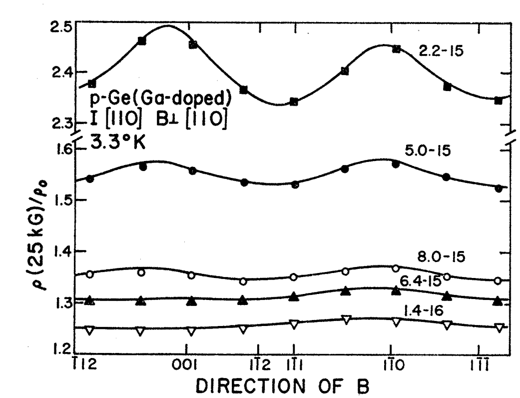

FIG. 6. Dependence of ρ/ρ_0 on the direction of the transverse magnetic field.

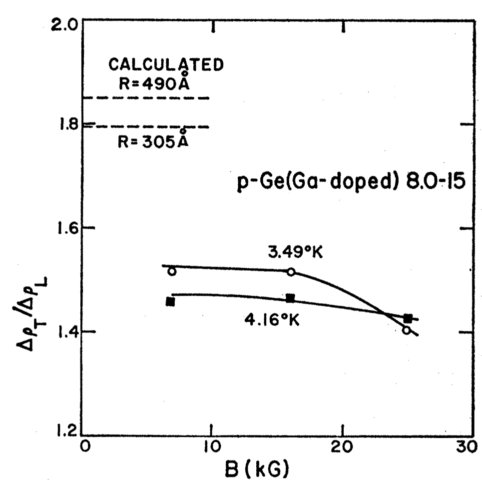

FIG. 8. Ratio of changes in resistivity due to transverse and due to longitudinal fields versus magnetic induction. Symbols and solid curves represent the data. The dashed lines give the values calculated from Mikoshiba's theory using $a=41$ Å and two different values of R (see text.)

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tion, which seems to cause some temperature dependence of ρ_T/ρ_0 in n-type Ge[2].

IV. CONCLUSION

From our data on samples of germanium doped with low concentrations of Ga, we conclude that a magnetic field affects phonon-assisted hop conduction in p-type Ge via its influence on the acceptor wave functions. Many of the features observed are similar to those occurring in n-type Ge, but some are not because of the difference between acceptor and donor impurity states.

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Path Variable Formulation of the Hot Carrier Problem

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A theoretical treatment of transport phenomena in strong electric fields is presented. Instead of the Legendre polynomial expansion of the distribution function usually employed in solving the transport equation, we transform the Boltzmann equation to a coordinate system determined by the collision-free trajectories of the particles and formulate an integral equation for the distribution function. This method is applied to hot carriers in nonpolar semiconductors, where the relevant transport equation is then reduced to a one-dimensional integral equation. This equation is solved numerically and energy distributions are calculated for n- and p-type germaniun. The calculations for heavy holes in germanium demonstrate the non-Maxwellian nature of the distribution function as well as its strong displacement in momentum space, and are in excellent agreement with experiment. The energy distributions for electrons show weaker deviations from Maxwellian and smaller ratios of drift to rms velocity, this being due to the weaker coupling to optical phonons for electrons as compared to holes.

I. INTRODUCTION

THE central theoretical problem associated with hot-carrier phenomena is the calculation of the steady-state particle distribution function in the presence of a strong electric field. Since the transport equation describing the distribution function is generally an integrodifferential equation, one is usually obliged to seek approximate solutions appropriate to specific physical situations.

The two major simplifications usually introduced in effecting such calculations are related to the role of carrier-carrier scattering and to the displacement of the distribution function in momentum space.

The role of carrier-carrier scattering in determining the form of the distribution function had first been discussed by Fröhlich[1] and has been examined in detail by Stratton.[2] The essential idea is that sufficiently strong carrier-carrier scattering results in a displaced Maxwellian distribution, characterized by an effective temperature greater than that of the lattice. Having fixed this form of the distribution function, it is then possible to calculate the displacement in momentum space and the effective temperature from momentum and energy balance equations, which are readily derived from the Boltzmann equation.

When carrier-carrier scattering is not sufficiently strong to justify this form of the distribution function, it is necessary to solve the Boltzmann equation with due regard to the detailed nature of the various scattering mechanisms. It is in carrying out this program that one usually introduces the assumption of a weakly displaced distribution function in momentum space, thus enabling one to represent the distribution function by only the two lowest-order terms in a Legendre polynomial expansion in the cosine of the angle between the electric field and momentum vector. This is roughly equivalent to assuming that the ratio of the averaged drift to root-mean-square velocity is small.

Recent experimental studies in p-type[3,4] germanium have demonstrated the inapplicability of this assumption in this case. In particular, the studies of Bray and co-workers[4] have emphasized the "streaming" character

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