

Meson-Baryon Couplings in a Quark Model*

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The widths for decay of low-lying baryons of negative parity into baryon plus pseudoscalar meson are determined in a quark model and extensively compared with experiment. One conclusion is that $\Xi^*(1816)$ is not in an octet with $N^*(1518)$, $Y_1^*(1660)$. A second prediction is a kinematical factor in s -wave decay enhancing the decay into high-mass mesons (K and η over π), which provides a qualitative reason for η peaks at threshold. Properties of the many missing baryon resonances are discussed. Channels appropriate for the search for some of these are indicated.

I. INTRODUCTION

IN this article we calculate the widths for strong decay of low-lying baryons, especially those of negative parity, into pseudoscalar meson plus baryon. Our baryon model is the three-quark system.¹ We first discuss the model and the form of the calculation, then we present the many results² and discuss implications of the current data.

Baryon states are assumed to be Gell-Mann-Zweig triplets of quarks governed by nonrelativistic dynamics.³ It is assumed in conformity with Dalitz's general analysis that the baryons form bases for representations of the group $SU(6) \times O_3$,⁴ with the lowest-lying representations being⁵:

(a) (56) even-parity states of $L=0$, corresponding to the well-known $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet;

(b) $(70) \times (3)$ odd-parity states of $L=1$ corresponding to two $\frac{1}{2}^-$ octets whose N^* components are known, two $\frac{3}{2}^-$ octets and a $\frac{5}{2}^-$ decuplet with $N^*(T=\frac{1}{2})(1518)$,

$Y_1^*(1660?)$, and $\Xi^*(1816)$ known, a $\frac{1}{2}^-$ decuplet whose N^* component is known (?), a $\frac{5}{2}^-$ octet with known N^* and $Y_1^*(1765)$, and the known $\frac{1}{2}^-$ and $\frac{3}{2}^-$ singlets $Y_0^*(1405)$ and $Y_0^*(1520)$, respectively.

These known resonances and the assignments we would like to make on the basis of our results are shown in Table I.

In a previous paper⁶ one of us demonstrated that this model can be justified by explicit dynamical calculation to determine the lowest-lying states. The main assumptions in this calculation were parastatistics⁷ and operation of s - and p -wave forces in Q - Q pairs.⁸ The most attractive states separated into three sets:

(a) and (b), above; and

(c) $(20) \times (3)$ even-parity states of $L=1$ corresponding to $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$ singlets. Possible candidates⁹ for this multiplet are $\frac{1}{2}^+$ $T=\frac{1}{2}$ $N^*(1450)$ (or Roper resonance), and $\Xi^*(1705)$ of unknown J^P . There are also Y^* possibilities. We will not present an extensive discussion of couplings to $(20) \times (3)$ states.

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¹ M. Gell-Mann, Phys. Letters 8, 214 (1963); G. Zweig, CERN (1964) (unpublished).

² Related recent calculations of meson-baryon couplings are R. J. Rivers, Phys. Letters 22, 514 (1966); J. J. Coyne, S. Meshkov, and G. B. Yodh, Phys. Rev. Letters 17, 666 (1966). Very closely related results have been obtained by R. Capps (private communication).

³ G. Morpurgo, Physics 2, 95 (1965); Y. Nambu, in *Symmetry Principles at High Energy*, edited by B. Kurşunoglu, A. Perlmutter, and I. A. Sakmar (W. H. Freeman and Company, San Francisco, 1965); R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966); A. N. Tavkhelidze, *Seminar on High Energy Physics and Elementary Particles* (International Atomic Energy Agency Vienna, 1965), p. 753.

⁴ K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 153 (1965).

⁵ In most cases our data are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 37, 633 (1965). For recently discovered N^* 's we have referred mainly to R. H. Dalitz (Ref. 3) and to P. Bareyre *et al.*, Phys. Letters 18, 342 (1965), obtaining widths by our own analysis.

We will show below that, in contrast to the mass spectrum, the baryon partial widths are essentially independent of the choice of quark statistics. The results as presented here, where certain spatial integrals are treated as parameters rather than being evaluated in detail, are the same for Fermi statistics and parastatistics and depend only on the assignments of the baryon states.

II. THE MODEL

Within each set (a), (b), (c), the states are degenerate in the presence of the spin- and unitary-spin-independent forces. The actual mass splittings are not of interest to us here: We assume that the wave functions are not influenced by the splitting.

⁶ A. N. Mitra, Phys. Rev. 151, 1168 (1966).

⁷ O. W. Greenberg, Phys. Rev. Letters 13, 598 (1964).

⁸ A. N. Mitra, Phys. Rev. 142, (4), 1119 (1966).

⁹ See Ref. 5; A. B. Clegg, Nucl. Phys. 76, 545 (1966); G. Smith, *Second Topical Conference on Resonant Particles*, edited by B. Munir (Ohio University Press, Athens, Ohio, 1965).

TABLE I. Negative-parity (70,3) baryon resonances. The notation in the left-hand column is $^{2S+1}(SU(3)$ dimensionality) $_J$, where S is the total quark intrinsic spin. Each dash indicates an unobserved resonance needed in the corresponding $SU(3)$ multiplet.

$(1)_{1/2}$	$Y_0^*(1405)$			
$(1)_{3/2}$	$Y_0^*(1520)$			
${}^2(8)_{1/2}$	$N^*(1700)$...		
${}^4(8)_{1/2}$	$N^*(1540)$	$Y_0^*(1670)$
${}^2(8)_{3/2}$	$N^*(1518)$...	$Y_1^*(1660)$...
${}^4(8)_{3/2}$
$(8)_{5/2}$	$N^*(1690)$...	$Y_1^*(1765)$...
$(10)_{1/2}$	$N^*(1680)$
$(10)_{3/2}$	$\Xi^*(1816)$...

The wave functions are sums of products of spatial functions $\psi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$, intrinsic spin functions $\chi(1, 2, 3)$, and $SU(3)$ functions $\phi(1, 2, 3)$. We distinguish different functions by their permutation symmetries.¹⁰ There are four permutation symmetries for functions of three variables: symmetric under any permutation (labeled, e.g., ψ^s), antisymmetric under any odd permutation (labeled, e.g., ψ^a), of mixed symmetry but symmetric with respect to, say, the 2,3 pair of coordinates (labeled e.g., ψ''), and of mixed symmetry but antisymmetric with respect to the 2,3 pair (labeled, e.g., ψ'). The rules for constructing products of these functions with definite permutation symmetry are well known. With particles of intrinsic spin $\frac{1}{2}$ only states χ^s (total spin $\frac{3}{2}$), and χ' and χ'' (each with total spin $\frac{1}{2}$) can be formed. The $SU(3)$ states $\phi^s, \phi'', \phi', \phi^a$ correspond to $SU(3)$ representations (10), (8), (8), (1), respectively.

Consider first the case of parastatistics where among the three quarks Bose statistics are obeyed. The (56) is characterized by a symmetric spatial wave function ψ^s with total angular momentum $L=0$. The over-all symmetric states which can be constructed are

$$(8): \quad \Psi = \psi^s(\chi' \phi' + \chi'' \phi''),$$

$$(10): \quad \Psi = \psi^s \chi^s \phi^s.$$

For the (70,3) the spatial function is of mixed symmetry with $L=1$. Thus Ψ contains terms

$$(\sum_{MSM_L} \langle LM_L SM_S | JM \rangle \psi_{L, M_L}^{\alpha} \chi_{S, M_S}^{\beta}) \phi^{\alpha}$$

where \mathbf{S} is the spin- $SU(3)$ matrix element. This spatial integral which occurs here has two distinct forms corresponding to s - and d -wave decays. Meanwhile, for all p wave decays within the (56) and within the (70 \times 3) the spatial integral involving the direct term q (which presumably dominates in this case) is just the normali-

¹⁰ See, e.g., M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170. For a similar construction of states of higher angular momenta ($L=1$ and 2), see A. N. Mitra, *Phys. Rev.* **151**, 1168 (1966) and (to be published).

which we abbreviate

$$[\psi^{\alpha} \chi^{\beta}]_J \phi^{\gamma},$$

where α, β, γ are the superscripts indicating the permutation symmetry. The over-all symmetric states which can be constructed are [the notation is ($SU(3)$ multiplicity) $_J$]:

$$(1)_{1/2, 3/2}: \quad \Psi = ([\psi' \chi']_{1/2, 3/2} - [\psi' \chi'']_{1/2, 3/2}) \phi^s,$$

$$(8)_{5/2}: \quad \Psi = [\psi' \chi^s]_{5/2} \phi' + [\psi'' \chi^s]_{5/2} \phi'',$$

$$(8)_{1/2, 3/2}: \quad \Psi = a \{ ([\psi' \chi']_{1/2, 3/2} + [\psi' \chi'']_{1/2, 3/2}) \phi' \\ + ([\psi' \chi']_{1/2, 3/2} - [\psi'' \chi'']_{1/2, 3/2}) \phi'' \} \\ + b ([\psi' \chi^s]_{1/2, 3/2} \phi' + [\psi'' \chi^s]_{1/2, 3/2} \phi''),$$

$$(10)_{1/2, 3/2}: \quad \Psi = ([\psi' \chi']_{1/2, 3/2} + [\psi'' \chi'']_{1/2, 3/2}) \phi^s.$$

As there are two $\frac{1}{2}$ - and two $\frac{3}{2}$ - octets, their wave functions are only determined to within one parameter.

We evaluate the Yukawa coupling of the baryons to the pseudoscalar meson octet, symbolically $B' \rightarrow B + \Pi$ (where Π stands for a member of the pseudoscalar octet), by assuming the basic interaction

$$Q' \rightarrow Q + \Pi. \quad (1)$$

On the basis of $SU(3)$ invariance and Galilean invariance, the interaction must have the form

$$\boldsymbol{\sigma} \cdot \left(\mathbf{q} - \frac{\boldsymbol{\omega}}{M_Q} \mathbf{p} \right) F_i \Pi_i, \quad (2)$$

where $i=1, \dots, 8$ runs over the SU_3 generators,¹¹ \mathbf{q} and \mathbf{p} are the respective meson and quark momenta, $\boldsymbol{\sigma}$ is the quark spin operator, ω is the meson relativistic mass, and M_Q is the quark mass.

The $Q' \rightarrow Q + \Pi_i$ amplitude can be calculated as an operation on quark 1:

$$(\bar{\Psi}(\mathbf{p}_1 - \frac{2}{3}\mathbf{q}), \boldsymbol{\sigma}_i(\mathbf{q} - \boldsymbol{\omega}\mathbf{p}_1/M_Q) F_i^{(1)} \bar{\Psi}'(\mathbf{p}_1)).$$

This matrix element can be factored into a spatial part times a spin- $SU(3)$ part. Consider, for example, decays from the (70 \times 3) into the (56). The matrix element has the form

$$\sum_{M_L} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 \psi_{L=0}^S(\mathbf{p}_1 - \frac{2}{3}\mathbf{q}, \mathbf{p}_2, \mathbf{p}_3)^* [\mathbf{q} - \boldsymbol{\omega}\mathbf{p}_1/M_Q] \psi''_{L=1, M_L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \cdot \mathbf{S}_{M_L}, \quad (3)$$

zation integral for the spatial wave function and can be done. The s , p , and d wave decays thus correspond to three distinct types of transitions. Within each type of transition the ratios of the coupling constants, or partial widths are meaningful. We do *not* need to calculate the spatial integrals. In (56) \rightarrow (56) + Π , where the standard $SU(6)$ coupling results apply,³ and (70,3)-

¹¹ See, e.g., M. Gell-Mann, in *The Eight-Fold Way* edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin and Company, Inc., New York, 1964), p. 11.

TABLE II. Order of magnitude of direct and recoil terms for $(70 \times 3) \rightarrow (56)$ transitions.

Decay	Direct	Recoil
<i>s</i> wave	Rq^2	$(1/R)\omega/M_Q$
<i>d</i> wave	Rq^2	$Rq^2\omega/M_Q$

$\rightarrow (70,3)^- + \Pi$, with *p*-wave mesons, we set the scale for all from, e.g.,

$$\Gamma(N^*(1238) \rightarrow N + \pi) = 120 \text{ MeV}.$$

For the *s*- and *d*-wave decays, the same spatial integrals occur for all transitions within each type, so within each type one of the partial widths Γ_i can be chosen as input.

The spatial integrals are energy-dependent. It is only here, and in the phase space, that we take into account the actual symmetry-violating masses of the baryons and mesons. The partial widths Γ_i are thus proportional to the square of a Yukawa coupling constant *g*, where this is the spin-*SU*(3) matrix element, times an energy dependence which would arise from the spatial integration. The *g*'s are algebraic quantities which could be readily calculated using appropriate group theory. The energy dependence is, however, intimately associated with the quark model and contains a surprise. We must examine both the direct and recoil contributions in the spatial integral in the $(70) \times (3) \rightarrow (56)$ matrix element [see Eq. (3)]. For the quark mass we can assume only that $\omega/M_Q \lesssim 1$, since some believe that the effective mass in a bound system is as low as $M_N/3$. Meanwhile we can assume that $(qR)^2 \ll 1$, where R^{-1} is the typical expectation value of a quark momentum operator. The order of magnitude of the direct and recoil terms for *s*- and *d*-wave decays is given in terms of these parameters in Table II.¹² It thus seems reasonable that the recoil term dominates the *s*-wave decay and the direct term dominates the *d*-wave decay. This is what we assume. The partial widths are then taken to have the forms:

$$s \text{ wave: } \Gamma = c_s g^2 q \omega^2, \quad (4a)$$

$$p \text{ wave: } \Gamma = c_p g^2 q^3 / (1 + \alpha q^2), \quad (4b)$$

$$d \text{ wave: } \Gamma = c_d g^2 q^5 / (1 + \alpha q^2)^2. \quad (4c)$$

Here $\alpha = (600 \text{ MeV})^{-2}$ introduces the finite range of the angular momentum barrier.¹³ In fact, the direct and recoil terms are very different for the *s* wave case, and experiment is quite inconsistent with the q^5 dependence associated with the former. We have the surprising prediction of a factor ω^2 which is very sensitive to the mass of the emitted meson for a low-*Q* decay. In the *d*-wave

¹² These estimates do not take into account the degree of overlap on the spatial integral. This will depend on the dynamics and could easily give rise to results at variance with Table II. In particular, in the *d*-wave case the quark dynamics of Ref. 6 suggest that the overlap may be much greater in the recoil than direct term.

¹³ The choice of α effectively determines the scale for the $Y_0^*(1520)$ decays. The results are not, in fact, very sensitive to α , considering the experimental errors.

case the difference between direct and recoil contributions is not large. One could not discriminate between these dependences by current experiment alone.

To complete the description of the calculation note that for all but the $(8) \times (8) \rightarrow (8)$ transitions, the quantity g_i is given by the Wigner-Eckart theorem for *SU*(3), which expresses it as a "reduced width" (independent of *I*, *Y*), times the appropriate isoscalar factor,¹⁴ viz.,

$$g_i = X_i (\Pi B | B'). \quad (5)$$

For the $(8) \times (8) \rightarrow (8)$ transition, on the other hand, g_i depends on two reduced widths, X_1 and X_2 , according to the formula

$$g_i = X_{i1} (88 | 8_1) + X_{i2} (88 | 8_2). \quad (6)$$

Let us digress to see that the partial widths will be the same whether we use Fermi statistics or Bose statistics in constructing the baryon wave functions. The over-all antisymmetric wave functions are, for the (56) (where the spatial state has $L=0$ and is antisymmetric):

$$(8): \quad \Psi = \psi^a(x' \phi' + x'' \phi''),$$

$$(10): \quad \Psi = \psi^a x^s \phi^s.$$

Meanwhile for the (70,3) (where the spatial states have $L=1$ and mixed symmetry) we have essentially

$$(1): \quad \Psi = ([\psi' x'] + [\psi'' x'']) \phi^a,$$

$$(8): \quad \Psi = a \{ ([\psi' x'] - [\psi'' x'']) \phi' - ([\psi'' x'] + [\psi' x'']) \phi'' \} + b ([\psi' x^s] \phi' - [\psi' x^s] \phi''),$$

$$(10): \quad \Psi = ([\psi'' x'] - [\psi' x'']) \phi^s.$$

Consider, for example, the transition from a (1) in the (70,3) to an (8) in the (56). The factorization of the matrix elements into spatial \times spin-*SU*(3) part has the form

$$\text{Bose: } (\psi^s, \mathbf{O}_R \psi'') \cdot (x' \phi', \mathbf{O}_S x' \phi^a),$$

$$\text{Fermi: } (\psi^a, \mathbf{O}_R \psi') \cdot (x' \phi', \mathbf{O}_S x' \phi^a).$$

Here the operator has been factored into space and spin-*SU*(3) parts using the notation $\mathbf{O}_R \cdot \mathbf{O}_S$. We see that the spin-*SU*(3) matrix elements are identical. This result holds for all cases. Indeed if we denote the spatial function of various symmetries by *A*, *S*, *M'*, *M''*, in obvious notation, the only modifications that the spatial integrals would suffer in going from parastatistics to Fermi statistics are $S \rightarrow A$, $A \rightarrow S$, $M' \rightarrow M''$, $M'' \rightarrow M'$. Within a given type of transition the spatial integral is the same for the transitions. The only difference between Bose and Fermi statistics is then that these spatial factors are formally different. Since we treat a given spatial integral as proportional to a free parameter (except in Sec. VI, below), our results are independent of the choice of statistics.

¹⁴ J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

TABLE III. Partial widths for decay of low-lying baryons. There are three major types of decay: The top group is d -wave decays. The second group is s -wave decays. The bottom group is p -wave decays. Masses are chosen to check the assignment of known particles. The plus superscript indicates a mass arbitrarily introduced for illustrative purposes in the case of an undiscovered particle. Other notations are explained in the text.

$SU(3)$ transition $B' \rightarrow B$	Reduced width X , for $(8) \times (8) = (8): (X_1, X_2)$	Transition	For $\frac{3}{2}^-, \frac{1}{2}^-$ octets ${}^2(8); {}^4(8)$	Γ_i (MeV) Theory ${}^2(8); {}^4(8)$	Experiment Γ_i (MeV)
$(8)_{5/2}^- \rightarrow (8)_{1/2}^+$	$(-\sqrt{\frac{5}{6}}, -\sqrt{\frac{1}{6}})$	$N^*(1690) \rightarrow N+\pi$	1/6	17	≈ 60
		$Y_1^*(1765) \rightarrow \Sigma+\pi$	-1/9	6	≤ 2
		$\Lambda+\pi$	-1/6	12 input	12
		$N+\bar{K}$	4/9	29	45
		$Y_0^*(?) \rightarrow N+\bar{K}$	0	0	} not observed
		$\Sigma+\pi$	1/2	...	
$\Xi^*(1930)^+ \rightarrow \Lambda+\bar{K}$	1/6	11			
$(8)_{5/2}^- \rightarrow (10)_{3/2}^+$	$\sqrt{(35/12)}$	$Y_1^*(1765) \rightarrow Y_1^*(1385)+\pi$	7/18	5	7
		$Y_0^*(?) \rightarrow Y_1^*(1385)+\pi$	7/4
		$\Xi^*(1930)^+ \rightarrow \Xi^*(1530)+\pi$	7/12
$(1)_{3/2}^- \rightarrow (8)_{1/2}^+$	$\sqrt{5}$	$Y_0^*(1520) \rightarrow N+E$	-5/4	5	5
		$\Sigma+\pi$	15/8	10	9
$(10)_{3/2}^- \rightarrow (8)_{1/2}^+$	$-\sqrt{(5/18)}$	$N_3^*(1585)^+ \rightarrow N+\pi$	5/36	9	...
		$Y_1^*(1660) \rightarrow N+\bar{K}$	5/108	1	} see below
		$\Sigma+\pi$	5/108	1	
		$\Lambda+\pi$	5/72	3	
		$\Xi^*(1816) \rightarrow \Lambda+\bar{K}$	5/72	2	} ...
		$\Xi+\pi$	5/72	2	
$\Omega(1990)^+ \rightarrow \Xi+\bar{K}$	5/72	0.4	6		
$(8)_{3/2}^- \rightarrow (8)_{1/2}^+$	$-\sqrt{(\frac{5}{3})} \frac{1}{6} \frac{1}{(a^2+b^2)^{1/2}}$ $(\sqrt{5}a-\sqrt{2}b, -5a-\sqrt{(\frac{5}{3})}b)$	$N^*(1518) \rightarrow N+\pi$	10/9 ; -1/36	52 ; 1	≈ 50
		$\Lambda+\bar{K}$	-5/8 ; 0
		$Y_1^*(1660) \rightarrow N+\bar{K}$	-5/108 ; -2/27	1.4 ; 2	7
		$\Sigma+\pi$	125/108 ; 1/54	29 ; 0.5	13
		$Y_0^*(1660)^+ \rightarrow N+\bar{K}$	-5/72 ; 1/36	3 ; 1	} not observed
		$\Lambda+\pi$	5/4 ; 0	37 ; 0	
$\Xi^*(1816) \rightarrow \Lambda+\bar{K}$	5/24 ; -1/12	5 ; 2	2		
$\Xi+\pi$	-5/18 ; -1/36	8 ; 1	10		
$\Sigma+\bar{K}$	5/72 ; 1/9	2 ; 4	1		
$\Sigma+\bar{K}$	10/9 ; -1/36	7 ; 0.2	< 1		
$(10)_{3/2}^- \rightarrow (10)_{3/2}^+$	4/9			0(1 MeV)	
$(8)_{3/2}^- \rightarrow (10)_{3/2}^+$	$\sqrt{(35/6)} \frac{1}{(a^2+b^2)^{1/2}}$ $1/3(\sqrt{(\frac{5}{3})}a+2b)$				0(1 MeV)
$(1)_{1/2}^- \rightarrow (8)_{1/2}^+$	-2/3	$Y_0^*(1405) \rightarrow N+\bar{K}$	1/9
		$\Sigma+\pi$	-1/6	21 input	35
$(10)_{1/2}^- \rightarrow (8)_{1/2}^+$	$-\sqrt{2}/9$	$\Lambda+\eta$	1/18
		$N_3^*(1680) \rightarrow N+\pi$	-1/81	48	≈ 50
		$\Sigma+\bar{K}$	1/81	0.06q	...
		$Y_1^*(1750)^+ \rightarrow N+\bar{K}$	1/243	19	} not observed
		$\Sigma+\pi$	-1/243	8	
		$\Lambda+\pi$	-1/162	17	
		$\Sigma+\eta$	1/162	0.038q	} ...
		$\Xi+\bar{K}$	-1/243	...	
		$\Xi^*(1820)^+ \rightarrow \Lambda+\bar{K}$	-1/162	26	} not observed
		$\Xi+\pi$	-1/162	13	
		$\Sigma+\bar{K}$	-1/162	19	
$\Xi+\eta$	1/162	0.038q	...		
$\Omega(?) \rightarrow \Xi+\bar{K}$	2/81				
$N^*(1540) \rightarrow N+\pi$	8/81 ; -2/81	240 ; 60	$\approx 70^a$		
$N+\eta$	-2/81 ; -2/81	0.17q ; 0.17q	$\approx 0.3q^a$		
$\Lambda+\bar{K}$	1/18 ; 0	0.27q ; 0	...		

TABLE III (continued)

$SU(3)$ transition $B' \rightarrow B$	Reduced width X for $(8) \times (8) = (8): (X_1, X_2)$	Transition	For $\frac{3}{2}^-, \frac{1}{2}^-$ octets ${}^2(8); {}^4(8)$	Γ_i (MeV) Theory ${}^2(8); {}^4(8)$	Γ_i (MeV) Experiment
$(8)_{1/2}^- \rightarrow (8)_{1/2}^+$	$\frac{1}{9\sqrt{2}} \frac{1}{(a^2+b^2)^{1/2}}$ $(- \sqrt{5}a + 2\sqrt{5}b, 5a+2b)$	$N^*(1700) \rightarrow N+\pi$	as above	410 ; 100	≈ 200
		$N+\eta$	as above	90 ; 90	...
		$\Lambda+K$	as above	80 ; 0	...
		$\Sigma+K$	$1/162; 8/81$	$0.03q; 0.48q$...
		$Y_1^*(1750)^+ \rightarrow N+\bar{K}$	$-1/243; -16/243$	19 ; 300	} not observed
		$\Sigma+\pi$	$5/243; 4/243$	42 ; 34	
		$\Lambda+\pi$	$-1/162; 2/81$	17 ; 68	} $\Gamma_i \approx 18^b$ $\Gamma_{\bar{K}N} \Gamma_{\eta\Lambda} \approx 0.2q$ (MeV) ²
		$\Sigma+\eta$	$-1/162; 2/81$	$0.038q; .15q$	
		$\Xi+K$	$32/243 - 8/243$...	
		$Y_0^*(1670) \rightarrow N+\bar{K}$	$1/9; 0$	370 ; 0	
$\Sigma+\pi$	$3/162; -6/81$	25 ; 100	} not observed		
$\Lambda+\eta$	$1/162; -2/81$	$0.038q; 0.15q$			
$\Xi+K$	$4/81; 4/81$...			
$\Xi^*(1880)^+ \rightarrow \Lambda+K$	$-2/81; -2/81$	102 ; 102			
$\Xi+\pi$	$1/162; 8/81$	13 ; 210	} not observed		
$\Sigma+\bar{K}$	$8/81; -2/81$	300 ; 75			
$\Xi+\eta$	$1/18; 0$	$0.34q; 0$			
$(10)_{3/2}^- \rightarrow (10)_{3/2}^+$	4/9	$N^*(1585)^+ \rightarrow N^*(1238)+\pi$	10/81	70	...
		$Y_1^*(1660) \rightarrow Y_1^*(1385)+\pi$	16/243	19	...
		$N^*(1238)+K$	16/243
		$\Xi^*(1816) \rightarrow \Xi^*(1530)+\pi$	2/81	8	...
		$\Omega(?) \rightarrow \Xi^*(1530)+K$	8/81	$0.5q$...
$(8)_{3/2}^- \rightarrow (10)_{3/2}^+$	$\frac{1}{(a^2+b^2)^{1/2}} \frac{\sqrt{10}}{9}$ $\times (-a + \sqrt{\frac{2}{3}}b)$	$N^*(1518) \rightarrow N^*(1238)+\pi$	$8/81; -20/81$	30 ; 74	≈ 50
		$Y_1^*(1660) \rightarrow Y_1^*(1385)+\pi$	$-4/243; 10/243$	5 ; 12	≤ 8
		$\rightarrow N^*(1238)+K$	$16/243; -40/243$	$0.3q; 0.8q$...
		$Y_0^*(1660)^+ \rightarrow Y_1^*(1385)+\pi$	$2/27; -5/27$	22 ; 54	...
		$\Xi^*(1816) \rightarrow \Xi^*(1530)+\pi$	$-2/81; 5/81$	8 ; 21	4
$\rightarrow Y_1^*(1385)+K$	$2/81; -5/81$	$0.1q; 0.3q$...		
$(10)_{3/2}^+ \rightarrow (8)_{1/2}^+$	$\frac{4}{3\sqrt{3}}$	$N^*(1238) \rightarrow N+\pi$	8/27	120 input	120
$(8)_{5/2}^- \rightarrow (1)_{3/2}^-$	$\frac{\sqrt{2}}{3\sqrt{3}}$	$Y_1^*(1765) \rightarrow Y_0^*(1520)+\pi$	-2/27	16	≈ 8
$(8)_{3/2}^- \rightarrow (1)_{3/2}^-$	$\frac{1}{\sqrt{(\frac{2}{3})}2/9} \frac{1}{(a^2+b^2)^{1/2}}$ $\times (a + \sqrt{2/5}b)$	$Y_1^*(1660) \rightarrow Y_0^*(1520)+\pi$	20/243; 8/243
		$Y_1^*(1660) \rightarrow Y_0^*(1405)+\pi$	-16/243; 10/243	16 ; 9	≤ 16

* See C. Michael, Phys. Letters 21, 93 (1966); A. W. Hendry and R. G. Moorhouse, *ibid.* 18, 171 (1965); A. T. Davies (private communication).

^b This guess is based on data of D. Berley *et al.*, Phys. Rev. Letters 15, 641 (1965).

III. RESULTS

The results of the calculations of partial widths Γ_i are given in Table III. For $(8)_{3/2}^-$ and $(8)_{1/2}^-$ states [the notation is: $(SU(3)$ multiplicity) _{r} ^{p}], the predictions are given for the doublet and quartet quark intrinsic spin (S) states. These octets are labeled ${}^2(8)$ and ${}^4(8)$, respectively. The reduced widths are given in terms of the doublet and quartet amplitudes, a and b , i.e., the wave function is

$$\Psi = a\Psi(S=\frac{1}{2}) + b\Psi(S=\frac{3}{2}), \quad (7)$$

where the Ψ 's are normalized. The threshold production

of K 's and η 's is given in the form: partial width $\Gamma = cq$, where q and Γ are both in MeV. The sign of g is entered before the values of g^2 . This column can be used to evaluate rates in case of mixtures.¹⁵

First some general remarks about the results: (1) The recoil factor ω^2 in the $(70,3) \rightarrow (56)$ s -wave rate strongly enhances K and η relative to π for low- Q decays. This means that the K and η partial widths will be much larger than indicated by $SU(3)$ and phase space, which

¹⁵ Relative values of these signs can also be observed directly. See, e.g., A. Kernan and W. M. Smart, Phys. Rev. Letters 17, 832, 1125 (1966). We note that our results disagree with theirs for the phase $\bar{K}N \rightarrow \Lambda\pi$ in $Y_1^*(1660)$, $Y_1^*(1765)$.

can explain the prominence of the $N+\eta$ and $\Lambda+\eta$ peaks near threshold.

(2) At present the sensitivity of experiments is such that many of the resonances will be unobserved. Many states have very small width and/or high inelasticity or are very broad such that they may not be observed for some time. It would be a mistake to assume that lack of observation of a resonance necessarily implies higher mass.

One final remark before detailed discussion of the results. There are three negative-parity Y_0^* 's, Y_1^* , and Ξ^* 's both for spin $\frac{3}{2}$ and spin $\frac{1}{2}$. Some of these probably overlap. For this reason presently quoted experimental partial widths for Y_1^* 's at 1660 and 1766 (as well as those for the N^* 's recently discovered in partial wave analyses) must be regarded as preliminary. For purposes of discussion we will, however, take the present numbers quite seriously.

A. Detailed Discussion of Table III

1. Decays of $\frac{5}{2}^-$ Baryons

The $(8)_{5/2}^-$ decays are in reasonable agreement with the 6 experimental numbers (including p -wave decay at bottom of Table III). The d -wave decay of $(8)_{5/2}^-$ baryons (into $(8)_{1/2}^+$ and $(10)_{3/2}^+$ plus pseudoscalar mesons) are the same as given by Coyne *et al.*² in their study of the (1134) representation of $SU(6)$. We determine the scale factor for d -wave decays [c_d of Eq. (4)] from the input width $Y_1^*(1765) \rightarrow \Lambda\pi$. Two members of the $(8)_{5/2}^-$, Y_0^* and Ξ^* , have not been observed. They are discussed below. [The decay of the $(1)_{3/2}^-$, the $Y_0^*(1520)$, is not the same as in the (1134) as given by Coyne *et al.*, but this fact is of little significance because of the factor α [see Eq. (4)], which can be adjusted to give the correct relation between the low- Q $Y_0^*(1520)$ decays and the high- Q $Y_1^*(1765)$ decays.] The agreement with current data for $Y_1^*(1765)$ and $N^*(1690)$ is satisfactory.

2. Decays of $\frac{3}{2}^-$ baryons

All other (70,3) states except the N^* decuplets have 2- or 3-fold degeneracy. Assuming $SU(3)$ is good and that the octet states divide into $^2(8)$ and $^4(8)$ we have to choose which of the assignments *most* suits experiment. For known $\frac{3}{2}^-$ baryons we have one of our most interesting results: they are not in the same octet.¹⁶ The $N^*(1518)$, $Y^*(1660)$ (assumed $\frac{3}{2}^-$), and $\Xi^*(1816)$ are fitted semiquantitatively by placing N^* and Y_1^* in the $^2(8)$ and Ξ^* in the (10). An excellent fit to data in the latter case can be obtained with 10% $^2(8)$ mixing. The state $(9/10)^{1/2}(10)+(1/10)^{1/2} \ ^2(8)$ yields satisfactory widths: $\Gamma_{\Lambda\bar{K}}$, $\Gamma_{\Xi\pi}$, $\Gamma_{\Sigma\bar{K}}$, $\Gamma_{\Xi^*\pi} = 5, 1, 0.0, 4$ MeV respectively. Small admixture to a dominant $^2(8)$ state does not work if we believe that $\Xi^*(1816) \rightarrow \Sigma+\bar{K}$ is very

¹⁶ Compare, for example, S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

small. For the $Y_1^*(1660)$ small admixture of $^4(8)$ to the dominant $^2(8)$ yields good agreement with current experiment. The many unobserved $\frac{3}{2}^-$ states are discussed below.

3. Decays of $\frac{1}{2}^-$ baryons

Among $\frac{1}{2}^-$ baryons the $Y_0^*(1405)$ $\frac{1}{2}^-$ observed decay width is relatively large. It would be convenient for the model if the actual width were about $\frac{1}{2}$ the quoted 35 MeV or if the theoretical widths were slightly damped compared with the expression (4a) for higher- Q decays. The only other observed $\frac{1}{2}^-$ states are N^* (nominal 1540) and $Y_0^*(1670)$ associated with η decays, which should be placed in the $^4(8)$, the probable $T = \frac{3}{2}N^*(1680)$ in the (10), and the very wide $N^*(1700)$ which should be placed in the $^2(8)$. Only fair agreement is achieved. Possible mixing which could remedy the disagreements is exemplified by the $Y_0^*(1670)$, whose observed narrowness is a serious difficulty. One needs to *adjust* the mixture *sensitively*. Just as an example,

$$(\sqrt{17/20}) \ ^4(8) - (1/\sqrt{10})(10) + (1/\sqrt{20}) \ ^2(8)$$

yields $\Gamma_{\bar{K}N} = 3$, $\Gamma_{\pi\Sigma} = 12$, $\Gamma_{\eta\Lambda} = 0.25q$. One of the most interesting results is that the $^2(8)_{1/2}^-$ lies above $^4(8)_{1/2}^-$. Several other $\frac{1}{2}^-$ states are indicated to have reasonable widths so that they should be observable in spite of the low statistical weight. These are discussed in Sec. V below.

We do not present here any model of Q Q forces which would lead to the mixtures mentioned above.

The properties of the missing resonances in the (70,3) are summarized in Table IV. In this table "input width" refers to the formation channel of the resonance relevant in most experiments. The three resonances which should be most visible are not inconsistent with observation. There is a possible Y_0^* around 1700 MeV.¹⁷ There is a $\Xi^*(1930)$ quite consistent with the 1870 state indicated in the table. Masses in the Ξ^* around 2040 MeV are not well explored.

IV. MISSING $\frac{3}{2}^-$ AND $\frac{5}{2}^-$ STATES

Now consider the as yet unobserved resonances in the (70,3) of spin $\frac{3}{2}$ and $\frac{5}{2}$. All reference below is to the $\frac{3}{2}^-$ states except when explicitly stated. Very crude mass guesses are indicated for these particles in Table IV.

N^* : There are a (10) and a $^4(8)$ with $I = \frac{3}{2}, \frac{1}{2}$, respectively. These N^* 's are relatively narrow and inelastic. The $I = \frac{3}{2} N^*$, in particular, should have low mass if our assignment to $\Xi^*(1816)$ is correct and is so narrow that it would likely be missed in the current mesh of accurate experiments.

Y_1^* : There are (10) and $^4(8)$ Y_1^* 's. They have very similar characteristics. They will be very difficult to observe, being highly inelastic. Perhaps indirect production, such as a missing-mass experiment, would reveal

¹⁷ J. D. Davies *et al.*, Phys. Rev. Letters **18**, 62 (1967).

TABLE IV. Properties of missing resonances in the (70,3). The assumed masses are crude guesses made in order to be definite. They are not generally the same masses considered in Table III. Those without (*) were chosen according to the rules: For octets $M = M_c + 175Y + A(1 - Y^2)(I - 1/4)$ MeV with A determined empirically in each multiplet, for decuplets $M = M_c + 175Y$. [There is considerable reason to believe, however, that the Gell-Mann-Okubo mass formula will not hold in the (70,3).^a] Those masses marked (*) had to be selected quite arbitrarily because of insufficient information. The total widths quoted are based only on the channels shown in Table III. Under remarks we note that the visibility of Y_1^* 's in ${}^4(8)_{3/2}$ and $(10)_{3/2}$ is sensitive to mixing which may affect the small input width Γ_{KN} . The ${}^4(8)_{1/2}\Xi^*$ may be effectively highly inelastic.

Y_I	${}^{2S+1}(SU(3))_J$	Assumed mass (MeV)	Γ (MeV)	Input widths (MeV)	Readily visible	Remarks
$N_{1/2}^*$	${}^4(8)_{3/2}$	1518*	75	$\Gamma_{\pi N} = 1$	No	Highly inelastic
$N_{3/2}^*$	$(10)_{3/2}$	1465	25	$\Gamma_{\pi N} = 4$	Perhaps	Narrow and inelastic
Y_0^*	${}^2(8)_{1/2}$	1875*	800	$\Gamma_{KN} = 750$	No	Too wide
Y_0^*	${}^2(8)_{3/2}$	1705	90	$\Gamma_{KN} = 45$	Yes	$\bar{K}N$ decay dominant
Y_0^*	${}^4(8)_{3/2}$	1660*	55	$\Gamma_{KN} = 0$	No	Highly inelastic
Y_0^*	$(8)_{5/2}$	1900	110	$\Gamma_{KN} = 0$	No	Highly inelastic
Y_1^*	${}^2(8)_{1/2}$	1875*	450	$\Gamma_{KN} = 30$	Perhaps	Large width mainly ΞK
Y_1^*	${}^4(8)_{1/2}$	1800*	550	$\Gamma_{KN} = 400$	No	Too wide
Y_1^*	$(10)_{1/2}$	1855	85	$\Gamma_{KN} = 30$	Perhaps	Low statistical weight
Y_1^*	${}^4(8)_{3/2}$	1660*	30	$\Gamma_{KN} = 2$	Perhaps	} May be highly inelastic
Y_1^*	$(10)_{3/2}$	1640	20	$\Gamma_{KN} = 1$	Perhaps	
$\Xi_{1/2}^*$	${}^2(8)_{1/2}$	2050	1000		No	Too wide
$\Xi_{1/2}^*$	${}^4(8)_{1/2}$	1890	400		No	Too wide
$\Xi_{1/2}^*$	$(10)_{1/2}$	2030	125		Perhaps	Low statistical weight
$\Xi_{1/2}^*$	${}^2(8)_{3/2}$	1870	50		Yes	$Y\bar{K}$ decay dominant
$\Xi_{1/2}^*$	${}^4(8)_{3/2}$	1816*	25		Perhaps	Very small $Y\bar{K}$ coupling
$\Xi_{1/2}^*$	$(8)_{5/2}$	2040	125		Yes	Large $\Xi\pi$ width
Ω^-	$(10)_{1/2}$	2205	160		No	Too wide
Ω^-	$(10)_{3/2}$	1990	10		Perhaps	Sensitive to Ξ^*K threshold

^a G. Kane, Phys. Rev. Letters 17, 719 (1966).

them. The $\Lambda\pi$ channel is also weak.¹⁸ A possible experiment is $K^-n \rightarrow Y_1^*\pi$.

Y_0^* : There are ${}^2(8)$ and ${}^4(8)$ and also the member of the $\frac{5}{2}^-$ octet. The ${}^2(8)$ is mainly $\bar{K}N$ and should be seen in total cross sections and in elastic scattering.¹⁷ In a heavy liquid one could look at $K^-p \rightarrow Y_0^*\pi^0$ which is characteristic of $I=0$ (the $\Sigma\pi$ coupling is small). The ${}^4(8)$ presents no obvious method of detection. The $(8)_{5/2}$ has very small $\bar{K}N$ coupling. It should be looked at in indirect production through the large $\Sigma\pi$ and $Y_1^*\pi$ modes.

Ξ^* : There are, as for Y_0^* , ${}^2(8)$, ${}^4(8)$, and $(8)_{5/2}^-$. The ${}^2(8)$ is characterized by the large $\Sigma\bar{K}$ (and $\Lambda\bar{K}$) mode (unless the mass is too low), while $\Xi\pi$ is small. The ${}^4(8)$ is dominated by $\Xi^*\pi$, and $\Xi\pi$ is significant. The $(8)_{5/2}^-$ is mainly $\Xi\pi$ and perhaps can be identified with the structure that has been observed⁵ at 1930 MeV.

V. DISCUSSION OF ENHANCED HEAVY MESON DECAY

The ω^2 recoil factor more than overcomes the suppression due to phase space of the s -wave decays into K or η compared to π , once one is a few MeV above the relevant threshold. Let us examine whether this effect is confirmed by experiment. We will say an inelastic

cross section has a "threshold peak" when in crossing the threshold (e.g., with increasing energy across the threshold in $\pi N \rightarrow \eta N$) the cross section rises (with infinite derivative as it must) to a large value and then drops down shortly above (say <100 MeV) threshold. The following conditions lead to a threshold peak: (1) The resonance mass is appropriately placed in the vicinity of the threshold. (2) The resonance is not too wide in other than the threshold channel. (3) The branching ratio into threshold channel is large (and not too small into input channel). Let us review the predictions of our model to see when conditions (2) and (3) are met [with some comments on (1)]. We first discuss $\frac{1}{2}^-$ to $\frac{1}{2}^+$ decay.

N^* 's: The relevant thresholds are $N\eta(1488)$, $\Lambda K(1611)$, and $\Sigma K(1689)$. The prediction for ${}^4(8)_{1/2}$ satisfies the conditions for (2) and (3) for $N\eta$. The ΛK branching ratio is very small. *The $N\eta$ threshold peak is observed.* The $(10)_{1/2}$ and ${}^2(8)_{1/2}$ are probably near the ΣK threshold, but ${}^2(8)$ is too wide and both have small ΣK branching ratio. A very accurate experiment could show a ΣK threshold peak due to the (10).

Y_1^* 's: The relevant thresholds are $\Sigma\eta(1742)$ and $\Xi K(1814)$. The ${}^4(8)$ is too wide for a threshold peak. The ${}^2(8)$ is narrow except for the possible ΞK channel, and the $\Sigma\eta$ branching ratio is significant. There could be an observable $\Sigma\eta$ threshold peak. If the mass is not too far below threshold *a large ΞK peak should be seen.* Experimental work to check this possibility would be of great interest. The (10) is narrow, and $\Sigma\eta$ branching ratio is large. There should be a $\Sigma\eta$ threshold peak if the mass is

¹⁸ A possible new Y_1^* at 1680 produced indirectly at high energy has been observed by M. Derrick *et al.* (private communication). Since the enhancement is strong in the $\Lambda\pi$ channel and appears rather wide, there is some doubt that this Y_1^* could be associated with either state under discussion here.

right. A Σ_η threshold peak is observed.¹⁹ It could be either (10) or ${}^2(8)$.

Y_0^* 's: The relevant thresholds are $\Lambda_\eta(1664)$ and $\Xi K(1814)$. The ${}^4(8)$ satisfies the conditions for a Λ_η threshold peak which is observed. (A little mixing is needed to fit the observation.) The ${}^2(8)$ is too wide.

Ξ^* 's: The relevant thresholds are $\Lambda\bar{K}(1611)$, $\Sigma\bar{K}(1689)$, and $\Xi_\eta(1867)$. The ${}^4(8)$ is rather wide but is strong in $\Sigma\bar{K}$. The ${}^2(8)$ is narrow except for $\Sigma\bar{K}$ and Ξ_η , which are quite strong; $\Lambda\bar{K}$ is significant. The (10) is narrow and relatively strong in these channels. *Threshold peaks in one or more of these channels should be seen, depending on mass. Threshold peaks will be difficult to see, however, for $Y = -2$.*

One can also consider the s wave decays: $\frac{3}{2}^- \rightarrow \frac{3}{2}^+$. The $N^* \rightarrow Y_1^* + K$ or $N^* + \eta$ thresholds are probably too high in mass. The decay $Y_1^* \rightarrow N^* + \bar{K}$ (threshold ≈ 1730) is predicted to be large if any of the ${}^2(8)_{3/2}$ or ${}^4(8)_{3/2}$ or $(10)_{3/2}$ has high enough mass. Presumably, according to results discussed above, the ${}^2(8)$ and (10) have too low a mass. Similarly, $\Xi^* \rightarrow Y_1^* + K$ would be large if any state overlaps the threshold.

We conclude that all threshold peaks seen are contained in the model. Further cases are predicted. We feel the evidence for the heavy meson enhancement is very good.

We should also discuss the theoretical justification of this recoil correction. The justification lies in the validity of the nonrelativistic quark model as opposed simply to the quark model. If quark model dynamics are taken seriously, the determining factor is the detailed form of the interaction. Roughly speaking, very strong QQ forces at the origin lead to large relativistic effects, making our Galilean invariance argument unreliable. If, on the other hand, the forces are not singular, most of the quark wave function will involve low momenta, and the Galilean invariance argument is good.

VI. DISCUSSION

It is seen that we obtain a semiquantitative agreement with baryon-decay rates, especially the d -wave decays, which is quite encouraging at the present time. The quark model specific kinematical effect of a factor (meson energy)² in the s -wave rates is helpful in obtaining agreement with data. Further experiments are needed to obtain definitive widths, and theoretical work²⁰ is needed to remove the degeneracies in order to properly test the model.

This discussion would be incomplete without reference to the possibly low-lying $\frac{1}{2}^+$ singlet Y_0^* in the

¹⁹ There is recent evidence for a Σ_η enhancement near threshold consistent with the total width of roughly MeV predicted for (10) or ${}^2(8)$ by the model. See D. Cline and J. Robinson, Phys. Letters 23, 509 (1966). Note that the model predicts that this Y_1^* is not in the same multiplet with the resonances found from their N_η and Λ_η decays.

²⁰ In addition to R. Capps (Ref. 2), O. W. Greenberg and M. Resnikoff (private communication) are considering mass splitting and mixing in the (70,3).

(20,3). Its mass depends on problematical symmetry-breaking effects. One estimate⁶ places it very low: $1280 < M < 1350$ MeV. If $M < 1330$ it will decay into $\Lambda + \gamma$. If $M > 1395$ it would decay predominantly into $\Lambda\pi\pi$. The coupling of (70,3) baryons to this Y_0^* involve several different radial integrals depending on dynamical details. We just remark that since coupling $(56,1)^+ \rightarrow (20,3)^+ + \Pi$ is small (it vanishes in our model), it is probably via decay of negative parity Y^* 's produced in $\bar{K}N$ reactions that the easiest experimental search could be made.

One of the interesting results is in the absolute rates. From our fit we already have $\alpha^{-1/2} = 0.6$ BeV. Presumably the baryon radius is $R = O(\alpha^{1/2})$. Quoting q, ω, Γ in BeV in the following, the empirically fitted rates in Table III obey

$$s \text{ wave: } \Gamma = 20g^2q\omega^2, \quad (8a)$$

$$p \text{ wave: } \Gamma = 42g^2q^3/(1+\alpha q^2), \quad (9a)$$

$$d \text{ wave: } \Gamma = 6g^2q^5/(1+\alpha q^2)^2. \quad (10a)$$

The coefficients of g^2 on the right-hand side are proportional to the squares of radial integrals:

$$s \text{ wave: } \frac{\omega}{M_Q} \int \psi^s \mathbf{p}_1 \cdot \psi' d\tau \propto O\left(\frac{\omega Y_0(\hat{q})}{RM_Q}\right), \quad (8b)$$

$$p \text{ wave: } q_z \int |\psi^s|^2 d\tau \propto q Y_1^0(\hat{q})/[3(1+\alpha q^2)]^{1/2}, \quad (9b)$$

d wave:

$$\int \psi^s [\frac{3}{2}q_z \psi_z'' - \frac{1}{2}\mathbf{q} \cdot \psi''] d\tau = O\left(\frac{Rq^2 Y_2^0(\hat{q})}{1+\alpha q^2}\right), \quad (10b)$$

where \mathbf{p}_1 is the momentum of quark 1, and ψ^s and ψ'' are spatial wave functions symmetric and of mixed symmetry (symmetric in quarks 2, 3), respectively, normalized:

$$\int |\psi^s|^2 d\tau = \int \psi'' \cdot \psi'' d\tau = 1.$$

The relations (8b) and (10b) are *exceedingly crude*, but are chosen optimistically to obtain large $1/R$ and M_Q . Comparing right-hand sides of Eqs. (8) and (9) we have $O(M_Q R) = (42 \times 3/20)^{1/2}$, and comparing (9) and (10) we have $O(1/R) = (42 \times 3/6)^{1/2}$. Thus

$$M_Q = O(10 \text{ BeV}), \quad (11)$$

$$1/R = O(5 \text{ BeV}).$$

These should be regarded as upper limits. If the overlap in both (8) and (10) is, say, $1/5$ we would obtain $1/R = 1$ BeV, $M_Q = 0.5$ BeV. The significance of these results is not obvious and we will not discuss it.

Finally, let us review the problem of quark statistics. To distinguish between the two forms of statistics one

must resort to more specific dynamical considerations. In particular the symmetry-breaking mass differences and mixing of wave functions, and the structure of the radial function (form factors) must be considered.

For example, the role of the $\mathbf{L} \cdot \mathbf{S}$ coupling is very different for the two. Clearly, a p -wave spin-orbit force, which one might expect to play the dominant role in bringing about the splitting among the various members of the $(70,3)$ super multiplet, has the $SU(3)$ -spin dependence P_u^- for parastatistics and P_u^+ for Fermi statistics, where P_u^\pm are respectively the projection operators for the (6) and (3^*) states of a Q - Q system. Since, on the other hand, P_u^- is a null operator for (10) states and likewise P_u^+ for (1) states, one would expect a p -wave spin-orbit force to split the (1) states but not (10) under parastatistics and vice versa under Fermi statistics. This would immediately explain the splitting between the $Y_0^*(1405)$ and $Y_0^*(1520)$ states under parastatistics, but not under Fermi statistics.

Our results also indicate that the $(10)_{3/2}$ and $(10)_{1/2}$ states are strongly split. In parastatistics this splitting might come about through $SU(3)$ violating terms, say of the form $\lambda_8^{(i)}\lambda_8^{(j)}$, in the Q - Q potential. The near degeneracy among several $SU(3)$ multiplets makes this hypothesis rather attractive. We have not yet worked out the detailed consequences of such a hypothesis. The splitting of (8) states by an $\mathbf{L} \cdot \mathbf{S}$ force is comparable under both forms of statistics.

A more interesting piece of evidence favoring parastatistics against Fermi statistics comes from the role of the positive parity states other than the (56) . Dynamically A functions of $L^P=1^+$ have strongly attractive kernels under p wave interaction. These functions give a total of $(20,3)$ states of $SU(6) \times O_3$ under parastatistics

and $(56,3)$ under Fermi statistics. The spin-orbit force splits these states into various $SU(3)$ multiplets, the lowest ones having $J^P=\frac{1}{2}^+$. This leaves for the states $J^P=\frac{1}{2}^+$ of lowest energies, a singlet and an octet under parastatistics, and a decuplet and an octet under Fermi statistics. Experimentally, it is tempting to identify the 1450-MeV Roper resonance with the $Y=1, I_3=\frac{1}{2}$ member of the above octet. Parastatistics therefore give a (more economical) prediction of a mere extra singlet, while Fermi statistics require a whole extra decuplet of low energy.

A third feature bearing on statistics concerns the shape of baryon form factors in relation to the kind of spatial symmetry (S or A) assumed.²¹ Thus an A function predicts nodal behavior for the form factor at $q^2 \approx 20 \text{ F}^{-2}$, in complete disharmony with experiment. An S function, on the other hand, predicts a smooth monotonic fall, which is at least consistent with experiment. This again favors parastatistics to Fermi statistics, as long as the (56) representation for baryons is not questioned.

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²¹A. N. Mitra and Rabi Majumdar, Phys. Rev. **150**, 1194 (1966).

Errata

Dynamics at Infinite Momentum, STEVEN WEINBERG [Phys. Rev. **150**, 1313 (1966)]. The following should be added at the end of Ref. 3: The relation between the old rules and the Feynman rules has been extensively examined by A. Ramakrishnan and his colleagues. See A. Ramakrishnan, J. K. Radha, and R. Thunga, Proc. Indian Acad. Sci. **52(A)**, 228 (1960); J. Math. Anal. Appl. **4**, 494 (1962); **5**, 225 (1962); A. Ramkrishnan, K. Venkatesan, and V. Devanathan, *ibid.* **8**, 345 (1964); and to be published.

Renormalization Constants in the Extended Lee Model ($W \rightleftharpoons V + \theta$), FEI S. CHEN-CHEUNG [Phys. Rev. **152**, 1408 (1966)]. J. B. Bronzan [Phys. Rev. **139**, B751 (1965)] has given a similar demonstration of the renormalization constants. The work

done by this author, however, was done independently [see Fei Shian Chen and C. M. Sommerfield, Bull. Am. Phys. Soc. **10**, 61 (1965)].

Possible Charge-Conjugation Noninvariance in the Photoproduction of Neutral ρ Mesons, SAUL BARSCHAY AND YUAN LI [Phys. Rev. **153**, 1657 (1967)]. In an error subsequent to the proof, the printer erroneously duplicated Fig. 8 as Fig. 9, leaving out entirely the figure referred to as Fig. 9 in the Appendix. The correct Fig. 9 appears below.

