Shift of Landau Levels in the Valence Band of Germanium in Crossed Electric and Magnetic Fields

Q. H. F. VREHEN,* W. ZAWADZKI,† AND M. REINE‡
National Magnet Laboratory,§ Massachusetts Institute of Technology, Cambridge, Massachusetts
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Optical-absorption and Voigt-effect measurements in germanium have been performed in crossed electric and magnetic fields for photon energies just below the direct gap. The electric fields were taken between 2.8×10^4 and 4.7×10^4 V/cm, the magnetic fields were chosen between 62.5 and 96.5 kOe. Three transitions were observed, two of which are allowed for $E=0$ ($\Delta n=0$), whereas the third is forbidden for $E=0$, but is induced by the electric field ($\Delta n=-1$). From the experimental data the energies of the three light-hole levels $|0a+\rangle$, $|1a+\rangle$, and $|0b+\rangle$ are deduced, under the assumption that the energies of the electron levels can be calculated according to the theory for a simple parabolic band in cross fields. A calculation is developed for the energies of the light-hole levels in cross fields for the range of high (E/H) values where a perturbation treatment is not sufficient. This calculation is within the framework of the effective-mass approximation and neglects the electric-field-induced coupling between light- and heavy-hole states. The theory shows that for higher (E/H) values the electric-field-induced shifts of the light-hole levels are comparable to those that would have been found in simple bands, and that the electric field removes the quantum effects in the energies of the light-hole levels. The theory is in satisfactory agreement with the experimental results.

I. INTRODUCTION

THE energies of the Landau levels of electrons and holes in a semiconductor in crossed electric and magnetic fields differ from their values for $E=0$. For simple parabolic bands the shift consists of two parts, one proportional to E and the other proportional[1] to $(E/H)^2$. Aronov[2] had shown theoretically that the latter part of the shift should be observable in the interband optical absorption for direct allowed transitions. This prediction was confirmed experimentally by Vrehen,[3] who observed a single absorption line below the direct gap in germanium in crossed fields. More recently, Komatsubara[4] has observed such shifts in transverse magnetoresistance in n-type InSb.

In the present paper we describe an extension of these measurements undertaken in a search for effects related to the complicated structure of the Landau levels in the valence bands of germanium. In absorption data for π- and σ-polarized radiation and in Voigt-effect data we found three peaks, all of which we attribute to light-hole–electron transitions on the basis of the dependence of their photon energies on $(E/H)^2$. Moreover, from the polarization characteristics we were able to determine which initial and final states are most probably involved in these transitions. Since the energy of the final states in crossed fields can be calculated (the conduction band is simple and fairly parabolic), the energies of three light-hole levels as functions of E and H can be derived from these measurements. Theoretically, we calculate these energies by neglecting the electric-field-induced coupling between light and heavy holes. Although the agreement between theory and experiment is not perfect, two significant features of the theoretical results are confirmed by the experiments. First, the electric field tends to remove the quantum effects near the top of the valence band, i.e., the spacings between Landau levels in the quantum region become more or less equal to those in the classical region. Secondly, once the quantum effects are removed the further shift of the light-hole levels with increasing electric field is nearly the same as it would be for holes in a simple parabolic band.

The experiment is described in Sec. II and the experimental results are presented in Sec. III. The theory is discussed in Sec. IV. In Sec. V theory is compared with experiment and a discussion of the results is given.

II. EXPERIMENT

Magnetic fields between 50 and 100 kOe were used throughout. To obtain measurable shifts one then needs electric fields between 10^4 and 10^5 V/cm. Such large electric fields are present in a reverse-biased p-n junction. Frova and Handler[5] have developed a method for the study of optical absorption in an electric field using p-n junctions. They kindly provided us with a few of their germanium diodes with which the experiments described in this paper have been performed. Since the diodes are relatively thick ($\sim200\ \mu$) and the absorption coefficient of germanium at photon energies above the direct gap is large ($\sim4\times10^3$ cm^{-1}), measurements could only be made for photon energies below the direct gap. For an ideal step junction the following relation holds[5]:

$$\Delta\alpha\equiv\alpha(E_{\max})-\alpha(0)=\frac{E_{\max}}{\Delta V}\frac{\Delta I}{I}. \quad (1)$$

* Present address: Philips Research Laboratories, Eindhoven, The Netherlands.

† On leave of absence from the Institute of Physics, Polish Academy of Sciences, Warsaw, Poland.

‡ Also Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts.

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E_{max} is the maximum electric field in the junction region, I is the transmitted intensity of infrared radiation and ΔI is the modulation of this intensity caused by a modulation ΔV in the bias voltage. The absorption coefficient for zero electric field is $\alpha(0)$ and $\alpha(E_{max})$ is the absorption coefficient in the presence of an electric field E_{max}. The actual junctions that we used are not ideal step junctions so that Eq. (1) is not strictly applicable.[5] However, we are mainly interested in the photon energy of certain absorption peaks and for the measurement of these peak positions Eq. (1) seems sufficiently accurate.

We found that the transitions could be observed much more clearly in a measurement of the Voigt phase shift than in an absorption measurement. The reason for this is the following. In the case of an absorption measurement in crossed fields the absorption peaks appear as relatively weak oscillations on top of a strong Franz-Keldysh absorption that is already present in an electric field alone. The Franz-Keldysh effect for $H=0$, however, does not lead to any phase difference between two linear polarizations at right angles, and certainly not in our experimental arrangement where the electric field is parallel to the direction of propagation of the infrared radiation. Therefore, the Voigt-effect measurement yields just the magnetic-field-induced oscillations that we are interested in.

The theory of dispersive effects in crossed fields has been considered recently by Zawadzki *et al.*[6] and Uritsky *et al.*[7] We define $\delta=\delta_\perp-\delta_\parallel$ as the phase difference between radiation polarized perpendicular and parallel to the magnetic field in rad/cm. Instead of Eq. (1), we now find a similar relation:

$$\delta(E_{max})-\delta(0)=(E_{max}/\Delta V)\Delta\varphi, \tag{2}$$

where

$$\sin\varphi=\left(\frac{I_--I_+}{I_-+I_+}\right)\left(\frac{I_\parallel+I_\perp}{2(I_\parallel I_\perp)^{1/2}}\right). \tag{3}$$

The second factor in the right-hand side of Eq. (3) was always very close to one, which allowed the use of the following approximate formula for $\Delta\varphi$:

$$\Delta\varphi\approx\frac{1}{\cos\varphi}\left[\frac{\Delta I_--\Delta I_+}{I_-+I_+}-\sin\varphi\left(\frac{\Delta I_-+\Delta I_+}{I_-+I_+}\right)\right]. \tag{4}$$

In the derivation of Eqs. (2)–(4) it is assumed that the sample is irradiated with a monochromatic infrared beam plane-polarized at 45° with the magnetic field. I_-, I_+ are the transmitted intensities for right- and left-handed circular polarization, $I_\parallel$ and $I_\perp$ those for plane polarization parallel and perpendicular to the magnetic field.

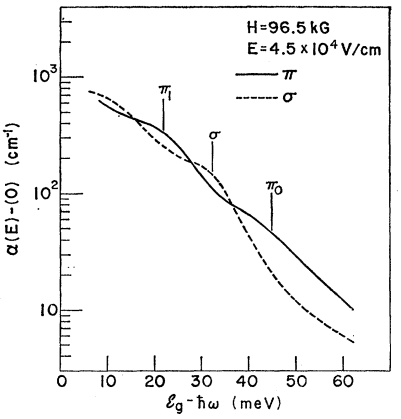

FIG. 1. Optical absorption in crossed fields for photon energies below the direct gap of germanium. $H=96.5$ kOe and $E=4.5\times10^4$ V/cm. For π polarization two peaks are observed; for σ radiation only one peak is present.

For photon energies below the gap, $\delta(0)$ is very small. The experiment therefore measures essentially $\delta(E_{max})$. The Voigt spectrum is then expected to consist of a number of lines which correspond to transitions shifted to energies below the gap by the presence of the electric field. The transitions are either those that were already allowed for $E=0$ ("allowed transitions," $\Delta n=0$ for simple parabolic bands) or those that were forbidden for $E=0$ but assume a finite transition probability for $E\neq0$ ("forbidden transitions," $\Delta n\neq0$ for simple parabolic bands). The individual lines have the same line shapes in crossed fields as they have for $E=0$. Voigt-effect line shapes have been studied by Halpern.[8] His results indicate that the Voigt effect corresponding to a given transition between Landau levels consists of a strong peak close to the energy of the transition. The peak is positive or negative depending on whether the transition occurs for π or σ polarization.

III. EXPERIMENTAL RESULTS

Earlier measurements by Vrehen[3] were performed with supposedly unpolarized radiation. Later tests showed that the radiation had been of predominantly σ polarization. In order to observe more structure in the optical absorption below the gap we decided to use plane-polarized radiation of π and σ type. Figure 1 shows absorption curves for π and σ radiation for $E=4.5\times10^4$ V/cm and $H=96.5$ kOe. One peak is observed for σ polarization. This is the same peak that was observed before with "unpolarized" radiation. Two peaks are present in the curve for π polarization, labeled π_0 and π_1 (for the lower- and higher-energy peak, respectively). Voigt-effect measurements are presented in Fig. 2 for $H=96.5$ kOe and various values of the electric field. In such a Voigt phase-shift plot, π peaks show up as positive peaks and σ peaks as negative peaks. With increasing electric field the peaks shift to lower energy and at the same time their intensity decreases. For simple parabolic bands the photon energy for a

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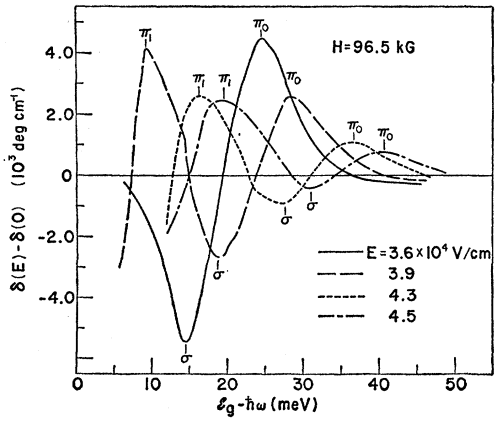

FIG. 2. Voigt effect in crossed fields for photon energies below the direct gap of germanium, for $H=96.5$ kOe and various values of the electric field strength. Two positive peaks correspond to π-polarized absorption peaks; one negative peak corresponds to a σ-polarized absorption peak. The transitions shift to lower energies and their intensities decrease as E increases.

transition from a Landau level n' in the valence band to a Landau level n in the conduction band is given by

$$\mathcal{E}=\mathcal{E}_g+(n+\tfrac{1}{2})\hbar\omega_c+(n'+\tfrac{1}{2})\hbar\omega_v -\tfrac{1}{2}(m_1+m_2)c^2E^2/H^2, \quad (5)$$

where $\mathcal{E}_g$ is the gap energy, ω_c and ω_v are the cyclotron frequencies for electrons and holes, m_1 and m_2 are the effective masses of electrons and holes, respectively, and c is the velocity of light.

Voigt-effect curves for $(E/H)^2=0.140$ and various values of H are given in Fig. 3. In agreement with Eq. (5) a linear shift proportional to the magnetic field is observed. This arrangement corresponds to the normal magneto-optical experiment with the transition energies shifted by a constant amount to energies below the gap by the electric field (with the possibility of additional, previously forbidden transitions). Figures 4 and 5 show that over the range of (E/H) values of our experiment the transition energies of the σ and π transitions can be described by a formula similar to Eq. (5):

$$\mathcal{E}=\mathcal{E}_g+\gamma H-\epsilon(E/H)^2, \quad (5')$$

where γ and ϵ depend on the particular transition involved. Figure 4 shows the transition energies as a function of $(E/H)^2$ for $H=96.5$ kOe. The experimental points fall on straight lines, which all have the same slope. Interpreting this slope with the help of Eq. (5), we find $m_1+m_2=0.076m$ (m is the free-electron mass). Since the electron effective mass is well known, $m_1=0.036m$, we find for the hole mass $m_2=0.040m$. This is very close to the light-hole mass and we conclude that all three peaks correspond to light-hole–electron transitions. Moreover, we see that in the region of high electric fields the electric-field-induced shifts are just those that theory predicts for simple parabolic bands. In Fig. 5 transition energies are given as a function of H for $(E/H)^2=0.140$. The transition energies are linear functions of H. Expressing the slopes in units of $(e\hbar/mc)$, we find $\gamma(\pi_0)=5.4$, $\gamma(\pi_1)=23.4$, and $\gamma(\sigma)=14.0$.

According to Aronov's theory,[2] "allowed" transitions decrease in intensity when an electric field is applied, whereas the intensity of "forbidden" transitions $\Delta n=\pm1$ first increases, then reaches a maximum and decreases again for still larger E. In the range of electric fields that we applied, the intensity of forbidden transitions $\Delta n=\pm1$ should already decrease. The rate of decrease as a function of electric field should be, however, slower than that for the allowed transitions. The exact behavior of the intensities of the three transitions was hard to determine because of the appreciable overlap of the lines. Nevertheless, it was definitely established that over the range of the measurements the intensities of the π_0 and σ transitions decreased more rapidly with increasing electric field than the intensity of the π_1 transition. This led us to conclude that π_1 corresponds to a forbidden transition $\Delta n=\pm1$, whereas π_0 and σ correspond to allowed transitions $\Delta n=0$. This assumption also leads to the best fit for the transition energies. Comparing the polarization characteristics of the three peaks with those for the transitions in interband magneto-optical absorption measurements at $E=0$,[9,10] we arrived at the following assignment for the transitions: The π_0 peak corresponds to the transition $|0a+\rangle\rightarrow|0\beta\rangle$, the π_1 peak to $|1a+\rangle\rightarrow|0\beta\rangle$, and the σ peak to $|0b+\rangle\rightarrow|0\beta\rangle$. Here we have used Roth's notation[9]; $|na+\rangle$ indicates the nth light-hole state in the a series, whereas $|n\beta\rangle$ stands for the nth spin-down conduction-band state (see also Ref. 11). All transitions thus occur to the same conduction-band level; the energy of this level in crossed fields can be calculated

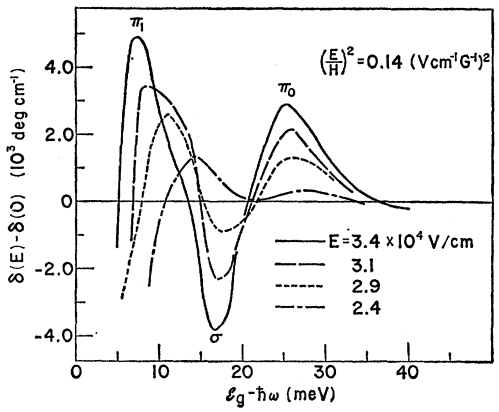

FIG. 3. Voigt effect in crossed fields for photon energies below the direct gap, for a constant value of $(E/H)^2$ and various values of E. With increasing magnetic field the peaks shift to higher photon energies.

[9] L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. 114, 90 (1959).
[10] E. Burstein, G. S. Picus, R. F. Wallis, and F. Blatt, Phys. Rev. 113, 15 (1959).
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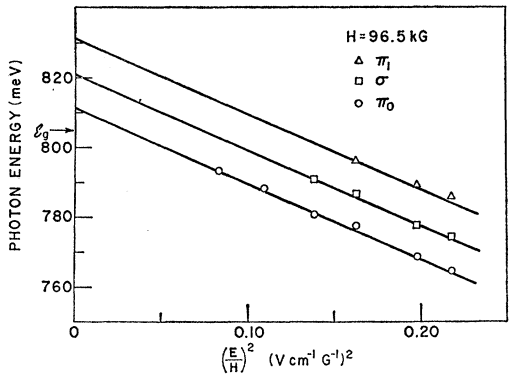

FIG. 4. Photon energies of Voigt-effect peaks in crossed fields for $H=96.5$ kOe, as a function of $(E/H)^2$. All three peaks undergo equally large shifts proportional to $(E/H)^2$.

from the theory for simple parabolic bands. Subtracting these values from the experimentally determined transition energies, we find the energies of the light-hole levels $|0a+\rangle$, $|1a+\rangle$, and $|0b+\rangle$ as a function of electric and magnetic field. A theoretical calculation of those energies is presented in the next section.

IV. THEORY

The Landau levels in the valence band of germanium have been calculated by Luttinger and Kohn[12,13] for $k_z=0$ (z is the direction of the magnetic field). Roth *et al.*[9] studied the interband magneto-optical absorption. Shifts of the Landau levels resulting from a small electric field applied perpendicular to the magnetic field have been calculated by Hensel and Peter,[14] Shindo,[15] and Vrehen[11] by perturbation theory. These perturbation theories are valid if $a_v \equiv (eEL_M/\hbar\omega_v) \ll 1$, where e is the absolute value of the electronic charge, E is the electric field strength, $L_M=(\hbar c/eH)^{1/2}$ is the radius of the cyclotron orbit for $n=0$, and ω_v is the cyclotron frequency of the holes under consideration. In the rest of this section we take ω_v to be the classical light-hole cyclotron frequency.

From the perturbation treatment we can understand qualitatively how the electric field influences the quantum effects. In first order all levels undergo the same shift. In second order a level n is coupled to the levels $n\pm1$; it is pushed to higher energy by the interaction with the level $n-1$, and to lower energy by the interaction with the level $n+1$. In simple parabolic bands this leads to a net downward shift that is the same for all the levels. In the case of degenerate bands, however, if two adjacent levels are much closer to each other than to any other levels, their mutual repulsion will dominate and the energy difference between them will increase. For example, in germanium the energy difference of the light-hole levels $|0a+\rangle$ and $|1a+\rangle$ is anomalously small. The perturbation theory shows that to second order the energy of $|0a+\rangle$ is decreased and the energy of $|1a+\rangle$ is increased by an electric field. This process will continue until the energy separations between the various levels are approximately equal. Thus we expect the electric field to remove the quantum effects. Further shifts will then be analogous to those in simple bands. The calculation in the present section confirms these qualitative arguments.

We now wish to calculate approximately the energies of the light-hole levels for electric field strengths so large that the perturbation treatment is not valid (a_v of the order of unity), yet small enough so that the effective-mass approximation still holds. According to Zak and Zawadzki,[16] this means that the condition $eEa_L/(\hbar\omega_v) \ll 1$ has to be fulfilled, where a_L is the lattice constant. Consequently we shall neglect the nonparabolicity in the light-hole bands. In the following we make use of the results published in a previous paper by one of us.[11] The wave functions of the light- and heavy-hole states in the a and b series for $E=0$ are taken as the basis set. The energy matrix elements in the presence of an electric field can then be calculated directly. The electric field adds terms to the diagonal elements, which can be neglected because they do not affect the optical transition energies, and induces off-diagonal elements $\mathcal{H}_{n,n\pm1}$. The complete matrix now consists of four infinite matrices, one for each ladder, coupled through the off-diagonal elements. The situation is simplified by the fact that the coupling between a- and b-series states can be neglected in germanium.[11] We now make the crucial approximation that we neglect the coupling between light- and heavy-hole states. This approximation, which we cannot justify rigorously, will be discussed in Sec. V. We are thus left with four uncoupled matrices, of which we study only the two for the light holes.

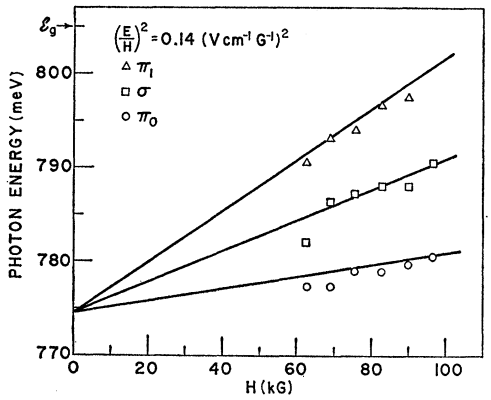

FIG. 5. Photon energies of Voigt-effect peaks in crossed fields for $(E/H)^2=0.14$ (V cm^{-1} Oe^{-1})^2, as a function of H. All three peaks undergo shifts proportional to the magnetic field.

[12] J. M. Luttinger and W. Kohn, Phys. Rev. **97**, 869 (1955).
[13] J. M. Luttinger, Phys. Rev. **102**, 1030 (1956).
[14] J. C. Hensel and M. Peter, Phys. Rev. **114**, 411 (1959).
[15] T. Shindo, J. Phys. Chem. Solids **26**, 1431 (1965).
[16] J. Zak and W. Zawadzki, Phys. Rev. **145**, 536 (1966).

If we had a convenient way of truncating the matrices we could then diagonalize them numerically and the problem would be solved. The magnitude of the off-diagonal elements, however, does not allow such a procedure. To see this, let us look at the matrix elements for a simple parabolic band. If all energies are expressed in units of $\hbar\omega_v$, we have $\mathcal{H}_{n,n}^0=(n+\frac{1}{2})$ and $\mathcal{H}_{n,n+1}^0=\mathcal{H}_{n+1,n}^0=[(n+1)/2]^{1/2}a_v$. The second-order shift in level n, due to coupling to level $n+1$ amounts to $-(n+1)a_v^2/2$, which cannot be made relatively unimportant by going to high n when a_v is of the order of unity. To proceed, we notice that for high n the matrix elements $\mathcal{H}_{n,n'}$ for a light-hole ladder approach those for a simple band of the same cyclotron frequency $\mathcal{H}_{n,n'}^0$, apart from a constant in the diagonal elements, which is not essential and may be added. We write

$$\mathcal{H}_{n,n'}=\mathcal{H}_{n,n'}^0+\mathcal{H}_{n,n'}', \tag{6}$$

where elements of $\mathcal{H}_{n,n'}'$ go to zero for large n. The diagonal elements of $\mathcal{H}'$ are essentially zero for $n>5$, whereas $\mathcal{H}_{n,n+1}'$ is of the order $\alpha[1/2(n+1)]^{1/2}$ with $\alpha=\frac{3}{4}$ for the a series and $\alpha=\frac{1}{4}$ for the b series. Thus it is reasonable to take only a finite number of columns and rows in $\mathcal{H}'$. $\mathcal{H}^0$ on the other hand can be diagonalized exactly, because the exact solution of the Schrödinger equation for simple parabolic bands is known. The transformation S which diagonalizes $\mathcal{H}^0$ is given by[2,11]

$$S_{n,n'}=(2^n n!2^{n'}n!)^{-1/2}e^{-a^2/4}\sum_{m=0}^{\leq n,n'} b_{nn'}(m)a_v^{n+n'-2m}, \tag{7}$$

where

$$b_{nn'}(m)=\frac{(-1)^{n-m}n!n'!2^m}{(n-m)!(n'-m)!m!}.$$

The matrix $S^{-1}\mathcal{H}S$ consists of the sum of $S^{-1}\mathcal{H}^0S$, which is diagonal, and $S^{-1}\mathcal{H}'S$, which is finite. $S^{-1}\mathcal{H}S$ can thus be truncated at once, and the resulting matrix diagonalized numerically.

We undertook such a computation[17] for the a and the b light-hole ladders in germanium for $\mathbf{H}\|[110]$ and with the valence-band parameters as given by Roth.[9] In the matrix $\mathcal{H}'$ eleven rows and columns were taken. The results of these calculations are shown in Figs. 6 and 7. In these figures the energies of the six lowest Landau levels (in units of $\hbar\omega_v$) are plotted as functions of a_v^2. (In Figs. 6, 7, and 8, energies are taken with respect to the valence band edge. Moreover, the energies are shown as increasing with increasing n.) For a simple parabolic band such a plot leads to straight lines with slope $-\frac{1}{2}$. It is seen from Fig. 7 that for the light holes of the b series the plot is very close to the one for simple bands. This explains why the behavior of the σ transition could be explained very well in terms of the simple theory.[3] More interesting is the case of the a-series light holes (Fig. 6). For small values of a_v^2 the slopes of the lines for $n=0$, 1, 2 are very different from $-\frac{1}{2}$, a fact also found in the perturbation theories.[11,14,15] For larger values of a_v^2 the slopes of all lines tend again to $-\frac{1}{2}$, in agreement with the experiments. The energy difference between the levels $n=0$ and $n=1$ is small for $a_v=0$, but for large values of a_v the levels become more or less equidistant, i.e. the quantum effects are to a large degree removed by the electric field.

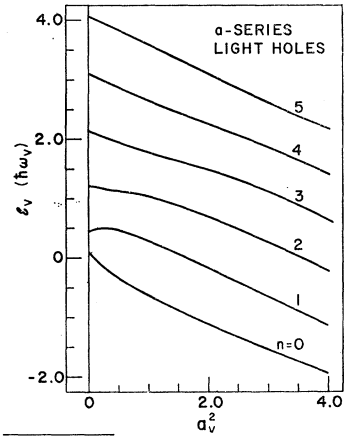

FIG. 6. Calculated energies of the a-series light-hole valence-band levels in germanium in crossed fields as a function of $a_v^2=(eEL_M/\hbar\omega_v)^2$. The energies are expressed in units of $\hbar\omega_v$.

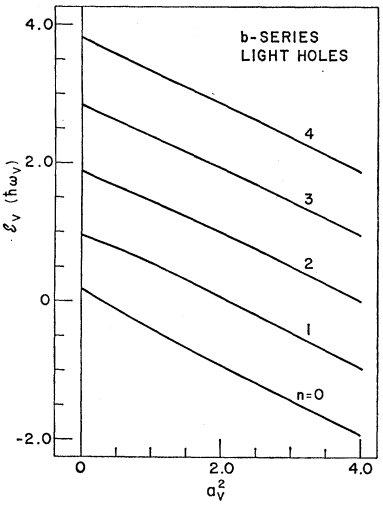

FIG. 7. Calculated energies of the b-series light-hole valence-band levels in germanium in crossed fields as a function of $a_v^2=(eEL_M/\hbar\omega_v)^2$. The energies are expressed in units of $\hbar\omega_v$.

V. DISCUSSION

In Fig. 8 the energies of the levels $|0a+\rangle$, $|1a+\rangle$, and $|0b+\rangle$ as observed experimentally are compared with the theoretical values for a_v^2 between 0 and 2. Although it has not been indicated in the figure, it should be realized that for $a_v=0$ the theoretical energies are in good agreement with the results of magneto-optical absorption measurements.[9,11] For the $|0b+\rangle$ level the theory developed in the previous section agrees quite well with experiment for all values of a_v^2. This is not surprising because for the b series the matrix elements of the electric potential between light- and heavy-hole states are small (see below), and it is these matrix elements that we neglected in the calculation. This

[17] The authors would like to thank Mrs. Ruth Sheshinski for programming the computations.

TABLE I. The off-diagonal matrix elements of the electric potential between the light-hole levels, and between light- and heavy-hole levels, expressed in units of $eEL_M = a_v\hbar\omega_v$.

n	$A_{n,n-1}{}^{++}$	$A_{n,n+1}{}^{++}$	$A_{n,n-1}{}^{+-}$	$A_{n,n+1}{}^{+-}$	$B_{n,n-1}{}^{++}$	$B_{n,n+1}{}^{++}$	$B_{n,n-1}{}^{+-}$	$B_{n,n+1}{}^{+-}$
0		0.707				0.707		
1	0.707	0.765		0.644	0.707	0.954		0.299
2	0.765	0.958		0.410	0.954	1.165		0.231
3	0.958	1.164	0.108	0.277	1.165	1.353	0.094	0.189
4	1.164	1.350	0.127	0.216	1.353	1.520	0.104	0.161
5	1.350	1.516	0.124	0.181	1.520	1.675	0.103	0.146
6	1.516	1.671	0.118	0.158	1.675	1.817	0.099	0.130
7	1.671	1.811	0.110	0.139	1.817	1.947	0.098	0.122
8	1.811	1.942	0.104	0.128	1.947	2.069	0.092	0.111

result also explains why in an earlier publication[3] a good fit to the data could be obtained with the theory for simple bands. In the a series the electric-field-induced coupling between light and heavy holes is stronger. Indeed, we see in Fig. 8 that there is a discrepancy between theory and experiment for the $|0a+\rangle$ and $|1a+\rangle$ levels. In two significant features, however, the theoretical predictions appear to be correct. First, for $1<a_v{}^2<2$ the slope of the theoretical curves agrees with experiment. Secondly, the theory predicts the difference in energy between the $|0a+\rangle$ and $|1a+\rangle$ levels to be larger for $1<a_v{}^2<2$ than for $a_v=0$, and this is observed experimentally. For $a_v=0$ the separation in energy is $0.34\hbar\omega_v$. For $a_v{}^2=2$ the theory predicts $0.93\hbar\omega_v$ and the experimental result is $0.78\hbar\omega_v$. Thus the electric field removes at least part of the quantum effects.

Our theoretical treatment has been based on two assumptions. First, that the electric-field-induced coupling between light and heavy holes can be neglected, and secondly, that the effective-mass approximation is valid. In Ref. 11 it has already been argued that the electric-field-induced coupling between light and heavy holes is relatively unimportant. In Table I we present the off-diagonal matrix elements $A_{n,n'}{}^{ij}$ and $B_{n,n'}{}^{ij}$, where $n'=n\pm1$ (these are the only off-diagonal elements different from zero) and i, j stand for $+$ and $-$, where $+$ indicates a light-hole level and $-$ indicates a heavy-hole level. Thus $A_{n,n+1}{}^{+-}$ is the matrix element of the electric field between the light-hole level $|na+\rangle$ and the heavy-hole level $|n+1, a-\rangle$. This matrix element is expressed in units of eEL_M (for a full exposition of the notation see Ref. 11). From Table I it is evident that matrix elements between light and heavy holes are smaller than those between light holes, and the former become relatively unimportant for larger n values. Moreover, the difference in energy between successive light-hole levels is generally smaller than the difference in energy between the light-hole level n and the heavy-hole level $(n+1)$ or $(n-1)$. Since, because of the electric field, the heavy-hole levels shift to lower energies faster than the light-hole levels, this difference is even more pronounced in the crossed-field configuration. These considerations suggest that it is a reasonable approximation to neglect the interaction between light and heavy holes, and from an inspection of Table I it is expected to be a better approximation for the b series than for the a series; this seems to be borne out by experiment.

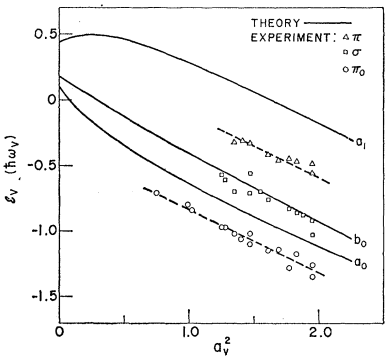

FIG. 8. Photon energies of the light-hole levels $|0a+\rangle$, $|1a+\rangle$, and $|0b+\rangle$ in germanium in crossed fields as calculated and as found experimentally. The energies are expressed in units of $\hbar\omega_v$ and plotted as functions of the parameter $a_v{}^2=(eEL_M/\hbar\omega_v)^2$. At $a_v=0$ the theory is in agreement with interband magneto-optical absorption measurements in zero electric field.

As to the applicability of the effective-mass approximation, Zak and Zawadzki[16] have shown that the requirement $(eEa_L/\hbar\omega_v)\ll1$ should be fulfilled. For larger values of (E/H) solutions can be obtained in a two-band model, as first suggested by Lax.[18] Progress along these lines has recently been made by several authors.[19–22] One particular aspect of the two-band model for nondegenerate bands is that it predicts the cyclotron frequency to decrease with electric field. For the maximum value of E/H in our experiments this decrease should amount to about 1%. For $a_v{}^2\approx2$ the experiment yielded an energy separation of the $|0a+\rangle$ and $|1a+\rangle$ levels about 16% smaller than we calculated from our theory. Apparently, only a minor part of this

[18] B. Lax, in *Proceedings of the Seventh International Conference on the Physics of Semiconductors* (Dunod Cie., Paris, 1964), p. 253.

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difference between theory and experiment can be ascribed to a breakdown of the effective-mass approximation. For the electric and magnetic fields that we applied, $(eEa_L/\hbar\omega_v)$ is smaller than 0.1 for the light holes, but it becomes of order 1 for the heavy holes. Therefore, the heavy holes cannot be treated in the effective-mass approximation. It is probable that for these high values of electric field, discrete levels no longer exist for the heavy holes, which would explain the fact that we do not observe any peaks in the absorption spectra that can be attributed to heavy-hole–electron transitions. It would certainly be meaningless to try to include the heavy holes in a calculation of the type developed in Sec. IV, which assumes the effective-mass approximation to hold.

Our results can be summarized as follows. Three interband magneto-optical transitions have been observed in germanium at energies below the direct gap in the presence of a large electric field perpendicular to the magnetic field. One of these transitions is a forbidden one, induced by the electric field, whereas others are allowed transitions. The shift of the transition energies due to the electric and magnetic fields as well as the intensity behavior have been studied. From the experimental data the energies of three light-hole levels in crossed electric and magnetic fields have been determined in a range of E/H values for which perturbation theories developed earlier are not valid (the requirement $eEL_M/\hbar\omega_v \ll 1$ is not fulfilled). The theory, which goes beyond the perturbation treatment, was developed within the framework of the effective-mass approximation by neglecting the electric-field-induced interaction between the light holes and heavy holes. The theory is in satisfactory agreement with the experimental data. It accounts in particular for the observation that the electric field removes the quantum effects from the light-hole Luttinger ladders.

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Nonlocal Effects in Low-Field Helicon Propagation in PbTe†

J. N. WALPOLE* AND A. L. MCWHORTER
Lincoln Laboratory‡ and Electrical Engineering Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

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Helicon propagation has been studied in the region of low magnetic fields, where band structure and nonlocal Landau damping effects produce sizable corrections to the high-field dispersion relation. Good agreement with both the phase and amplitude of 9-GHz helicons in n-PbTe at 4.2°K is obtained by a semiclassical treatment of a degenerate, many-valley, ellipsoidal model. The calculation is carried out by expanding the conductivity tensor in powers of $1/B_0$, treating ω/ω_c, $1/\omega_c\tau$, and qv_F/ω_c as small quantities, where B_0 is the static magnetic field, ω_c is the cyclotron frequency, τ is the collision time, q is the wave number, and v_F is the Fermi velocity. Because of the anisotropic energy surfaces in PbTe, Landau damping occurs for propagation along the field as well as at an angle. The corrections to the phase are quite sensitive to the transverse effective mass; comparison with experiment yields a value of $0.020m_0$ for the band-edge transverse mass in PbTe. It is also found that both relaxation to the local equilibrium and anisotropic scattering are important effects, although only the former is included in the theoretical calculations.

I. INTRODUCTION

IN a sufficiently large static magnetic field, it is possible for circularly polarized electromagnetic waves to propagate with negligible damping in conducting solids. These waves, known as helicons, have been studied in many metals and semiconductors.[1] Provided that displacement current and quantum effects can be neglected, the helicon dispersion relation in high magnetic fields is independent of band structure and depends only on the carrier concentration, the strength of the magnetic field, and the angle of the field with

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[1] For reviews of theoretical and experimental work on helicons see articles by S. J. Buchsbaum, in *Plasma Effects in Solids* (Dunod Cie., Paris, 1965), p. 3; R. Bowers, *ibid.*, p. 19. Alfvén waves propagate instead of helicons if the concentration of holes and electrons are equal.